GECCO 2021 Tutorial on Benchmarking Multiobjective Optimizers 2.0

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GECCO 2021 Tutorial on Benchmarking Multiobjective Optimizers 2.0

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The final slides are available at
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Isn't Benchmarking Trivial?

Isn't Benchmarking Trivial?

- Choose some algorithms
- Choose some test functions
- Discuss the results

Isn't Benchmarking Trivial?

In principle: yes

But many details and pitfalls

Isn't Benchmarking Trivial?

In principle: yes

But many details and pitfalls

and in addition

- we compare *sets* of solutions
- which might be random (in size, position, dimension, ...)
- ...

We Typically Start with Convergence Graphs

(...in single-objective optimization)

We Typically Start with Convergence Graphs

(...in single-objective optimization)

We'll come back to this later again

Overview

Our plan

Discuss history, present and future of multiobjective benchmarking

With respect to different topics

- performance assessment / methodology
- test functions

Finally, recommendations on good algorithms

Disclaimer

This is not an introductory tutorial to multiobjective optimization!

We assume you know basic definitions like

- Objective function
- Pareto dominance/Pareto front/Pareto set
- Ideal/Nadir points

Disclaimer II

We only consider continuous search spaces

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We only consider unconstrained problems

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What we present is highly subjective & selective

- how important do we find each milestone?
- use version numbering and branches
- what have we learned from the past?

- ① Performance Assessment
- ② Test Problems and Their Visualizations
- ③ Recommendations from Numerical Results

v0.0.1alpha

In The Early Beginnings...

... multiobjective EAs were mainly compared visually:

ZDT6 benchmark problem: **IBEA**, **SPEA2**, **NSGA-II**

v0.1beta

Problem	MOCSA				NSGA2			
	$\langle d \rangle$	S	GD	ER	$\langle d \rangle$	S	GD	ER
ZDT1	0.0404	0.0055	0.0000	0	0.0270	0.0156	0.0011	0.04
ZDT2	0.0404	0.0082	0.0000	0	0.0292	0.0146	0.0212	0.02
ZDT3	0.0438	0.0148	0.0001	0	0.0329	0.0201	0.0020	0.02
ZDT4	0.0404	0.0097	0.0000	0	0.0328	0.0159	0.0006	0.02
ZDT6	0.0327	0.0150	0.0000	0	0.0216	0.0119	0.0000	0
DTLZ1	0.1114	0.0068	0.0000	0	0.0615	0.0319	0.0000	0
DTLZ2	0.2319	0.0646	0.0021	0.02	0.1361	0.0683	0.0020	0.04
DTLZ3	0.2770	0.0225	0.0000	0	0.1139	0.0739	0.0000	0
DTLZ4	0.2478	0.0424	0.0009	0	0.1630	0.0898	0.0019	0.02
DTLZ5	0.0487	0.0059	0.0000	0	0.0309	0.0176	0.0610	0.06
DTLZ6	0.0484	0.0156	0.0000	0	0.0306	0.0135	0.0000	0
DTLZ7	0.2897	0.0510	0.0011	0.04	0.1880	0.1322	0.0071	0.22

arXiv, 2012

Table 4: Influence of different κ values on the performance of the cone-MOEA on test problems Deb52, ZDT1, and DTLZ2 with three and four objective functions. Median (M) and standard deviation (SD) over 30 independent runs are shown. Intermediate values for κ seem to yield reasonably good performance values for all metrics. A more appropriate study is required in order to formally characterize the effect of this parameter.

Metric		κ ; Deb52										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0006	0.0006	0.0005	0.0006	0.0005	0.0006	0.0006	0.0006	0.0006	0.0001	0.0001
	SD	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	0.0001	0.0001
Δ	M	0.6766	0.6813	0.5244	0.2991	0.2552	0.2432	0.2648	0.2892	0.3147	0.3194	0.3199
	SD	0.0004	0.0021	0.0025	0.0027	0.0034	0.0039	0.0017	0.0019	0.0016	0.0042	0.0066
HV	M	0.2735	0.2779	0.2794	0.2802	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806
	SD	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
H	M	19.00	51.00	74.00	93.00	101.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	$< 10^{-4}$	$< 10^{-4}$	0.2537	0.3457	0.4842	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	0.1826	0.1826	0.1826

Metric		κ ; ZDT1										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0103	0.0069	0.0055	0.0059	0.0074	0.0040	0.0042	0.0051	0.0053	0.0050	0.0038
	SD	0.0072	0.0038	0.0057	0.0047	0.0049	0.0042	0.0060	0.0058	0.0040	0.0050	0.0034
Δ	M	0.3046	0.5543	0.3678	0.2084	0.1818	0.1812	0.1898	0.1937	0.1934	0.1956	0.1891
	SD	0.0122	0.0607	0.0480	0.0408	0.0235	0.0220	0.0234	0.0251	0.0240	0.0232	0.0155
HV	M	0.8435	0.8561	0.8602	0.8607	0.8598	0.8652	0.8650	0.8636	0.8633	0.8638	0.8657
	SD	0.0115	0.0066	0.0094	0.0079	0.0082	0.0069	0.0099	0.0096	0.0066	0.0083	0.0057
H	M	37.00	63.00	84.50	98.00	100.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	0.6397	5.7211	2.8730	5.0901	3.8201	0.5467	0.8584	0.9371	0.9377	1.3515	0.7112

