# GECCO 2021 Tutorial on Benchmarking Multiobjective Optimizers 2.0

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# GECCO 2021 Tutorial on Benchmarking Multiobjective Optimizers 2.0

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# Isn’t Benchmarking Trivial?

# Isn’t Benchmarking Trivial?

- Choose some algorithms
- Choose some test functions
- Discuss the results

# Isn’t Benchmarking Trivial?

In principle: yes

But many details and pitfalls

# Isn’t Benchmarking Trivial?

In principle: yes

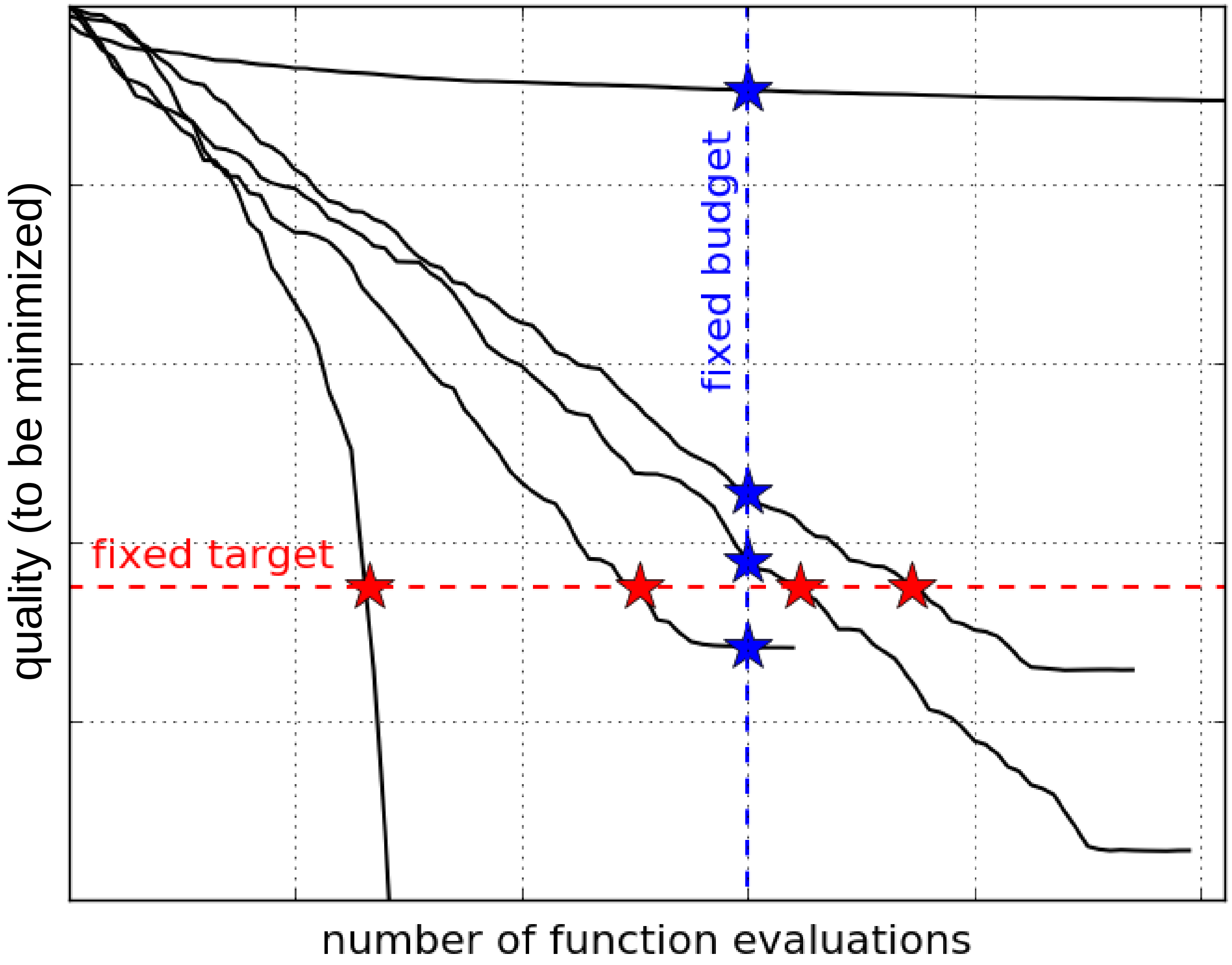
But many details and pitfalls

and in addition

- we compare *sets* of solutions
- which might be random (in size, position, dimension, …)
- …

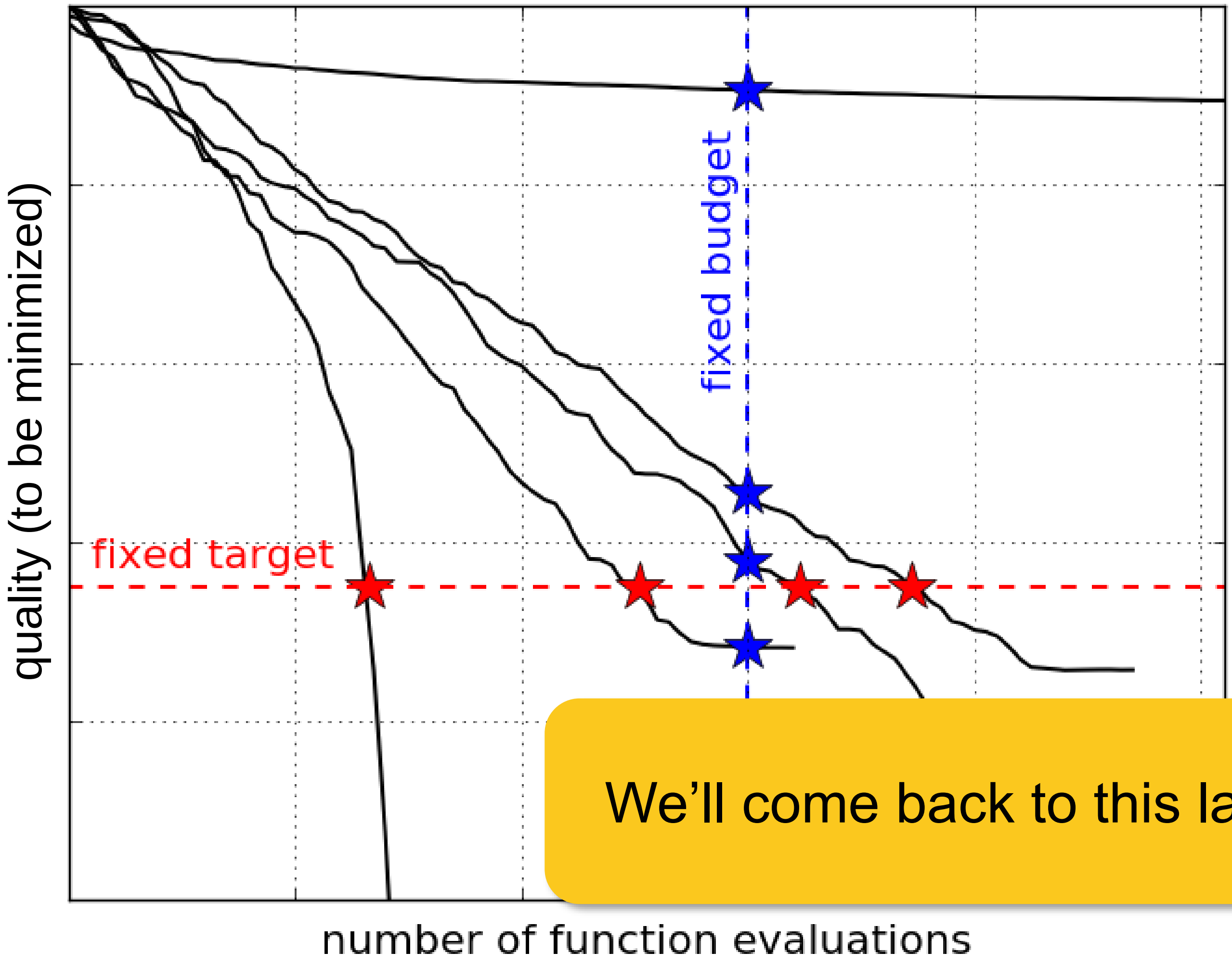
# We Typically Start with Convergence Graphs

(…in single-objective optimization)



# We Typically Start with Convergence Graphs

(…in single-objective optimization)



We’ll come back to this later again

# Overview

## Our plan

Discuss history, present and future of multiobjective benchmarking

## With respect to different topics

- performance assessment / methodology
- test functions

## Finally, recommendations on good algorithms

# Disclaimer

This is not an introductory tutorial to multiobjective optimization!

We assume you know basic definitions like

- Objective function
- Pareto dominance/Pareto front/Pareto set
- Ideal/Nadir points

# Disclaimer II

We only consider continuous search spaces

# Disclaimer II

We only consider continuous search spaces

We only consider unconstrained problems

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We only consider unconstrained problems

What we present is highly subjective & selective

- how important do we find each milestone?
- use version numbering and branches
- what have we learned from the past?

# Overview

➊ Performance Assessment

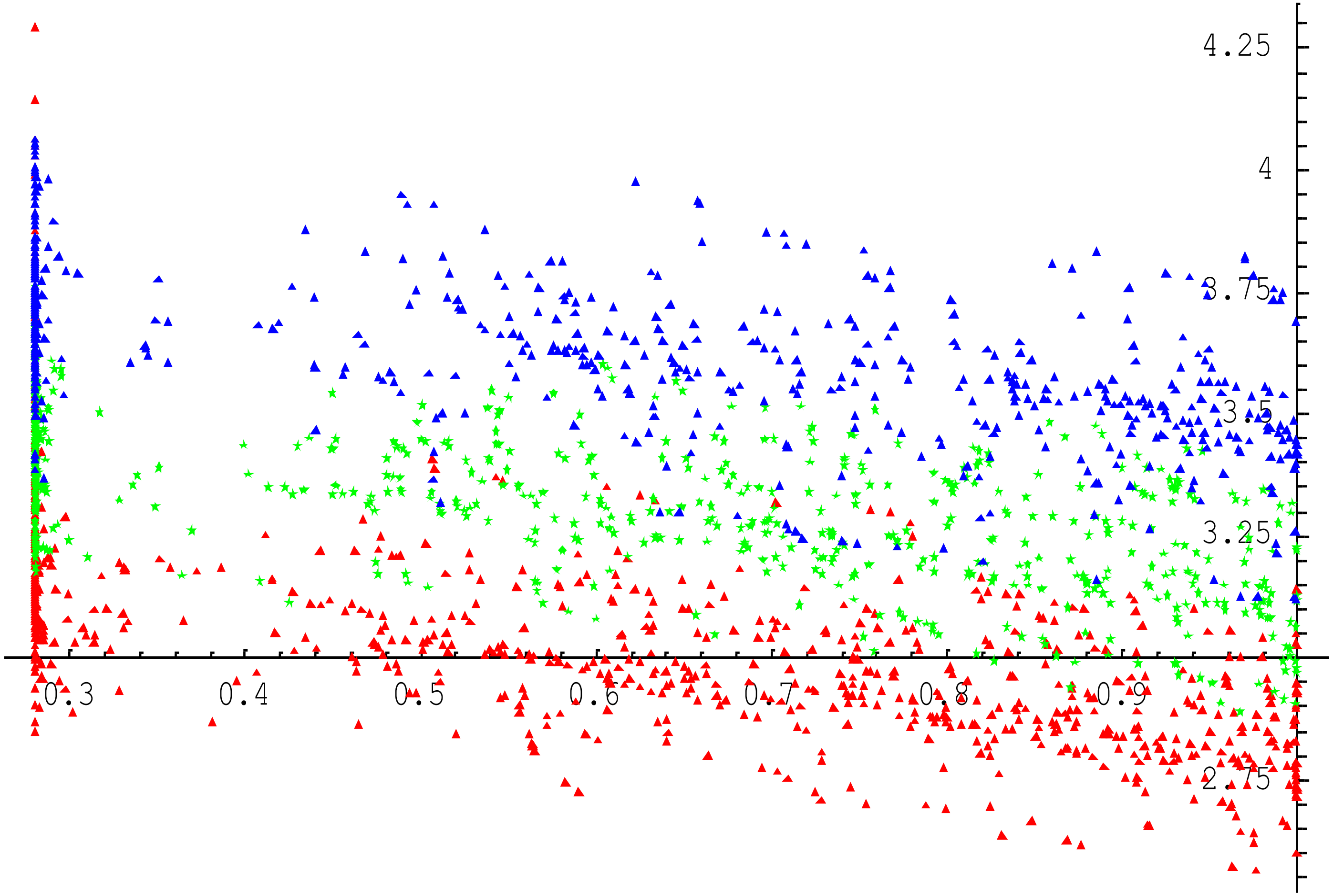
➋ Test Problems and Their Visualizations

➌ Recommendations from Numerical Results

# v0.0.1alpha

# In The Early Beginnings…

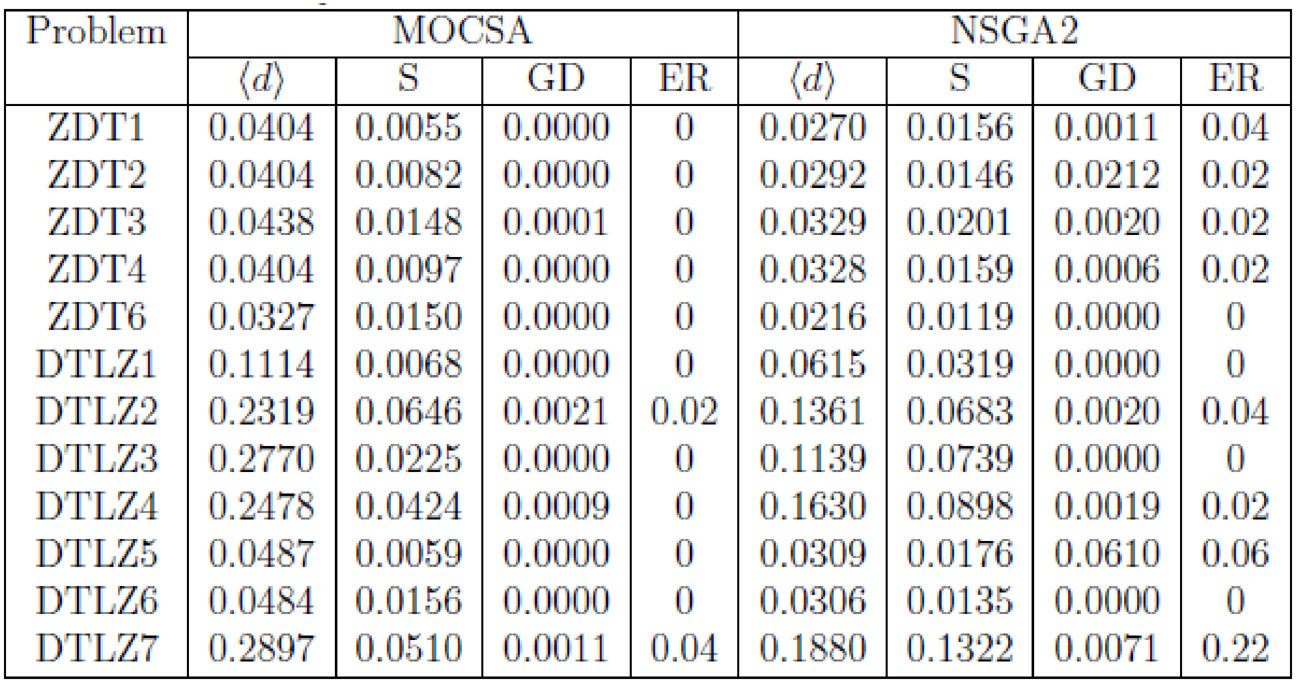
... multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

# v0.1beta

# Tables



| Problem | MOCSA | | | | NSGA2 | | | |
|---|---|---|---|---|---|---|---|---|
| | $\langle d \rangle$ | S | GD | ER | $\langle d \rangle$ | S | GD | ER |
| ZDT1 | 0.0404 | 0.0055 | 0.0000 | 0 | 0.0270 | 0.0156 | 0.0011 | 0.04 |
| ZDT2 | 0.0404 | 0.0082 | 0.0000 | 0 | 0.0292 | 0.0146 | 0.0212 | 0.02 |
| ZDT3 | 0.0438 | 0.0148 | 0.0001 | 0 | 0.0329 | 0.0201 | 0.0020 | 0.02 |
| ZDT4 | 0.0404 | 0.0097 | 0.0000 | 0 | 0.0328 | 0.0159 | 0.0006 | 0.02 |
| ZDT6 | 0.0327 | 0.0150 | 0.0000 | 0 | 0.0216 | 0.0119 | 0.0000 | 0 |
| DTLZ1 | 0.1114 | 0.0068 | 0.0000 | 0 | 0.0615 | 0.0319 | 0.0000 | 0 |
| DTLZ2 | 0.2319 | 0.0646 | 0.0021 | 0.02 | 0.1361 | 0.0683 | 0.0020 | 0.04 |
| DTLZ3 | 0.2770 | 0.0225 | 0.0000 | 0 | 0.1139 | 0.0739 | 0.0000 | 0 |
| DTLZ4 | 0.2478 | 0.0424 | 0.0009 | 0 | 0.1630 | 0.0898 | 0.0019 | 0.02 |
| DTLZ5 | 0.0487 | 0.0059 | 0.0000 | 0 | 0.0309 | 0.0176 | 0.0610 | 0.06 |
| DTLZ6 | 0.0484 | 0.0156 | 0.0000 | 0 | 0.0306 | 0.0135 | 0.0000 | 0 |
| DTLZ7 | 0.2897 | 0.0510 | 0.0011 | 0.04 | 0.1880 | 0.1322 | 0.0071 | 0.22 |

arXiv, 2012

# Tables

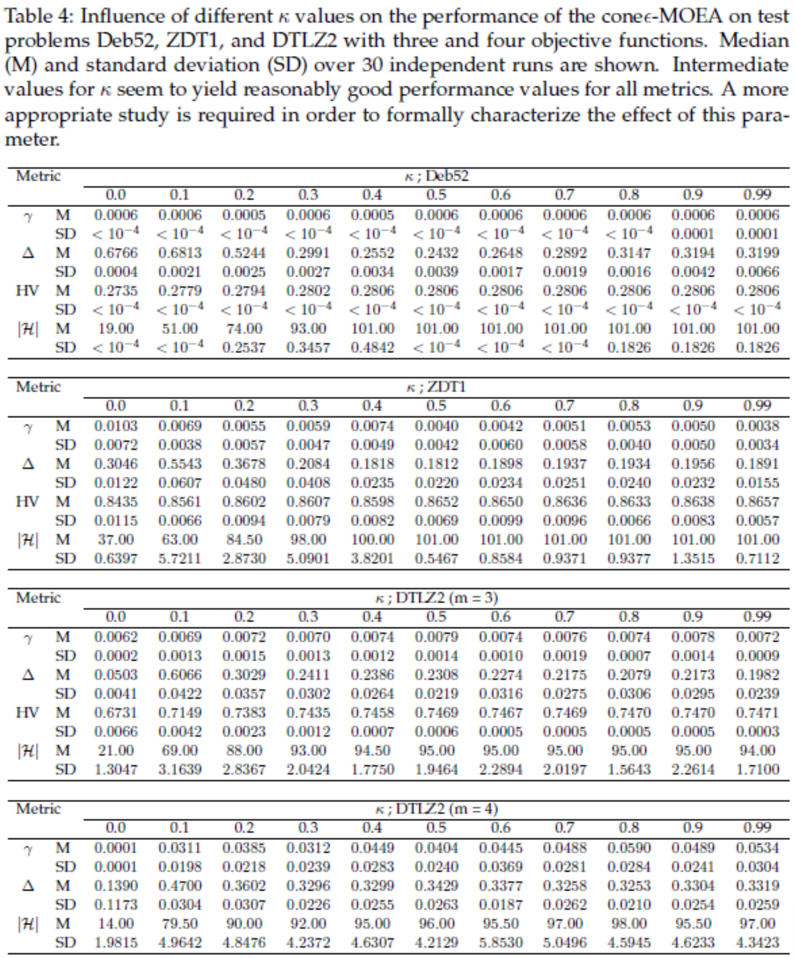


Table 4: Influence of different $\kappa$ values on the performance of the cone$\epsilon$-MOEA on test problems Deb52, ZDT1, and DTLZ2 with three and four objective functions. Median (M) and standard deviation (SD) over 30 independent runs are shown. Intermediate values for $\kappa$ seem to yield reasonably good performance values for all metrics. A more appropriate study is required in order to formally characterize the effect of this parameter.

| Metric | | $\kappa$ ; Deb52 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.99 |
| $\gamma$ | M | 0.0006 | 0.0006 | 0.0005 | 0.0006 | 0.0005 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 |
| | SD | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | 0.0001 | 0.0001 |
| $\Delta$ | M | 0.6766 | 0.6813 | 0.5244 | 0.2991 | 0.2552 | 0.2432 | 0.2648 | 0.2892 | 0.3147 | 0.3194 | 0.3199 |
| | SD | 0.0004 | 0.0021 | 0.0025 | 0.0027 | 0.0034 | 0.0039 | 0.0017 | 0.0019 | 0.0016 | 0.0042 | 0.0066 |
| HV | M | 0.2735 | 0.2779 | 0.2794 | 0.2802 | 0.2806 | 0.2806 | 0.2806 | 0.2806 | 0.2806 | 0.2806 | 0.2806 |
| | SD | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ |
| $\lvert\mathcal{H}\rvert$ | M | 19.00 | 51.00 | 74.00 | 93.00 | 101.00 | 101.00 | 101.00 | 101.00 | 101.00 | 101.00 | 101.00 |
| | SD | $< 10^{-4}$ | $< 10^{-4}$ | 0.2537 | 0.3457 | 0.4842 | $< 10^{-4}$ | $< 10^{-4}$ | $< 10^{-4}$ | 0.1826 | 0.1826 | 0.1826 |

| Metric | | $\kappa$ ; ZDT1 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.99 |
| $\gamma$ | M | 0.0103 | 0.0069 | 0.0055 | 0.0059 | 0.0074 | 0.0040 | 0.0042 | 0.0051 | 0.0053 | 0.0050 | 0.0038 |
| | SD | 0.0072 | 0.0038 | 0.0057 | 0.0047 | 0.0049 | 0.0042 | 0.0060 | 0.0058 | 0.0040 | 0.0050 | 0.0034 |
| $\Delta$ | M | 0.3046 | 0.5543 | 0.3678 | 0.2084 | 0.1818 | 0.1812 | 0.1898 | 0.1937 | 0.1934 | 0.1956 | 0.1891 |
| | SD | 0.0122 | 0.0607 | 0.0480 | 0.0408 | 0.0235 | 0.0220 | 0.0234 | 0.0251 | 0.0240 | 0.0232 | 0.0155 |
| HV | M | 0.8435 | 0.8561 | 0.8602 | 0.8607 | 0.8598 | 0.8652 | 0.8650 | 0.8636 | 0.8633 | 0.8638 | 0.8657 |
| | SD | 0.0115 | 0.0066 | 0.0094 | 0.0079 | 0.0082 | 0.0069 | 0.0099 | 0.0096 | 0.0066 | 0.0083 | 0.0057 |
| $\lvert\mathcal{H}\rvert$ | M | 37.00 | 63.00 | 84.50 | 98.00 | 100.00 | 101.00 | 101.00 | 101.00 | 101.00 | 101.00 | 101.00 |
| | SD | 0.6397 | 5.7211 | 2.8730 | 5.0901 | 3.8201 | 0.5467 | 0.8584 | 0.9371 | 0.9377 | 1.3515 | 0.7112 |

| Metric | | $\kappa$ ; DTLZ2 (m = 3) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.99 |
| $\gamma$ | M | 0.0062 | 0.0069 | 0.0072 | 0.0070 | 0.0074 | 0.0079 | 0.0074 | 0.0076 | 0.0074 | 0.0078 | 0.0072 |
| | SD | 0.0002 | 0.0013 | 0.0015 | 0.0013 | 0.0012 | 0.0014 | 0.0010 | 0.0019 | 0.0007 | 0.0014 | 0.0009 |
| $\Delta$ | M | 0.0503 | 0.6066 | 0.3029 | 0.2411 | 0.2386 | 0.2308 | 0.2274 | 0.2175 | 0.2079 | 0.2173 | 0.1982 |
| | SD | 0.0041 | 0.0422 | 0.0357 | 0.0302 | 0.0264 | 0.0219 | 0.0316 | 0.0275 | 0.0306 | 0.0295 | 0.0239 |
| HV | M | 0.6731 | 0.7149 | 0.7383 | 0.7435 | 0.7458 | 0.7469 | 0.7467 | 0.7469 | 0.7470 | 0.7470 | 0.7471 |
| | SD | 0.0066 | 0.0042 | 0.0023 | 0.0012 | 0.0007 | 0.0006 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0003 |
| $\lvert\mathcal{H}\rvert$ | M | 21.00 | 69.00 | 88.00 | 93.00 | 94.50 | 95.00 | 95.00 | 95.00 | 95.00 | 95.00 | 94.00 |
| | SD | 1.3047 | 3.1639 | 2.8367 | 2.0424 | 1.7750 | 1.9464 | 2.2894 | 2.0197 | 1.5643 | 2.2614 | 1.7100 |

| Metric | | $\kappa$ ; DTLZ2 (m = 4) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.99 |
| $\gamma$ | M | 0.0001 | 0.0311 | 0.0385 | 0.0312 | 0.0449 | 0.0404 | 0.0445 | 0.0488 | 0.0590 | 0.0489 | 0.0534 |
| | SD | 0.0001 | 0.0198 | 0.0218 | 0.0239 | 0.0283 | 0.0240 | 0.0369 | 0.0281 | 0.0284 | 0.0241 | 0.0304 |
| $\Delta$ | M | 0.1390 | 0.4700 | 0.3602 | 0.3296 | 0.3299 | 0.3429 | 0.3377 | 0.3258 | 0.3253 | 0.3304 | 0.3319 |
| | SD | 0.1173 | 0.0304 | 0.0307 | 0.0226 | 0.0255 | 0.0263 | 0.0187 | 0.0262 | 0.0210 | 0.0254 | 0.0259 |
| $\lvert\mathcal{H}\rvert$ | M | 14.00 | 79.50 | 90.00 | 92.00 | 95.00 | 96.00 | 95.50 | 97.00 | 98.00 | 95.50 | 97.00 |
| | SD | 1.9815 | 4.9642 | 4.8476 | 4.2372 | 4.6307 | 4.2129 | 5.8530 | 5.0496 | 4.5945 | 4.6233 | 4.3423 |

