Phase Plane Analysis of Traffic Flow Evolution Based on a macroscopic traffic flow model

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Abstract. In this paper, a new phase plane analysis method is proposed to study the nonlinear phenomena of traffic flow. Most of the papers describe only one or several traffic phenomena and do not analyze all of them from the perspective of system stability. Therefore, this paper studies the phase plane analysis of traffic flow phenomenon from the perspective of traffic system stability, and describes various complicated nonlinear traffic phenomena through phase plane analysis.

Keywords: Phase Plane Diagrams, Stop-and-go Waves, Stability Analysis, Phase Plane Analysis.

1 Introduction

At present, traffic congestion is getting more and more serious in most cities and regions in China. In order to solve the problem of heavy traffic, we cannot simply increase the number of traffic roads and limit the driving of vehicles, which can only solve the temporary problem on the surface. Only by strengthening the research on the intrinsic nature of traffic flow can we fundamentally prevent and solve the problem of traffic congestion.

In a large number of studies on traffic flow phenomena, we find that although many scholars have studied and proposed many research methods, the results only describe one or several traffic phenomena. In this paper, from the perspective of system stability, traffic congestion and system instability are related, and a phase plane analysis method is proposed to transform the traffic flow problem into a system stability analysis problem.

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The core content of phase plane analysis is variable substitution. Substitute the density and speed variables in traditional traffic models. It can be described as $\sigma = 1/v$ and $\eta = 1/(\rho_m - \rho)$. The physical meaning of variable substitution is that from $\sigma = 1/v$ it can be found that as long as traffic congestion occurs and the vehicle speed approaches zero, the state variable σ tends to infinity and the system becomes unstable. It can be found from $\eta = 1/(\rho_m - \rho)$ that as long as there is traffic congestion, the density of vehicles tends to saturation density, the state variable η tends to infinity, and the system becomes unstable. This means that the substitution expands the scope of the variable to infinity, so the correspondence between traffic congestion and system instability can be established on the phase plan.

2 Variable substitution based on a macroscopic traffic flow model

Tang et al. proposed a macroscopic traffic flow model considering the driver's prediction effect [1]. he expression is as follows:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial(\rho v)}{\partial x} = 0 \\ \dfrac{\partial v}{\partial t} + v\dfrac{\partial v}{\partial x} = \dfrac{(1+\beta)(Ve(\rho) - v)}{T + \beta\tau} - \beta\tau c_0 \rho^2 V_e^{'}(\rho)\dfrac{\partial v}{\partial x} \end{cases} \tag{1}$$

Type:
ρ —Average vehicle density;
v —Average vehicle speed;
τ —Time interval;
β —Nonnegative constants;
$V_e[\rho(x,t)]$ —Equilibrium speed;
c_0 —Velocity of disturbance propagation;
$V_e[\rho(x,t)]$ has the following form:

$$V_e[\rho] = v_f \left\{ \left[1 + \exp\left(\frac{\rho/\rho_m - 0.25}{0.06} \right) \right]^{-1} - 3.72 \times 10^{-6} \right\} \tag{2}$$

From the formula (2) :

$$V_e'(\rho) = -\frac{v_f \exp\left(\dfrac{\rho - 0.25\rho_m}{0.06\rho_m}\right)}{0.06\rho_m \left[1 + \exp\left(\dfrac{\rho - 0.25\rho_m}{0.06\rho_m}\right)\right]^2} \tag{3}$$

Here v_f is the free flow velocity, ρ_m is the maximum or congestion density.

In the present paper, a simple transformation is employed as follows,

$$\begin{cases} \sigma = \dfrac{1}{v} \\ \eta = \dfrac{1}{\rho_m - \rho} \end{cases} \tag{4}$$

Substituting the variables into equation (1), we have a new traffic flow model as follows:

$$\begin{cases} \sigma^2 \dfrac{\partial \eta}{\partial t} + \left(\eta - \rho_m \eta^2\right) \dfrac{\partial \sigma}{\partial x} + \sigma \dfrac{\partial \eta}{\partial x} = 0 \\ -1/\sigma^2 \dfrac{\partial \sigma}{\partial t} - 1/\sigma^3 \dfrac{\partial \sigma}{\partial x} - \dfrac{(1+\beta)\left(v_e(\eta) - \dfrac{1}{\sigma}\right)}{T + \beta\tau} - \beta\tau c_0 \left(\rho_m - \dfrac{1}{\eta}\right)^2 v_e'(\eta)\left(-\dfrac{1}{\sigma^2}\dfrac{\partial \sigma}{\partial x}\right) = 0 \end{cases} \tag{5}$$