Metric		κ ; DTLZ2 (m = 3)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0062	0.0069	0.0072	0.0070	0.0074	0.0079	0.0074	0.0076	0.0074	0.0078	0.0072
	SD	0.0002	0.0013	0.0015	0.0013	0.0012	0.0014	0.0010	0.0019	0.0007	0.0014	0.0009
Δ	M	0.0503	0.6066	0.3029	0.2411	0.2386	0.2308	0.2274	0.2175	0.2079	0.2173	0.1982
	SD	0.0041	0.0422	0.0357	0.0302	0.0264	0.0219	0.0316	0.0275	0.0306	0.0295	0.0239
HV	M	0.6731	0.7149	0.7383	0.7435	0.7458	0.7469	0.7467	0.7469	0.7470	0.7470	0.7471
	SD	0.0066	0.0042	0.0023	0.0012	0.0007	0.0006	0.0005	0.0005	0.0005	0.0005	0.0003
H	M	21.00	69.00	88.00	93.00	94.50	95.00	95.00	95.00	95.00	95.00	94.00
	SD	1.3047	3.1639	2.8367	2.0424	1.7750	1.9464	2.2894	2.0197	1.5643	2.2614	1.7100

Metric		κ ; DTLZ2 (m = 4)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
γ	M	0.0001	0.0311	0.0385	0.0312	0.0449	0.0404	0.0445	0.0488	0.0590	0.0489	0.0534
	SD	0.0001	0.0198	0.0218	0.0239	0.0283	0.0240	0.0369	0.0281	0.0284	0.0241	0.0304
Δ	M	0.1390	0.4700	0.3602	0.3296	0.3299	0.3429	0.3377	0.3258	0.3253	0.3304	0.3319
	SD	0.1173	0.0304	0.0307	0.0226	0.0255	0.0263	0.0187	0.0262	0.0210	0.0254	0.0259
H	M	14.00	79.50	90.00	92.00	95.00	96.00	95.50	97.00	98.00	95.50	97.00
	SD	1.9815	4.9642	4.8476	4.2372	4.6307	4.2129	5.8530	5.0496	4.5945	4.6233	4.3423

Table 7: Problemwise comparison of the algorithms on the four performance metrics used, for problems Deb52, Pol and the ZDT family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ -MOEA	cone-MOEA	C-NSGA-II	SPEA2	NSGA-II*
Deb52	γ	0.58±7e-3	0.56±3e-3	0.56±2e-3	0.58±1e-2	0.55±7e-3	0.55±6e-3
	Δ	0.53±8e-3	0.99±6e-4	0.32±3e-4	0.33±6e-3	0.20±3e-3	0.41±8e-3
	HV	0.99±7e-5	0.99±6e-5	0.99±0	0.99±1e-4	0.99±3e-5	0.99±7e-5
	CS	0.02±7e-4	0.03±10e-4	0.03±8e-4	0.02±8e-4	0.03±9e-4	0.02±8e-4
Pol	γ	0.20±2e-2	0.13±6e-4	0.19±2e-2	0.19±9e-3	0.15±2e-3	0.16±2e-3
	Δ	0.58±1e-2	0.98±9e-4	0.29±6e-3	0.38±7e-3	0.24±3e-3	0.36±8e-3
	HV	1.00±1e-5	1.00±5e-6	1.00±4e-6	1.00±3e-5	1.00±4e-6	1.00±8e-6
	CS	0.04±1e-3	0.04±1e-3	0.04±2e-3	0.04±10e-4	0.06±1e-3	0.05±1e-3
Zdt1	γ	0.16±2e-2	0.30±3e-2	0.23±3e-2	0.18±2e-2	0.30±2e-2	0.19±2e-2
	Δ	0.79±1e-2	0.70±4e-3	0.37±6e-3	0.50±8e-3	0.29±7e-3	0.56±1e-2
	HV	0.99±10e-4	0.98±2e-3	0.98±2e-3	0.98±9e-4	0.98±1e-3	0.98±1e-3
	CS	0.33±2e-2	0.20±3e-2	0.27±3e-2	0.30±2e-2	0.15±2e-2	0.27±2e-2
Zdt2	γ	0.43±9e-3	0.65±8e-3	0.30±4e-3	0.80±1e-2	0.41±8e-3	0.41±8e-3
	Δ	0.76±1e-2	0.56±3e-3	0.38±4e-3	0.50±7e-3	0.28±4e-3	0.58±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.99±4e-5	0.98±1e-4	0.99±8e-5	0.99±9e-5
	CS	0.07±3e-3	0.07±3e-3	0.07±3e-3	0.07±3e-3	0.07±3e-3	0.07±3e-3

