Stochastic Resource Allocation for Outage Minimization in Random Access with Correlated Activation

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***Abstract*—A key challenge for random access communications arising in the monitoring of physical phenomena is optimizing the access policy. This is particularly the case when the activity of each sensor is correlated, contrasting with the independence assumption underpinning standard slotted ALOHA schemes. In this paper, we propose a stochastic resource allocation algorithm to reduce outages via maximization of the expected number of sensors that are able to reliably communicate with an access point. Allowing for devices to transmit data over multiple consecutive frames, we show that the proposed algorithm converges with probability one to a locally optimal solution. Moreover, our algorithm significantly outperforms existing methods in terms of the average number of successful transmissions when utilizing successive interference cancellation.**

I. Introduction

To reduce energy consumption in industrial fault detection and monitoring of physical phenomena, it is highly desirable for sensor networks to be event triggered. That is, transmissions occur only when a sensor observes important information. As faults and physical phenomena typically form stochastic processes, the corresponding sensor network relies on random access communications.

A fundamental problem in random access is to design efficient resource allocation strategies in networks synchronized via a beacon sent by an access point. In the typical case that control signaling from the access point is limited, each sensor must independently determine which time and frequency slots should be utilized. One approach is for each sensor to transmit data whenever it is obtained, corresponding to slotted ALOHA [1]. An alternative method is for sensors to use a randomly selected code over the time and frequency resources that are utilized within each frame, as developed in coded slotted ALOHA [2], [3].

A key assumption in standard ALOHA-based schemes is that each sensor is activated independently. However, the processes observed by different sensors are often correlated (e.g. due to alarm event [4]) . For example, in earthquake or flood detection [5], [6], nearby sensors will be more likely to transmit than sensors further away. In industrial fault detection, sensors for components on the same machine are more likely to simultaneously detect a fault than sensors on different machines. As a consequence, activity of the sensors will also be correlated.

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In order to improve decoding of sensor data at the access point, it is desirable to incorporate the presence of correlation in the design of the access policy. In many protocols, such as NB-IoT, a frame is divided into multiple time slots. In this case, the design of the access policy corresponds to the rule that each active sensor uses to decide which time slot to utilize.

In this paper, we design a new random access policy tailored to sensor networks with correlated activation. In our scheme, we limit transmission of control information for access policy updates from the access point to the sensors to every L frames, where L is typically large resulting in low levels of control signaling. Moreover, the objective is to optimize the random access policy so that the average number of active sensors can transmit their data with a sufficiently high signal to interference and noise ratio, accounting for successive interference cancellation. While an access policy has been developed in [7] for correlated activity, the scheme required control information to be sent within each frame.

To design the random access policy, we optimize the probability each active sensor transmits within a given time slot. A key challenge in solving this problem is that the objective function is not available in closed form due to the impact of random fading, a random set of active sensors, and correlation in the activity of sensors [8]. In order to solve the optimization problem, we observe that it is in the form of a stochastic optimization problem and apply methods of stochastic approximation [9]. A key feature of this approach is that the only required data can be obtained from channel and activity estimates, which are given by standard multi-user detection algorithms (e.g., [10] tailored for correlated activity). Updated probabilities for each sensor to utilize a given time slot are then sent from the access point to each sensor every L frames.

A key question is whether our algorithm yields locally optimal solutions. This is a non-trivial problem as it is often the case that sensors can transmit for more than one consecutive frame, leading to time-correlation in the data obtained from multi-user detection. We show that the algorithm converges to a locally optimal solution with probability one as $L \to \infty$ when the activity of each sensor in each frame forms an irreducible Markov chain.

To evaluate our policy, we compare its performance to policies obtained via maximization of the throughput and the expected sum-rate. These objectives naturally arise when control information is sent every frame (rather than every L frames) [7]. We show that for sufficiently high signal to interference and noise ratio (SINR) decoding thresholds, our algorithm can lead to significant performance improvements over the algorithms proposed in [7], [8], even when activity of sensors is correlated between each frame.

II. System Model

Consider N sensors that aim to transmit data to a common access point utilizing a single common subcarrier. In each frame, a sensor may transmit within at most one of K time slots, with the possibility that a given sensor may not seek to transmit any signal. The activity vector is defined by $\mathbf{X} = (X_1, \ldots, X_N) \in \{0,1\}^N$, where sensor i is active if $X_i = 1$ and inactive otherwise.