arXiv, 2020

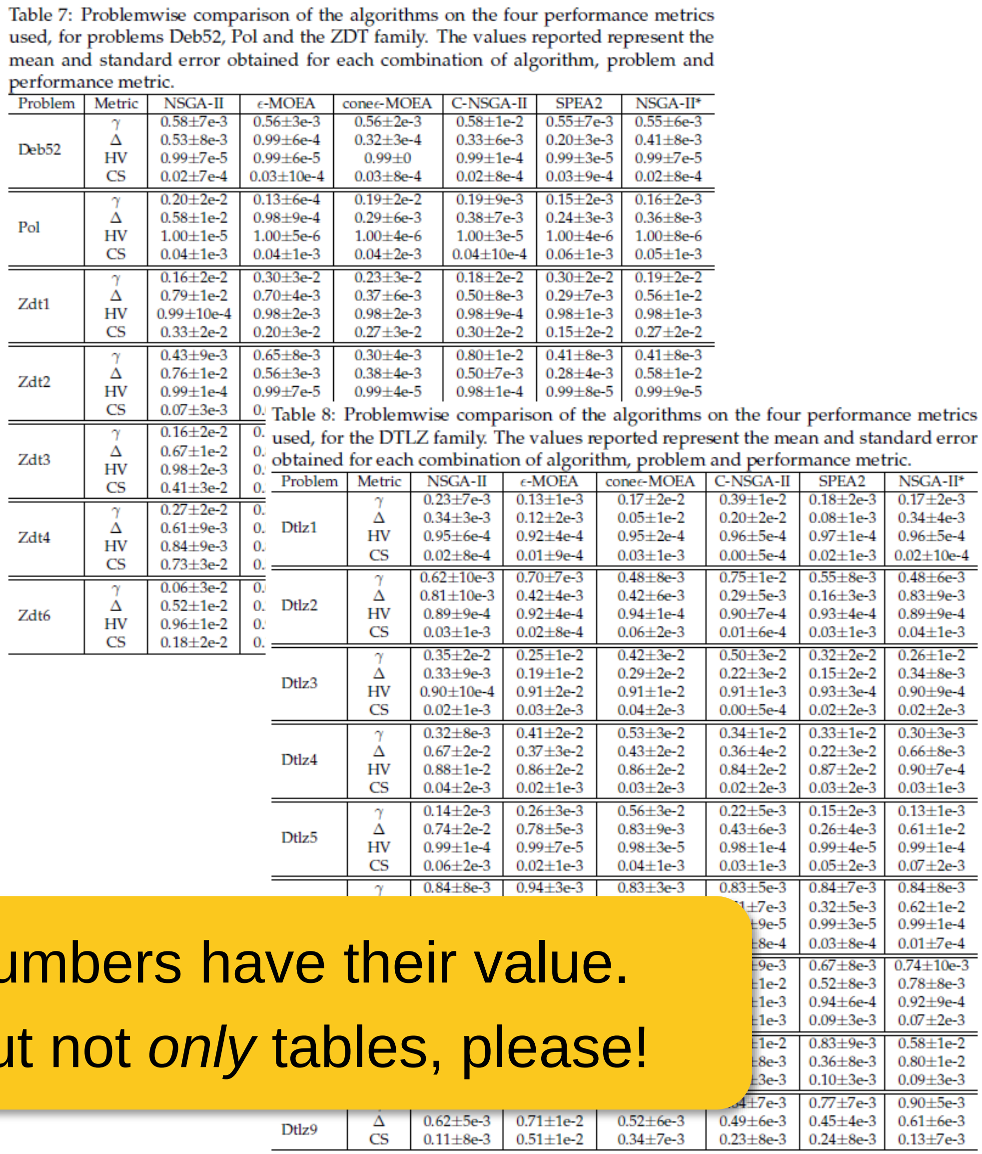


Table 7: Problemwise comparison of the algorithms on the four performance metrics used, for problems Deb52, Pol and the ZDT family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

| Problem | Metric | NSGA-II | $\epsilon$-MOEA | cone$\epsilon$-MOEA | C-NSGA-II | SPEA2 | NSGA-II* |
|---|---|---|---|---|---|---|---|
| Deb52 | $\gamma$ | 0.58±7e-3 | 0.56±3e-3 | 0.56±2e-3 | 0.58±1e-2 | 0.55±7e-3 | 0.55±6e-3 |
| | $\Delta$ | 0.53±8e-3 | 0.99±6e-4 | 0.32±3e-4 | 0.33±6e-3 | 0.20±3e-3 | 0.41±8e-3 |
| | HV | 0.99±7e-5 | 0.99±6e-5 | 0.99±0 | 0.99±1e-4 | 0.99±3e-5 | 0.99±7e-5 |
| | CS | 0.02±7e-4 | 0.03±10e-4 | 0.03±8e-4 | 0.02±8e-4 | 0.03±9e-4 | 0.02±8e-4 |
| Pol | $\gamma$ | 0.20±2e-2 | 0.13±6e-4 | 0.19±2e-2 | 0.19±9e-3 | 0.15±2e-3 | 0.16±2e-3 |
| | $\Delta$ | 0.58±1e-2 | 0.98±9e-4 | 0.29±6e-3 | 0.38±7e-3 | 0.24±3e-3 | 0.36±8e-3 |
| | HV | 1.00±1e-5 | 1.00±5e-6 | 1.00±4e-6 | 1.00±3e-5 | 1.00±4e-6 | 1.00±8e-6 |
| | CS | 0.04±1e-3 | 0.04±1e-3 | 0.04±2e-3 | 0.04±10e-4 | 0.06±1e-3 | 0.05±1e-3 |
| Zdt1 | $\gamma$ | 0.16±2e-2 | 0.30±3e-2 | 0.23±3e-2 | 0.18±2e-2 | 0.30±2e-2 | 0.19±2e-2 |
| | $\Delta$ | 0.79±1e-2 | 0.70±4e-3 | 0.37±6e-3 | 0.50±8e-3 | 0.29±7e-3 | 0.56±1e-2 |
| | HV | 0.99±10e-4 | 0.98±2e-3 | 0.98±2e-3 | 0.98±9e-4 | 0.98±1e-3 | 0.98±1e-3 |
| | CS | 0.33±2e-2 | 0.20±3e-2 | 0.27±3e-2 | 0.30±2e-2 | 0.15±2e-2 | 0.27±2e-2 |
| Zdt2 | $\gamma$ | 0.43±9e-3 | 0.65±8e-3 | 0.30±4e-3 | 0.80±1e-2 | 0.41±8e-3 | 0.41±8e-3 |
| | $\Delta$ | 0.76±1e-2 | 0.56±3e-3 | 0.38±4e-3 | 0.50±7e-3 | 0.28±4e-3 | 0.58±1e-2 |
| | HV | 0.99±1e-4 | 0.99±7e-5 | 0.99±4e-5 | 0.98±1e-4 | 0.99±8e-5 | 0.99±9e-5 |
| | CS | 0.07±3e-3 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| Zdt3 | $\gamma$ | 0.16±2e-2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| | $\Delta$ | 0.67±1e-2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| | HV | 0.98±2e-3 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| | CS | 0.41±3e-2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| Zdt4 | $\gamma$ | 0.27±2e-2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| | $\Delta$ | 0.61±9e-3 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| | HV | 0.84±9e-3 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| | CS | 0.73±3e-2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| Zdt6 | $\gamma$ | 0.06±3e-2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| | $\Delta$ | 0.52±1e-2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| | HV | 0.96±1e-2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| | CS | 0.18±2e-2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

Table 8: Problemwise comparison of the algorithms on the four performance metrics used, for the DTLZ family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

| Problem | Metric | NSGA-II | $\epsilon$-MOEA | cone$\epsilon$-MOEA | C-NSGA-II | SPEA2 | NSGA-II* |
|---|---|---|---|---|---|---|---|
| Dtlz1 | $\gamma$ | 0.23±7e-3 | 0.13±1e-3 | 0.17±2e-2 | 0.39±1e-2 | 0.18±2e-3 | 0.17±2e-3 |
| | $\Delta$ | 0.34±3e-3 | 0.12±2e-3 | 0.05±1e-2 | 0.20±2e-2 | 0.08±1e-3 | 0.34±4e-3 |
| | HV | 0.95±6e-4 | 0.92±4e-4 | 0.95±2e-4 | 0.96±5e-4 | 0.97±1e-4 | 0.96±5e-4 |
| | CS | 0.02±8e-4 | 0.01±9e-4 | 0.03±1e-3 | 0.00±5e-4 | 0.02±1e-3 | 0.02±10e-4 |
| Dtlz2 | $\gamma$ | 0.62±10e-3 | 0.70±7e-3 | 0.48±8e-3 | 0.75±1e-2 | 0.55±8e-3 | 0.48±6e-3 |
| | $\Delta$ | 0.81±10e-3 | 0.42±4e-3 | 0.42±6e-3 | 0.29±5e-3 | 0.16±3e-3 | 0.83±9e-3 |
| | HV | 0.89±9e-4 | 0.92±4e-4 | 0.94±1e-4 | 0.90±7e-4 | 0.93±4e-4 | 0.89±9e-4 |
| | CS | 0.03±1e-3 | 0.02±8e-4 | 0.06±2e-3 | 0.01±6e-4 | 0.03±1e-3 | 0.04±1e-3 |
| Dtlz3 | $\gamma$ | 0.35±2e-2 | 0.25±1e-2 | 0.42±3e-2 | 0.50±3e-2 | 0.32±2e-2 | 0.26±1e-2 |
| | $\Delta$ | 0.33±9e-3 | 0.19±1e-2 | 0.29±2e-2 | 0.22±3e-2 | 0.15±2e-2 | 0.34±8e-3 |
| | HV | 0.90±10e-4 | 0.91±2e-2 | 0.91±1e-2 | 0.91±1e-3 | 0.93±3e-4 | 0.90±9e-4 |
| | CS | 0.02±1e-3 | 0.03±2e-3 | 0.04±2e-3 | 0.00±5e-4 | 0.02±2e-3 | 0.02±2e-3 |
| Dtlz4 | $\gamma$ | 0.32±8e-3 | 0.41±2e-2 | 0.53±3e-2 | 0.34±1e-2 | 0.33±1e-2 | 0.30±3e-3 |
| | $\Delta$ | 0.67±2e-2 | 0.37±3e-2 | 0.43±2e-2 | 0.36±4e-2 | 0.22±3e-2 | 0.66±8e-3 |
| | HV | 0.88±1e-2 | 0.86±2e-2 | 0.86±2e-2 | 0.84±2e-2 | 0.87±2e-2 | 0.90±7e-4 |
| | CS | 0.04±2e-3 | 0.02±1e-3 | 0.03±2e-3 | 0.02±2e-3 | 0.03±2e-3 | 0.03±1e-3 |
| Dtlz5 | $\gamma$ | 0.14±2e-3 | 0.26±3e-3 | 0.56±3e-2 | 0.22±5e-3 | 0.15±2e-3 | 0.13±1e-3 |
| | $\Delta$ | 0.74±2e-2 | 0.78±5e-3 | 0.83±9e-3 | 0.43±6e-3 | 0.26±4e-3 | 0.61±1e-2 |
| | HV | 0.99±1e-4 | 0.99±7e-5 | 0.98±3e-5 | 0.98±1e-4 | 0.99±4e-5 | 0.99±1e-4 |
| | CS | 0.06±2e-3 | 0.02±1e-3 | 0.04±1e-3 | 0.03±1e-3 | 0.05±2e-3 | 0.07±2e-3 |
| [illegible] | $\gamma$ | 0.84±8e-3 | 0.94±3e-3 | 0.83±3e-3 | 0.83±5e-3 | 0.84±7e-3 | 0.84±8e-3 |
| | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.32±5e-3 | 0.62±1e-2 |
| | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.99±3e-5 | 0.99±1e-4 |
| | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.03±8e-4 | 0.01±7e-4 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.67±8e-3 | 0.74±10e-3 |
| | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.52±8e-3 | 0.78±8e-3 |
| | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.94±6e-4 | 0.92±9e-4 |
| | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.09±3e-3 | 0.07±2e-3 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.83±9e-3 | 0.58±1e-2 |
| | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.36±8e-3 | 0.80±1e-2 |
| | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.10±3e-3 | 0.09±3e-3 |
| Dtlz9 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.77±7e-3 | 0.90±5e-3 |
| | $\Delta$ | 0.62±5e-3 | 0.71±1e-2 | 0.52±6e-3 | 0.49±6e-3 | 0.45±4e-3 | 0.61±6e-3 |
| | CS | 0.11±8e-3 | 0.51±1e-2 | 0.34±7e-3 | 0.23±8e-3 | 0.24±8e-3 | 0.13±7e-3 |

Numbers have their value.

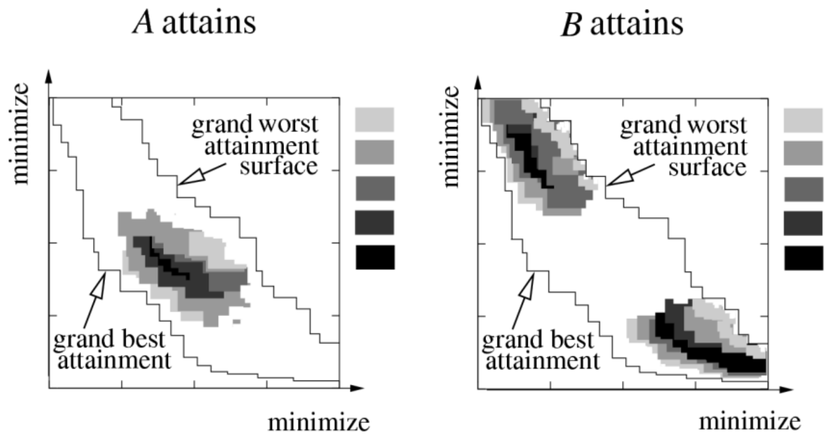
But not *only* tables, please!

# v1.0

# v1.0: Two Approaches for Empirical Studies

## Attainment function approach

- applies statistical tests directly to the approximation set
- detailed information about how and where performance differences occur



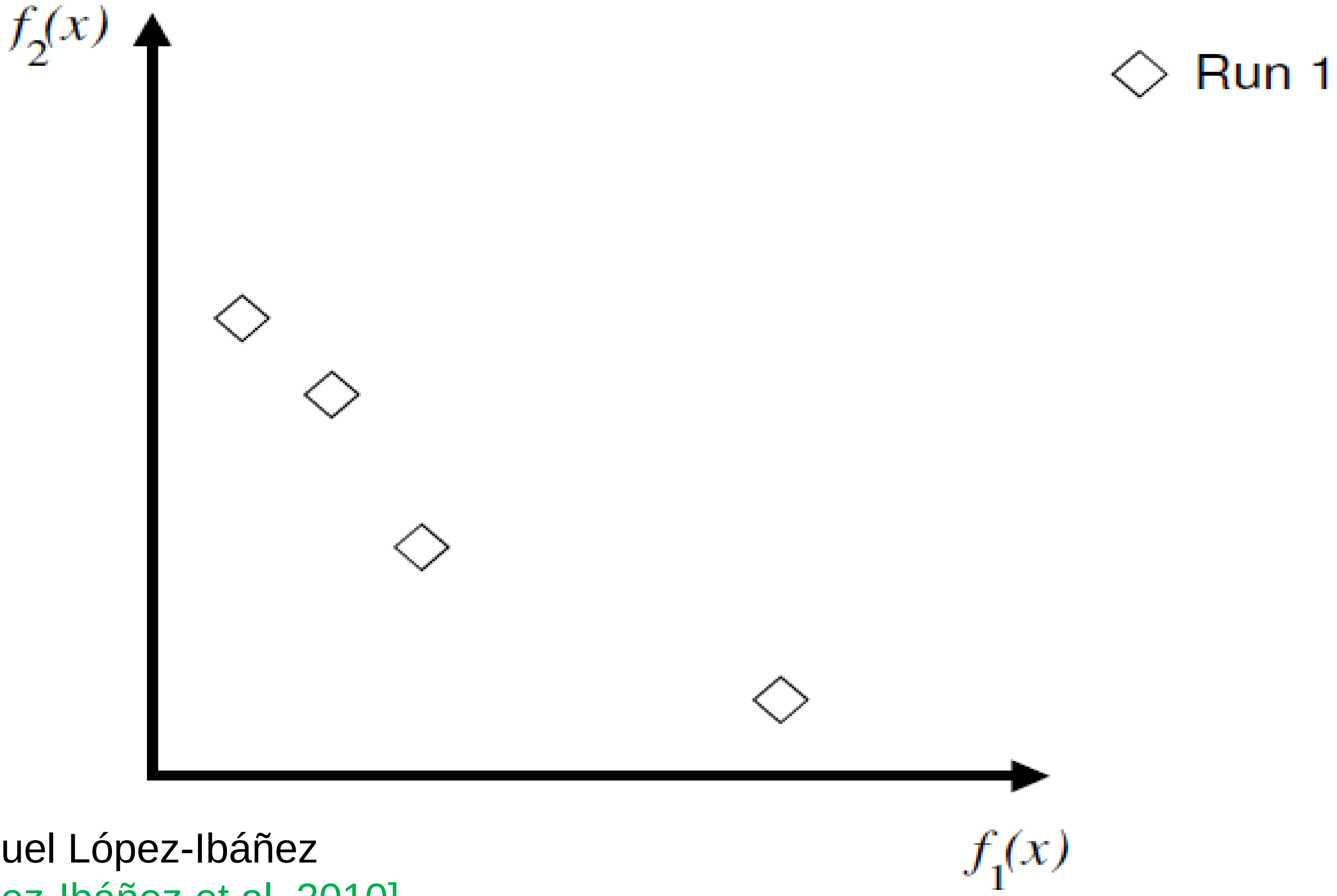
## Quality indicator approach

- reduces each approximation set to a single quality value
- applies statistical tests to the quality values

| *Indicator* | A | B |
|---|---|---|
| Hypervolume indicator | 6.3431 | 7.1924 |
| $\epsilon$-indicator | 1.2090 | 0.12722 |
| $R_2$ indicator | 0.2434 | 0.1643 |
| $R_3$ indicator | 0.6454 | 0.3475 |

see e.g. [Zitzler et al. 2003]

# Empirical Attainment Functions



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[López-Ibáñez et al. 2010]

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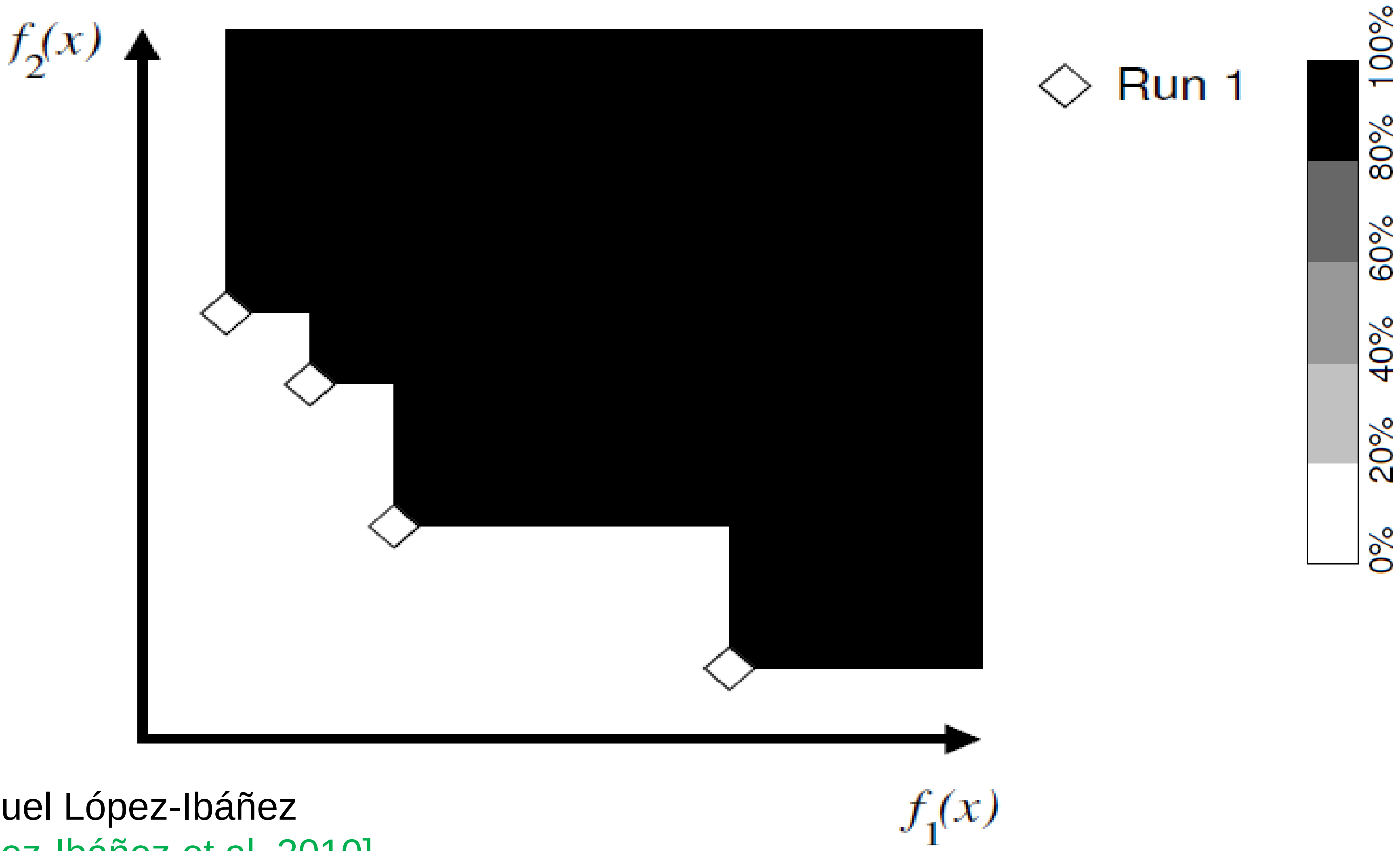

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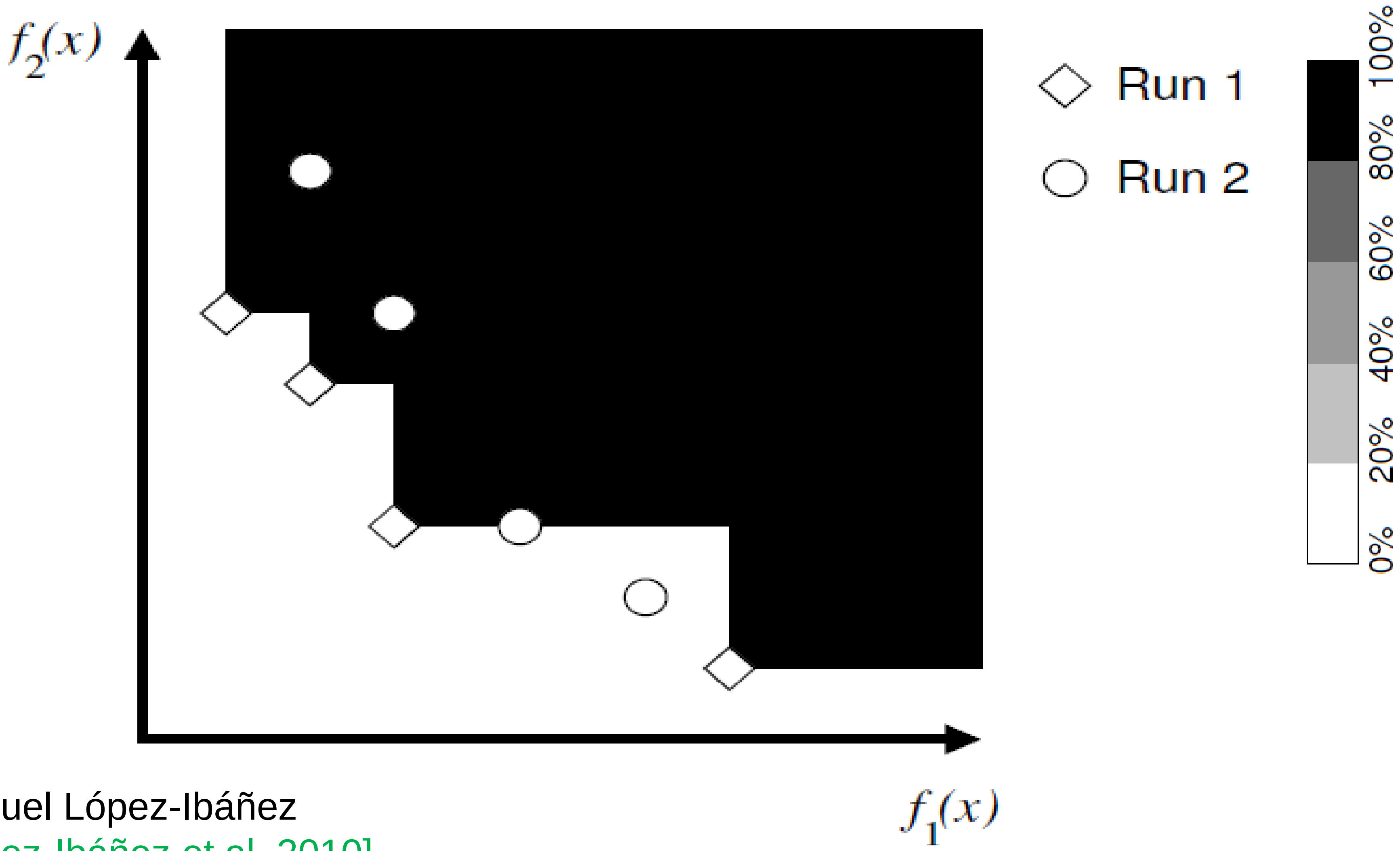

© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

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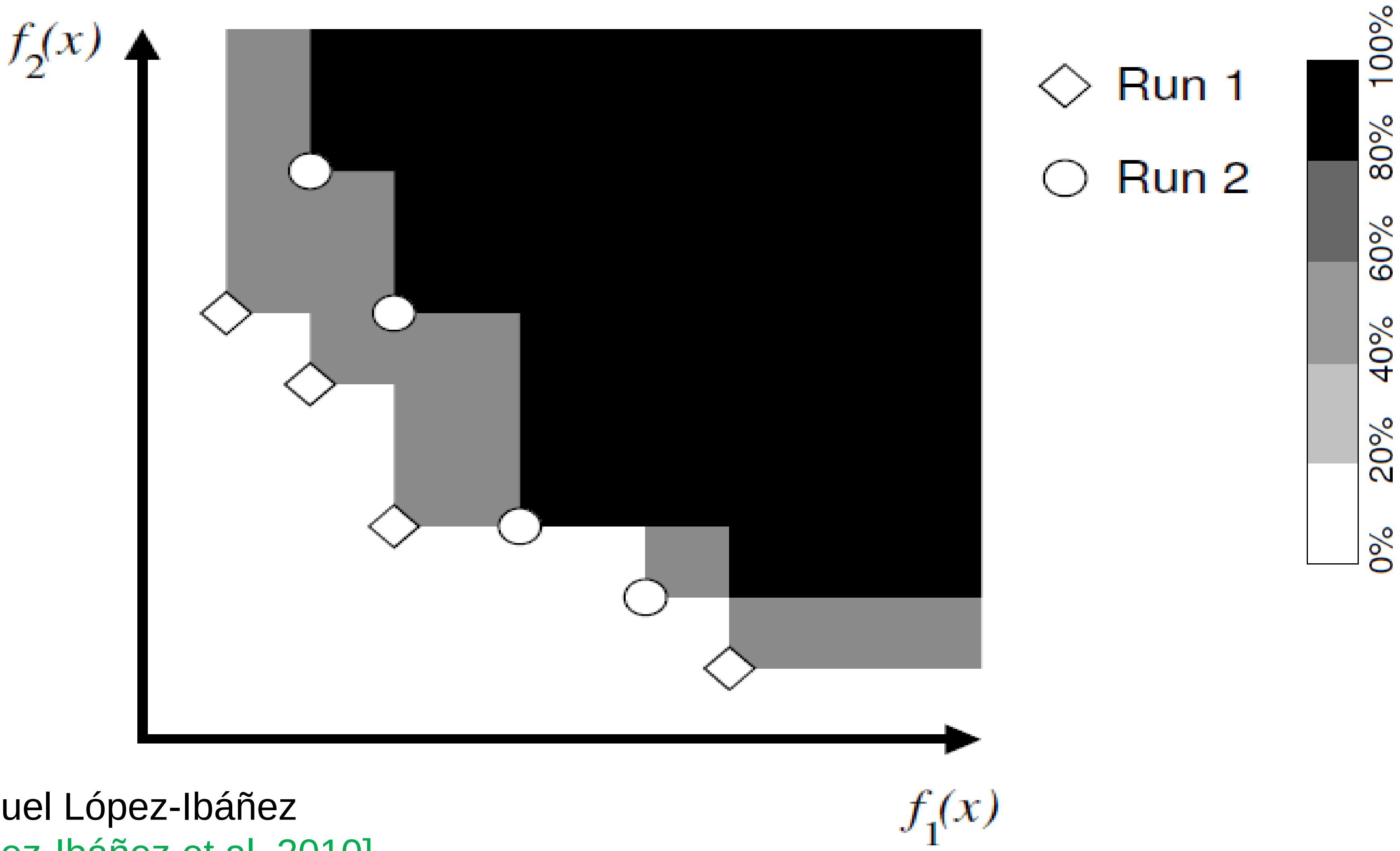