Table 8: Problemwise comparison of the algorithms on the four performance metrics used, for the DTLZ family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	ϵ -MOEA	cone-MOEA	C-NSGA-II	SPEA2	NSGA-II*
Dtlz1	γ	0.23±7e-3	0.13±1e-3	0.17±2e-2	0.39±1e-2	0.18±2e-3	0.17±2e-3
	Δ	0.34±3e-3	0.12±2e-3	0.05±1e-2	0.20±2e-2	0.08±1e-3	0.34±4e-3
	HV	0.95±6e-4	0.92±4e-4	0.95±2e-4	0.96±5e-4	0.97±1e-4	0.96±5e-4
	CS	0.02±8e-4	0.01±9e-4	0.03±1e-3	0.00±5e-4	0.02±1e-3	0.02±10e-4
Dtlz2	γ	0.62±10e-3	0.70±7e-3	0.48±8e-3	0.75±1e-2	0.55±8e-3	0.48±6e-3
	Δ	0.81±10e-3	0.42±4e-3	0.42±6e-3	0.29±5e-3	0.16±3e-3	0.83±9e-3
	HV	0.89±9e-4	0.92±4e-4	0.94±1e-4	0.93±4e-4	0.89±9e-4	0.89±9e-4
	CS	0.03±1e-3	0.02±8e-4	0.06±2e-3	0.01±6e-4	0.03±1e-3	0.04±1e-3
Dtlz3	γ	0.35±2e-2	0.25±1e-2	0.42±3e-2	0.50±3e-2	0.32±2e-2	0.26±1e-2
	Δ	0.33±9e-3	0.19±1e-2	0.29±2e-2	0.22±3e-2	0.15±2e-2	0.34±8e-3
	HV	0.90±10e-4	0.91±2e-2	0.91±1e-2	0.91±1e-3	0.93±3e-4	0.90±9e-4
	CS	0.02±1e-3	0.03±2e-3	0.04±2e-3	0.00±5e-4	0.02±2e-3	0.02±2e-3
Dtlz4	γ	0.32±8e-3	0.41±2e-2	0.53±3e-2	0.34±1e-2	0.33±1e-2	0.30±3e-3
	Δ	0.67±2e-2	0.37±3e-2	0.43±2e-2	0.36±4e-2	0.22±3e-2	0.66±8e-3
	HV	0.88±1e-2	0.86±2e-2	0.86±2e-2	0.84±2e-2	0.87±2e-2	0.90±7e-4
	CS	0.04±2e-3	0.02±1e-3	0.03±2e-3	0.02±2e-3	0.03±2e-3	0.03±1e-3
Dtlz5	γ	0.14±2e-3	0.26±3e-3	0.56±3e-2	0.22±5e-3	0.15±2e-3	0.13±1e-3
	Δ	0.74±2e-2	0.78±5e-3	0.83±9e-3	0.43±6e-3	0.26±4e-3	0.61±e-2
	HV	0.99±1e-4	0.99±7e-5	0.98±3e-5	0.98±1e-4	0.99±4e-5	0.99±1e-4
	CS	0.06±2e-3	0.02±1e-3	0.04±1e-3	0.03±1e-3	0.05±2e-3	0.07±2e-3
Dtlz9	γ	0.84±8e-3	0.94±3e-3	0.83±3e-3	0.83±5e-3	0.84±7e-3	0.84±8e-3
	Δ	0.62±5e-3	0.71±1e-2	0.52±6e-3	0.44±7e-3	0.77±7e-3	0.90±5e-3
	HV	0.99±1e-4	0.99±1e-4	0.99±1e-4	0.99±1e-4	0.99±1e-4	0.99±1e-4
	CS	0.11±8e-3	0.51±1e-2	0.34±7e-3	0.23±8e-3	0.24±8e-3	0.13±7e-3

Numbers have their value.
But not *only* tables, please!

v1.0

v1.0: Two Approaches for Empirical Studies

Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur

Quality indicator approach

- reduces each approximation set to a single quality value
- applies statistical tests to the quality values

<i>Indicator</i>	A	B
Hypervolume indicator	6.3431	7.1924
ϵ -indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

see e.g. [\[Zitzler et al. 2003\]](#)

Empirical Attainment Functions

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[López-Ibáñez et al. 2010]

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Empirical Attainment Functions: Definition

The Empirical Attainment Function $\alpha(z)$ "counts" how many solution sets $\mathcal{X}_i$ attain or dominate a vector z at time T :

$$\alpha_T(z) = \frac{1}{N} \sum_{i=1}^N \mathbf{1}_{\{\mathcal{X}_i \preceq_T z\}}$$

with $\preceq_T$ being the weak dominance relation between a solution set and an objective vector at time T .

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with $\preceq_T$ being the weak dominance relation between a solution set and an objective vector at time T .

Note that $\alpha_T(z)$ is the **empirical cumulative distribution function of the achieved objective function distribution at time T** in the single-objective case ("fixed budget scenario").

Empirical Attainment Functions in Practice

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[López-Ibáñez et al. 2010]

latest implementation online at
<http://eden.dei.uc.pt/~cmfonsec/software.html>
R package: <http://lopez-ibanez.eu/eaftools>
see also [López-Ibáñez et al. 2010, Fonseca et al. 2011]

Quality Indicator Approach

Idea:

- transfer multiobjective problem into a set problem
- define an objective function (“unary quality indicator”) on sets
- use the resulting total (pre-)order (on the quality values)

Quality Indicator Approach

Idea:

- transfer multiobjective problem into a set problem
- define an objective function (“unary quality indicator”) on sets
- use the resulting total (pre-)order (on the quality values)

Question:

Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?

⇒ Underlying dominance relation
should be reflected!

$$A \preceq B: \Leftrightarrow \forall_{b \in B} \exists_{a \in A} a \preceq b$$

Monotonicity and Strict Monotonicity

- Monotonicity when quality indicator does not contradict relation

$$A \preceq B \Rightarrow I(A) \geq I(B)$$

Monotonicity and Strict Monotonicity

- Monotonicity when quality indicator does not contradict relation

$$A \preceq B \Rightarrow I(A) \geq I(B)$$

- Strict monotonicity: better = higher indicator

$$A \preceq B \text{ and } A \neq B \Rightarrow I(A) > I(B)$$

Example: Refinements Using Indicators

$I(A)$ = volume of the weakly dominated area in objective space

unary hypervolume indicator

$I(A, R)$ = how much needs A to be moved to weakly dominate R

unary epsilon indicator

v1.0.1 – v1.0.100 and counting

Performance Assessment of Multiobjective Optimizers: An Analysis and Review

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22 indicators

[Zitzler et al. 2003]

Even More Indicators...