Then substitute the variable into formula (2), and the expression of the equivalent velocity is as follows:

$$V_e(\eta) = v_f \left\{ \left[1 + \exp\left(\dfrac{0.75 - \dfrac{1}{\eta\rho_m}}{0.06} \right) \right]^{-1} - 3.72 \times 10^{-6} \right\} \tag{6}$$

Then variable substitution $\eta = \dfrac{1}{\rho_m - \rho}$ is substituted into $V_e'(\rho)$, we can get:

$$V_e'(\eta) = -\dfrac{v_f \exp\left(12.5 - \dfrac{1}{0.06\rho_m \eta}\right)}{0.06\rho_m \eta^2 \left[1 + \exp\left(12.5 - \dfrac{1}{0.06\rho_m \eta}\right)\right]^2} \tag{7}$$

Specifically, the range of variable ρ in the original traffic flow mode is 0-$0.25 veh/m$ and the range of variable v is 0-$30 m/s$, so the range of variation in density or velocity is very limited. At this time, the variable substitution in the phase plane analysis method plays a vital role, it can expand the state variables ρ and v in the original model to infinity, which means that it breaks through the limitations. It can be found that when the speed approaches zero, the state variable σ is close to infinity. Similarly, the state variable η approaches infinity when the density approaches the congestion density. The increase of state variables σ and η indicates that the average vehicle speed decreases, vehicle density increases, and the system becomes unstable.

Therefore, changes in state variables can be used to judge the stability of the system. In phase plane analysis, traffic congestion can be equated with system instability.

3 Study on the stop-and-go phenomenon based on the phase plane analysis

In this section, we mainly do a comparative analysis. The comparison objects are the new model with variable substitution and the traditional model without variable substitution. The method of analysis is to compare the phase plane diagram drawn by the new model with the density space-time diagram [4][5] of the traditional model through specific numerical simulation. It turns out that the new method is consistent with the traditional method in describing the stop-and-go traffic phenomenon. But phase diagrams can more clearly describe changes in density or velocity at any time or in any part.

We can find the stop-and-go traffic phenomena in the phenomenon of small disturbance amplification. We simulate stop-and-go traffic phenomena in an enlarged local disturbance on the initial uniform traffic flow. The expression of the initial density is as follows:

$$\rho(x,0)=\rho_0+\Delta\rho_0\left\{\cosh^{-2}\left[\frac{160}{L}\left(x-\frac{5L}{16}\right)\right]-\frac{1}{4}\cosh^{-2}\left[\frac{40}{L}\left(x-\frac{11L}{32}\right)\right]\right\}\quad x\in[0,L] \tag{8}$$

Type:

ρ_0 —Initial uniform density;

$\Delta\rho_0$ —Disturbance density;

L —Section length;

Among them, $\Delta\rho_0$=0.01veh/m is the amplitude of localized perturbation, the path length L investigated in this section is 32.2km. The expression of the initial velocity is as follows:

$$v(x,0)=V\left(\rho(x,0)\right)\quad x\in[0,L] \tag{9}$$

The dynamic adjacent boundary conditions are given by the following formula:

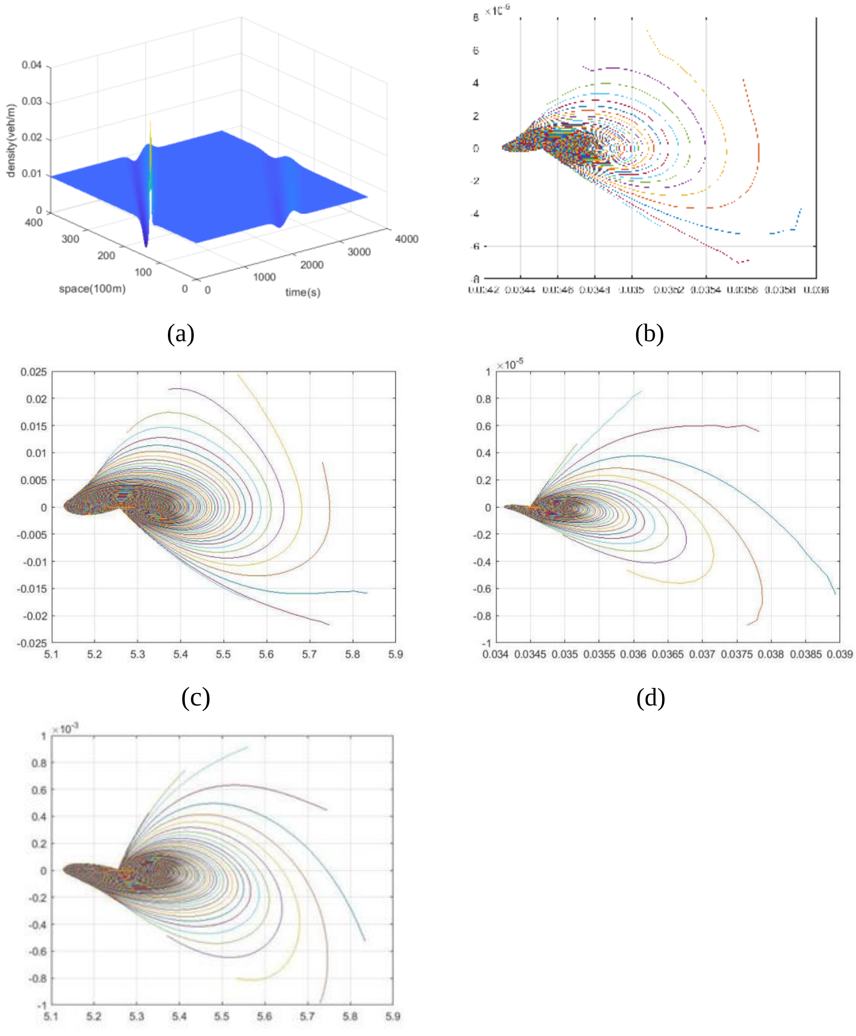
$$\rho(1,t)=\rho(2,t),\rho(L,t)=\rho(L-1,t),v(1,t)=v(2,t),v(L,t)=v(L-1,t) \tag{10}$$