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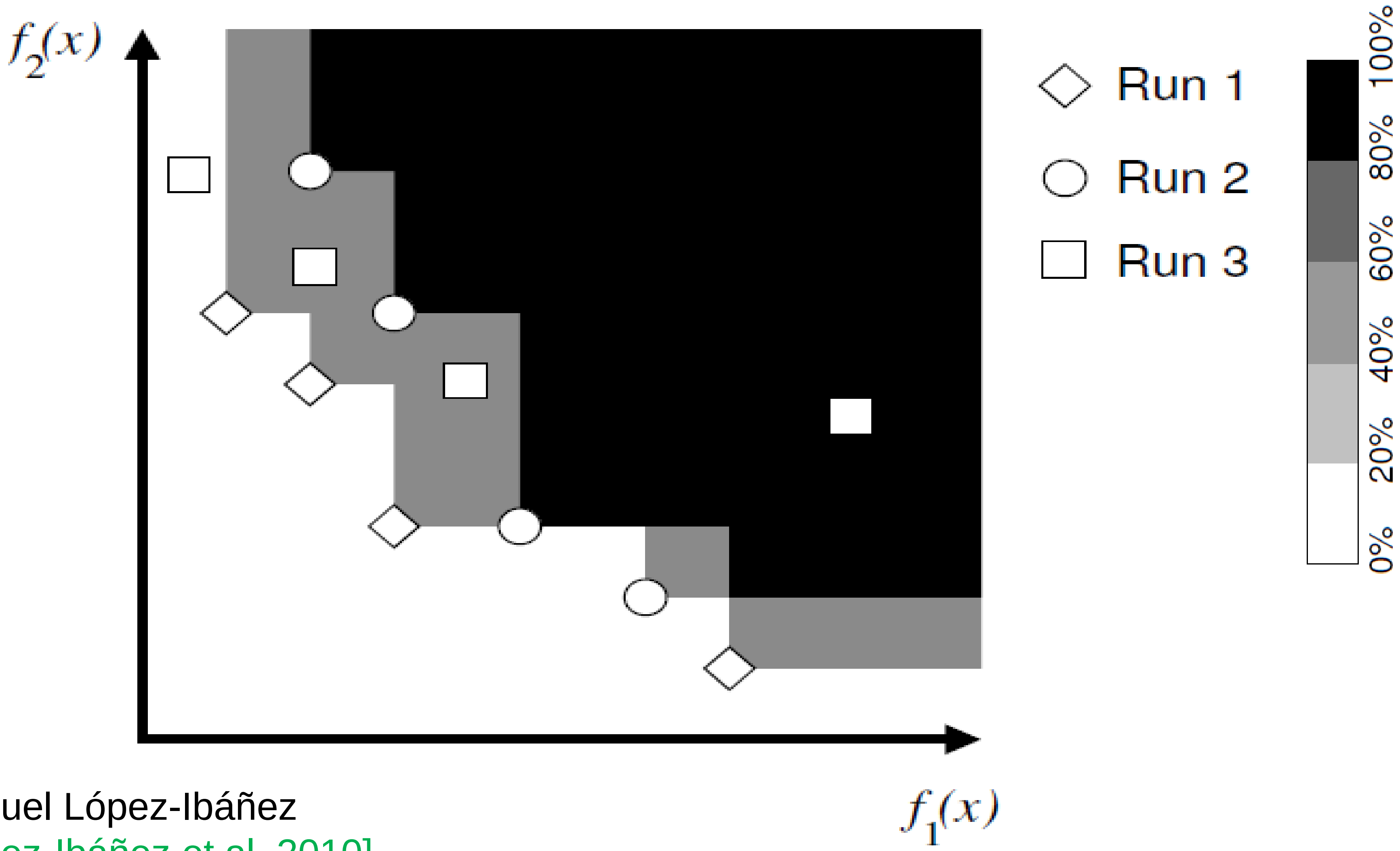

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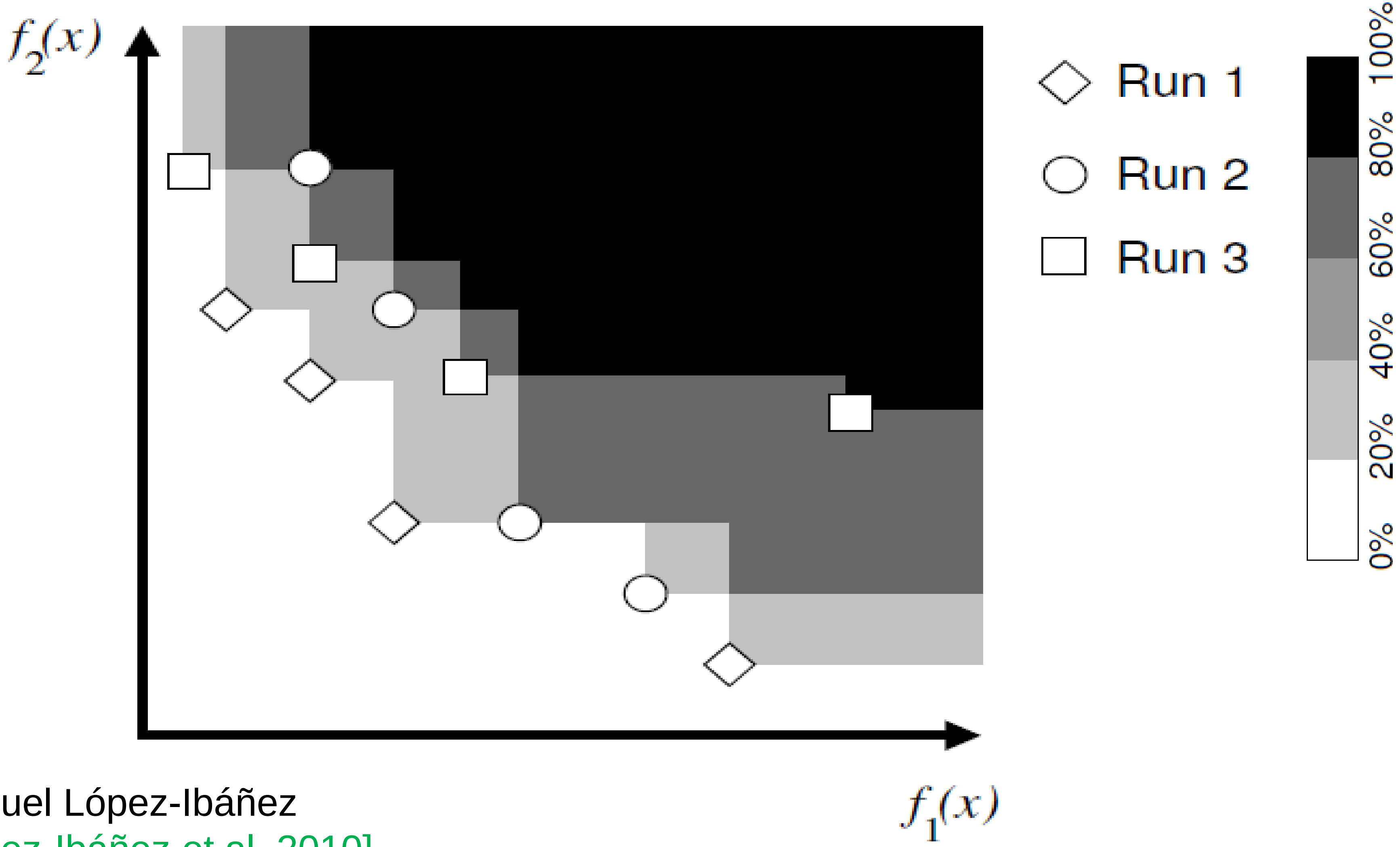

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[López-Ibáñez et al. 2010]

# Empirical Attainment Functions: Definition

The Empirical Attainment Function $\alpha(z)$ "counts" how many solution sets $\mathcal{X}_i$ attain or dominate a vector $z$ at time $T$:

$$\alpha_T(z) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_{\{\mathcal{X}_i \trianglelefteq_T z\}}$$

with $\trianglelefteq_T$ being the weak dominance relation between a solution set and an objective vector at time $T$.

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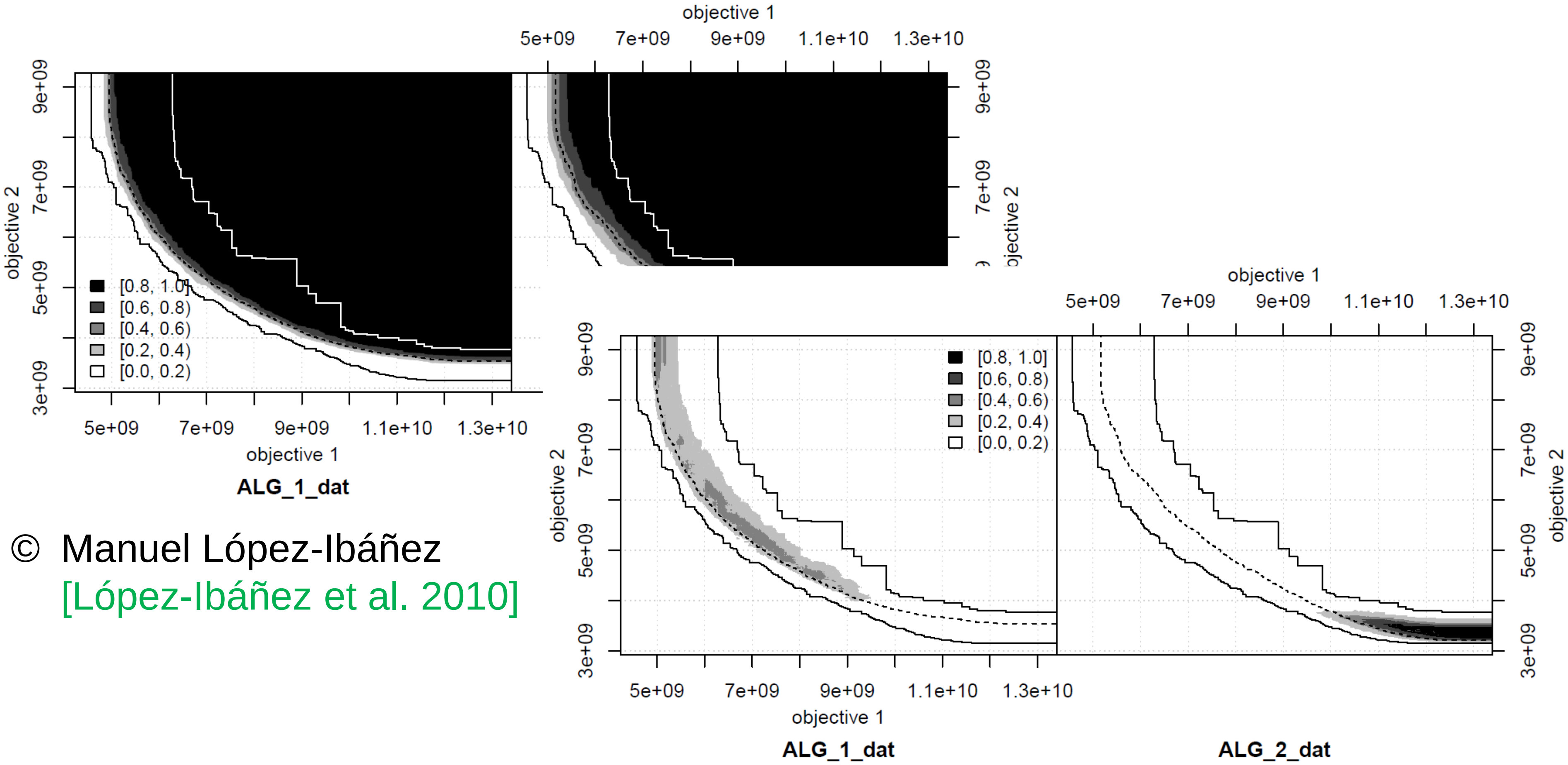
Note that $\alpha_T(z)$ is the empirical cumulative distribution function of the achieved objective function distribution at time T in the single-objective case ("fixed budget scenario").

# Empirical Attainment Functions in Practice



© Manuel López-Ibáñez
[López-Ibáñez et al. 2010]

latest implementation online at
`http://eden.dei.uc.pt/~cmfonsec/software.html`
R package: `http://lopez-ibanez.eu/eaftools`
see also [López-Ibáñez et al. 2010, Fonseca et al. 2011]

# Quality Indicator Approach

**Idea:**

- transfer multiobjective problem into a set problem
- define an objective function (“unary quality indicator”) on sets
- use the resulting total (pre-)order (on the quality values)

# Quality Indicator Approach

**Idea:**

- transfer multiobjective problem into a set problem
- define an objective function (“unary quality indicator”) on sets
- use the resulting total (pre-)order (on the quality values)

**Question:**

Can any total (pre-)order be used or are there any requirements concerning the resulting preference relation?

$\Rightarrow$ Underlying dominance relation should be reflected!

$$A \preccurlyeq B :\Leftrightarrow \forall_{b \in B} \exists_{a \in A}\, a \preccurlyeq b$$

# Monotonicity and Strict Monotonicity

- Monotonicity when quality indicator does not contradict relation

$$A \preccurlyeq B \Rightarrow I(A) \geq I(B)$$

# Monotonicity and Strict Monotonicity

- Monotonicity when quality indicator does not contradict relation

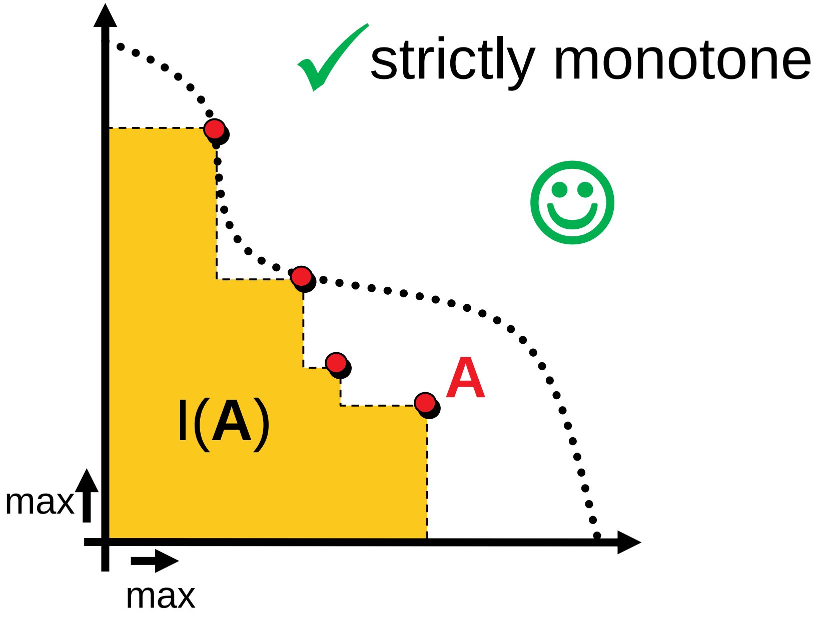
$$A \preccurlyeq B \Rightarrow I(A) \geq I(B)$$

- Strict monotonicity: better = higher indicator

$$A \preccurlyeq B \text{ and } A \neq B \Rightarrow I(A) > I(B)$$

# Example: Refinements Using Indicators

I(A) = volume of the weakly dominated area in objective space

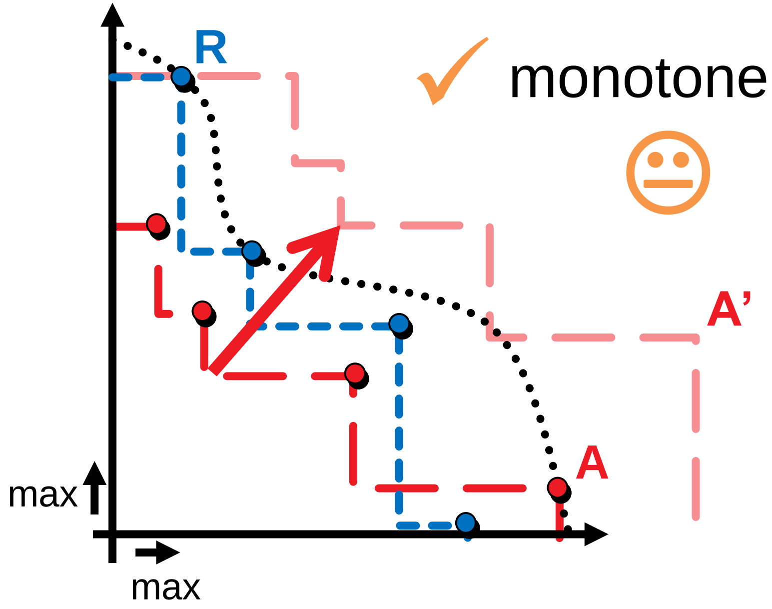


✓ strictly monotone

☺

unary hypervolume indicator

I(A,R) = how much needs A to be moved to weakly dominate R



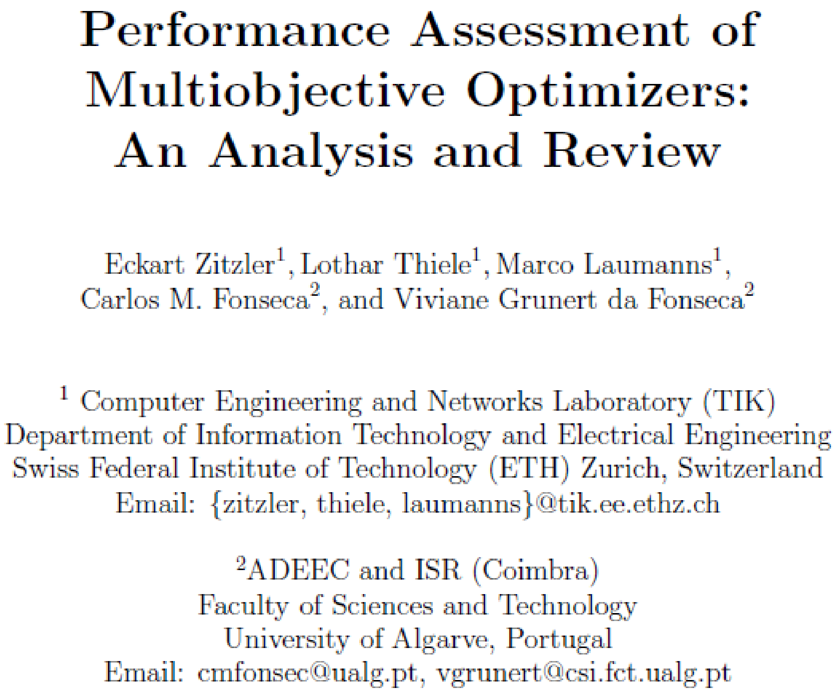
✓ monotone

😐

unary epsilon indicator

# v1.0.1 – v1.0.100 and counting

# Many Indicators Available



## Performance Assessment of Multiobjective Optimizers: An Analysis and Review

Eckart Zitzler[1], Lothar Thiele[1], Marco Laumanns[1],
Carlos M. Fonseca[2], and Viviane Grunert da Fonseca[2]

[1] Computer Engineering and Networks Laboratory (TIK)
Department of Information Technology and Electrical Engineering
Swiss Federal Institute of Technology (ETH) Zurich, Switzerland
Email: {zitzler, thiele, laumanns}@tik.ee.ethz.ch

[2]ADEEC and ISR (Coimbra)
Faculty of Sciences and Technology
University of Algarve, Portugal
Email: cmfonsec@ualg.pt, vgrunert@csi.fct.ualg.pt

[Zitzler et al. 2003]

22 indicators

# Even More Indicators…

Performance indicators in multiobjective optimization

Charles Audet[a], Jean Bigeon[b], Dominique Cartier[c], Sébastien Le Digabel[a],
Ludovic Salomon[a,1]

*[a]GERAD and Département de mathématiques et génie industriel, École Polytechnique de Montréal,*
*C.P. 6079, Succ. Centre-ville, Montréal, Québec, H3C 3A7, Canada.*
*[b]Univ. Grenoble Alpes, CNRS, Grenoble INP, G-SCOP, 38000 Grenoble, France.*
*[c]Collège de Maisonneuve, 3800 Rue Sherbrooke E, Montréal, Québec, H1X 2A2, Canada.*

[Audet et al 2021]

63 indicators

**Quality Evaluation of Solution Sets in**
**Multiobjective Optimisation: A Survey**

Miqing Li, and Xin Yao[1]

[1]CERCIA, School of Computer Science, University of Birmingham, Birmingham B15 2TT, U. K.
*Email: limitsing@gmail.com, x.yao@cs.bham.ac.uk

[Li and Yao 2019]

100 indicators

# Many Indicators: What Do We Do?

**Focus on indicators which are (strictly) monotone**

- all hypervolume-based indicators
- unary epsilon indicator
- R2
- IGD+

# Many Indicators: What Do We Do?

## Focus on indicators which are (strictly) monotone

- all hypervolume-based indicators
- unary epsilon indicator
- R2
- IGD+

## Why is monotonicity important?

- Pareto dominance is the lowest form of preference
- If dominance relation does not hold, we have not defined a true multiobjective problem.

# v2.0

# Benchmarking Multiobjective Optimizers 2.0

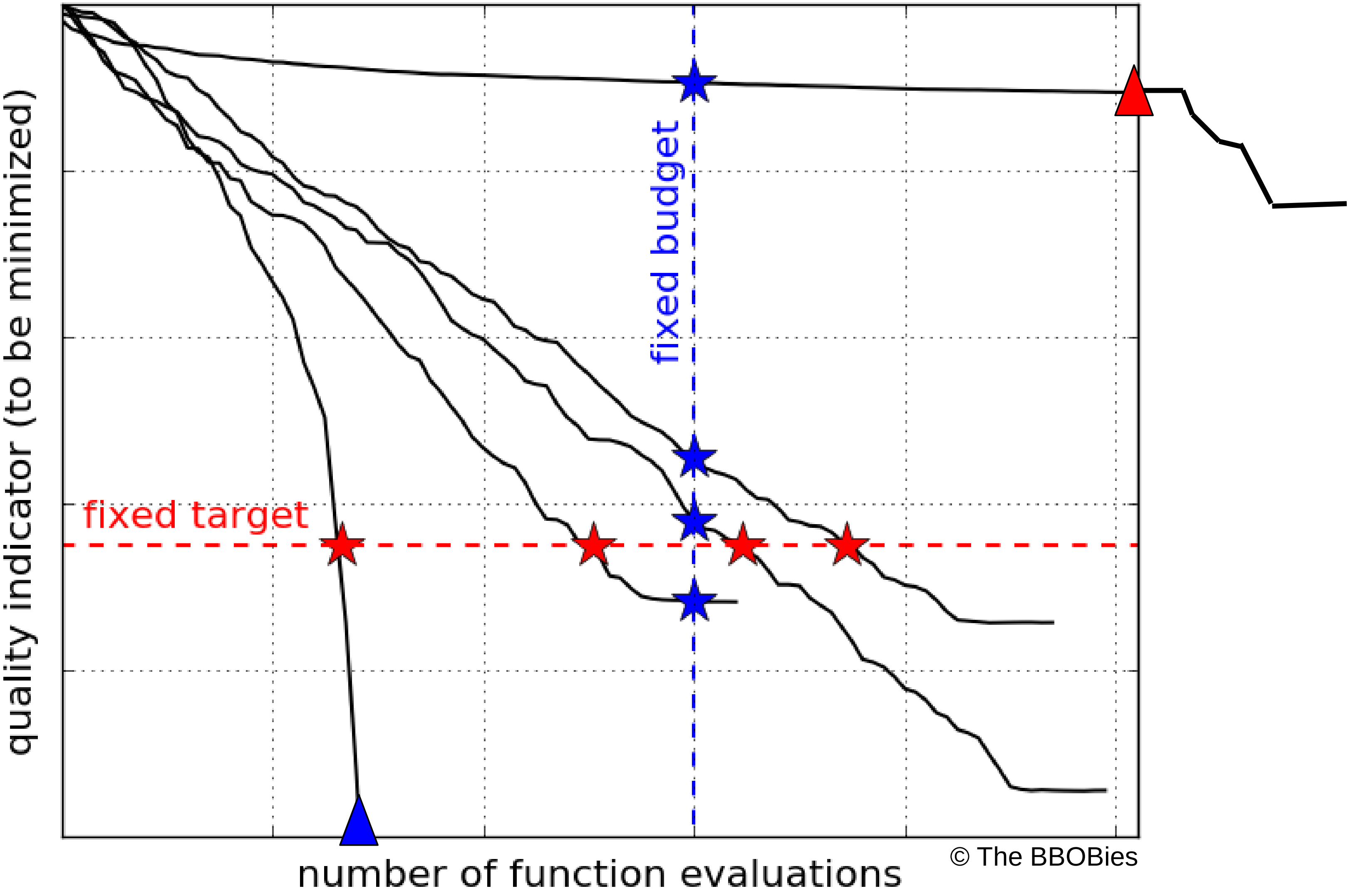
- With the right (strictly) monotone indicator, multiobjective optimization is not different from single-objective optimization (!)

We can use our normal tools from single-objective optimization, including

- reporting of target-based runtimes
- ECDFs of runtimes, performance profiles, data profiles
- statistical tests, box plots, …

# Measuring Performance Empirically

convergence graphs is all we have to start with...