Performance indicators in multiobjective optimization

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63 indicators

[Audet et al 2021]

Quality Evaluation of Solution Sets in Multiobjective Optimisation: A Survey

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100 indicators

[Li and Yao 2019]

Many Indicators: What Do We Do?

Focus on indicators which are (strictly) monotone

- all hypervolume-based indicators
- unary epsilon indicator
- R2
- IGD+

Many Indicators: What Do We Do?

Focus on indicators which are (strictly) monotone

- all hypervolume-based indicators
- unary epsilon indicator
- R2
- IGD+

Why is monotonicity important?

- Pareto dominance is the lowest form of preference
- If dominance relation does not hold, we have not defined a true multiobjective problem.

v2.0

Benchmarking Multiobjective Optimizers 2.0

- With the right (strictly) monotone indicator, multiobjective optimization is not different from single-objective optimization (!)

We can use our normal tools from single-objective optimization, including

- reporting of target-based runtimes
- ECDFs of runtimes, performance profiles, data profiles
- statistical tests, box plots, ...

Measuring Performance Empirically

convergence graphs is all we have to start with...

Measuring Performance Empirically

convergence graphs is all we have to start with...

ECDF:

Empirical Cumulative Distribution Function of the Runtime

A Convergence Graph

First Hitting Time is Monotonous

15 Runs

15 Runs $\leq$ 15 Runtime Data Points

Empirical Cumulative Distribution

the **ECDF** of run lengths to reach the target

- has for each data point a **vertical step of constant size**
- displays for each x-value (budget) the count of observations to the left (first hitting times)

Reconstructing A Single Run

Reconstructing A Single Run

50 equally
spaced targets

Reconstructing A Single Run

Reconstructing A Single Run

Reconstructing A Single Run

the **empirical CDF** makes a step for each star, is monotonous and displays for each budget the fraction of targets achieved within the budget

Reconstructing A Single Run

the ECDF
recovers the
monotonous
graph,
discretised and
flipped

Reconstructing A Single Run

the ECDF
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Aggregation

Aggregation

15 runs

50 targets

Aggregation

15 runs

50 targets

Aggregation

15 runs

50 targets

ECDF with
750 steps

Aggregation

50 targets from
15 runs
integrated in a
single graph

Interpretation

50 targets from
15 runs
integrated in a
single graph

area over the
ECDF curve

=

average log
runtime

(or geometric avg.
runtime) over all
targets (difficult and
easy) and all runs

ECDF graphs

- should never aggregate over dimension
dimension is input parameter to algorithm

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- but often over targets and functions
- can show data of more than 1 algorithm at a time

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 - but for multiple and absolute targets

ECDF graphs

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- can show data of more than 1 algorithm at a time
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 - introduced by Moré and Wild [Moré and Wild 2009]
 - but for multiple and absolute targets
- are COCO's main performance visualization tool
<https://github.com/numbbo/coco>

Example ECDF (later more)

Mostly Overlooked: Scaling with Dimension

- In single-objective optimization: scaling behavior mandatory to investigate

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 - actually two dimensions: search and objective space

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~10 papers from EMO'21 and PPSN/GECCO/CEC'21 change dimension but 50+ papers have a “fixed” dimension

Mostly Overlooked: Scaling with Dimension

- In single-objective optimization: scaling behavior mandatory to investigate
- In multiobjective optimization:
 - actually two dimensions: search and objective space
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~10 papers from EMO'21 and PPSN/GECCO/CEC'21 change dimension
but 50+ papers have a “fixed” dimension

- but in practice search space scalability almost more important
number of objectives often fixed

A Few General Recommendations

- always **display everything** you have
- look at **single runs**
- do each experiment **at least twice**
(= look at the *variance* of your results)

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or any indicator which is at least monotone

A Few General Recommendations

- always **display everything** you have
- look at **single runs**
- do each experiment **at least twice**
(= look at the *variance* of your results)
- as quality indicators, use hypervolume, R2, or epsilon indicator
or any indicator which is at least monotone
- see also the tutorial slides by Nikolaus Hansen on this topic (not restricted to single-objective optimization!)

<http://www.cmap.polytechnique.fr/~nikolaus.hansen/gecco2018-experimentation-guide-slides.pdf>

Recommended Experimental Setup (w/ or w/o COCO)

① Benchmarking Experiment

② Choosing Algorithms for Comparison

see <https://numbbo.github.io/data-archive/>

③ Postprocessing

```
python -m cocopp resultfolder/ ALG2 ALG3
```

④ Displaying and Discussing Summary Results

⑤ Investigating and Discussing Complementary Results

⑥ Processed Data Sharing

provide html output somewhere

⑦ Raw Data Sharing

easy with COCO archive module & through issue tracker

- ❶ Performance Assessment
- ❷ Test Problems and Their Visualizations
- ❸ Recommendations from Numerical Results

Test Problems and Their Visualizations

Introduction

Test Problems (1)

Artificial problems (continuous and unconstrained)

- v0.1:** Individual problems
- v0.2:** MOP suite (unscalable problems)
- v0.5:** ZDT suite (scalable number of variables)
- v1.0:** DTLZ suite (scalable number of variables and objectives)
- v1.2:** WFG suite
- v1.3:** Other suites with a bottom-up construction
- v1.5:** Suites of distance-based problems
- v2.0:** The bbob-biobj(-ext) suite

Visualization of multiobjective landscapes

Low-dimensional search spaces

Dominance ratio

Local dominance

Gradient path length

PLOT

Any-dimensional search spaces

Line cuts

Optima network

Test Problems and Their Visualizations

Test Problems (2)

Artificial problems (other)

Constrained problems

Mixed-integer problems

Real-world problems

v0.1: Individual problems

v0.2: Suites of unscalable problems

v0.5: Suites of scalable problems (number of variables)

Conclusions

Introduction

Why use test problems?