It is often the case that elements of $\mathbf{X}$, namely $X_i,\ i = 1, \ldots, N$, are assumed to be mutually independent; however, in many scenarios dependence naturally arises. For example, consider the case where sensors detect earthquakes. In this setting, sensors closest to the epicenter of the earthquake are more likely to detect its presence. As a consequence, we should expect in general that the elements of $\mathbf{X}$ are statistically dependent.

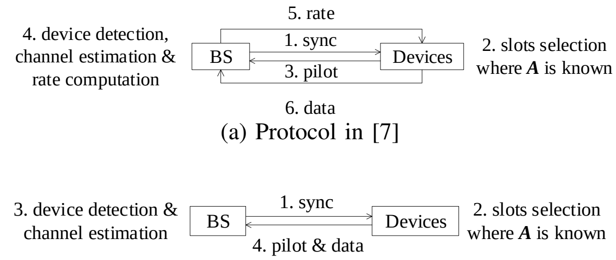
Moreover, sensor data transmission may require more than one consecutive frame. As a consequence, the activity random vectors in each frame, denoted by $\mathbf{X}^1, \mathbf{X}^2, \ldots$, will in general be also statistically dependent.

To capture such dependence between sensors and over frames, we assume that the stochastic process $(\mathbf{X}^n,\ n = 1, 2, \ldots)$ forms an irreducible Markov chain admitting a stationary distribution $P_{\mathbf{X}}$. As individual sensor activations are also not independent, the stationary distribution *does not* factor as $P_{\mathbf{X}}(\mathbf{x}) = \prod_{i=1}^{N} P_{X_i}(x_i),\ \mathbf{x} \in \{0,1\}^N$.

We consider the transmission protocol given in Algorithm 1. Note that Algorithm 1 differs from the protocol in [7] as no control signaling between the access point and sensors is available, precluding the possibility of rate-adaptation (illustrated in Fig.1). In **Step 1**, sensors randomly select a single slot within the frame according to an allocation matrix $\mathbf{A} \in \mathbb{R}^{N \times K}$, where A_{ij} is the probability that sensor i transmits in slot j conditioned on activation, and $\sum_{j=1}^{K} A_{ij} = 1$. In the sequel, we address how the allocation matrix $\mathbf{A}$ can be optimized.

Algorithm 1 Transmission Protocol

1: (Downlink): Sync signal sent by the access point.
2: (Local at Sensors): Each active sensor selects a transmission slot in $\{1, \ldots, K\}$ randomly according to the allocation matrix $\mathbf{A}$.
3: (Uplink): Sensors that have selected slot 1 transmit pilots and data on a single common subcarrier at a fixed rate.
4: (Access Point): The access point detects active sensors, estimates channel coefficients, and decodes data.
5: Repeat Steps 2-3 for slots $2, 3, \ldots, K$.



(a) Protocol in [7]

(b) Protocol in Algorithm 1

Fig. 1: Comparison of protocols.

For each slot $j \in \{1, \ldots, K\}$, detailed in **Step 2** for the first slot in Algorithm 1, each sensor that has selected slot j to transmit sends a pilot signal and data at a fixed rate R and transmit power P. At this point, as detailed in **Step 3**, the access point performs multi-user detection—assumed to be error free—in order to detect active sensors in slot j and their corresponding channel gains for the common subcarrier utilized by all sensors, denoted by $\{|g_i|^2\}_{i \in \mathcal{S}_j}$, where $\mathcal{S}_j$ is the set of active sensors in slot j.

Once sensors have been identified and the channel estimated, the access point decodes the transmitted data. To perform data decoding, the access point exploits successive interference cancellation (SIC). In more detail, suppose that each slot j consists of T channel uses and the received signal on the common subcarrier is given by

$$\mathbf{y}_j = \mathbf{D}_j \mathbf{g} + \mathbf{w}_j, \tag{1}$$

where $\mathbf{D}_j \in \mathbb{C}^{T \times N}$ is the data to be transmitted by all sensors. The coefficients $g_i = \sqrt{P} r_i^{-\eta/2} h_i,\ i \in \{1, \ldots, N\}$ with $h_i \in \mathbb{C}$, and $r_i^{-\eta/2}$ denoting the block fading coefficient, and path loss for the i-th sensor, respectively, assumed to be constant for each sensor over the T channel uses. For $i \notin \mathcal{S}_j$, we set the i-th column of $\mathbf{D}_j$ to zero. The term $\mathbf{w}_j \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{T \times T})$ is additive white Gaussian noise.