# Measuring Performance Empirically

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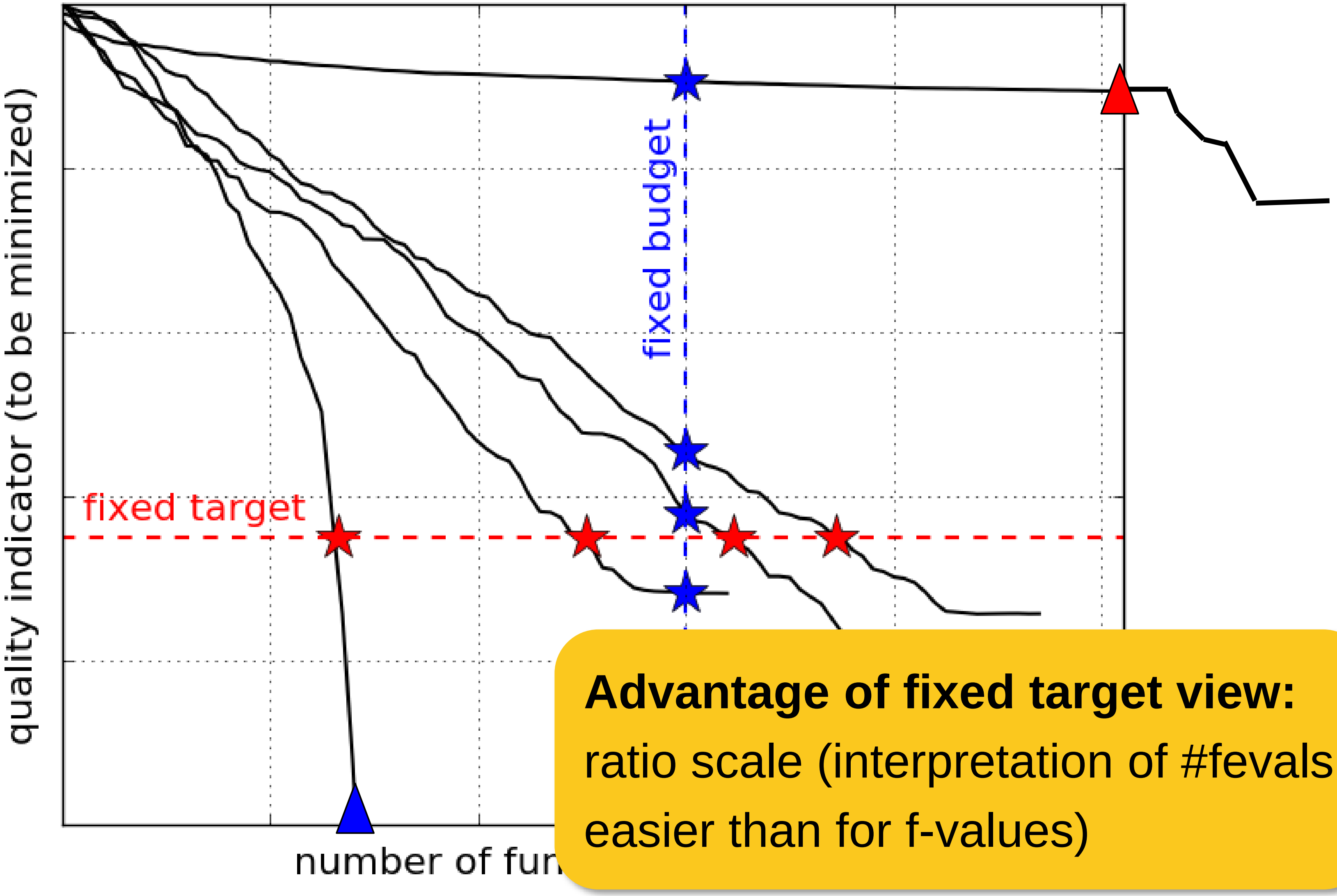
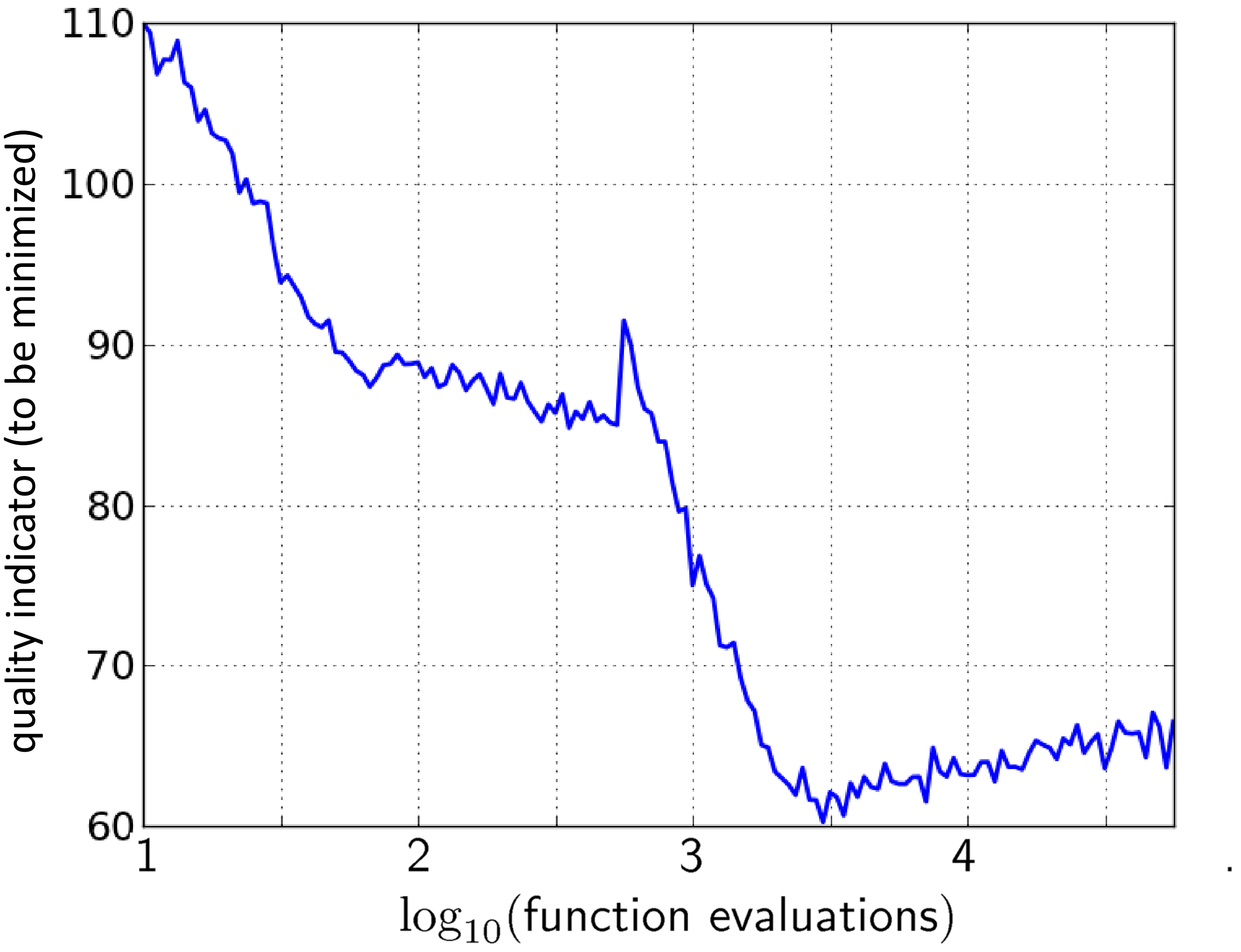


**Advantage of fixed target view:**
ratio scale (interpretation of #fevals easier than for f-values)

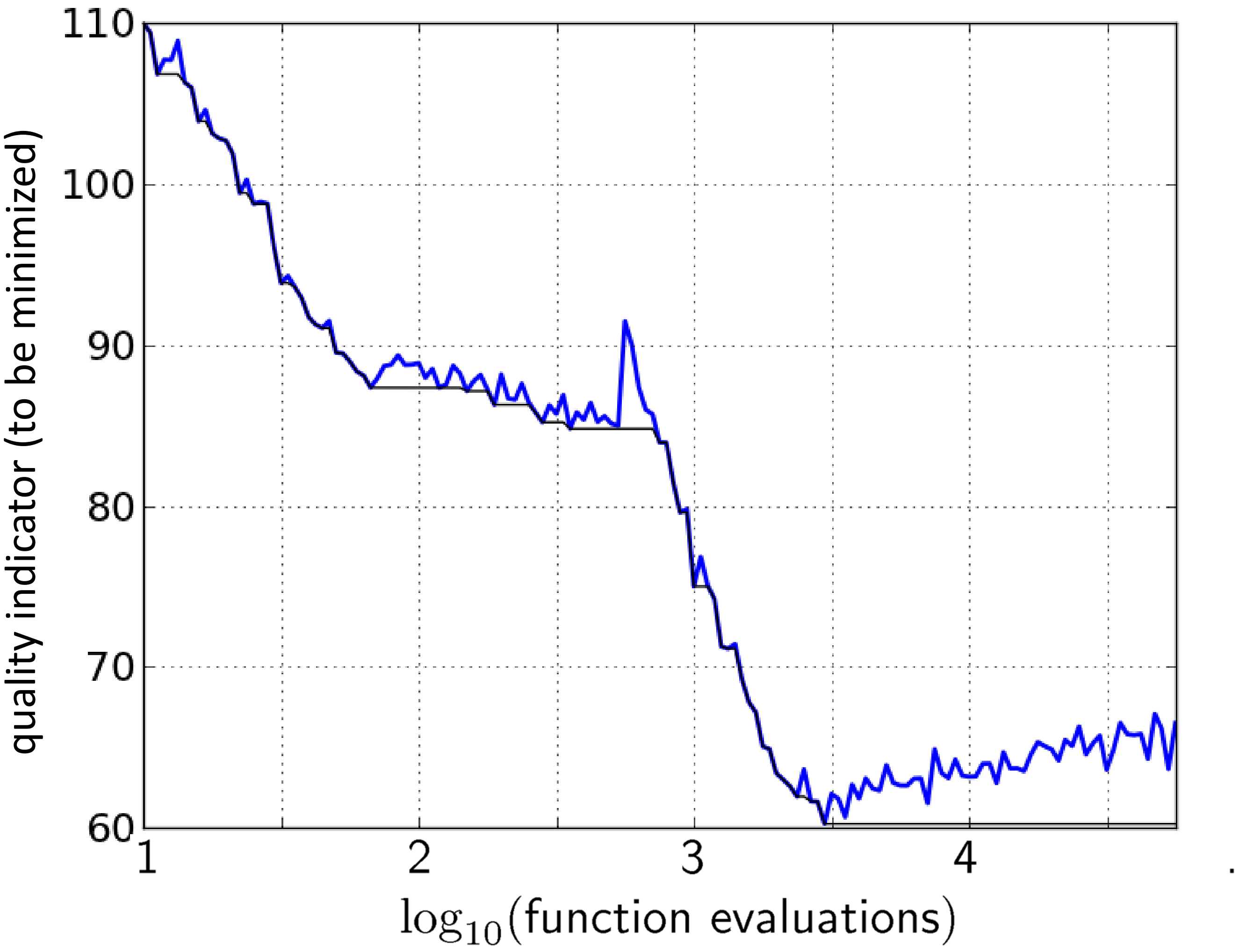
# ECDF:

Empirical Cumulative Distribution Function of the Runtime

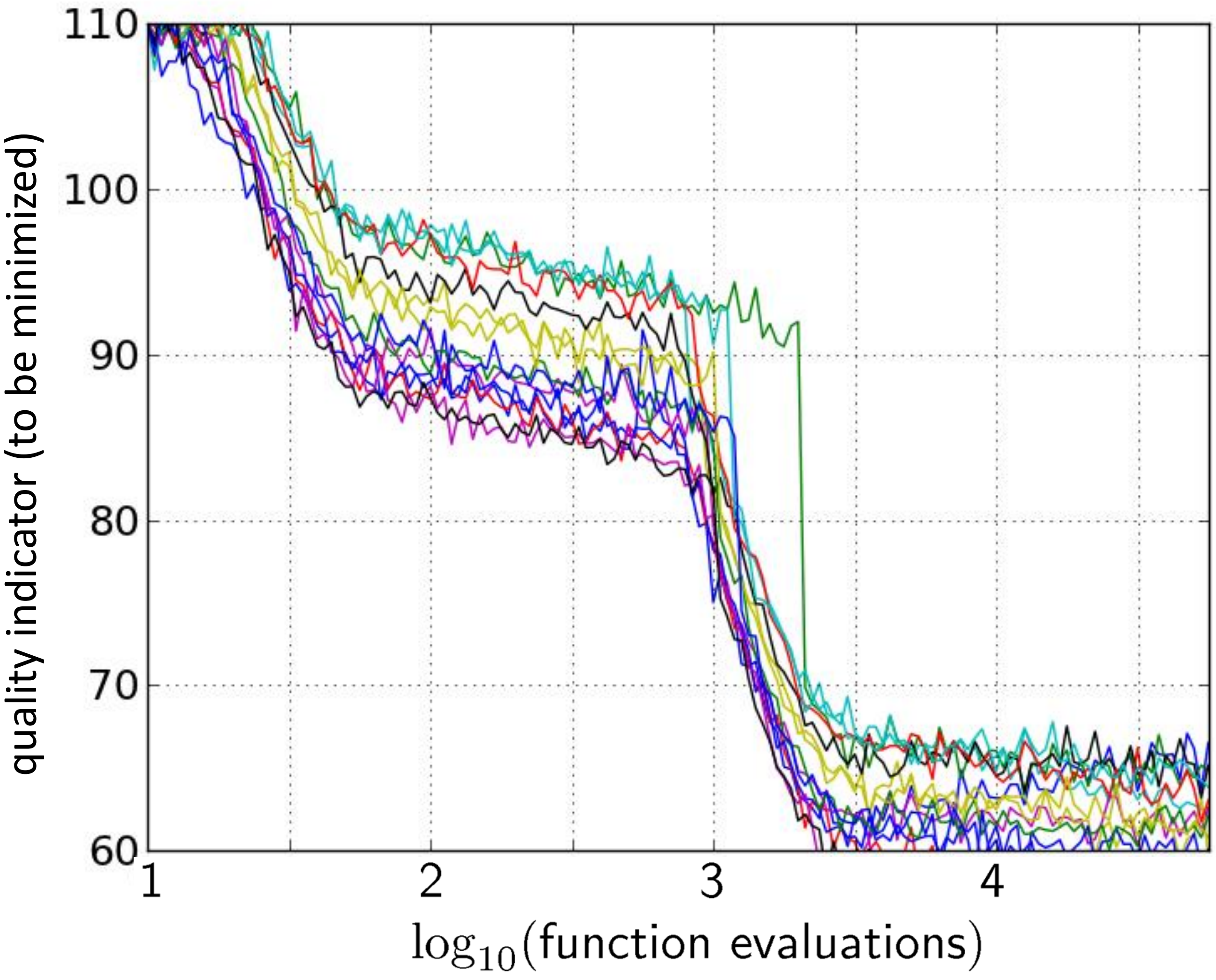
# A Convergence Graph



# First Hitting Time is Monotonous

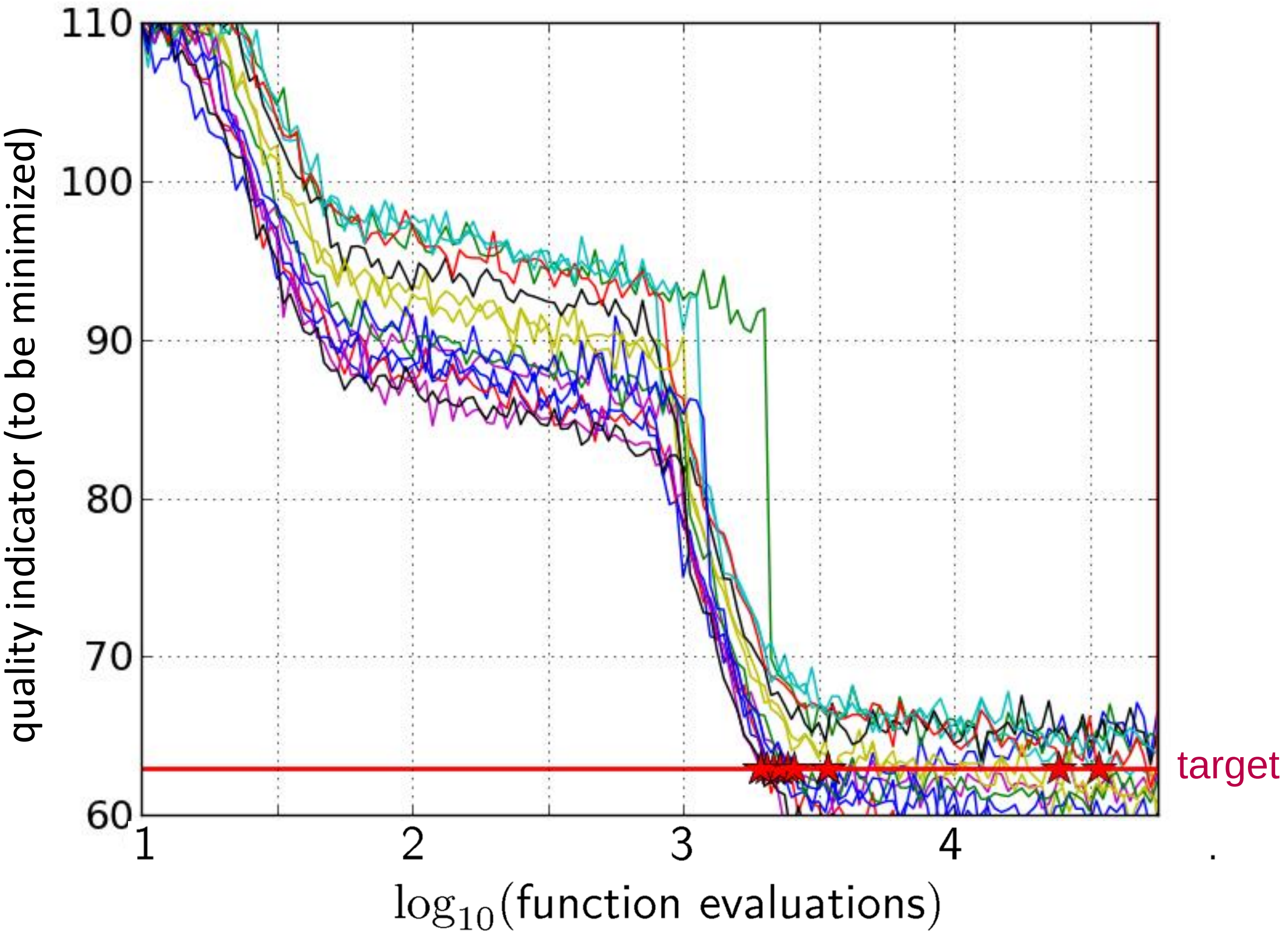


# 15 Runs


110
100
90
80
70
60
1
2
3
4
quality indicator (to be minimized)
$\log_{10}$(function evaluations)

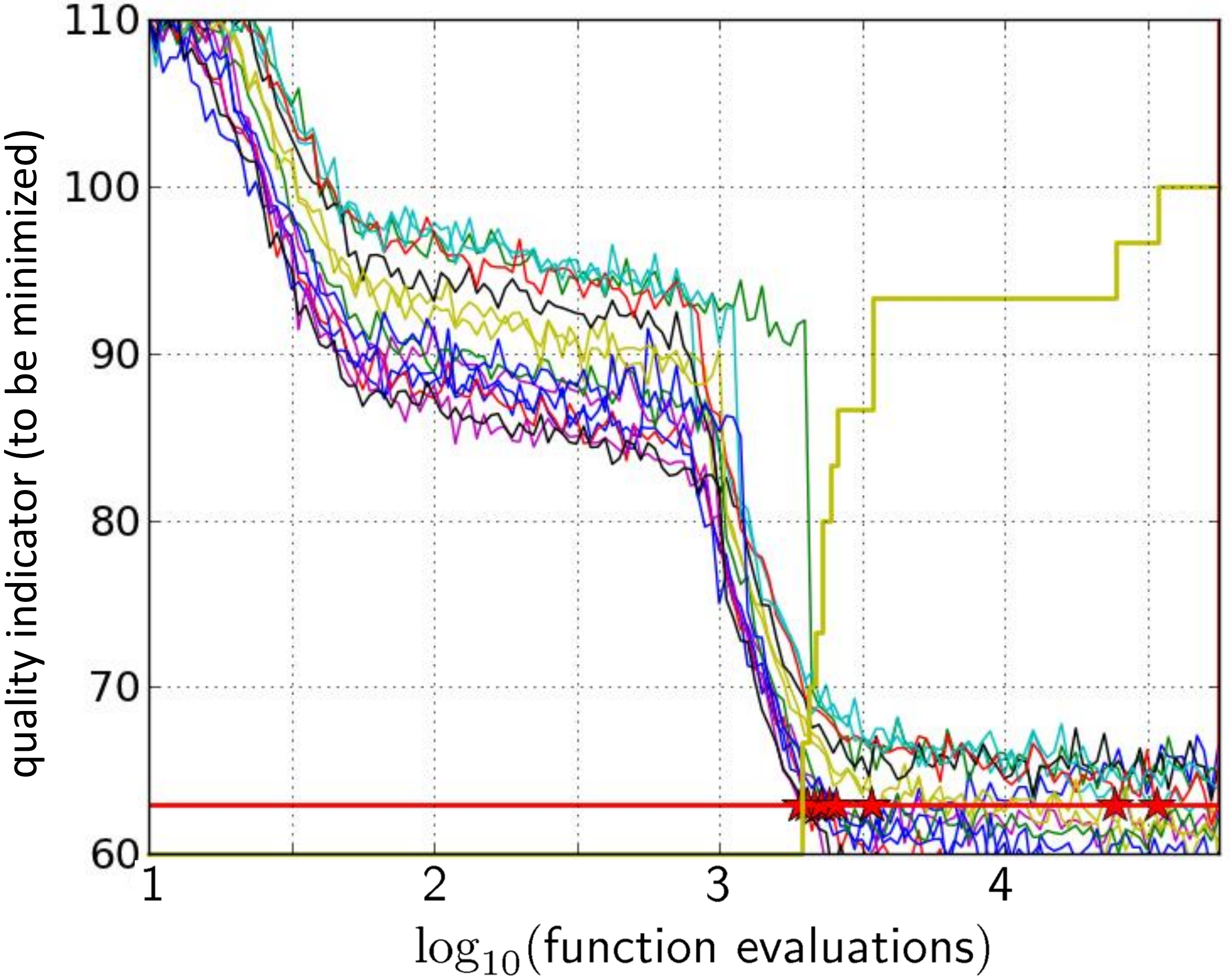
.

# 15 Runs ≤ 15 Runtime Data Points



.

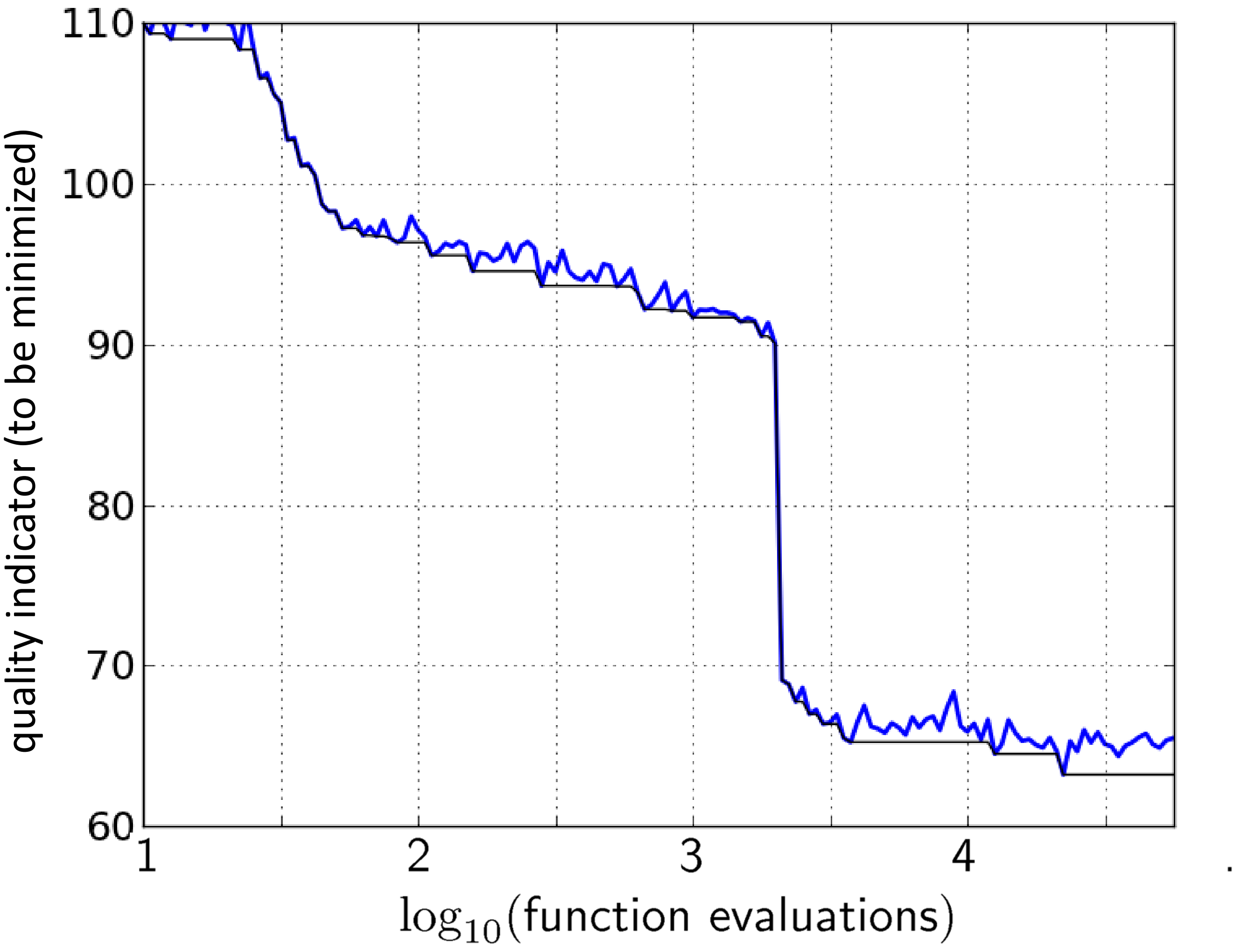
# Empirical Cumulative Distribution



the ECDF of run lengths to reach the target

- has for each data point a vertical step of constant size
- displays for each x-value (budget) the count of observations to the left (first hitting times)

# Reconstructing A Single Run



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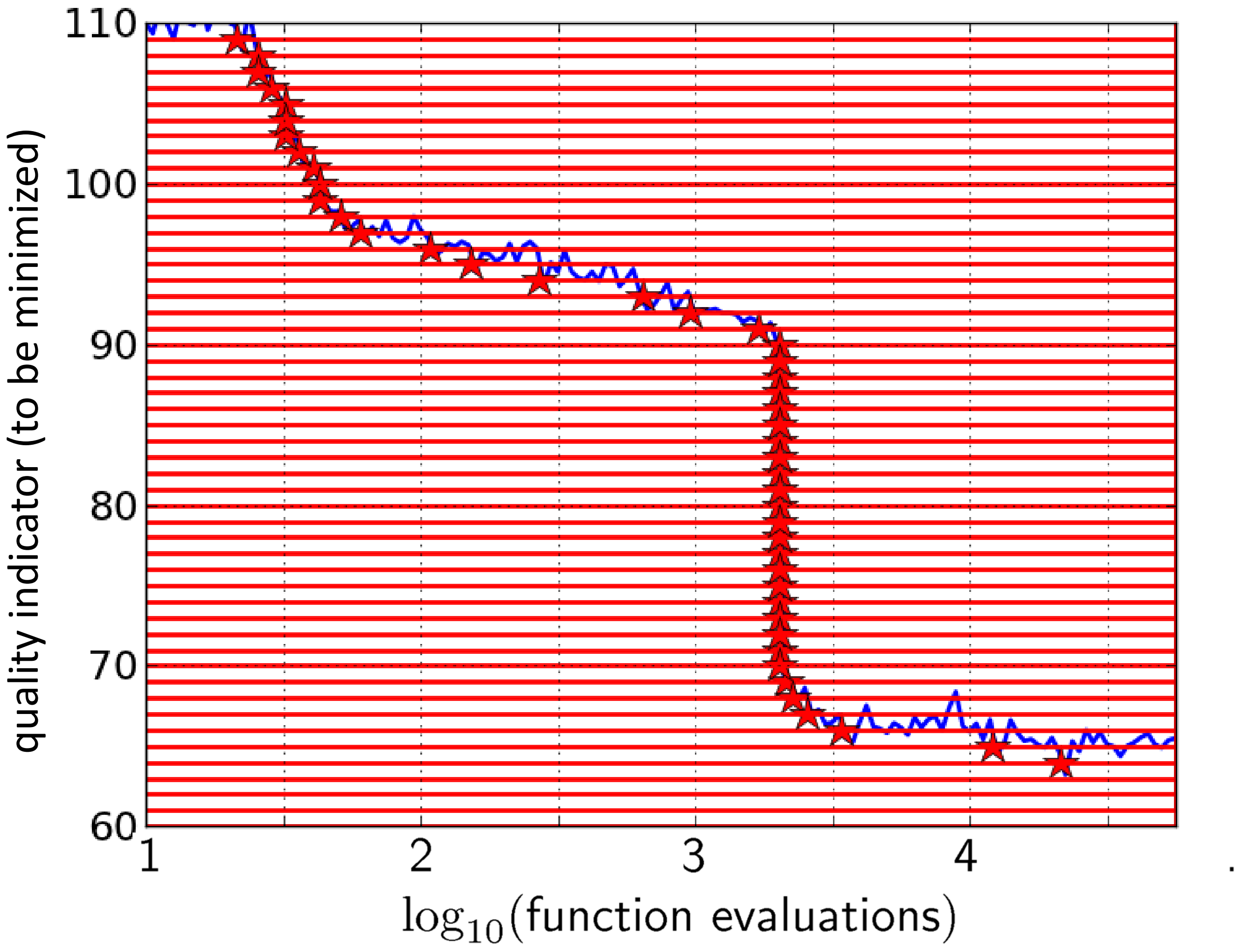
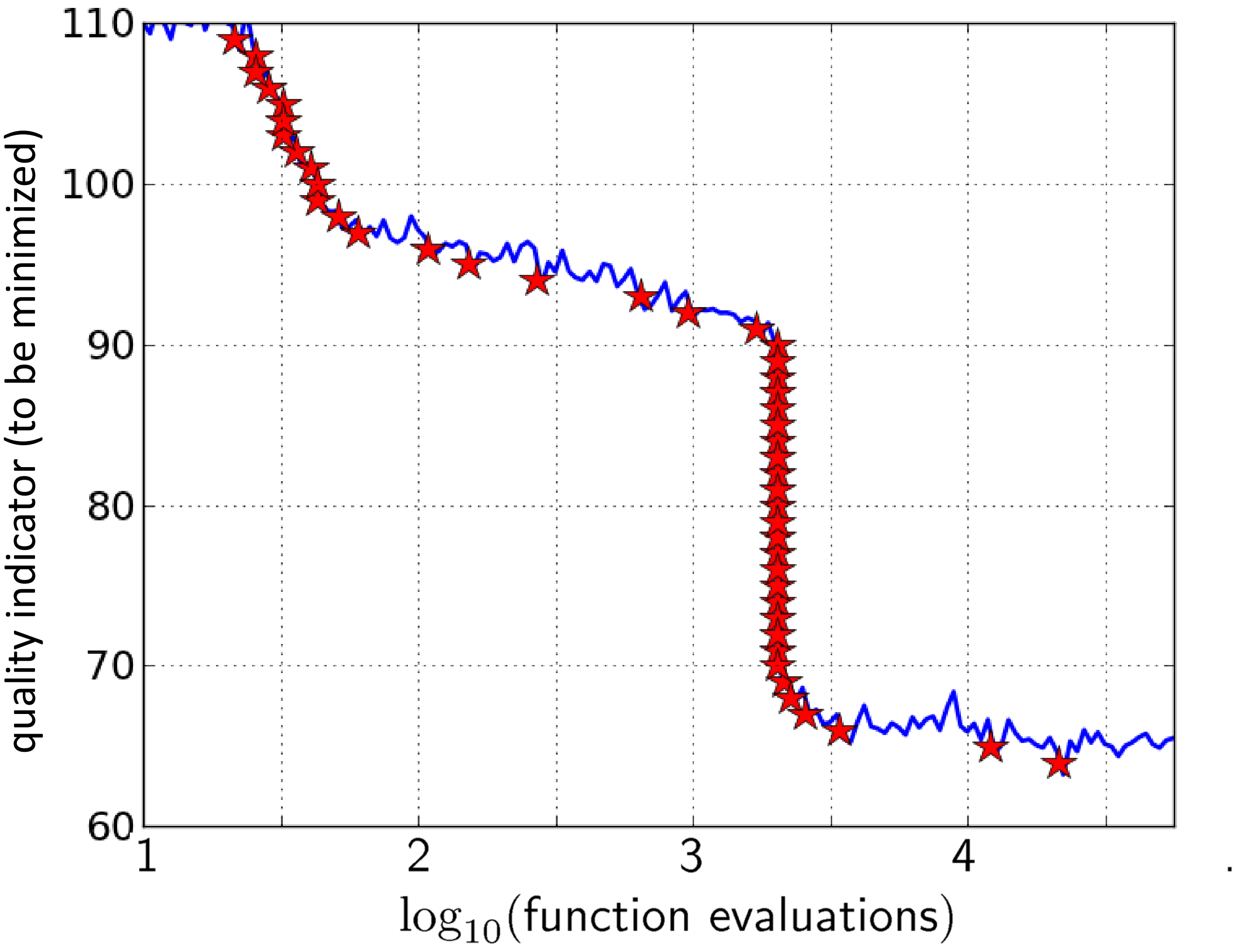