In order to facilitate the simulation, the space interval is equal to 100m, the time interval is 1s, and the values of other parameters in the model are as follows:

$$c_0=11m/s,\tau=5s,v_f=30m/s,\rho_m=0.2veh/m,\beta=0.3$$

Corresponding to the above parameters, according to the stability condition, the critical density of the prediction model is 0.037veh/m and 0.091veh/m, the initial density is set at 0.037veh/m< ρ_0 <0.091veh/m, the traffic flow is linearly unstable in this range, small perturbations at this initial density will diverge and lead to stop-go phenomenon.

In this section, in order to draw the model replaced the new prediction model of the floor plan, we should first of all, for a given value, the discrete model of finite difference method, and numerical solving state variables, take $\left(\eta,\frac{\partial\eta}{\partial t}\right),\left(\eta,\frac{\partial\eta}{\partial x}\right),\left(\sigma,\frac{\partial\sigma}{\partial t}\right),\left(\sigma,\frac{\partial\sigma}{\partial x}\right)$ as the coordinates, draw four phase plans in proper order. Through the four figures, we can study more clearly the corresponding density or velocity or displacement change at any time, thus the fluctuations of traffic flow completely into the stability of the system analysis.



(a) (b) (c) (d) (e)

Fig. 1. β =0.2 initial density ρ_0 = 0.01veh/m (a) is the density space-time diagram, (b) is the phase plan with coordinates (σ, $\partial\sigma/\partial t$), (c) is the phase plan with coordinates (η, $\partial\eta/\partial t$), (d) is the phase plan with coordinates (σ, $\partial\sigma/\partial x$)), (e) is the phase plan with coordinates (η, $\partial\eta/\partial x$).

Figs. 1(b)-(e) are the phase plane diagrams of variables η and σ. We draw the stage process of the curve and find the starting point and the route of the whole run. We can see all the curve changes from outside to inside, gradually approaching the center of the ring. No curve approaches infinity, which indicates that the initial small disturbance disappears with time. On the whole, Figs.1 (b) - (c) have no curve at infinity. This means the system is stable and there are no traffic jams. This is consistent with the density space-time diagram in Fig.1 (a). The disturbance eventually disappears and the traffic flow eventually converges to the initial uniform density.Fig.1 (a) is the density spatiotemporal diagram where the initial density of the model is set within the stable range.Fig.1(d)-(e) reflects the fluctuation of vehicle density and speed in the whole section at each moment.

We found that its trajectory is composed of multiple overlapping circle structure, this means that all running curve is outside-in change over time. It shows that the density fluctuations gradually reduce over time on the whole road. The initial small disturbance is disappeared with the time and the transportation system is stable. If it's unstable, it's the opposite.

Fig.2 shows an unstable traffic flow phenomenon, which diverges when a small disturbance is applied. As can be seen from the density spatiotemporal diagram in-Fig.2-2(a), due to the initial density taken within the unstable range of the model, the small disturbance imposed on the initial uniform density is gradually amplified with the increase of time, resulting in the instability of the traffic flow and the formation of the traffic phenomenon of walking and stopping, that is, the traffic cluster. According to curve sections, draw Fig.2-2 (b) andFig.2-2 (d). We can find that there are many curves approaching infinity, and the further up the outer ring goes, which indicates that the density fluctuation is increasing at the top, the vehicle speed is decreasing at the top, the state variable is approaching infinity, and the system becomes unstable, which is consistent with the tendency of the system to become unstable as shown in Fig.2-2 (a).From the Fig.2-2 (d) and Fig.2-2 (e), we can find that there are many curves tend to infinity, which shows that the density fluctuations are enlarged with time, and the phenomenon of small disturbances diverging with time is displayed intuitively. The entire system is unstable. Compared with the density spatiotemporal map, the phase plan can more clearly reflect the density change at the current time and the next time on the entire road section. Through the phase plan, we can directly convert the traffic congestion phenomenon into the instability curve of the system. The more unstable the traffic system is, the more obvious the effect is.