Under SIC and sufficiently large T, the achievable rate for sensor $m \in \mathcal{S}_j$ with the m-th largest channel gain in $\{|g_i|^2\}_{i \in \mathcal{S}_j}$, denoted by $|g_m|^2$, is well approximated by

$$R_{j,m} = W \log\left(1 + \frac{|g_m|^2}{\sigma^2 + \sum_{m < l \leq |\mathcal{S}_j|} |g_l|^2}\right), m = 1, \ldots, |\mathcal{S}_j|, \tag{2}$$

where W is available bandwidth and assumed in the sequel, without loss of generality, to be $W = 1$. The data from sensor $m \in \mathcal{S}_j$ can therefore only be reliable decoded if $R_{j,m} > R$, where R is the fixed rate used by each sensor.

III. Optimization Criteria

The goal of this paper is to reduce the outages in sensor data transmissions by optimizing the allocation matrix $\mathbf{A}$, which assigns the probability each sensor accesses each time slot. A natural criterion for optimization of $\mathbf{A}$ is the expected number of active sensors that transmit without an outage, given by

$$T_1(\mathbf{A}) = \mathbb{E}_{\mathbf{X}}[T_1^{\mathbf{X}}(\mathbf{A})], \tag{3}$$

with

$$T_1^{\mathbf{X}}(\mathbf{A}) = \sum_{k=1}^{K} \sum_{\mathcal{S}\in\mathcal{P}(N)} Q_k(\mathcal{S}|\mathbf{X}) \cdot \sum_{m=1}^{|\mathcal{S}|} \mathbf{1}\left\{\log\left(1 + \frac{|g_{\mathcal{S}(m)}|^2}{\sigma^2 + \sum_{m<l\leq|\mathcal{S}|}|g_{\mathcal{S}(l)}|^2}\right) > R\right\}, \tag{4}$$

where $\mathcal{P}(N)$ denotes the power set of $\{1,\ldots,N\}$ and $\mathbf{1}\{\cdot\}$ is an indicator function and

$$Q_k(\mathcal{S}|\mathbf{X}) = \prod_{i\in\mathcal{S}} X_i A_{ik} \prod_{j\in\mathcal{S}^c}(1 - X_j A_{jk}), \tag{5}$$

which corresponds to the probability the sensors in $\mathcal{S}$ transmit in slot k conditioned on the activity vector $\mathbf{X}$. The expectation with respect to the activity vector $\mathbf{X}$ corresponds to expectation with respect to the stationary distribution, $P_{\mathbf{X}}$, of the Markov chain $\mathbf{X}^1, \mathbf{X}^2, \ldots$ (detailed in Sec. II).

An alternative criterion is the expected fraction of slots in which exactly one sensor transmits, or the expected throughput, which was considered in [7], [8] when the activity process $\mathbf{X}^1, \mathbf{X}^2, \ldots$ is independent. Formally, the expected throughput is given by

$$T_2(\mathbf{A}) = \mathbb{E}_{\mathbf{X}}\left[T_2^{\mathbf{X}}(\mathbf{A})\right] \tag{6}$$

where

$$T_2^{\mathbf{X}}(\mathbf{A}) = \sum_{n=1}^{N}\sum_{k=1}^{K} X_n A_{nk} \prod_{\substack{m=1\\ m\neq n}}^{N}(1 - X_m A_{mk}). \tag{7}$$

As for the criterion (3), the expectation with respect to the activity vector $\mathbf{X}$ corresponds to expectation with respect to the stationary distribution, $P_{\mathbf{X}}$, of the Markov chain $\mathbf{X}^1, \mathbf{X}^2, \ldots$.

A further criterion that may be considered is

$$\Pr\left(\min_{\substack{k=1,\ldots,K\\ m=1,\ldots,|\mathcal{S}_k|}} R_{k,m} > R\right), \tag{8}$$

which is the probability that the data sent by all active sensors can be decoded error free. Note that (8) can be bounded by a criterion based on the sum-rate. Indeed,

$$\Pr\left(\min_{\substack{k=1,\ldots,K\\ m=1,\ldots,|\mathcal{S}_k|}} R_{k,m} > R\right) \leq \Pr\left(\sum_{k=1}^{K}\sum_{m=1}^{|\mathcal{S}_k|} R_{k,m} > R\right) \leq \frac{\mathbb{E}_{\mathbf{X},\mathbf{h}}\left[\sum_{k=1}^{K}\sum_{m=1}^{|\mathcal{S}_k|} R_{k,m}\right]}{R}, \tag{9}$$