50 equally spaced targets

# Reconstructing A Single Run



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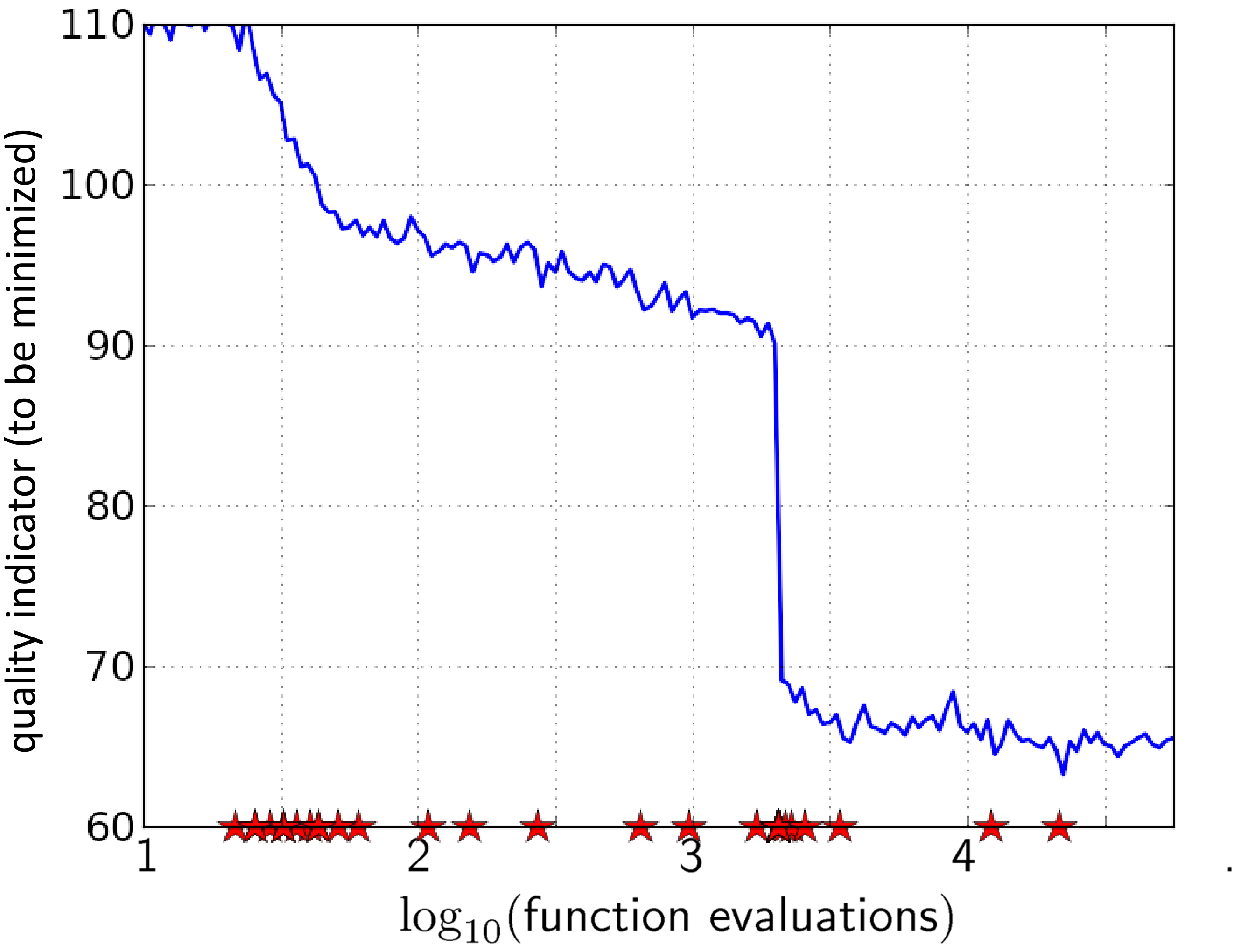

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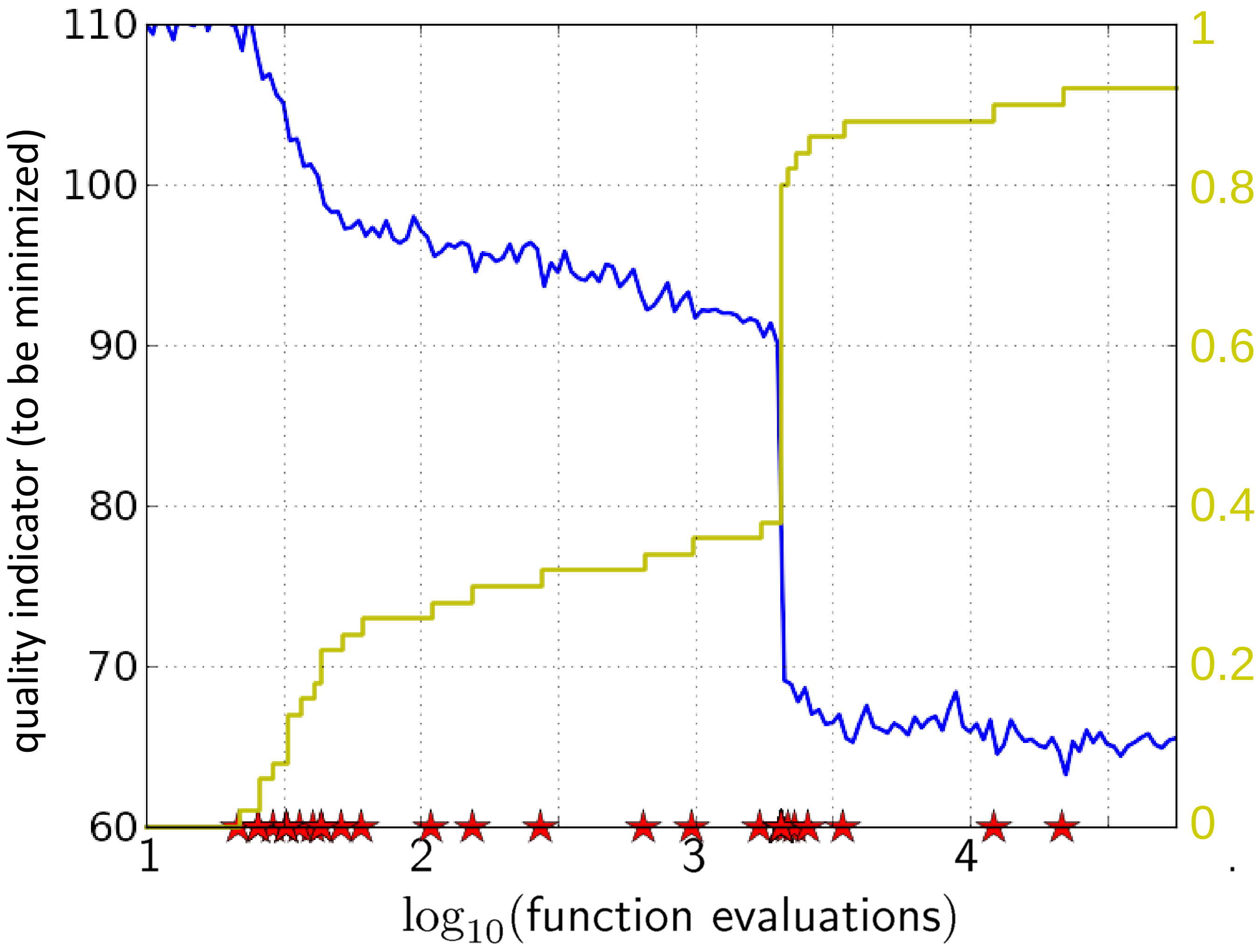
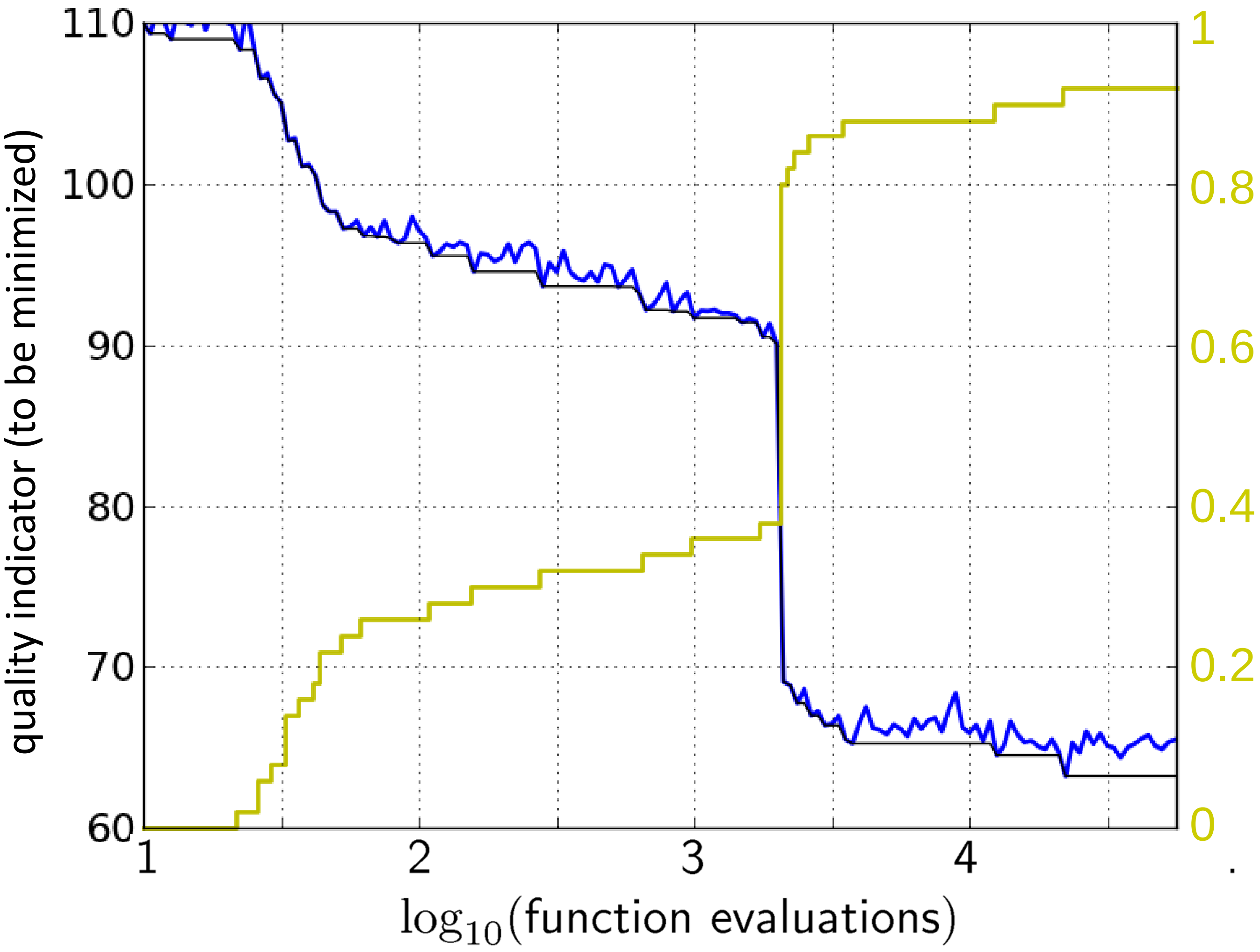


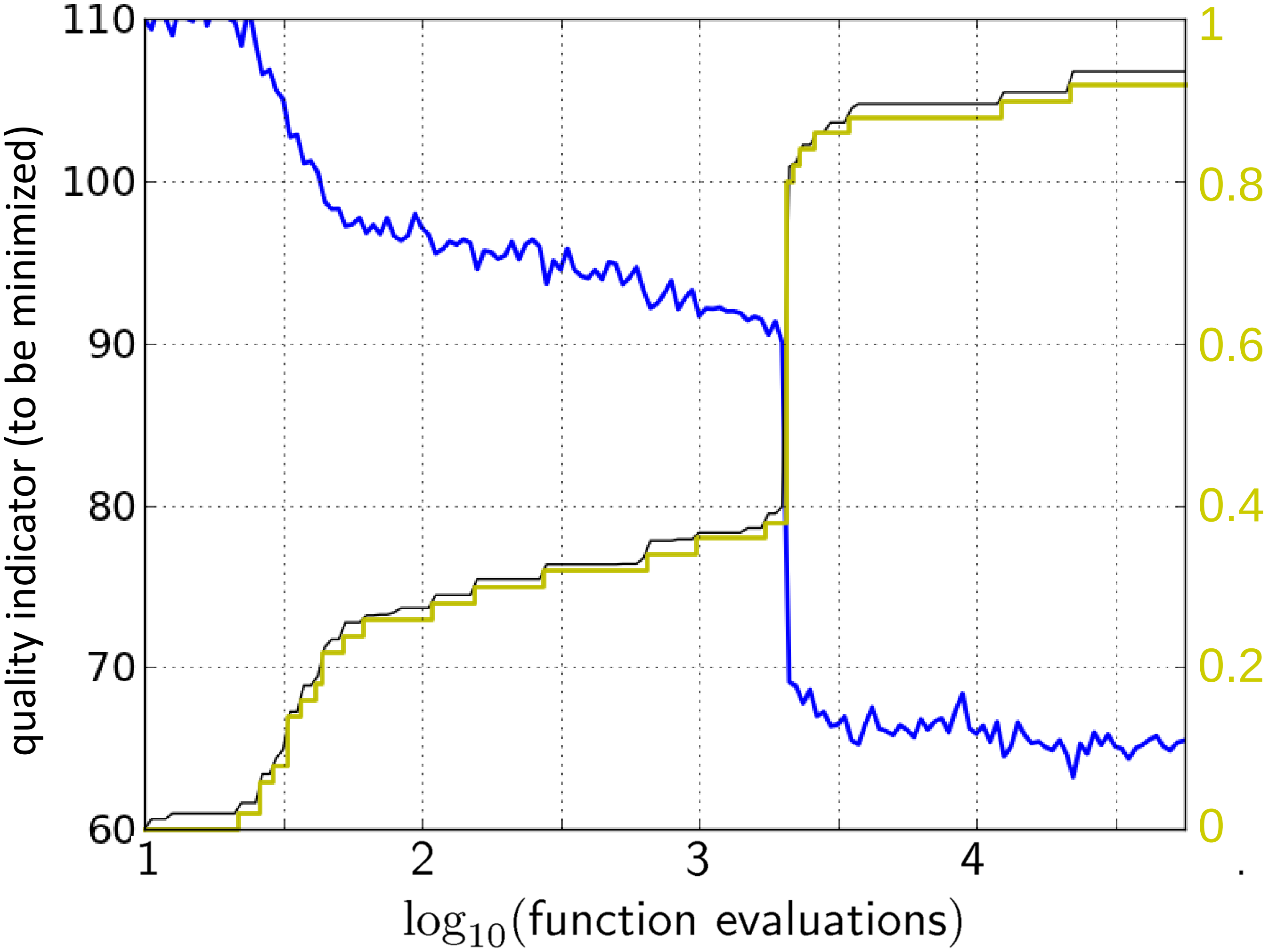
the empirical CDF makes a step for each star, is monotonous and displays for each budget the fraction of targets achieved within the budget

# Reconstructing A Single Run



the ECDF
recovers the monotonous graph, discretised and flipped

# Reconstructing A Single Run



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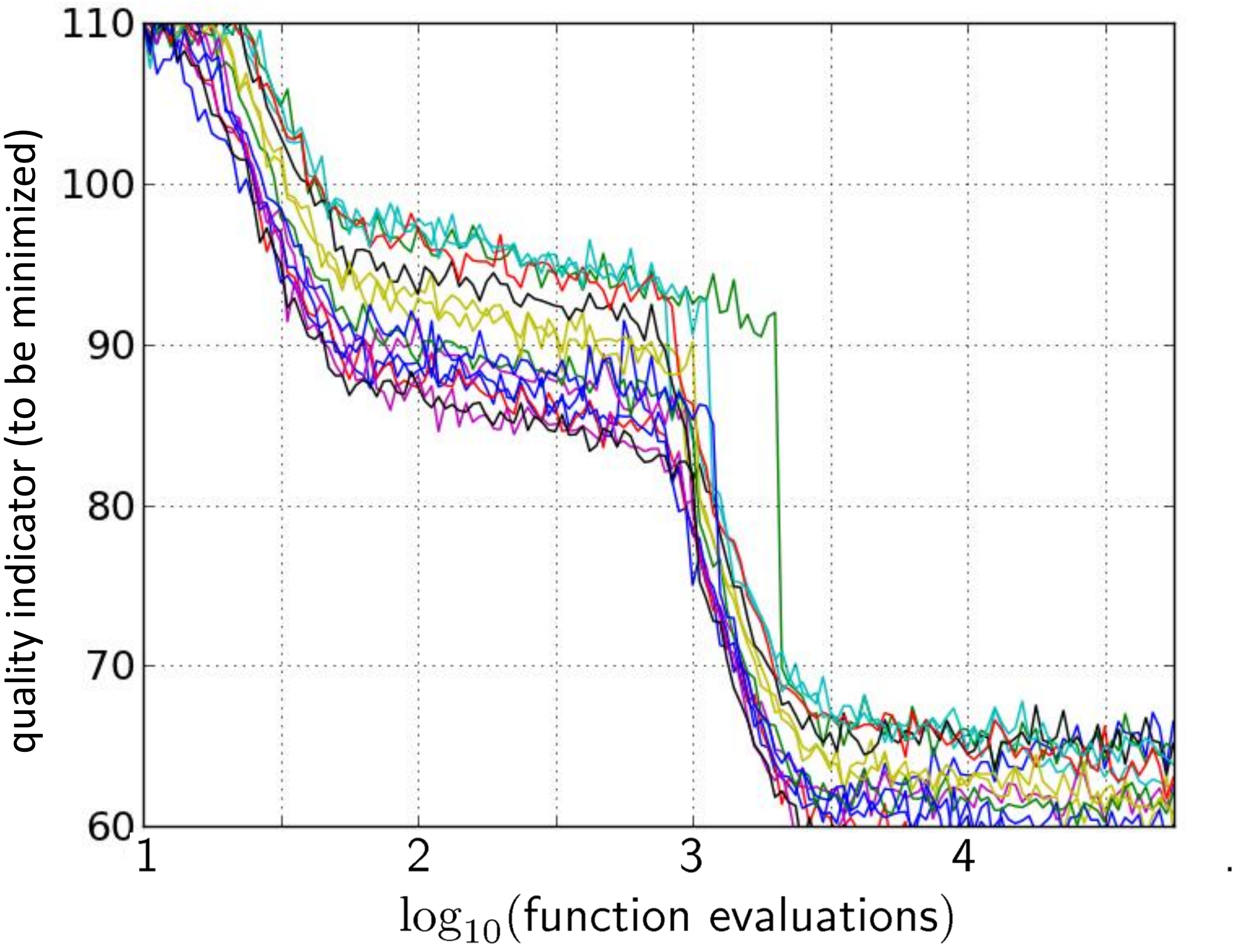
# Aggregation



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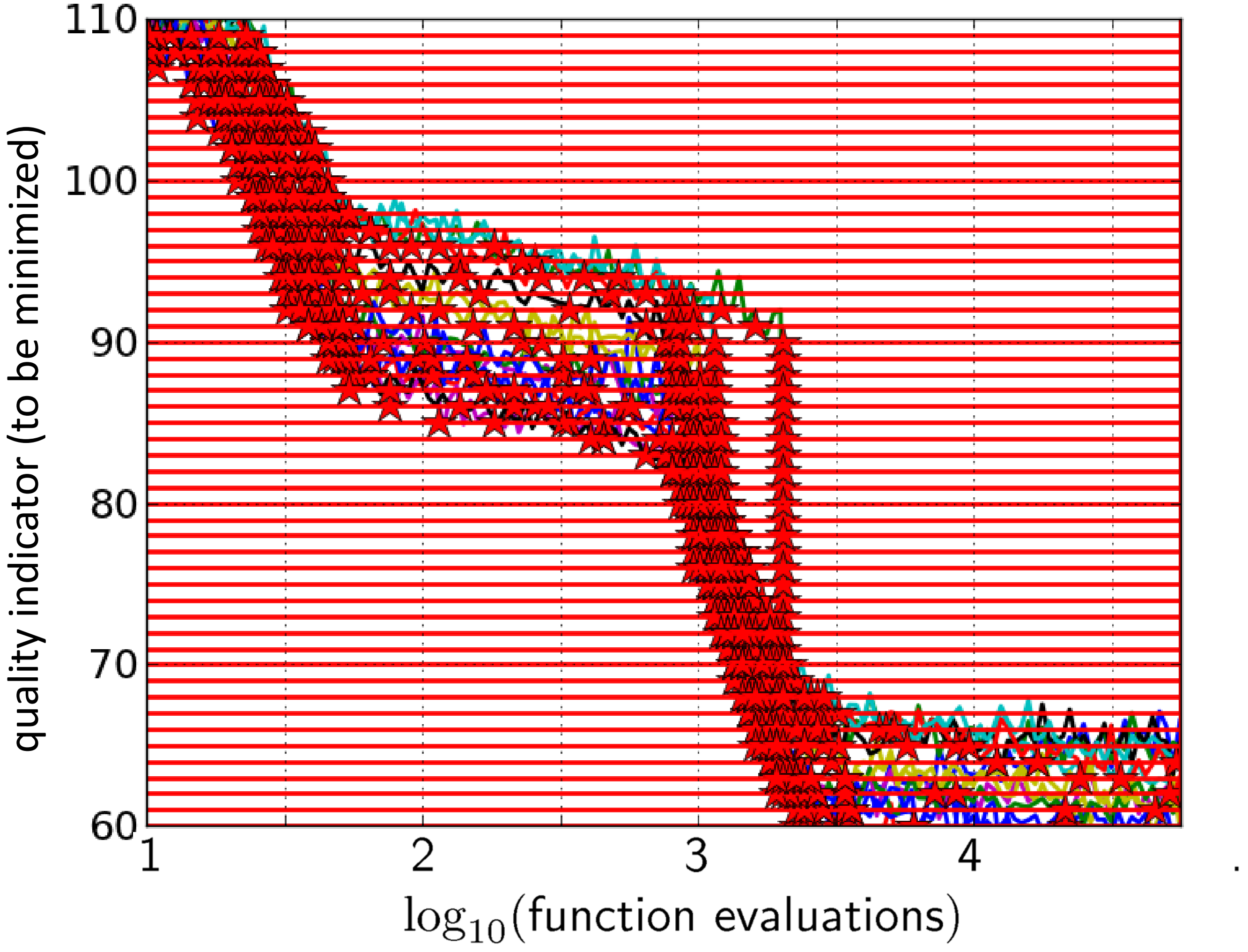
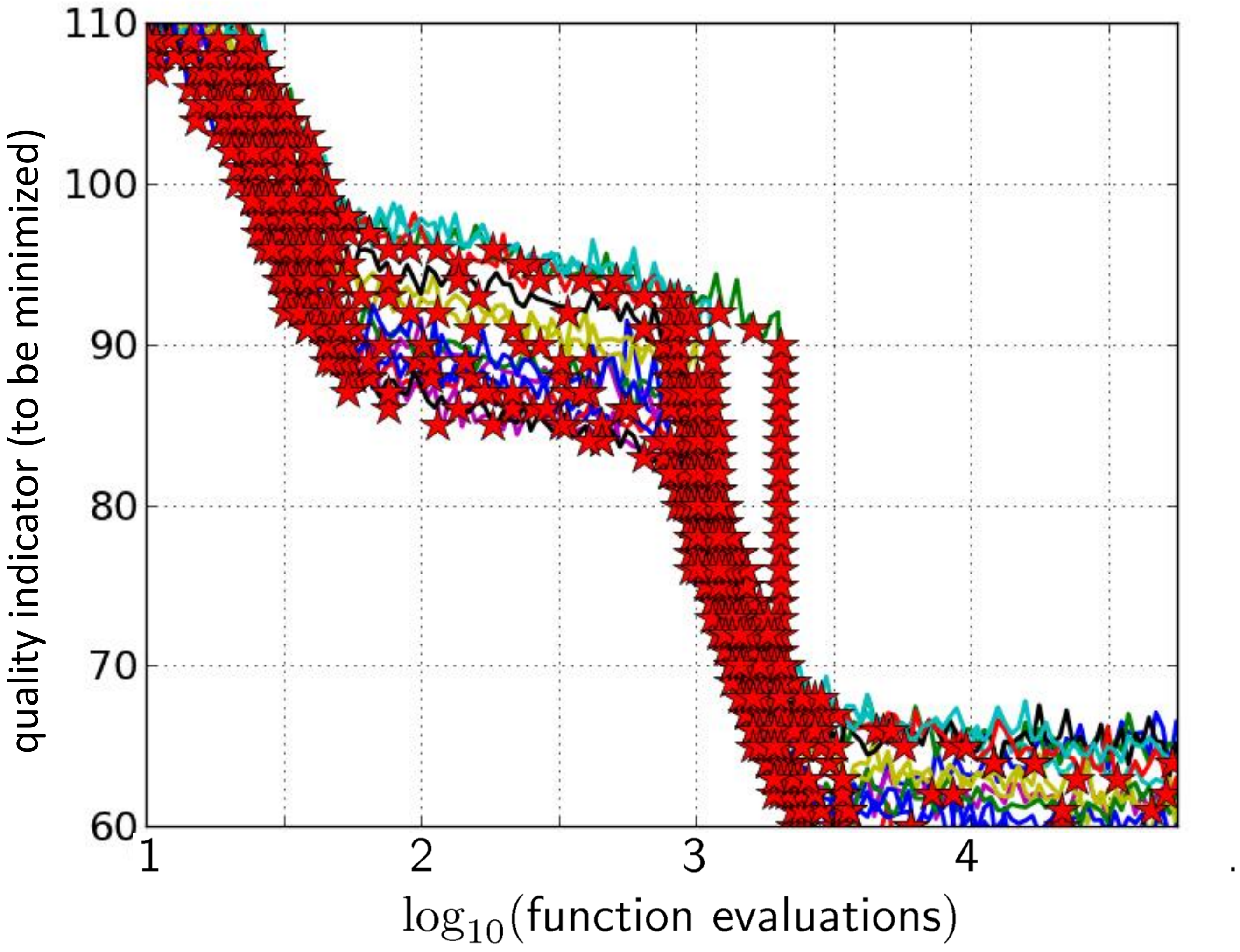