Desirable Characteristics of a Problem Set

[Bartz-Beielstein et al. 2020]

1. Diverse
2. Representative
3. Scalable and tunable
4. Known optima / best performance
5. [Continually updated]

Artificial problems (continuous and unconstrained)

v0.1

Individual problems

$$\text{Minimize} = \begin{cases} f_1(x) = x^2 \\ f_2(x) = (x - 2)^2 \end{cases}$$

[Schaffer 1985]

$$\text{Minimize} = \begin{cases} f_1(\mathbf{x}) = \sum_{i=1}^2 \left[-10 \exp \left(-0.2 \sqrt{x_i^2 + x_{i+1}^2} \right) \right] \\ f_2(\mathbf{x}) = \sum_{i=1}^3 \left[|x_i|^{0.8} + 5 \sin(x_i^3) \right] \end{cases}$$

[Kursawe 1991]

Individual problems

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$$\text{Minimize} = \begin{cases} f_1(\mathbf{x}) = 1 - \exp\left[-\sum_{i=1}^n \left(x_i - \frac{1}{\sqrt{n}}\right)^2\right] \\ f_2(\mathbf{x}) = 1 - \exp\left[-\sum_{i=1}^n \left(x_i + \frac{1}{\sqrt{n}}\right)^2\right] \end{cases}$$

[Fonseca and Fleming 1995]

$$\text{Minimize} = \begin{cases} f_1(x, y) = \left[1 + (A_1 - B_1(x, y))^2 + (A_2 - B_2(x, y))^2\right] \\ f_2(x, y) = (x + 3)^2 + (y + 1)^2 \end{cases}$$

$$\text{where} = \begin{cases} A_1 = 0.5 \sin(1) - 2 \cos(1) + \sin(2) - 1.5 \cos(2) \\ A_2 = 1.5 \sin(1) - \cos(1) + 2 \sin(2) - 0.5 \cos(2) \\ B_1(x, y) = 0.5 \sin(x) - 2 \cos(x) + \sin(y) - 1.5 \cos(y) \\ B_2(x, y) = 1.5 \sin(x) - \cos(x) + 2 \sin(y) - 0.5 \cos(y) \end{cases}$$

[Poloni et al. 1996]

Individual problems

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$$\text{Minimize} = \begin{cases} f_1(x, y) = 0.5(x^2 + y^2) + \sin(x^2 + y^2) \\ f_2(x, y) = \frac{(3x-2y+4)^2}{8} + \frac{(x-y+1)^2}{27} + 15 \\ f_3(x, y) = \frac{1}{x^2+y^2+1} - 1.1 \exp(-(x^2 + y^2)) \end{cases}$$

[Viennet et al. 1996]

v0.2

MOP = Multi-Objective Problem

[Van Veldhuizen 1999]

Properties

- A collection of 7 test problems from the literature (including the 5 shown before)
- Most problems have 2 or 3 variables
- Not scalable in the number of objectives
- Many problems have optimal solutions on the boundary or middle of the search space
- Some problems are both nonseparable and multimodal
- A collection of various Pareto front geometries
- The Pareto set is hard to compute for some problems

v0.5

ZDT = Zitzler, Deb, Thiele

[Zitzler et al. 2000]

The same construction for all problems
(following Deb's toolkit [Deb 1999])

Given $\mathbf{x} = \{x_1, \dots, x_n\}$
Distribution f.
Minimise $f_1(\mathbf{y})$
Distance f. Front shape
 $f_2(\mathbf{y}, \mathbf{z}) = g(\mathbf{z})h(f_1(\mathbf{y}), g(\mathbf{z}))$

where $\mathbf{y} = \{x_1, \dots, x_j\}$ Position variable(s) ($j = 1$ for ZDT)

$\mathbf{z} = \{x_{j+1}, \dots, x_n\}$ Distance variables

The separation of variables was done to simplify problem construction

ZDT1

ZDT2

ZDT3

Properties

- 6 test problems, but ZDT5 is regularly omitted, because it has a binary encoding
- Scalable in the number of (distance) variables
- All problems have 2 objectives
- 4 problems have optimal solutions on the boundary and 1 in the middle of the search space
- All problems are separable (the first objective depends only on the first variable)
- Some problems are multimodal
- Convex, concave and disconnected Pareto fronts
- The Pareto sets and fronts are known

v1.0

DTLZ = Deb, Thiele, Laumanns, Zitzler

[Deb et al. 2005]

Desired Features of Test Problems

1. Have controllable difficulty to converge to the Pareto front, a widely-distributed set of Pareto-optimal solutions
2. Scalable number of variables
3. Scalable number of objectives
4. Simple to construct
5. Pareto front easy to comprehend, both the Pareto set and front known
6. Similar difficulties to those present in real-world problems

Problem Design Approaches

1. Multiple single-objective functions approach
2. Bottom-up approach
 1. Choose a Pareto front
 2. Build the objective space
 3. Construct the search space (add difficulties using the function g)
3. Constraint surface approach (for constrained problems)