by the Markov inequality. This suggests a useful criterion is given by the expected sum-rate

$$T_3(\mathbf{A}) = \mathbb{E}_{\mathbf{h},\mathbf{X}}\left[T_3^{\mathbf{h},\mathbf{X}}(\mathbf{A})\right] \tag{10}$$

where

$$T_3^{\mathbf{h},\mathbf{X}}(\mathbf{A}) = \sum_{k=1}^{K}\sum_{\mathcal{S}\in\mathcal{P}(N)} Q_k(\mathcal{S}|\mathbf{X}) \cdot \sum_{m=1}^{|\mathcal{S}|} \log\left(1 + \frac{|g_{\mathcal{S}(m)}|^2}{\sigma^2 + \sum_{m<l\leq|\mathcal{S}|}|g_{\mathcal{S}(l)}|^2}\right) \tag{11}$$

with $Q_k(\mathcal{S}|\mathbf{X})$ given in (5). As for the previous criteria, the expectation with respect to the activity vector $\mathbf{X}$ corresponds to expectation with respect to the stationary distribution, $P_{\mathbf{X}}$, of the Markov chain $\mathbf{X}^1, \mathbf{X}^2, \ldots$. We remark that the expected sum-rate was also considered in [7], albeit in the case that the access point could perform rate-adaptation and the activity process $\mathbf{X}^1, \mathbf{X}^2, \ldots$ is independent.

The criteria in (3), (6) and (10) do not in general admit closed-form expressions—partly due to the fact that $P_{\mathbf{X}}$ is not known to the access point—and are non-convex functions of the continuous variables in $\mathbf{A}$. In the following section, we propose new algorithms for the optimization of the allocation matrix $\mathbf{A}$ with respect to these criteria, with provable convergence guarantees. In particular, we seek solutions

$$\mathbf{A}_1^* = \arg \max_{\substack{\mathbf{A}\in\mathbb{R}_+^{N\times K}:\\ \sum_j A_{ij}=1,\ i=1,\ldots,N}} T_1(\mathbf{A}), \tag{12}$$

$$\mathbf{A}_2^* = \arg \max_{\substack{\mathbf{A}\in\mathbb{R}_+^{N\times K}:\\ \sum_j A_{ij}=1,\ i=1,\ldots,N}} T_2(\mathbf{A}). \tag{13}$$

and

$$\mathbf{A}_3^* = \arg \max_{\substack{\mathbf{A}\in\mathbb{R}_+^{N\times K}:\\ \sum_j A_{ij}=1,\ i=1,\ldots,N}} T_3(\mathbf{A}). \tag{14}$$

At present, solutions based on tractable bounds of (6) have been obtained for the problem (12) in [8]. We have recently introduced stochastic optimization methods to solve (13) and (14) under the assumption that the sequence of activity vectors $\mathbf{X}^1, \mathbf{X}^2, \ldots$ is independent. In the following section, we develop a new algorithm to optimize (12) and show that this algorithm along with those proposed in [7] to solve (13) and (14) all converge with probability one, *even when the activity vector sequence forms a Markov process*.

IV. Optimization of the Allocation Matrix

A key observation is that the optimization problems in Sec. III are stochastic optimization problems with $P_{\mathbf{X}}$ unknown. As a consequence, (12), (13) and (14) can be solved via stochastic approximation algorithms [9], which exploit samples $\mathbf{X}^1, \mathbf{X}^2, \ldots$. In particular, since the functions inside the expectations arising in the objectives are continuously differentiable in $\mathbf{A}$, stochastic gradient ascent (SGA) can be applied. In this section, we derive the SGA algorithms for the problems in Sec. III and show that they converge almost surely to a local maximum.

Suppose that for a sequence of training frames, sensors access slots according to an initial allocation matrix $\mathbf{A}^1$. The access point can then obtain knowledge of sensor activity via multi-user detection; namely, in each frame t, the activity vector $\mathbf{X}^t,\ t=1,2,\ldots$. Note that data can be transmitted during the training frames, as the same multi-user detection algorithm is required for data decoding.