15 runs

50 targets

# Aggregation



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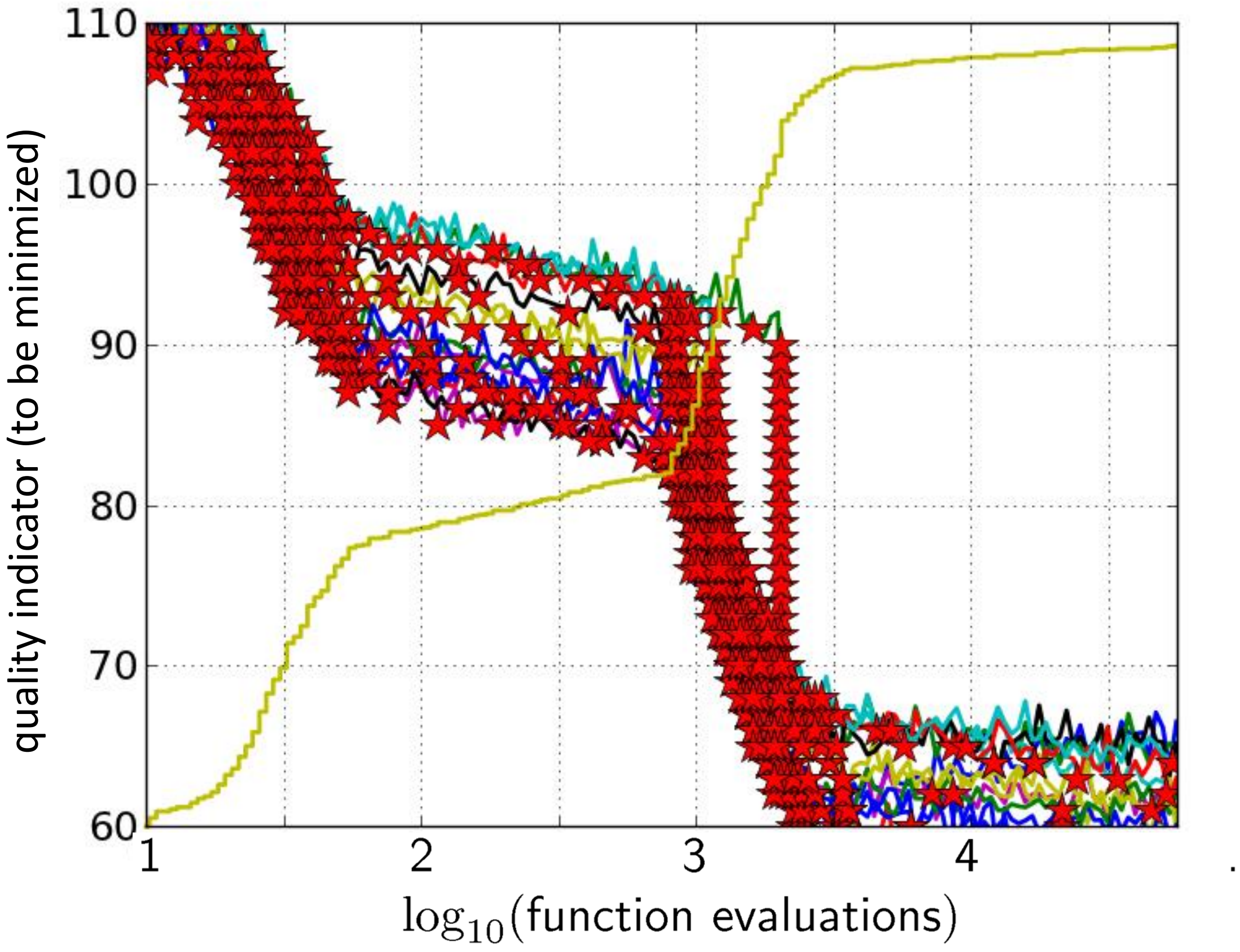
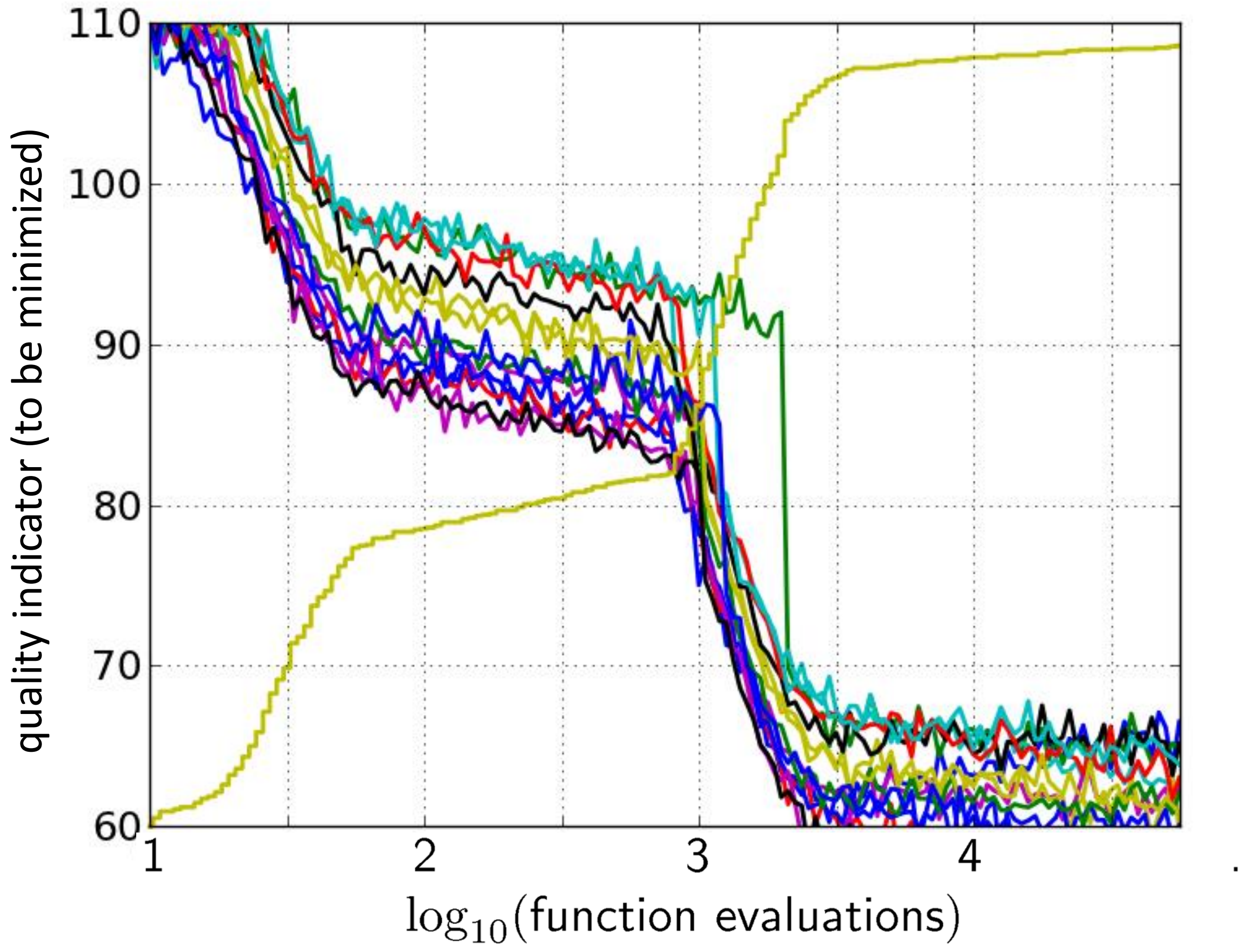


15 runs

50 targets

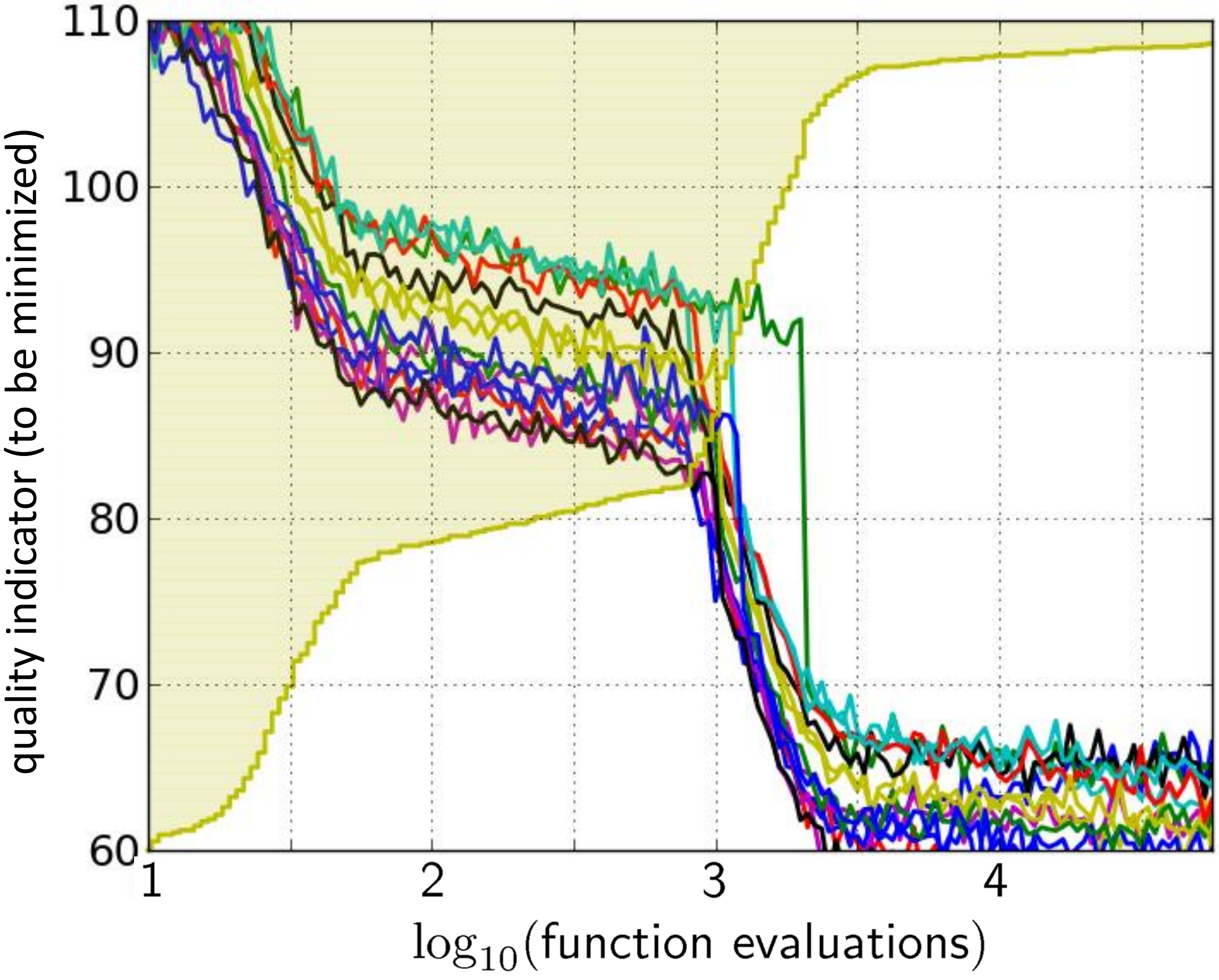
ECDF with
750 steps

# Aggregation



50 targets from
15 runs
integrated in a
single graph

# Interpretation



50 targets from
15 runs
integrated in a
single graph

area over the
ECDF curve
=
average log
runtime
(or geometric avg. runtime) over all targets (difficult and easy) and all runs

# Worth to Note

ECDF graphs

- should never aggregate over dimension

dimension is input parameter to algorithm

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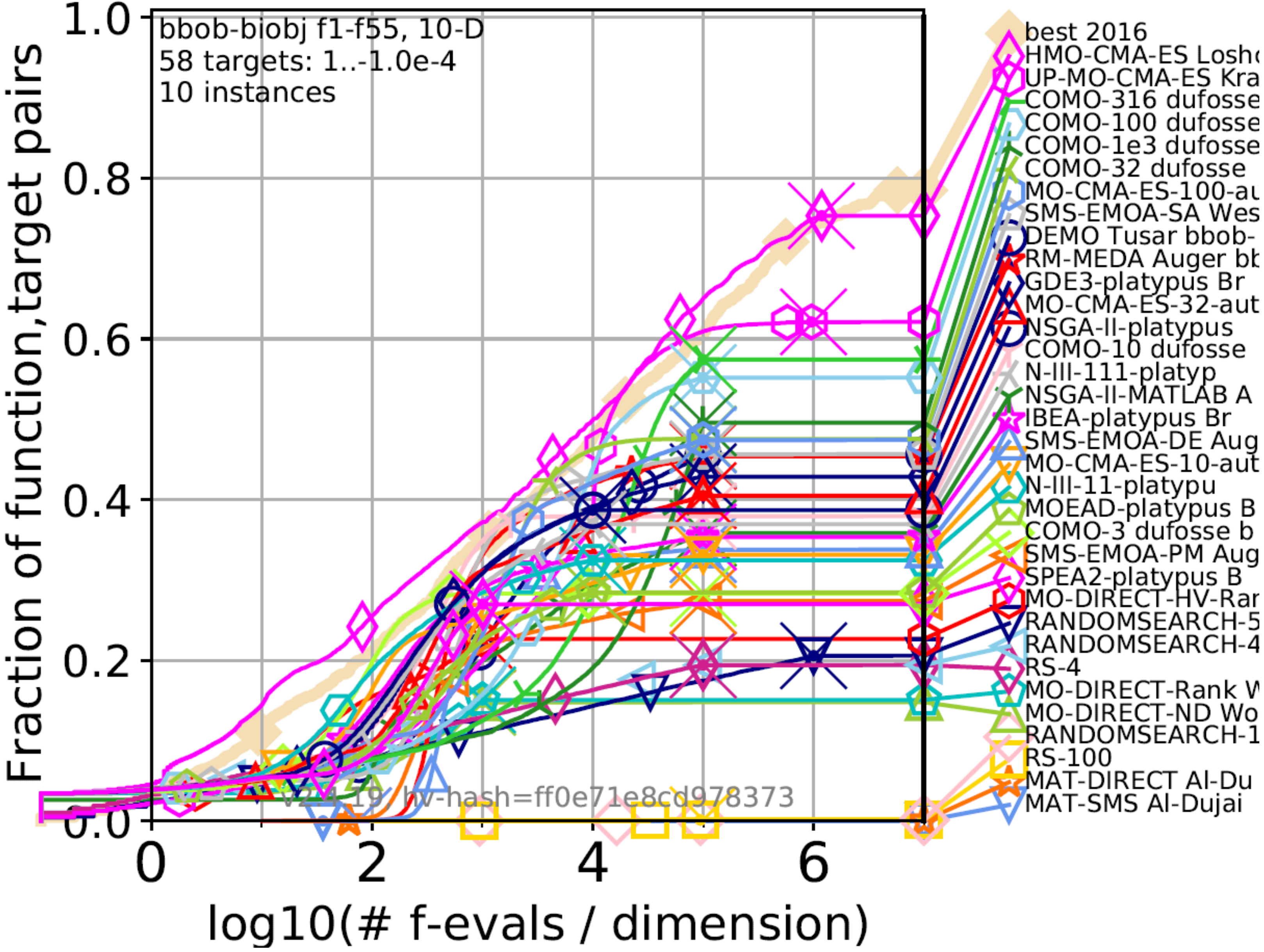
- are an extension of data profiles
  - introduced by Moré and Wild [Moré and Wild 2009]
  - but for multiple and absolute targets
- are COCO's main performance visualization tool

  `https://github.com/numbbo/coco`

# Example ECDF (later more)



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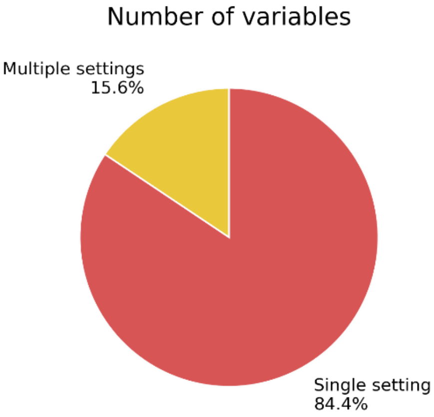

~10 papers from EMO’21 and PPSN/GECCO/CEC’21 change dimension

but 50+ papers have a “fixed” dimension

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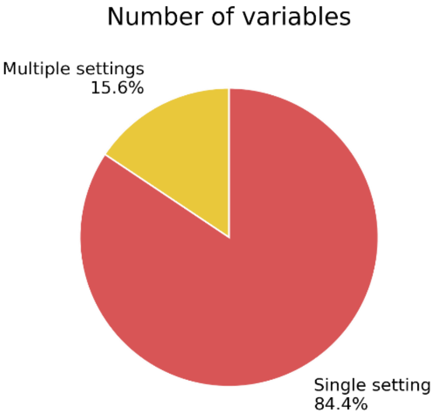

~10 papers from EMO'21 and PPSN/GECCO/CEC'21 change dimension

but 50+ papers have a "fixed" dimension

  - but in practice search space scalability almost more important

number of objectives often fixed

# A Few General Recommendations

- always display everything you have
- look at single runs
- do each experiment at least twice

(= look at the *variance* of your results)

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# A Few General Recommendations

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- look at single runs
- do each experiment at least twice

  (= look at the *variance* of your results)
- as quality indicators, use hypervolume, R2, or epsilon indicator

  or any indicator which is at least monotone
- see also the tutorial slides by Nikolaus Hansen on this topic (not restricted to single-objective optimization!)

**`http://www.cmap.polytechnique.fr/~nikolaus.hansen/gecco2018-experimentation-guide-slides.pdf`**

# Recommended Experimental Setup (w/ or w/o COCO)

❶ **Benchmarking Experiment**

❷ **Choosing Algorithms for Comparison**

see `https://numbbo.github.io/data-archive/`

❸ **Postprocessing**

```
python -m cocopp resultfolder/ ALG2 ALG3
```

❹ **Displaying and Discussing Summary Results**

❺ **Investigating and Discussing Complementary Results**

❻ **Processed Data Sharing**

provide html output somewhere

❼ **Raw Data Sharing**

easy with COCO archive module & through issue tracker

# Overview

❶ Performance Assessment

❷ Test Problems and Their Visualizations

❸ Recommendations from Numerical Results

# Test Problems and Their Visualizations

## Introduction

## Test Problems (1)

**Artificial problems (continuous and unconstrained)**

- **v0.1:** Individual problems
- **v0.2:** MOP suite (unscalable problems)
- **v0.5:** ZDT suite (scalable number of variables)
- **v1.0:** DTLZ suite (scalable number of variables and objectives)
- **v1.2:** WFG suite
- **v1.3:** Other suites with a bottom-up construction
- **v1.5:** Suites of distance-based problems
- **v2.0:** The bbob-biobj(-ext) suite

# Test Problems and Their Visualizations

## Visualization of multiobjective landscapes

**Low-dimensional search spaces**

- Dominance ratio
- Local dominance
- Gradient path length
- PLOT

**Any-dimensional search spaces**

- Line cuts
- Optima network

# Test Problems and Their Visualizations

## Test Problems (2)

**Artificial problems (other)**

- Constrained problems
- Mixed-integer problems

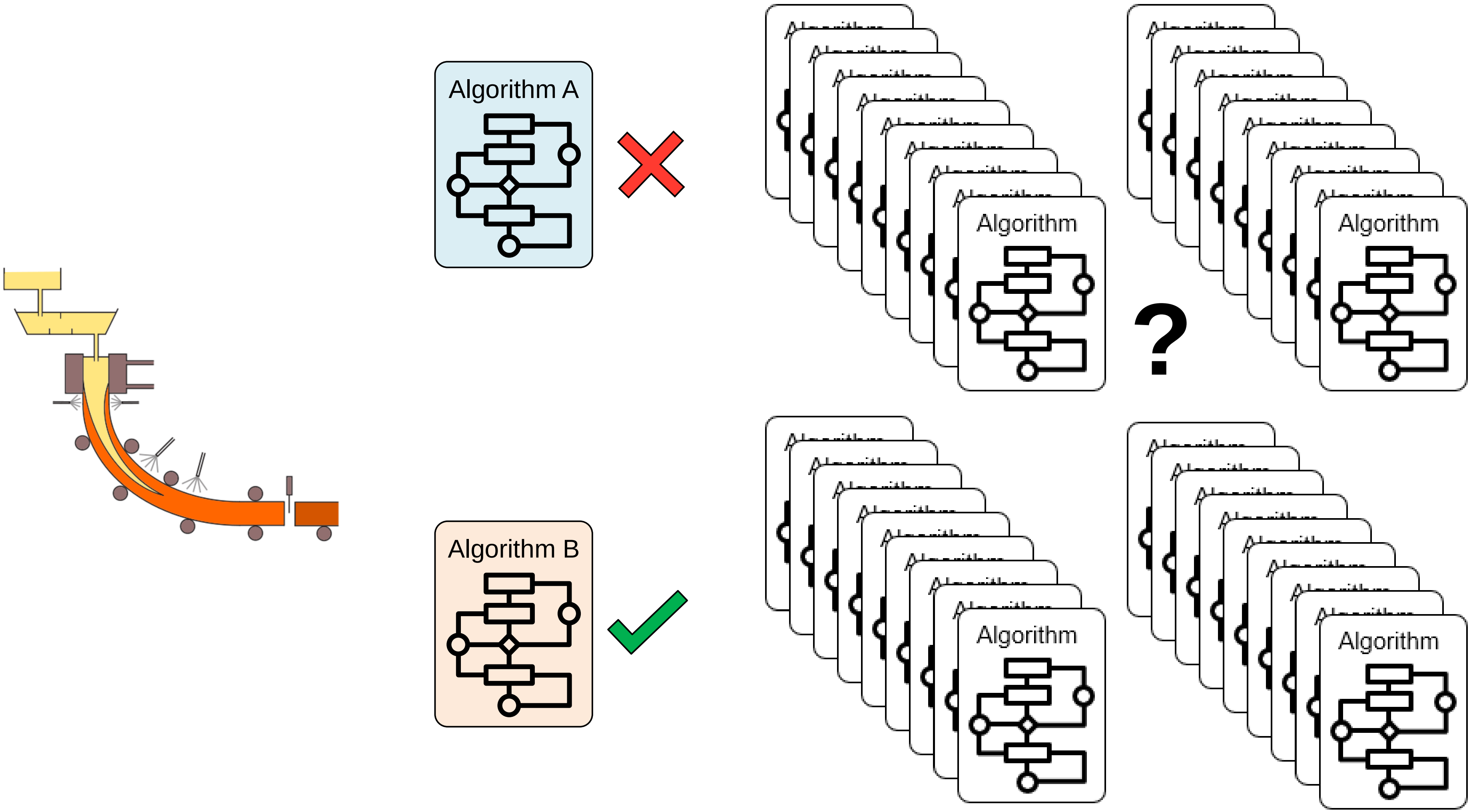
**Real-world problems**

- **v0.1:** Individual problems
- **v0.2:** Suites of unscalable problems
- **v0.5:** Suites of scalable problems (number of variables)

## Conclusions

# Introduction

## Why use test problems?



# Introduction

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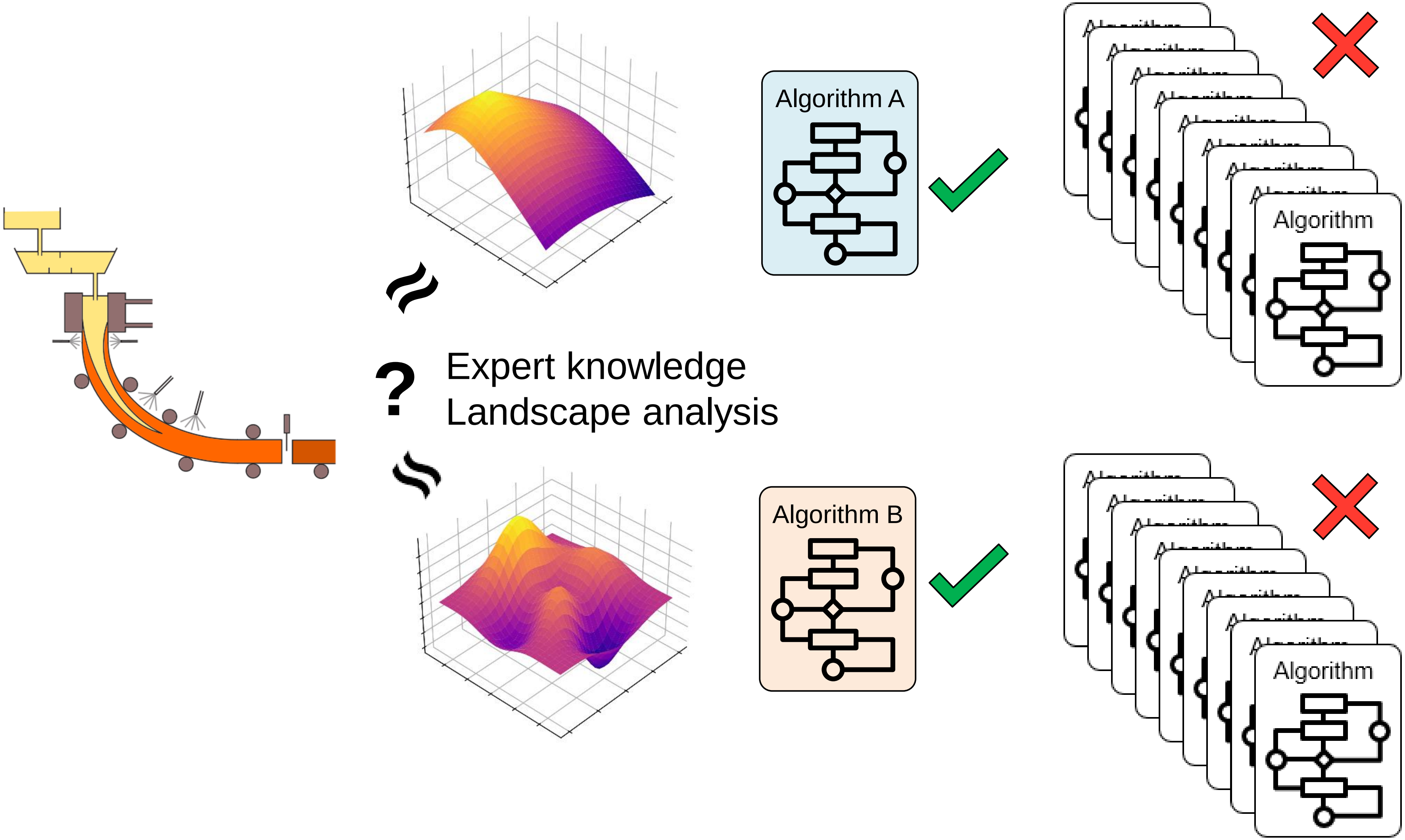


Expert knowledge
Landscape analysis

# Introduction

## Desirable Characteristics of a Problem Set

[Bartz-Beielstein et al. 2020]

1. Diverse
2. Representative
3. Scalable and tunable
4. Known optima / best performance
5. [Continually updated]

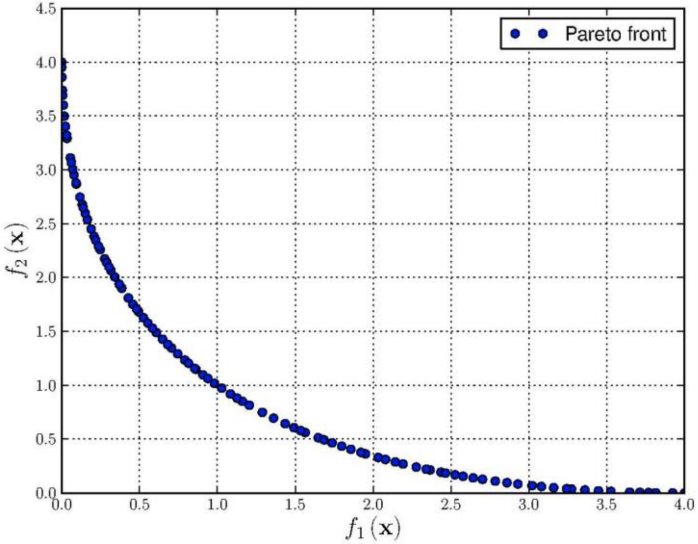
# Artificial problems (continuous and unconstrained)