Properties

- Originally 9 problems, but then 2 were dropped
- Scalable number of distance variables, $M - 1$ position variables
- Scalable number of objectives
- Objectives separable in practice (optimizing one variable at a time will yield at least one global optimum)
- Linear, concave and disconnected Pareto fronts
- The Pareto sets and fronts are known
- Most problems have the Pareto set in the middle of the search space

Note that although the suite is scalable in the number of variables, this is rarely used in benchmark studies

v1.2

WFG = Walking Fish Group

[Huband et al. 2006]

Recommendations for multiobjective test problems

1. No extremal variables
2. No medial variables
3. Scalable number of variables
4. Scalable number of objectives
5. Dissimilar variable domains
6. Dissimilar objective ranges
7. Pareto set and front known

Recommendations for multiobjective test suites

[Huband et al. 2006]

1. A few unimodal test problems to test convergence velocity relative to different Pareto optimal geometries and bias conditions
2. Cover the three core types of geometries: degenerate Pareto fronts, disconnected Pareto fronts, and disconnected Pareto sets
3. The majority of problems should be multimodal with a few deceptive problems
4. The majority of problems should be nonseparable
5. Contain problems that are both nonseparable and multimodal to be representative of real-world problems

Properties

- 9 problems constructed from a combination of shape functions and several transformations
- Scalable number of variables (2 variables are not supported for some of the problems)
- Scalable number of objectives
- Includes also nonseparable, multimodal, deceptive and biased problems
- Convex, linear, concave, mixed, disconnected and degenerate Pareto fronts
- The Pareto sets and fronts are known
- Optimal solutions do not lie on the boundary or the middle of the search space, but the Pareto set is linear for 8 of the 9 problems
- Still rely on distance and position variables

v1.3

Problems constructed with the bottom-up approach

[Zapotecas et al. 2019]

- LZ test suite of 9 problems with complicated Pareto sets [Li and Zhang 2009]
- SZDT test suite of 7 scalable problems with complicated Pareto sets [Saxena et al. 2011]
- Convex DTLZ problem [Deb and Jain 2014]
- Inverted DTLZ problem [Jain and Deb 2014]
- MNI test suite of 2 test problems with diverse shapes of the Pareto front [Masuda et al. 2016]
- LSMOP test suite of 9 test problems for large-scale optimization with variable dependencies [Cheng et al. 2017b]
- Minus-DTLZ and Minus-WFG test suites [Ishibuchi et al. 2017]
- MMF test problems with diverse landscapes [Yue et al. 2019]

CEC Competition Suites

Information about all CEC competitions:

https://www3.ntu.edu.sg/home/EPNSugan/index_files/cec-benchmarking.htm

- 13 test problems for CEC 2007 [Huang et al. 2007]
 - OKA [Okabe et al. 2004], SYM-PART [Rudolph et al. 2007]
 - 4 shifted ZDT, 1 rotated ZDT
 - 2 shifted DTLZ, 1 rotated DTLZ
 - 3 WFG
- 13 test problems for CEC 2009 (UF suite) [Zhang et al. 2009]
 - 10 with complicated Pareto sets (4 from the LZ suite)
 - 2 extended rotated DTLZ
 - 1 WFG

CEC Competition Suites

- 15 test problems for CEC 2017 (MaF suite) [Cheng et al. 2017a]
 - 7 modified DTLZ problems
 - 2 distance minimization problems
 - 3 WFG problems
 - 1 SZDT problem
 - 2 LSMOP problem
- 22 test problems for CEC 2019 [Liang et al. 2019]
 - 2 SYM-PART
 - Omni-test [Deb and Tiwari 2008]
 - 19 MMF problems
- 24 test problems for CEC 2020 [Liang et al. 2020]
 - 24 MMF problems

Survey of Recent Papers

- 64 papers on unconstrained continuous multiobjective optimization from recent conferences (without application papers)
 - CEC 2020
 - GECCO 2020
 - PPSN 2020
 - EMO 2021

v1.5

Distance-Based Problems

General idea

[Ishibuchi et al. 2010]

Minimize $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x}))$

$f_i(\mathbf{x}) = \min\{\text{dis}(\mathbf{x}, \mathbf{a}_{i1}), \text{dis}(\mathbf{x}, \mathbf{a}_{i2}), \dots, \text{dis}(\mathbf{x}, \mathbf{a}_{im})\}$

- 2-D test problems that are inherently visualizable
- Pareto set easy to characterize
- Scalable in the number of objectives
- Useful for visualizing the distribution of solutions
- Unlikely to be relevant for real-world problems
- Based on earlier work [Köppen et al. 2005, Rudolph et al. 2007]

Extensions

- High-dimensional search spaces [Masuda et al. 2014]
- Distance to lines (instead of points) [Li et al. 2014, 2018]
- Dominance resistance regions [Fieldsend 2016]
- Local Pareto fronts [Liu et al. 2018]
- Problem generator for scalable problems with various properties (local fronts, disconnected Pareto sets and fronts, dominance resistance regions, uneven ranges of objective values, varying density of solutions) [Fieldsend et al. 2019]

v2.0

Motivation

[Brockhoff et al. 2016]

- Most other suites are constructed based on the desired Pareto front properties
- Consequently, problems have artificial properties not likely to exist in real-world problems
 - Distance and position parameters (DTLZ-like problems)
 - Linear objectives (distance-based problems)
- In real-world problems each objective is a separate function
- Go back to basics – use single-objective functions for each objective
 - Idea not new [Schaffer 1985, Igel et al. 2007, Emmerich and Deutz 2007, Kerschke et al. 2016]