A. SGA Algorithms for (13) and (14)

We first recall the algorithms proposed in [7] to optimize the allocation matrix $\mathbf{A}$ for the criterion in (13) and (14). We highlight that these algorithms were developed for random access with feedback allowing for rate-adaptation, which is not the case in the scenario studied in this paper. Moreover, convergence of the algorithms was restricted to independent activity vector sequences $\mathbf{X}^1,\mathbf{X}^2,\ldots$, which will be generalized in Sec. IV-C for the Markov case.

Algorithm 2 provides details of the throughput maximization algorithm to solve (13), where $\Pi_H(\mathbf{A})$ denotes the closest point (w.r.t. $\|\mathbf{A}\|_F=\sum_{i=1}^N\sum_{j=1}^K A_{ij}^2$ is the Forbenius Norm) in the constraint set $\mathcal{H}=\{\mathbf{A}\in\mathbb{R}_+^{N\times K}:\sum_j A_{ij}=1,\ i=1,\ldots,N\}$ from $\mathbf{A}$. The term $\mathbf{Y}_2^t(\mathbf{A}_2^t)$ is an unbiased estimate of the gradient of the throughput T_2 at $\mathbf{A}_2^t$. To compute the estimate, first observe that

$$\frac{\partial T_2}{\partial A_{ql}}(\mathbf{A}) =\mathbb{E}_{\mathbf{X}}\left[X_q\prod_{\substack{m=1\\m\neq q}}^{N}(1-X_mA_{ml})-\sum_{\substack{n=1\\n\neq q}}^{N}X_qX_nA_{nl}\prod_{\substack{m=1\\m\neq n\\m\neq q}}^{N}(1-X_mA_{ml})\right]. \tag{15}$$

Contrary to gradient ascent, SGA need only use one training sample in each iteration. Given a realization $\mathbf{X}^i$, the unbiased estimate of the gradient in the t-th iteration is then given by

$$Y_{2,ql}^t(\mathbf{A}) =X_q^t\prod_{\substack{m=1\\m\neq q}}^{N}(1-X_m^tA_{ml})-\sum_{\substack{n=1\\n\neq q}}^{N}X_q^tX_n^tA_{nl}\prod_{\substack{m=1\\m\neq n\\m\neq q}}^{N}(1-X_m^tA_{ml}). \tag{16}$$

Algorithm 2 SGA Algorithm for (13)

1: **Input:** Choose an initial iterate $\mathbf{A}_2^1$ and step-size $\alpha_2^1>0$.
2: $t\leftarrow 0$.
3: **While** not converged
4: Using $\mathbf{X}^t$, compute an unbiased estimate $\mathbf{Y}_2^t(\mathbf{A}_2^t)$ of $\nabla\mathrm{T}_2\ (\mathbf{A}^t)$, given in (16).
5: Set $\mathbf{A}_2^{t+1}\leftarrow\Pi_{\mathcal{H}}[\mathbf{A}_2^t+\alpha_2^t\mathbf{Y}_2^t(\mathbf{A}_2^t)]$.
6: $t\leftarrow t+1$.
7: End **While**
8: **Output:** $\mathbf{A}_2^t$.

We now turn to the expected sum-rate maximization problem in (14), for which the allocation $\mathbf{A}$ is obtained via Algorithm 3.

In this case, we have

$$\frac{\partial T_3}{\partial A_{ql}}(\mathbf{A})=\mathbb{E}_{\mathbf{X},\mathbf{h}}\left[\sum_{\mathcal{S}\in\mathcal{P}(N)}\frac{\partial Q_q(\mathcal{S}|\mathbf{X})}{\partial A_{ql}}\cdot\sum_{m=1}^{|\mathcal{S}|}\log\left(1+\frac{|g_{\mathcal{S}(m)}|^2}{\sigma^2+\sum_{m<p\leq|\mathcal{S}|}|g_{\mathcal{S}(p)}|^2}\right)\right], \tag{17}$$

where

$$\frac{\partial Q_q(\mathcal{S}|\mathbf{X})}{\partial A_{ql}}=\begin{cases}X_q\prod_{i\in\mathcal{S}\setminus\{q\}}X_iA_{il}\prod_{j\in\mathcal{S}^c}(1-X_jA_{jl}), & q\in\mathcal{S}\\ -X_q\prod_{i\in\mathcal{S}}X_iA_{il}\prod_{j\in\mathcal{S}^c\setminus\{q\}}(1-X_jA_{jl}), & q\in\mathcal{S}^c.\end{cases} \tag{18}$$