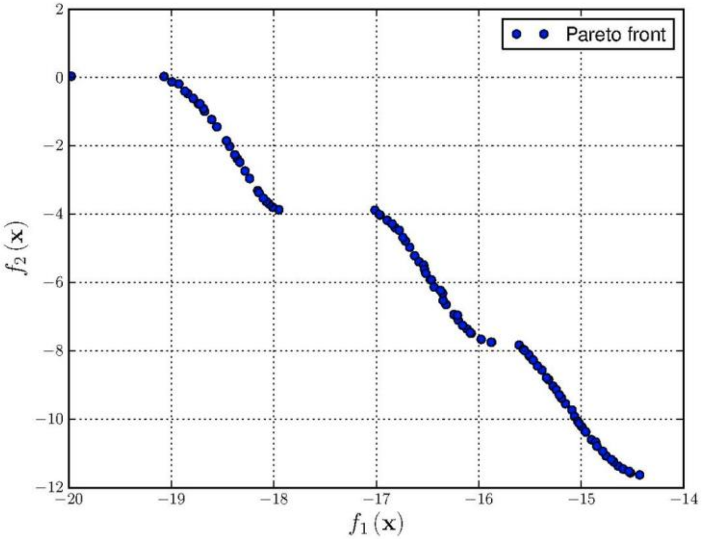
# v0.1

# Individual problems

$$\text{Minimize} = \begin{cases} f_1(x) = x^2 \\ f_2(x) = (x-2)^2 \end{cases}$$

[Schaffer 1985]





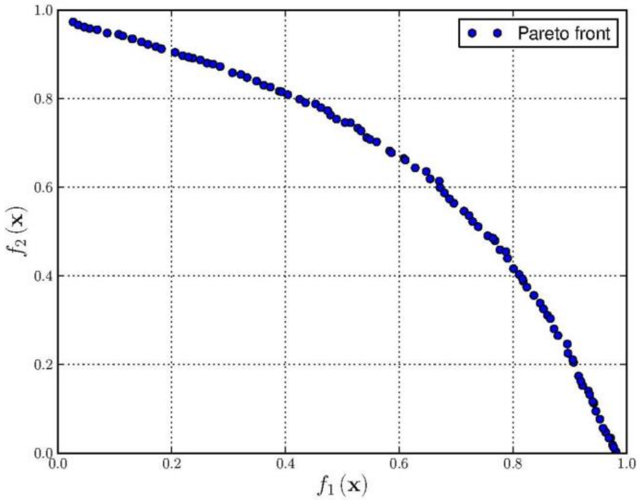
$$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = \sum_{i=1}^{2}\left[-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right] \\ f_2(\boldsymbol{x}) = \sum_{i=1}^{3}\left[|x_i|^{0.8} + 5\sin\left(x_i^3\right)\right] \end{cases}$$

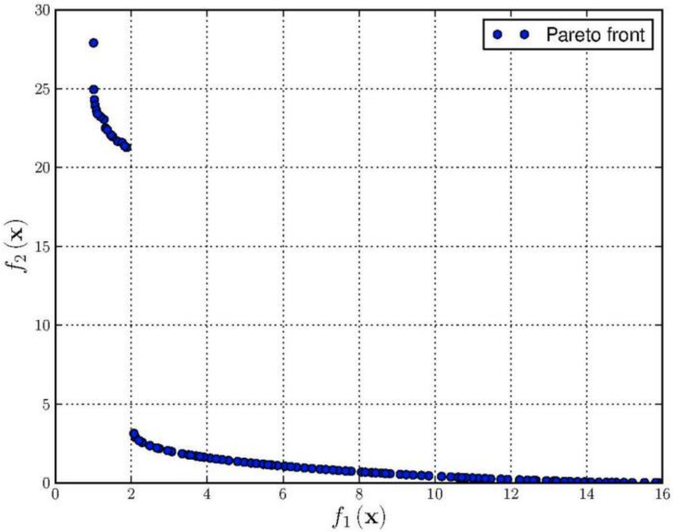
[Kursawe 1991]

# Individual problems

$$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i - \frac{1}{\sqrt{n}}\right)^2\right] \\ f_2(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i + \frac{1}{\sqrt{n}}\right)^2\right] \end{cases}$$

[Fonseca and Fleming 1995]





$$\text{Minimize} = \begin{cases} f_1(x,y) = \left[1 + (A_1 - B_1(x,y))^2 + (A_2 - B_2(x,y))^2\right] \\ f_2(x,y) = (x+3)^2 + (y+1)^2 \end{cases}$$

$$\text{where} = \begin{cases} A_1 = 0.5\sin(1) - 2\cos(1) + \sin(2) - 1.5\cos(2) \\ A_2 = 1.5\sin(1) - \cos(1) + 2\sin(2) - 0.5\cos(2) \\ B_1(x,y) = 0.5\sin(x) - 2\cos(x) + \sin(y) - 1.5\cos(y) \\ B_2(x,y) = 1.5\sin(x) - \cos(x) + 2\sin(y) - 0.5\cos(y) \end{cases}$$

[Poloni et al. 1996]

# Individual problems

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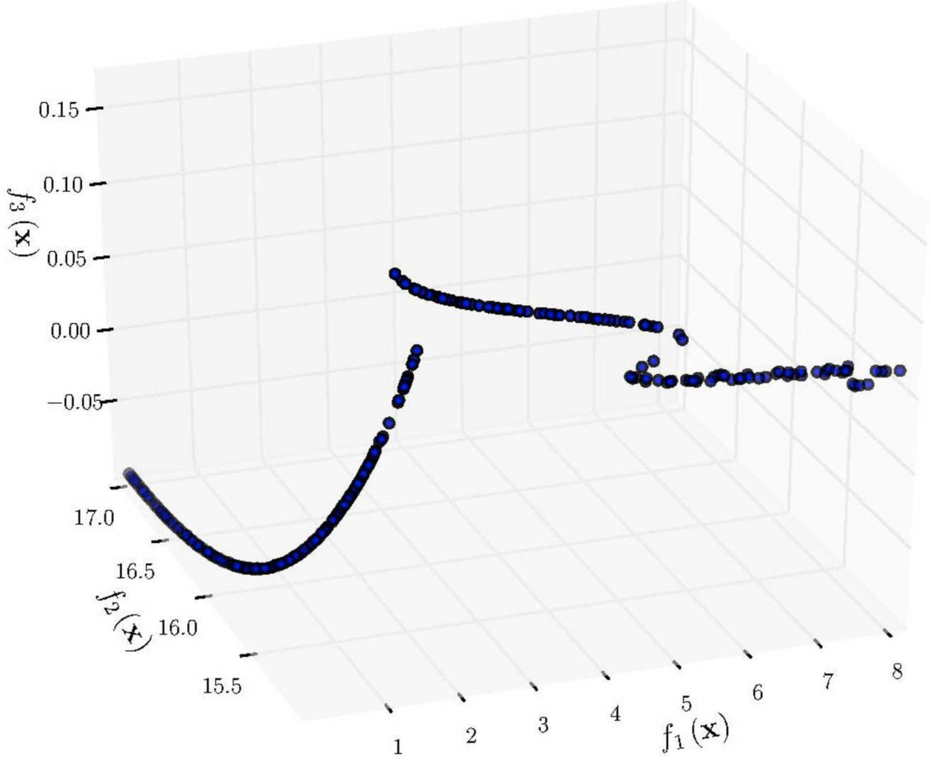
$$\text{Minimize} = \begin{cases} f_1(x,y) = 0.5\left(x^2+y^2\right) + \sin\left(x^2+y^2\right) \\ f_2(x,y) = \frac{(3x-2y+4)^2}{8} + \frac{(x-y+1)^2}{27} + 15 \\ f_3(x,y) = \frac{1}{x^2+y^2+1} - 1.1\exp\left(-\left(x^2+y^2\right)\right) \end{cases}$$

[Viennet et al. 1996]



# v0.2

# MOP Suite

MOP = Multi-Objective Problem

[Van Veldhuizen 1999]

## Properties

- A collection of 7 test problems from the literature (including the 5 shown before)
- Most problems have 2 or 3 variables
- Not scalable in the number of objectives
- Many problems have optimal solutions on the boundary or middle of the search space
- Some problems are both nonseparable and multimodal
- A collection of various Pareto front geometries
- The Pareto set is hard to compute for some problems

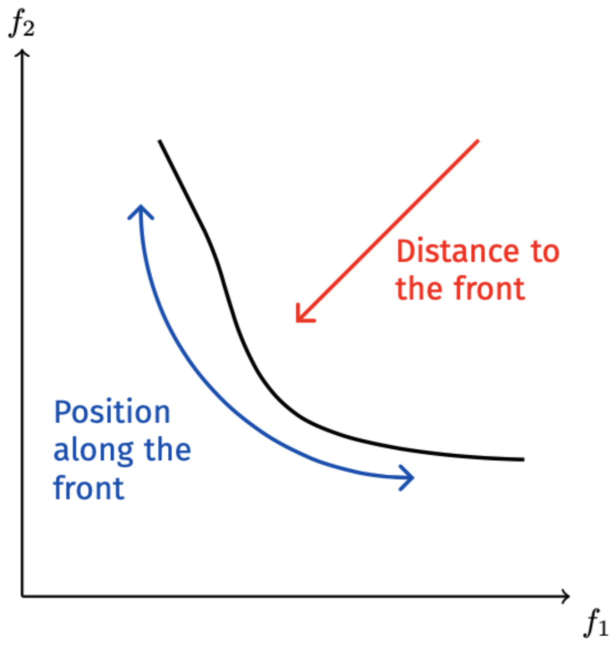
# v0.5

# ZDT Suite

ZDT = Zitzler, Deb, Thiele

[Zitzler et al. 2000]

The same construction for all problems (following Deb's toolkit [Deb 1999])



Given $\mathbf{x} = \{x_1, \ldots, x_n\}$

Distribution f.

Minimise $\boxed{f_1}(\mathbf{y})$

Distance f. Front shape

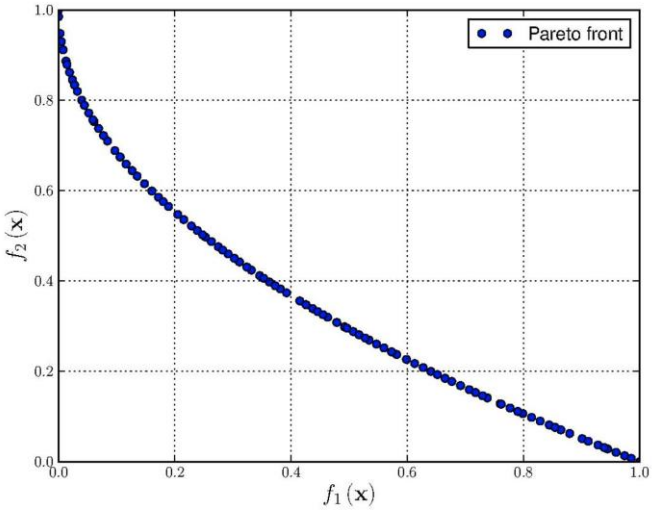
$$f_2(\mathbf{y}, \mathbf{z}) = \boxed{g}(\mathbf{z})\boxed{h}(f_1(\mathbf{y}), g(\mathbf{z}))$$

where $\mathbf{y} = \{x_1, \ldots, x_j\}$ Position variable(s) (*j* = 1 for ZDT)

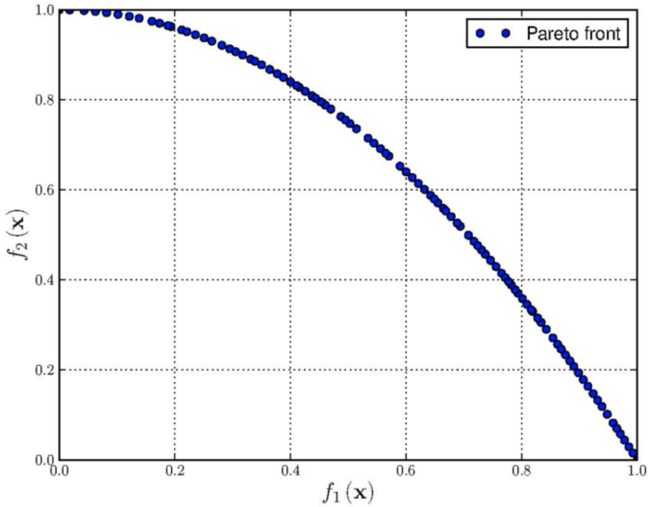
$\mathbf{z} = \{x_{j+1}, \ldots, x_n\}$ Distance variables

The separation of variables was done to simplify problem construction

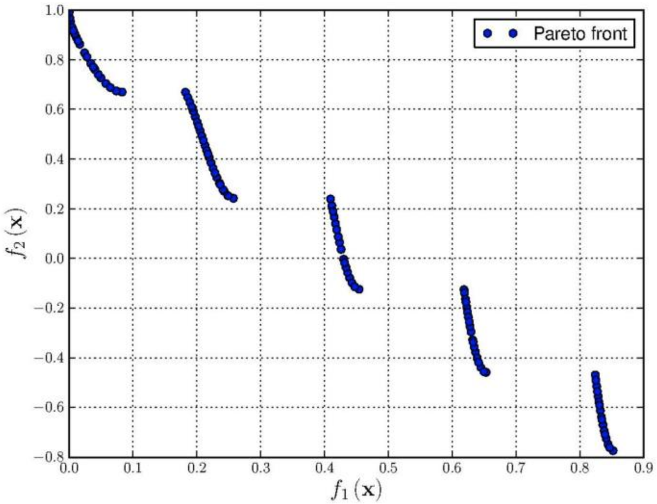
# ZDT Suite



ZDT1



ZDT2



ZDT3

# ZDT Suite

## Properties

- 6 test problems, but ZDT5 is regularly omitted, because it has a binary encoding
- Scalable in the number of (distance) variables
- All problems have 2 objectives
- 4 problems have optimal solutions on the boundary and 1 in the middle of the search space
- All problems are separable (the first objective depends only on the first variable)
- Some problems are multimodal
- Convex, concave and disconnected Pareto fronts
- The Pareto sets and fronts are known

# v1.0

# DTLZ Suite

DTLZ = Deb, Thiele, Laumanns, Zitzler

[Deb et al. 2005]

## Desired Features of Test Problems

1. Have controllable difficulty to converge to the Pareto front, a widely-distributed set of Pareto-optimal solutions
2. Scalable number of variables
3. Scalable number of objectives
4. Simple to construct
5. Pareto front easy to comprehend, both the Pareto set and front known
6. Similar difficulties to those present in real-world problems

# DTLZ Suite

## Problem Design Approaches

1. Multiple single-objective functions approach
2. Bottom-up approach
    1. Choose a Pareto front
    2. Build the objective space
    3. Construct the search space (add difficulties using the function *g*)
3. Constraint surface approach (for constrained problems)

# DTLZ Suite

## Properties

- Originally 9 problems, but then 2 were dropped
- Scalable number of distance variables, $M - 1$ position variables
- Scalable number of objectives
- Objectives separable in practice (optimizing one variable at a time will yield at least one global optimum)
- Linear, concave and disconnected Pareto fronts
- The Pareto sets and fronts are known
- Most problems have the Pareto set in the middle of the search space

Note that although the suite is scalable in the number of variables, this is rarely used in benchmark studies

# v1.2

# WFG Suite

WFG = Walking Fish Group

[Huband et al. 2006]

## Recommendations for multiobjective test problems

1. No extremal variables
2. No medial variables
3. Scalable number of variables
4. Scalable number of objectives
5. Dissimilar variable domains
6. Dissimilar objective ranges
7. Pareto set and front known

# WFG Suite

## Recommendations for multiobjective test suites

[Huband et al. 2006]

1. A few unimodal test problems to test convergence velocity relative to different Pareto optimal geometries and bias conditions
2. Cover the three core types of geometries: degenerate Pareto fronts, disconnected Pareto fronts, and disconnected Pareto sets
3. The majority of problems should be multimodal with a few deceptive problems
4. The majority of problems should be nonseparable
5. Contain problems that are both nonseparable and multimodal to be representative of real-world problems

# WFG Suite

## Properties

- 9 problems constructed from a combination of shape functions and several transformations
- Scalable number of variables (2 variables are not supported for some of the problems)
- Scalable number of objectives
- Includes also nonseparable, multimodal, deceptive and biased problems
- Convex, linear, concave, mixed, disconnected and degenerate Pareto fronts
- The Pareto sets and fronts are known
- Optimal solutions do not lie on the boundary or the middle of the search space, but the Pareto set is linear for 8 of the 9 problems
- Still rely on distance and position variables

# v1.3

# Other Suites and Problems

## Problems constructed with the bottom-up approach

[Zapotecas et al. 2019]

- LZ test suite of 9 problems with complicated Pareto sets [Li and Zhang 2009]
- SZDT test suite of 7 scalable problems with complicated Pareto sets [Saxena et al. 2011]
- Convex DTLZ problem [Deb and Jain 2014]
- Inverted DTLZ problem [Jain and Deb 2014]
- MNI test suite of 2 test problems with diverse shapes of the Pareto front [Masuda et al. 2016]
- LSMOP test suite of 9 test problems for large-scale optimization with variable dependencies [Cheng et al. 2017b]
- Minus-DTLZ and Minus-WFG test suites [Ishibuchi et al. 2017]
- MMF test problems with diverse landscapes [Yue et al. 2019]

# CEC Competition Suites

Information about all CEC competitions: https://www3.ntu.edu.sg/home/EPNSugan/index_files/cec-benchmarking.htm

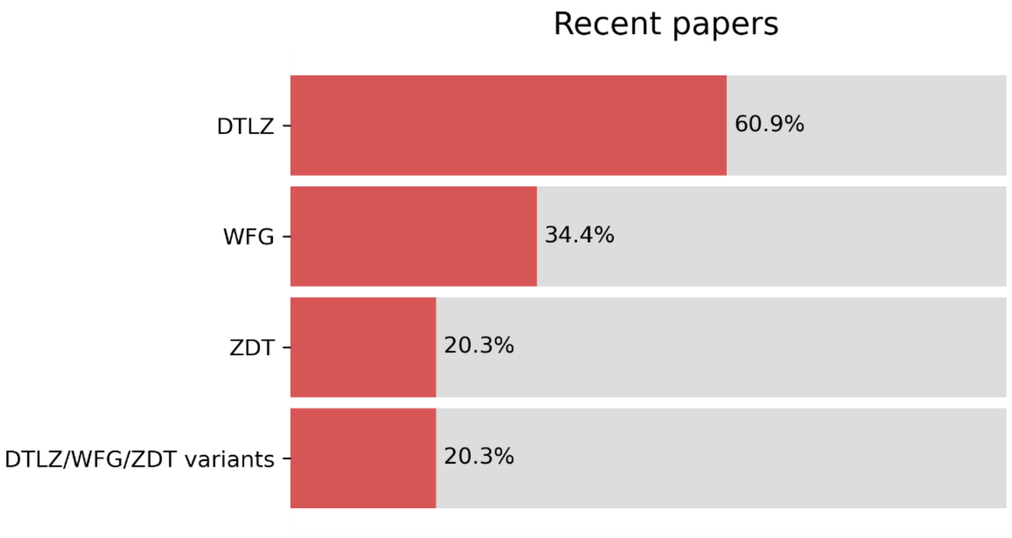
- 13 test problems for CEC 2007 [Huang et al. 2007]
  - OKA [Okabe et al. 2004], SYM-PART [Rudolph et al. 2007]
  - 4 shifted ZDT, 1 rotated ZDT
  - 2 shifted DTLZ, 1 rotated DTLZ
  - 3 WFG
- 13 test problems for CEC 2009 (UF suite) [Zhang et al. 2009]
  - 10 with complicated Pareto sets (4 from the LZ suite)
  - 2 extended rotated DTLZ
  - 1 WFG

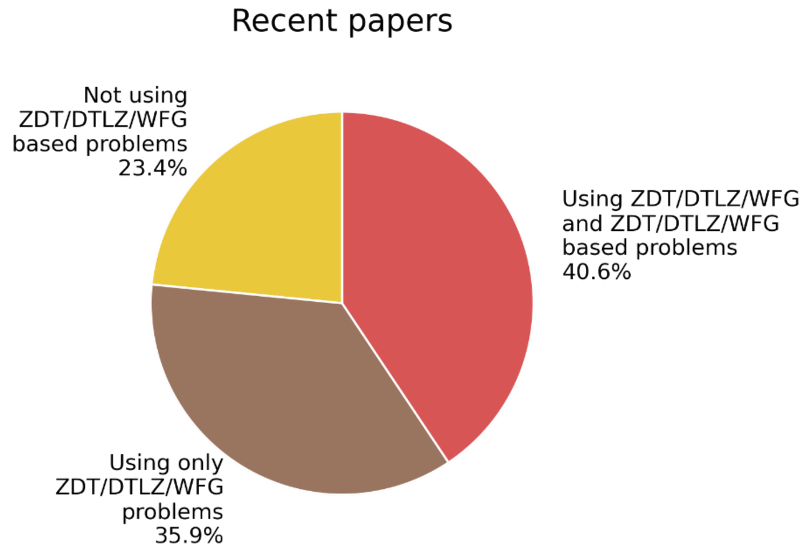
# CEC Competition Suites

- 15 test problems for CEC 2017 (MaF suite) [Cheng et al. 2017a]
  - 7 modified DTLZ problems
  - 2 distance minimization problems
  - 3 WFG problems
  - 1 SZDT problem
  - 2 LSMOP problem
- 22 test problems for CEC 2019 [Liang et al. 2019]
  - 2 SYM-PART
  - Omni-test [Deb and Tiwari 2008]
  - 19 MMF problems
- 24 test problems for CEC 2020 [Liang et al. 2020]
  - 24 MMF problems

# Survey of Recent Papers

- 64 papers on unconstrained continuous multiobjective optimization from recent conferences (without application papers)
  - CEC 2020
  - GECCO 2020
  - PPSN 2020
  - EMO 2021





# v1.5

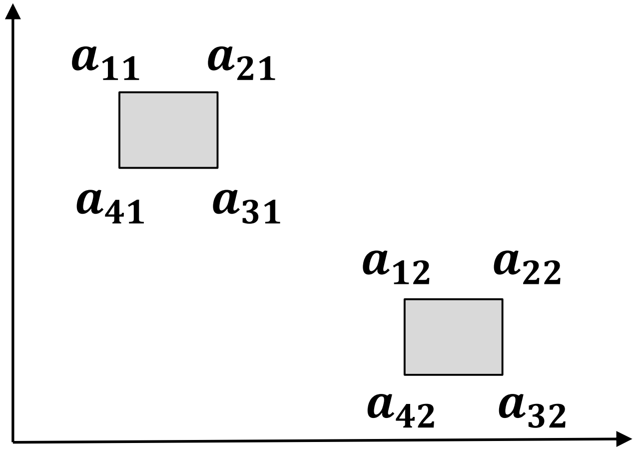
# Distance-Based Problems

## General idea

[Ishibuchi et al. 2010]

$$\text{Minimize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}),\ \ldots,\ f_k(\mathbf{x}))$$

$$f_i(\mathbf{x}) = \min\{\text{dis}(\mathbf{x}, \mathbf{a}_{i1}), \text{dis}(\mathbf{x}, \mathbf{a}_{i2}), \ldots, \text{dis}(\mathbf{x}, \mathbf{a}_{im})\}$$



- 2-D test problems that are inherently visualizable
- Pareto set easy to characterize
- Scalable in the number of objectives
- Useful for visualizing the distribution of solutions
- Unlikely to be relevant for real-world problems
- Based on earlier work [Köppen et al. 2005, Rudolph et al. 2007]

# Distance-Based Problems

## Extensions

- High-dimensional search spaces [Masuda et al. 2014]
- Distance to lines (instead of points) [Li et al. 2014, 2018]
- Dominance resistance regions [Fieldsend 2016]
- Local Pareto fronts [Liu et al. 2018]
- Problem generator for scalable problems with various properties (local fronts, disconnected Pareto sets and fronts, dominance resistance regions, uneven ranges of objective values, varying density of solutions) [Fieldsend et al. 2019]

# v2.0

# bbob-biobj Suite

## Motivation

[Brockhoff et al. 2016]

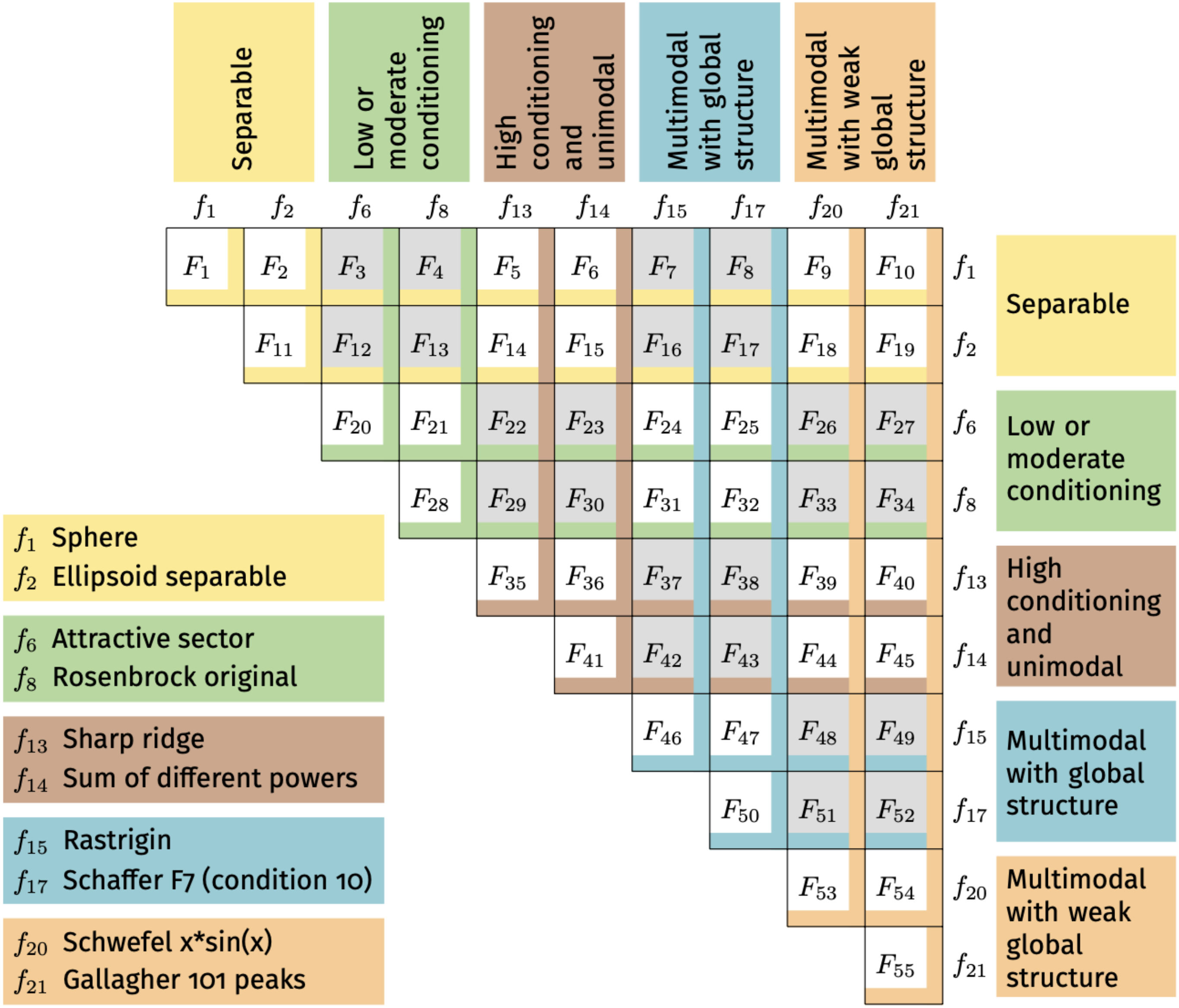
- Most other suites are constructed based on the desired Pareto front properties
- Consequently, problems have artificial properties not likely to exist in real-world problems
  - Distance and position parameters (DTLZ-like problems)
  - Linear objectives (distance-based problems)
- In real-world problems each objective is a separate function
- Go back to basics – use single-objective functions for each objective
  - Idea not new [Schaffer 1985, Igel et al. 2007, Emmerich and Deutz 2007, Kerschke et al. 2016]

# bbob-biobj Suite

## Construction

- Use the functions from the **bbob** suite
  - Well-understood
  - Scalable in the number of variables and *parametrized*
  - 24 functions categorized in 5 groups based on their properties
    - Separable
    - Low or moderate conditioning
    - High conditioning and unimodal
    - Multimodal with global structure
    - Multimodal with weak global structure
- How to avoid an explosion in the number of problems?