Construction

- Use the functions from the **bbob** suite
 - Well-understood
 - Scalable in the number of variables and *parametrized*
 - 24 functions categorized in 5 groups based on their properties
 - Separable
 - Low or moderate conditioning
 - High conditioning and unimodal
 - Multimodal with global structure
 - Multimodal with weak global structure
- How to avoid an explosion in the number of problems?

bbob-biobj Suite

bbob-biobj-ext Suite

Separable					Low or moderate conditioning				High conditioning and unimodal					Multimodal with global structure					Multimodal with weak global structure						
f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}	f_{17}	f_{18}	f_{19}	f_{20}	f_{21}	f_{22}	f_{23}	f_{24}		
F_1	F_2	F_{56}	F_{57}	F_{58}	F_3		F_4						F_5	F_6	F_7		F_8			F_9	F_{10}				f_1
	F_{11}	F_{59}	F_{60}	F_{61}	F_{12}		F_{13}						F_{14}	F_{15}	F_{16}		F_{17}			F_{18}	F_{19}				f_2
			F_{62}	F_{63}																					f_3
				F_{64}																					f_4
																									f_5
					F_{20}	F_{65}	F_{21}	F_{66}					F_{22}	F_{23}	F_{24}		F_{25}			F_{26}	F_{27}				f_6
							F_{67}	F_{68}																	f_7
								F_{28}	F_{69}				F_{29}	F_{30}	F_{31}		F_{32}			F_{33}	F_{34}				f_8
																									f_9
													F_{70}	F_{71}	F_{72}	F_{73}									f_{10}
														F_{74}	F_{75}	F_{76}									f_{11}
															F_{77}	F_{78}									f_{12}
															F_{35}	F_{36}	F_{37}		F_{38}			F_{39}	F_{40}		f_{13}
															F_{41}	F_{42}		F_{43}			F_{44}	F_{45}			f_{14}
																F_{46}		F_{47}	F_{79}	F_{80}	F_{48}	F_{49}			f_{15}
																									f_{16}
																									f_{17}
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																									f_{20}
																									f_{21}
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																									f_{23}
																									f_{24}
																									f_{24}

f_1 Sphere
 f_2 Ellipsoid separable
 f_3 Rastrigin separable
 f_4 Skew Rastrigin-Bueche
 f_5 Linear slope

f_6 Attractive sector
 f_7 Step-ellipsoid
 f_8 Rosenbrock original
 f_9 Rosenbrock rotated

f_{10} Ellipsoid
 f_{11} Discus
 f_{12} Bent cigar
 f_{13} Sharp ridge
 f_{14} Sum of different powers

f_{15} Rastrigin
 f_{16} Weierstrass
 f_{17} Schaffer F7 (condition 10)
 f_{18} Schaffer F7 (condition 1000)
 f_{19} Griewank-Rosenbrock F8F2

f_{20} Schwefel $x^*\sin(x)$
 f_{21} Gallagher 101 peaks
 f_{22} Gallagher 21 peaks
 f_{23} Katsuuras
 f_{24} Lunacek bi-Rastrigin

Separable

Low or moderate conditioning

High conditioning and unimodal

Multimodal with global structure

Multimodal with weak global structure

Properties

- Construction similar as in real-world problems
- Scalability in the number of variables
- Various problem properties (more diverse than existing multiobjective test suites)
- Problem instances more diverse than for the single-objective suite
- Currently limited to 2 objectives
- Unknown Pareto set and front (but known single-objective optima)
- Available approximations of the Pareto fronts (and sets for lower-dimensional problems)

Visualization of multiobjective landscapes

Visualization in Multiobjective Optimization

[Filipič and Tušar 2018, 2020]

Visualization of Multiobjective Problem Landscapes

Low-dimensional search spaces

Dominance ratio [Fonseca 1995]

Gradient path length (inspired by gradient plots [Kerschke and Grimme 2017])

Local dominance [Fieldsend et al. 2019]

PLOT [Shaepermeier et al. 2020]

Any-dimensional search spaces

Line cuts [Brockhoff et al. 2016, Volz et al. 2019]

Optima network [Liefvooghe et al. 2018, Fieldsend and Alyahya 2019]

Various visualizations of bbob-biobj-ext problems:

<https://numbbo.github.io/bbob-biobj/>

Visualizations of bbob-biobj and other multi-objective suites using PLOT:

<https://schaepermeier.shinyapps.io/moPLOT/>

Visualization of Multiobjective Problem Landscapes

Problems for demonstration

- Double sphere problem bbob-biobj $F_1 = (f_1, f_1)$, instance 1
- Sphere-Gallagher problem bbob-biobj $F_{10} = (f_1, f_{21})$, instance 1
- Double Gallagher problem bbob-biobj $F_{55} = (f_{21}, f_{21})$, instance 1

Gallagher = Gallagher's Gaussian
101-me Peaks Function

Dominance Ratio

[Fonseca 1995]

- Discretized search space (501 x 501 grid)
- Dominance ratio = the ratio of grid points that dominate the current point
- All nondominated points have a ratio of zero
- Visualize dominance ratios in logarithmic scale

Dominance ratio ($F_1 = (f_1, f_1)$, 2-D, inst. 1)

Dominance ratio ($F_{10} = (f_1, f_{21})$, 2-D, inst. 1)

Dominance ratio ($F_{55} = (f_{21}, f_{21})$, 2-D, inst. 1)