Given realizations of $\mathbf{X}^t$ and $\mathbf{h}^t$, an unbiased estimate of the gradient in the t-th iteration is then given by

$$Y_{3,ql}^t(\mathbf{A})=\sum_{\mathcal{S}\in\mathcal{P}(N)}\frac{\partial Q_q(\mathcal{S}|\mathbf{X}^t)}{\partial A_{ql}}\sum_{m=1}^{|\mathcal{S}|}\log\left(1+\frac{|g_{\mathcal{S}(m)}^t|^2}{\sigma^2+\sum_{m<p\leq|\mathcal{S}|}|g_{\mathcal{S}(p)}^t|^2}\right). \tag{19}$$

Algorithm 3 SGA Algorithm for (14)

1: **Input:** Choose an initial iterate $\mathbf{A}_3^1$ and step-size $\alpha_3^1>0$.
2: $t\leftarrow 0$.
3: **While** not converged
4: Using $\mathbf{X}^t,\mathbf{h}^t$, compute an unbiased estimate $\mathbf{Y}_3^t(\mathbf{A}_3^t)$ of $\nabla\mathrm{T}_3(\mathbf{A}_3^t)$, given in (21).
5: Set $\mathbf{A}_3^{t+1}\leftarrow\Pi_{\mathcal{H}}[\mathbf{A}_3^t+\alpha_3^t\mathbf{Y}_3^t(\mathbf{A}_3^t)]$.
6: $t\leftarrow t+1$.
7: End **While**
8: **Output:** $\mathbf{A}_3^t$.

B. New SGA Algorithm for (12)

We now introduce a stochastic gradient ascent algorithm for the problem (12), detailed in Algorithm 4. Convergence is established in Sec. IV-C.

For the criterion in (3), the gradient is given by

$$\frac{\partial T_1}{\partial A_{ql}}(\mathbf{A})=\mathbb{E}_{\mathbf{X},\mathbf{h}}\left[\sum_{\mathcal{S}\in\mathcal{P}(N)}\frac{\partial Q_q(\mathcal{S}|\mathbf{X})}{\partial A_{ql}}\cdot\sum_{m=1}^{|\mathcal{S}|}\mathbf{1}\left\{\log\left(1+\frac{|g_{\mathcal{S}(m)}|^2}{\sigma^2+\sum_{m<p\leq|\mathcal{S}|}|g_{\mathcal{S}(p)}|^2}\right)>R\right\}\right], \tag{20}$$

where $\frac{\partial Q_q(\mathcal{S}|\mathbf{X})}{\partial A_{ql}}$ is given in (18).

Given realizations of $\mathbf{X}^t$ and $\mathbf{h}^t$, an unbiased estimate of the gradient in the t-th iteration is then given by

$$\mathbf{Y}_{1,ql}^t(\mathbf{A})=\sum_{\mathcal{S}\in\mathcal{P}(N)}\frac{\partial Q_q(\mathcal{S}|\mathbf{X}^t)}{\partial A_{ql}}\cdot\sum_{m=1}^{|\mathcal{S}|}\mathbf{1}\left\{\log\left(1+\frac{|g_{\mathcal{S}(m)}|^2}{\sigma^2+\sum_{m<p\leq|\mathcal{S}|}|g_{\mathcal{S}(p)}|^2}\right)>R\right\}. \tag{21}$$

Algorithm 4 SGA Algorithm for (12)

Input: Choose an initial iterate $\mathbf{A}_1^1$ and step-size $\alpha_1^1 > 0$.
$t \leftarrow 0$.
While not converged
Using $\mathbf{X}^t, \mathbf{h}^t$, compute an unbiased estimate $\mathbf{Y}_1^t(\mathbf{A}_1^t)$ of $\nabla \mathrm{T}_1(\mathbf{A}_1^t)$, given in (21).
Set $\mathbf{A}_1^{t+1} \leftarrow \Pi_{\mathcal{H}}[\mathbf{A}_1^t + \alpha_1^t \mathbf{Y}_1^t(\mathbf{A}_1^t)]$.
$t \leftarrow t + 1$.
End **While**
Output: $\mathbf{A}_1^t$.

C. Convergence

We now show that Algorithm 4 converges with probability one to a local maximum of the problem (12).