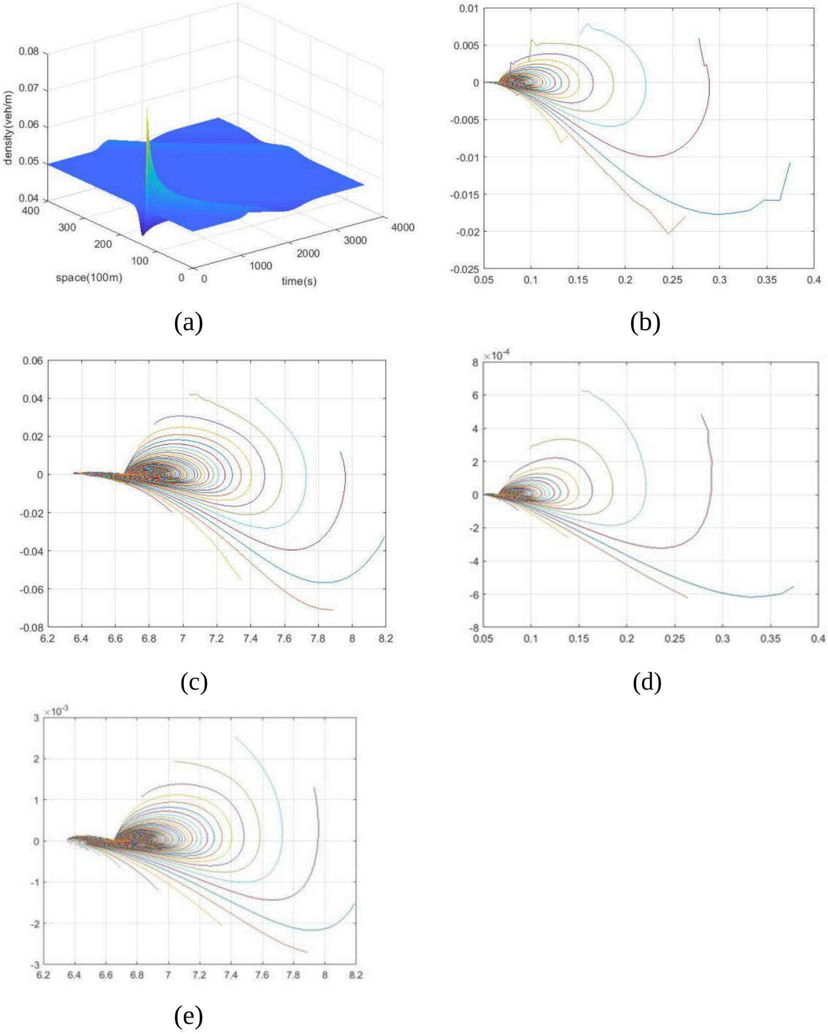


Fig. 2. β =0.3Initial density ρ_0 = 0.042 veh/m (a) is the density space-time diagram, (b) is the phase plan with coordinates (σ, $\partial\sigma/\partial t$), (c) is the phase plan with coordinates (η, $\partial\eta/\partial t$),(d) is the phase plan with coordinates (σ, $\partial\sigma/\partial x$), (e) is the phase plan with coordinates (η, $\partial\eta/\partial x$).

4 Conclusions

In this article, the original model is transformed into a new model by using a variable method. Using the phase plan, we can also describe the various non-linear phenomena observed in the traffic flow. We first build a new traffic flow model by replacing variables in the model. By analyzing and comparing the space-time diagrams and phase diagrams of different initial densities, it is found that when the traffic system changes from gentle to congested, you can immediately see some curves that tend to infinity from the phase plane diagram, and these curves that tend to infinity are throughout the whole The phase plan accounts for a very large proportion. If the traffic is always smooth, it can be found from the phase plan that the curves with small oscillations are concentrated in the part of the initial position. This shows that the performance of the new method on stop phenomenon is consistent with the traditional method. This new method allows for a clearer description of changes in density or velocity over time on a phase plan. This method changes the limitation of space-time density and transforms the study of traffic flow phenomenon into the study of system stability. It can be better applied to the study of traffic flow phenomenon.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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