Gradient Path Length

Adjusted from [Kerschke and Grimme 2017]

- Compute the **bi-objective gradient** for all grid points

$$v = \frac{\nabla f_1(x)}{\|\nabla f_1(x)\|} + \frac{\nabla f_2(x)}{\|\nabla f_2(x)\|}$$

- From a grid point, follow the path in the direction of this gradient
- Visualize the length of the path to the local optimum

Path length ($F_1 = (f_1, f_1)$, 2-D, inst. 1)

Path length ($F_{10} = (f_1, f_{21})$, 2-D, inst. 1)

Path length ($F_{55} = (f_{21}, f_{21})$, 2-D, inst. 1)

Local Dominance

[Fieldsend et al. 2019]

- Green: Dominance-neutral local optima regions
 - Points that are mutually nondominated with all their 8 neighbors (not equal to Pareto sets)
- Pink: Basins of attraction
 - Points that are dominated by at least one neighbor and whose dominating paths lead to the same green region
- White: Boundary regions
 - Points whose dominating paths lead to different green regions

Line Cuts

Search space projection ($F_1 = (f_1, f_1)$, 5-D, inst. 2)

Search space projection ($F_{10} = (f_1, f_{21})$, 5-D, inst. 2)

Search space projection ($F_{55} = (f_{21}, f_{21})$, 5-D, inst. 2)

Unscaled obj. space ($F_1 = (f_1, f_1)$, 5-D, inst. 2)

Unscaled obj. space ($F_{10} = (f_1, f_{21})$, 5-D, inst. 2)

Unscaled obj. space ($F_{55} = (f_{21}, f_{21})$, 5-D, inst. 2)

Comparison of Problem Landscapes

ZDT4

Two problems where both objectives are separable, first is unimodal and second is multimodal

bbob-biobj-ext F_{56}
 f_1 Sphere function
 f_3 Rastrigin function

Comparison of Problem Landscapes

DTLZ1

Two problems where both objectives are separable and multimodal

bbob-biobj-ext F_{62}

f_3 Rastrigin function

f_4 Skew Rastrigin-Bueche

Comparison of Problem Landscapes

WFG9

Two problems where both objectives are nonseparable and multimodal

bbob-biobj-ext F_{82}

f_{17} Schaffer F7

f_{19} Griewank-Rosenbrock

Other artificial problems

Suites of multiobjective problems with constraints

- CTP [Deb et al. 2001]
- CF [Zhang et al. 2009]
- C-DTLZ [Jain and Deb 2014]
- NCTP [Li et al. 2016]
- DC-DTLZ [Li et al. 2019]
- LIR-CMOP [Fan et al. 2019a]
- DAS-CMOP [Fan et al. 2019b]
- MW [Ma and Wang 2019]

Analysis and visualization of multiobjective problems with constraints:

<https://vodopijaaljosa.github.io/cmop-web/>

Tutorial on Multiobjective optimization in the presence of constraints:

<https://dis.ijs.si/filipic/cec2021tutorial/>

Other Artificial Problems

Suites of multiobjective mixed-integer problems

- Exeter suite of 6 problems constructed with the bottom-up approach [McClymont and Keedwell 2011]
- bbob-biobj-mixint suite of 92 bi-objective problems [Tušar et al. 2019]

Pareto set and front approximations for three different instances of the double sphere function

Real-world problems

v0.1

Individual problems

- Radar waveform design problem with a varying number of variables and 9 objectives [Hughes 2007]
- HBV problem of calibrating the HBV rainfall-runoff model with 14 variables and 4 objectives [Reed et al. 2013]
- MAZDA car structure design problem with 222 integer variables, 2 objectives and 54 constraints [Kohira et al. 2018]

v0.2

Suites of unscalable problems

[Tanabe and Ishibuchi 2020]

- RE suite of 16 test problems with a different number of variables (2–7) and objectives (2–9)
 - 11 continuous problems
 - 1 integer problem
 - 4 mixed-integer problems
- CRE suite of 8 test problems with constraints a different number of variables (3–7) and objectives (2–5)
 - 6 continuous problems
 - 1 integer problem
 - 1 mixed-integer problem
- Both suites consist of previously published problems

v0.5

Suites of scalable problems

- Heat exchanger design problem with scalable variables and 1 or 2 objectives [Daniels et al. 2018]
- Suite of 3 bi-objective TopTrumps problems in multiple dimensions and instances [Volz et al. 2019]
- Suite of 26 bi-objective MarioGAN problems in multiple dimensions and instances [Volz et al. 2019]

Conclusions

We should think about the usefulness of our research

Results of a questionnaire on the properties of real-world problems has shown their diversity [van der Blom et al. 2020]

<https://sites.google.com/view/maco-da-rwp/home>

Most research is done on continuous unconstrained problems

Although the test problems are scalable, most studies use a fixed number of variables

Conclusions

Problem suites constructed with the bottom-up approach have unrealistic properties

Algorithms are overfitting to these problems (especially the overused DTLZ and WFG)

[Ishibuchi et al. 2017]

Using separate functions for the objectives looks like a step in the right direction

- ① Performance Assessment
- ② Test Problems and Their Visualizations
- ③ Recommendations from Numerical Results


```
python -m cocopp bbob-biobj*
```

Aggregated Results Over All 55 Functions

Aggregated Results Over All 55 Functions

Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions

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- benchmarking results from more classical approaches

Three “New Year” Resolutions

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anything else than tables for fixed budget

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Thank you!

Supplementary Material

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