Theorem 1. *Suppose that $\{\mathbf{X}^i\}$ forms an irreducible Markov chain admitting a stationary distribution $P_{\mathbf{X}}$ and $\alpha^t = \alpha^1/t,\ t = 1, 2, \ldots$ with $0 < \alpha^1 < \infty$. Then, the iterates $\mathbf{A}^t$ in Algorithm 4 converge almost surely as $t \to \infty$ to a local maximum of the problem (12).*

Proof. (Sketch) We use the notation defined in [9, Chap 6]. To establish the theorem, it is necessary to verify (A6.1.1)-(A6.1.7) and (A4.3.3) in [9] and apply [9, Theorem 6.1.1]. Due to the fact that (3) is bounded and Lipschitz continuous on $\mathcal{H}$, Assumptions (A6.1.1)-(A6.1.7) immediately hold with the exception of (A6.1.3). Assumption (A4.3.3) holds since the constraint set $\mathcal{H}$ forms a linear connected compact surface and hence has a continuously differentiable outer normal.

All that remains is to prove that (A6.1.3) holds. We first identify the "effective memory" variables ξ_n in [9, Sec. 6.1.1] with $\mathbf{X}^{n-1}$. It is straightforward to show that the bias variables β_n are zero with probability one and hence the conditional mean (defined in [9, Chap 6]) satisfies $g_n(\theta_n, \xi_n) = g(\theta_n, \xi_n)$ for all n; i.e., independent of n. Using the remark in [9, Sec. 6.2] and applying the strong law of large numbers for irreducible Markov chains on countable state spaces [11], it follows that (A6.1.3) holds. □

A similar argument also establishes that Algorithm 2 and Algorithm 3 also converge almost surely to a locally optimal solution of their respective problems. This generalizes the convergence results proven in [7] to the case where the activity vectors form a Markov process.

V. Simulation Results

To evaluate our algorithm, we consider the following event-triggered scenario based on the model introduced in [8]. In particular, events are generated according to a homogeneous Poisson point process (HPPP) on $[0, L]^2$ with rate λ and a given sensor transmits if an event occurs within a radius R of the sensor. Throughout our numerical study, we assume that $L = 50$, $\lambda = 10/(L + 2R)^2$ and $\sigma^2 = 1$. In this model, each sensor has the same activity probability given by

$$\mathbf{E}[X_i] = 1 - \exp\{-\lambda \pi R^2\}. \tag{22}$$

For the purpose of evaluating the relative performance of each of the criteria, the activity vectors $\mathbf{X}^1, \mathbf{X}^2, \ldots$ form a Markov chain induced by active devices transmitting in the subsequent frame with probability 0.1.

In [7], it was observed that the throughput and expected sum-rate maximization algorithms were sensitive to initial conditions. We therefore consider five choices of the initial value of the allocation matrix $\mathbf{A}^1$:

$$\mathbf{A}_1 = \begin{bmatrix} 1/K & 1/K & \ldots & 1/K \\ 1/K & 1/K & \ldots & 1/K \\ \vdots & \vdots & \ddots & \vdots \\ 1/K & 1/K & \ldots & 1/K \end{bmatrix}_{N \times K}$$

$$\mathbf{A}_2 = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \ldots & 0 \end{bmatrix}_{N \times K}, \quad \mathbf{A}_3 = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & & 1 \\ 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}_{N \times K} \tag{23}$$

and $\mathbf{A}_4$ and $\mathbf{A}_5$, which correspond to the allocations obtained via $\mathbf{Alg_{upper}}$ and $\mathbf{Alg_{lower}}$ in [8], respectively. In the following results, we select the initial value of the allocation matrix leading to the highest performance.

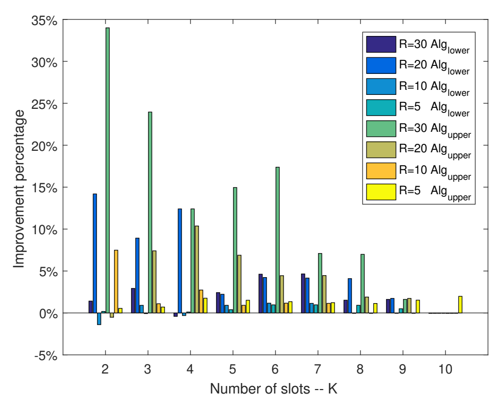


Fig. 2: Improvement of Algorithm 4 over $\mathbf{Alg_{lower}}$ and $\mathbf{Alg_{upper}}$ [8] for $\tau = 0$ dB with $N = 10$ sensors for varying K, where $\lambda = 10/(50 + 2R)^2$ and $p = 0.1$.

We first compare Algorithm 4 based on (3) with the algorithms for throughput maximization developed in [8]. In order to establish reasonable rate thresholds, we express the rate R in (3) in terms of the SINR $\tau = e^R - 1$. We consider $N = 10$ sensors in the network, uniformly distributed on the circles with distances from the access point given by

$$\begin{aligned} \mathbf{d} = &[0.1743, 0.8433, 1.2320, 1.5303, 5.8221, \\ &6.5653, 7.1598, 8.0840, 8.4006, 9.7600]. \end{aligned} \tag{24}$$

Fig. 2 and Fig. 3 show the percentage improvement in the expected number of successful transmissions (i.e., criterion (3)) using Algorithm 3 compared with the methods in [8], with

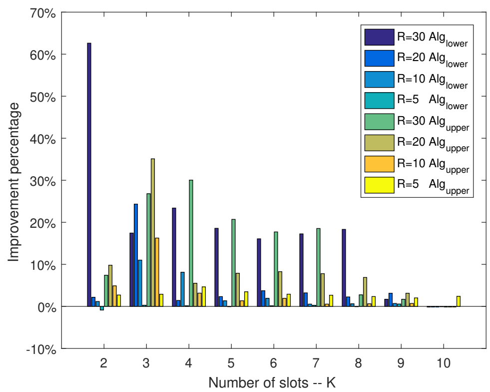


Fig. 3: Improvement of Algorithm 4 over $\mathbf{Alg_{lower}}$ and $\mathbf{Alg_{upper}}$ [8] for $\tau = 20$ dB with $N = 10$ sensors for varying K, where $\lambda = 10/(50 + 2R)^2$ and $p = 0.1$.

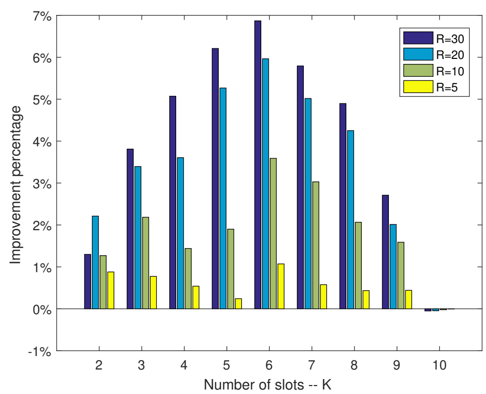


Fig. 4: Improvement of Algorithm 4 over Algorithm 3 based on the expected sum-rate for $\tau = 0$ dB with $N = 10$ for varying K, where $\lambda = 10/(50 + 2R)^2$ and $p = 0.1$.

$\tau = 0$ dB and $\tau = 20$ dB, respectively. For both $\tau = 0$ dB and $\tau = 20$ dB significant improvements over the algorithms in [8] on the order of 20% are evident for low numbers of available time slots.

We now compare Algorithm 4 based on the criterion in (3) with Algorithm 3. Fig. 4 shows that performance improvement with $\tau = 0$ dB is limited. Fig. 5 shows the performance improvement with $\tau = 20$ dB. In this case, we observe performance improvements of over 20% for low numbers of slots (on the order of $20\% - 30\%$).

VI. Conclusions

A key problem in the design of random access communications for event-triggered sensor networks is minimizing outages in the presence of correlated activation. In this paper, we developed a new stochastic resource optimization algorithm, which can outperform existing methods when a limited number of slots are available per frame. The proposed algorithm is also shown to converge with probability one, even when activity vectors in each frame form a Markov process. A further benefit of our approach is that parameters of the

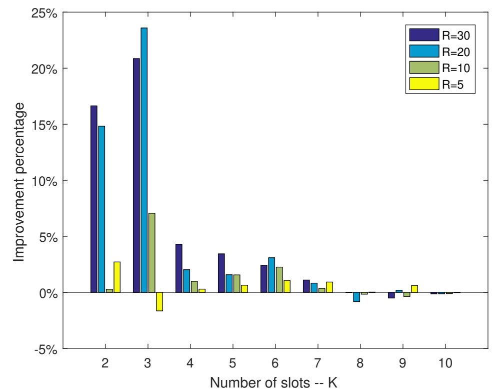


Fig. 5: Improvement of Algorithm 4 over Algorithm 3 based on the expected sum-rate for $\tau = 20$ dB with $N = 10$ for varying K, where $\lambda = 10/(50 + 2R)^2$ and $p = 0.1$.

distribution governing activation do not in general need to be directly estimated.

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