# bbob-biobj Suite



| | Separable | | Low or moderate conditioning | | High conditioning and unimodal | | Multimodal with global structure | | Multimodal with weak global structure | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $f_1$ | $f_2$ | $f_6$ | $f_8$ | $f_{13}$ | $f_{14}$ | $f_{15}$ | $f_{17}$ | $f_{20}$ | $f_{21}$ | |
| $f_1$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $F_7$ | $F_8$ | $F_9$ | $F_{10}$ | Separable |
| $f_2$ | | $F_{11}$ | $F_{12}$ | $F_{13}$ | $F_{14}$ | $F_{15}$ | $F_{16}$ | $F_{17}$ | $F_{18}$ | $F_{19}$ | |
| $f_6$ | | | $F_{20}$ | $F_{21}$ | $F_{22}$ | $F_{23}$ | $F_{24}$ | $F_{25}$ | $F_{26}$ | $F_{27}$ | Low or moderate conditioning |
| $f_8$ | | | | $F_{28}$ | $F_{29}$ | $F_{30}$ | $F_{31}$ | $F_{32}$ | $F_{33}$ | $F_{34}$ | |
| $f_{13}$ | | | | | $F_{35}$ | $F_{36}$ | $F_{37}$ | $F_{38}$ | $F_{39}$ | $F_{40}$ | High conditioning and unimodal |
| $f_{14}$ | | | | | | $F_{41}$ | $F_{42}$ | $F_{43}$ | $F_{44}$ | $F_{45}$ | |
| $f_{15}$ | | | | | | | $F_{46}$ | $F_{47}$ | $F_{48}$ | $F_{49}$ | Multimodal with global structure |
| $f_{17}$ | | | | | | | | $F_{50}$ | $F_{51}$ | $F_{52}$ | |
| $f_{20}$ | | | | | | | | | $F_{53}$ | $F_{54}$ | Multimodal with weak global structure |
| $f_{21}$ | | | | | | | | | | $F_{55}$ | |

$f_1$ Sphere
$f_2$ Ellipsoid separable

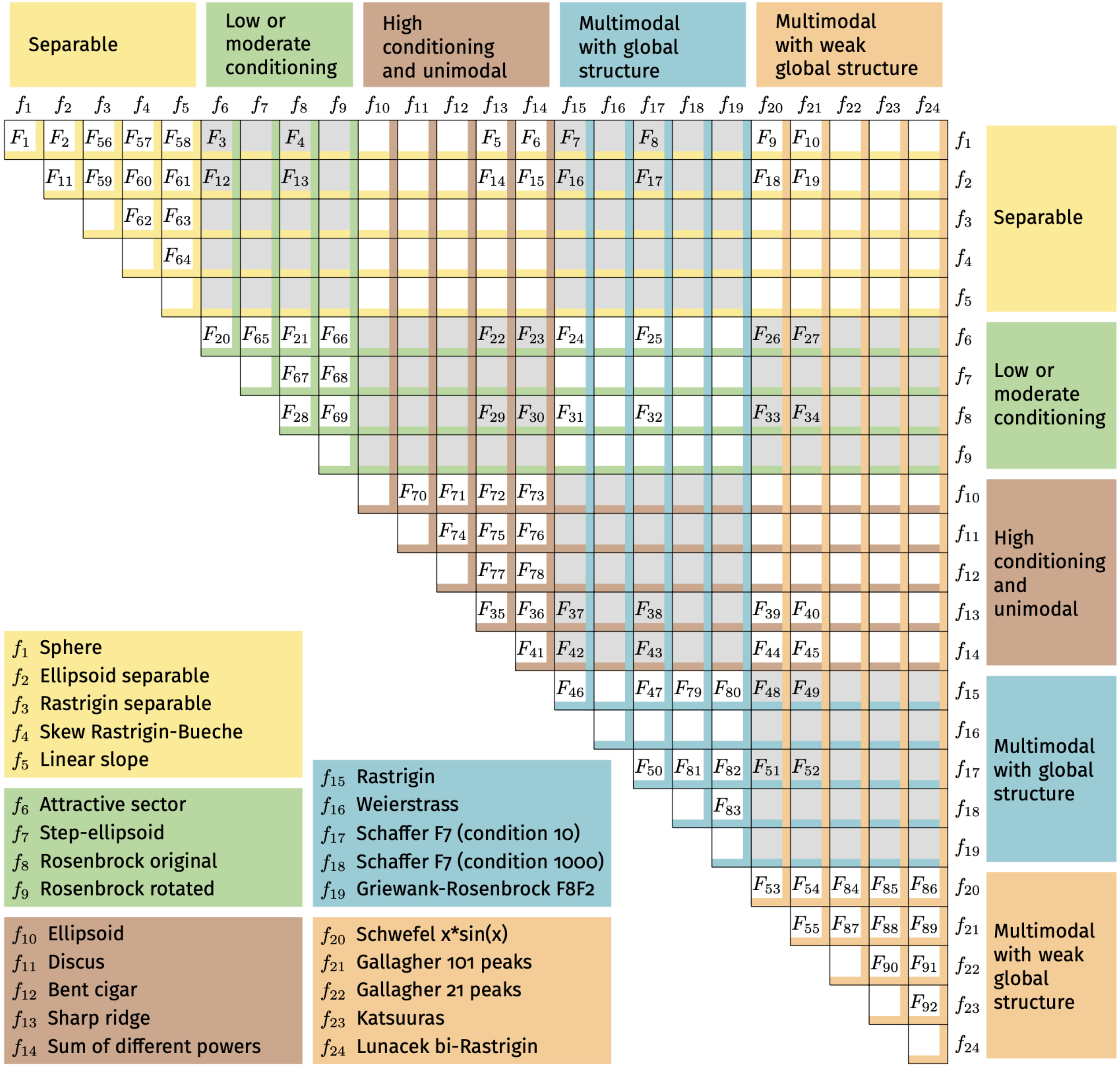
$f_6$ Attractive sector
$f_8$ Rosenbrock original

$f_{13}$ Sharp ridge
$f_{14}$ Sum of different powers

$f_{15}$ Rastrigin
$f_{17}$ Schaffer F7 (condition 10)

$f_{20}$ Schwefel x*sin(x)
$f_{21}$ Gallagher 101 peaks

# bbob-biobj-ext Suite



Separable: $f_1$–$f_5$; Low or moderate conditioning: $f_6$–$f_9$; High conditioning and unimodal: $f_{10}$–$f_{14}$; Multimodal with global structure: $f_{15}$–$f_{19}$; Multimodal with weak global structure: $f_{20}$–$f_{24}$

| | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ | $f_{10}$ | $f_{11}$ | $f_{12}$ | $f_{13}$ | $f_{14}$ | $f_{15}$ | $f_{16}$ | $f_{17}$ | $f_{18}$ | $f_{19}$ | $f_{20}$ | $f_{21}$ | $f_{22}$ | $f_{23}$ | $f_{24}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f_1$ | $F_1$ | $F_2$ | $F_{56}$ | $F_{57}$ | $F_{58}$ | $F_3$ | | $F_4$ | | | | | $F_5$ | $F_6$ | $F_7$ | | $F_8$ | | | $F_9$ | $F_{10}$ | | | |
| $f_2$ | | $F_{11}$ | $F_{59}$ | $F_{60}$ | $F_{61}$ | $F_{12}$ | | $F_{13}$ | | | | | $F_{14}$ | $F_{15}$ | $F_{16}$ | | $F_{17}$ | | | $F_{18}$ | $F_{19}$ | | | |
| $f_3$ | | | | $F_{62}$ | $F_{63}$ | | | | | | | | | | | | | | | | | | | |
| $f_4$ | | | | | $F_{64}$ | | | | | | | | | | | | | | | | | | | |
| $f_5$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $f_6$ | | | | | | $F_{20}$ | $F_{65}$ | $F_{21}$ | $F_{66}$ | | | | $F_{22}$ | $F_{23}$ | $F_{24}$ | | $F_{25}$ | | | $F_{26}$ | $F_{27}$ | | | |
| $f_7$ | | | | | | | | $F_{67}$ | $F_{68}$ | | | | | | | | | | | | | | | |
| $f_8$ | | | | | | | | $F_{28}$ | $F_{69}$ | | | | $F_{29}$ | $F_{30}$ | $F_{31}$ | | $F_{32}$ | | | $F_{33}$ | $F_{34}$ | | | |
| $f_9$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $f_{10}$ | | | | | | | | | | | $F_{70}$ | $F_{71}$ | $F_{72}$ | $F_{73}$ | | | | | | | | | | |
| $f_{11}$ | | | | | | | | | | | | $F_{74}$ | $F_{75}$ | $F_{76}$ | | | | | | | | | | |
| $f_{12}$ | | | | | | | | | | | | | $F_{77}$ | $F_{78}$ | | | | | | | | | | |
| $f_{13}$ | | | | | | | | | | | | | $F_{35}$ | $F_{36}$ | $F_{37}$ | | $F_{38}$ | | | $F_{39}$ | $F_{40}$ | | | |
| $f_{14}$ | | | | | | | | | | | | | | $F_{41}$ | $F_{42}$ | | $F_{43}$ | | | $F_{44}$ | $F_{45}$ | | | |
| $f_{15}$ | | | | | | | | | | | | | | | $F_{46}$ | | $F_{47}$ | $F_{79}$ | $F_{80}$ | $F_{48}$ | $F_{49}$ | | | |
| $f_{16}$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $f_{17}$ | | | | | | | | | | | | | | | | | $F_{50}$ | $F_{81}$ | $F_{82}$ | $F_{51}$ | $F_{52}$ | | | |
| $f_{18}$ | | | | | | | | | | | | | | | | | | | $F_{83}$ | | | | | |
| $f_{19}$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $f_{20}$ | | | | | | | | | | | | | | | | | | | | $F_{53}$ | $F_{54}$ | $F_{84}$ | $F_{85}$ | $F_{86}$ |
| $f_{21}$ | | | | | | | | | | | | | | | | | | | | | $F_{55}$ | $F_{87}$ | $F_{88}$ | $F_{89}$ |
| $f_{22}$ | | | | | | | | | | | | | | | | | | | | | | | $F_{90}$ | $F_{91}$ |
| $f_{23}$ | | | | | | | | | | | | | | | | | | | | | | | | $F_{92}$ |
| $f_{24}$ | | | | | | | | | | | | | | | | | | | | | | | | |

Separable: $f_1$–$f_5$; Low or moderate conditioning: $f_6$–$f_9$; High conditioning and unimodal: $f_{10}$–$f_{14}$; Multimodal with global structure: $f_{15}$–$f_{19}$; Multimodal with weak global structure: $f_{20}$–$f_{24}$

| | |
|---|---|
| $f_1$ | Sphere |
| $f_2$ | Ellipsoid separable |
| $f_3$ | Rastrigin separable |
| $f_4$ | Skew Rastrigin-Bueche |
| $f_5$ | Linear slope |

| | |
|---|---|
| $f_6$ | Attractive sector |
| $f_7$ | Step-ellipsoid |
| $f_8$ | Rosenbrock original |
| $f_9$ | Rosenbrock rotated |

| | |
|---|---|
| $f_{10}$ | Ellipsoid |
| $f_{11}$ | Discus |
| $f_{12}$ | Bent cigar |
| $f_{13}$ | Sharp ridge |
| $f_{14}$ | Sum of different powers |

| | |
|---|---|
| $f_{15}$ | Rastrigin |
| $f_{16}$ | Weierstrass |
| $f_{17}$ | Schaffer F7 (condition 10) |
| $f_{18}$ | Schaffer F7 (condition 1000) |
| $f_{19}$ | Griewank-Rosenbrock F8F2 |

| | |
|---|---|
| $f_{20}$ | Schwefel x*sin(x) |
| $f_{21}$ | Gallagher 101 peaks |
| $f_{22}$ | Gallagher 21 peaks |
| $f_{23}$ | Katsuuras |
| $f_{24}$ | Lunacek bi-Rastrigin |

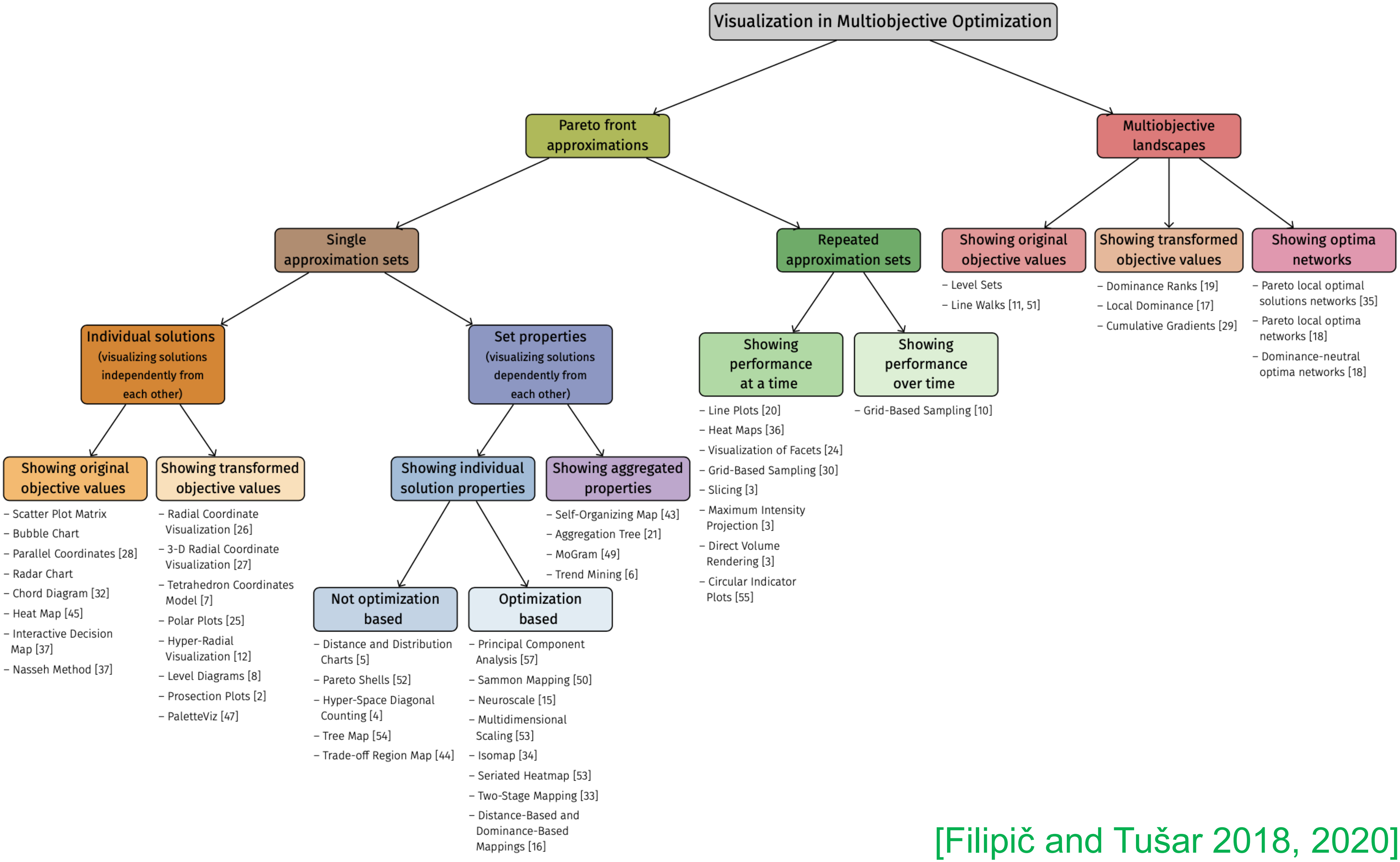
# bbob-biobj(-ext) Suite

## Properties

- Construction similar as in real-world problems
- Scalability in the number of variables
- Various problem properties (more diverse than existing multiobjective test suites)
- Problem instances more diverse than for the single-objective suite
- Currently limited to 2 objectives
- Unknown Pareto set and front (but known single-objective optima)
- Available approximations of the Pareto fronts (and sets for lower-dimensional problems)

# Visualization of multiobjective landscapes

# Visualization in Multiobjective Optimization



- Visualization in Multiobjective Optimization
  - Pareto front approximations
    - Single approximation sets
      - Individual solutions (visualizing solutions independently from each other)
        - Showing original objective values
          - Scatter Plot Matrix
          - Bubble Chart
          - Parallel Coordinates [28]
          - Radar Chart
          - Chord Diagram [32]
          - Heat Map [45]
          - Interactive Decision Map [37]
          - Nasseh Method [37]
        - Showing transformed objective values
          - Radial Coordinate Visualization [26]
          - 3-D Radial Coordinate Visualization [27]
          - Tetrahedron Coordinates Model [7]
          - Polar Plots [25]
          - Hyper-Radial Visualization [12]
          - Level Diagrams [8]
          - Prosection Plots [2]
          - PaletteViz [47]
      - Set properties (visualizing solutions dependently from each other)
        - Showing individual solution properties
          - Not optimization based
            - Distance and Distribution Charts [5]
            - Pareto Shells [52]
            - Hyper-Space Diagonal Counting [4]
            - Tree Map [54]
            - Trade-off Region Map [44]
          - Optimization based
            - Principal Component Analysis [57]
            - Sammon Mapping [50]
            - Neuroscale [15]
            - Multidimensional Scaling [53]
            - Isomap [34]
            - Seriated Heatmap [53]
            - Two-Stage Mapping [33]
            - Distance-Based and Dominance-Based Mappings [16]
        - Showing aggregated properties
          - Self-Organizing Map [43]
          - Aggregation Tree [21]
          - MoGram [49]
          - Trend Mining [6]
    - Repeated approximation sets
      - Showing performance at a time
        - Line Plots [20]
        - Heat Maps [36]
        - Visualization of Facets [24]
        - Grid-Based Sampling [30]
        - Slicing [3]
        - Maximum Intensity Projection [3]
        - Direct Volume Rendering [3]
        - Circular Indicator Plots [55]
      - Showing performance over time
        - Grid-Based Sampling [10]
  - Multiobjective landscapes
    - Showing original objective values
      - Level Sets
      - Line Walks [11, 51]
    - Showing transformed objective values
      - Dominance Ranks [19]
      - Local Dominance [17]
      - Cumulative Gradients [29]
    - Showing optima networks
      - Pareto local optimal solutions networks [35]
      - Pareto local optima networks [18]
      - Dominance-neutral optima networks [18]

[Filipič and Tušar 2018, 2020]

# Visualization of Multiobjective Problem Landscapes

## Low-dimensional search spaces

Dominance ratio [Fonseca 1995]

Gradient path length (inspired by gradient plots [Kerschke and Grimme 2017])

Local dominance [Fieldsend et al. 2019]

PLOT [Shaepermeier et al. 2020]

## Any-dimensional search spaces

Line cuts [Brockhoff et al. 2016, Volz et al. 2019]

Optima network [Liefooghe et al. 2018, Fieldsend and Alyahya 2019]

Various visualizations of bbob-biobj-ext problems:
https://numbbo.github.io/bbob-biobj/

Visualizations of bbob-biobj and other multi-objective suites using PLOT:
https://schaepermeier.shinyapps.io/moPLOT/

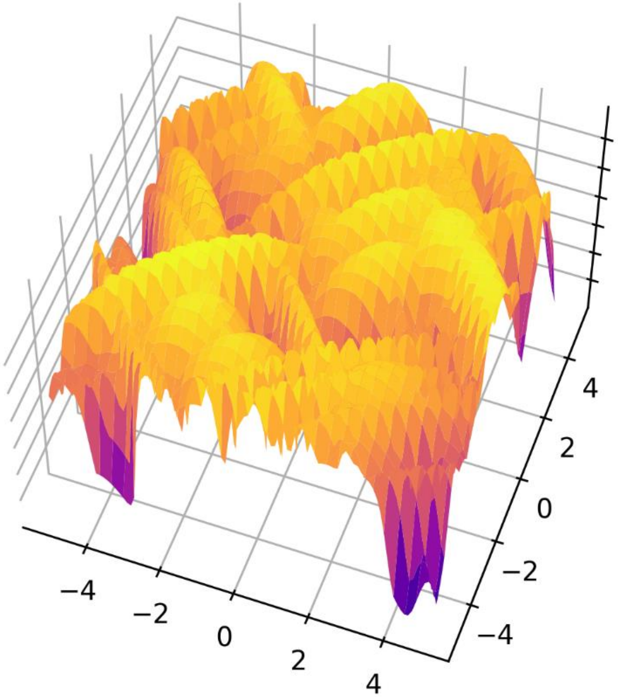
# Visualization of Multiobjective Problem Landscapes

## Problems for demonstration

- Double sphere problem bbob-biobj $F_1 = (f_1, f_1)$, instance 1
- Sphere-Gallagher problem bbob-biobj $F_{10} = (f_1, f_{21})$, instance 1
- Double Gallagher problem bbob-biobj $F_{55} = (f_{21}, f_{21})$, instance 1

Gallagher = Gallagher's Gaussian

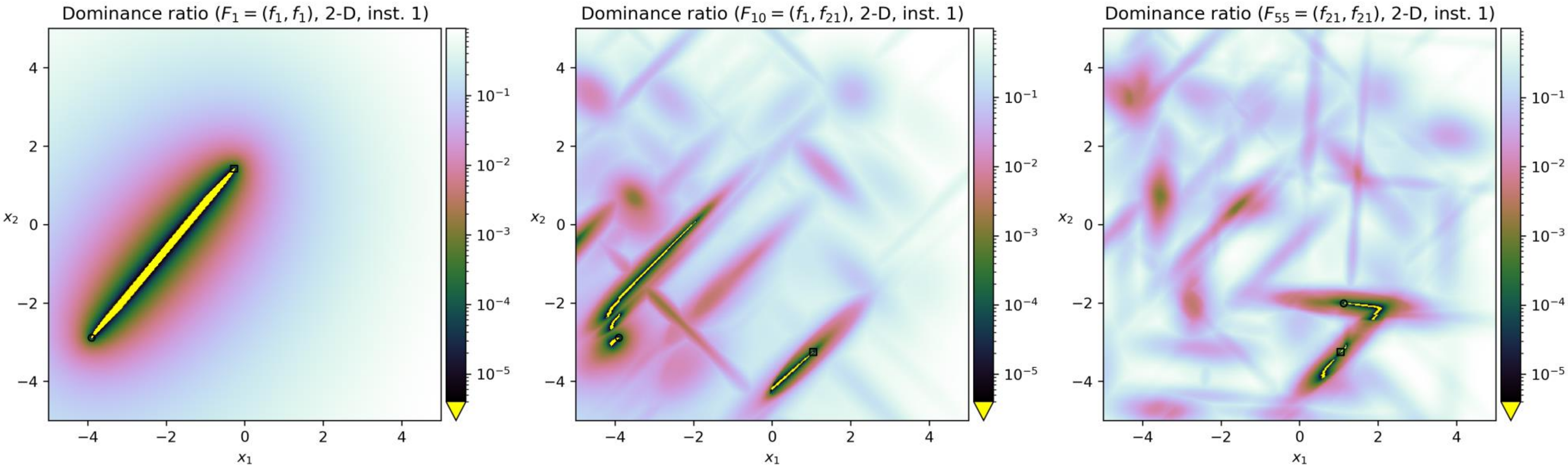
101-me Peaks Function



# Dominance Ratio

[Fonseca 1995]

- Discretized search space (501 x 501 grid)
- Dominance ratio = the ratio of grid points that dominate the current point
- All nondominated points have a ratio of zero
- Visualize dominance ratios in logarithmic scale



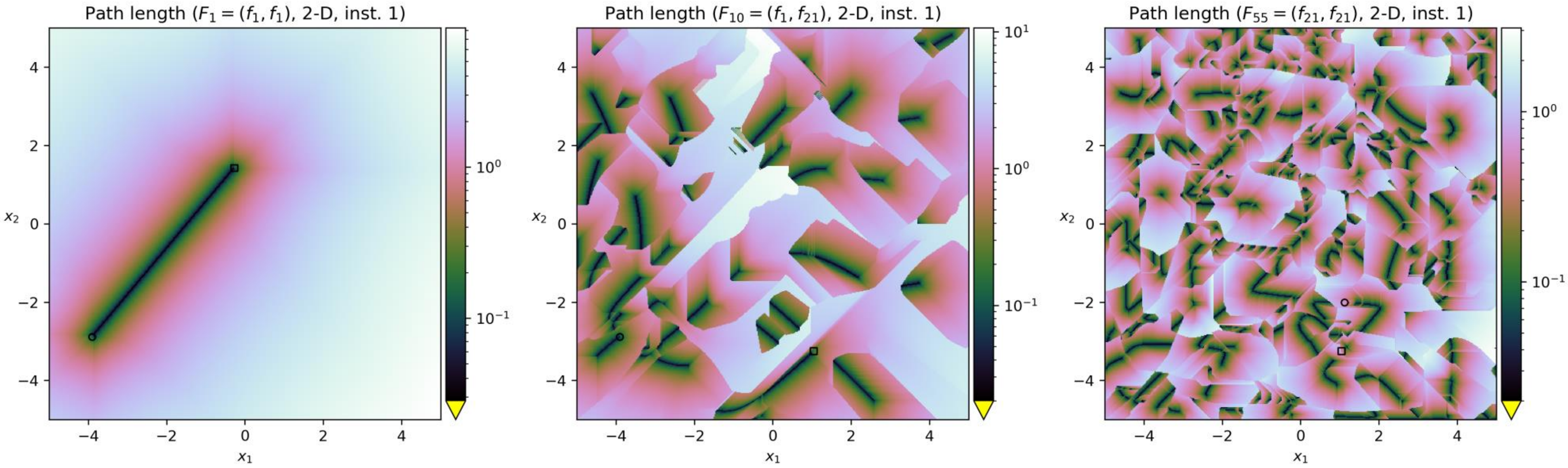
# Gradient Path Length

Adjusted from [Kerschke and Grimme 2017]

- Compute the ***bi-objective gradient*** for all grid points

$$v = \frac{\nabla f_1(x)}{\|\nabla f_1(x)\|} + \frac{\nabla f_2(x)}{\|\nabla f_2(x)\|}$$

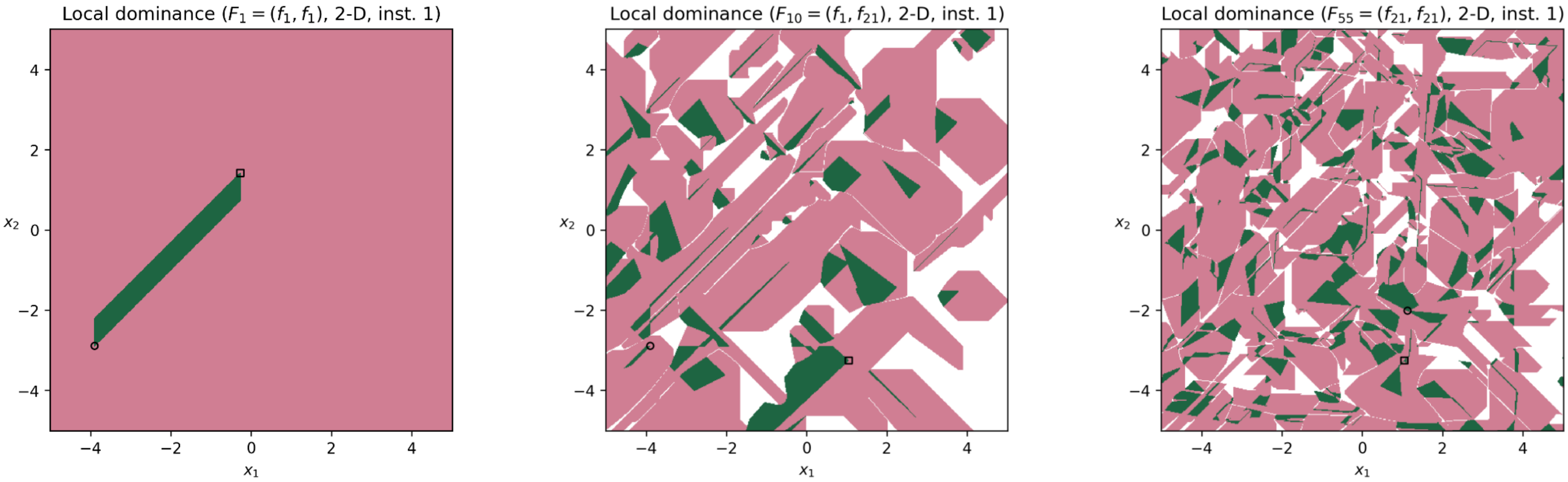
- From a grid point, follow the path in the direction of this gradient
- Visualize the length of the path to the local optimum



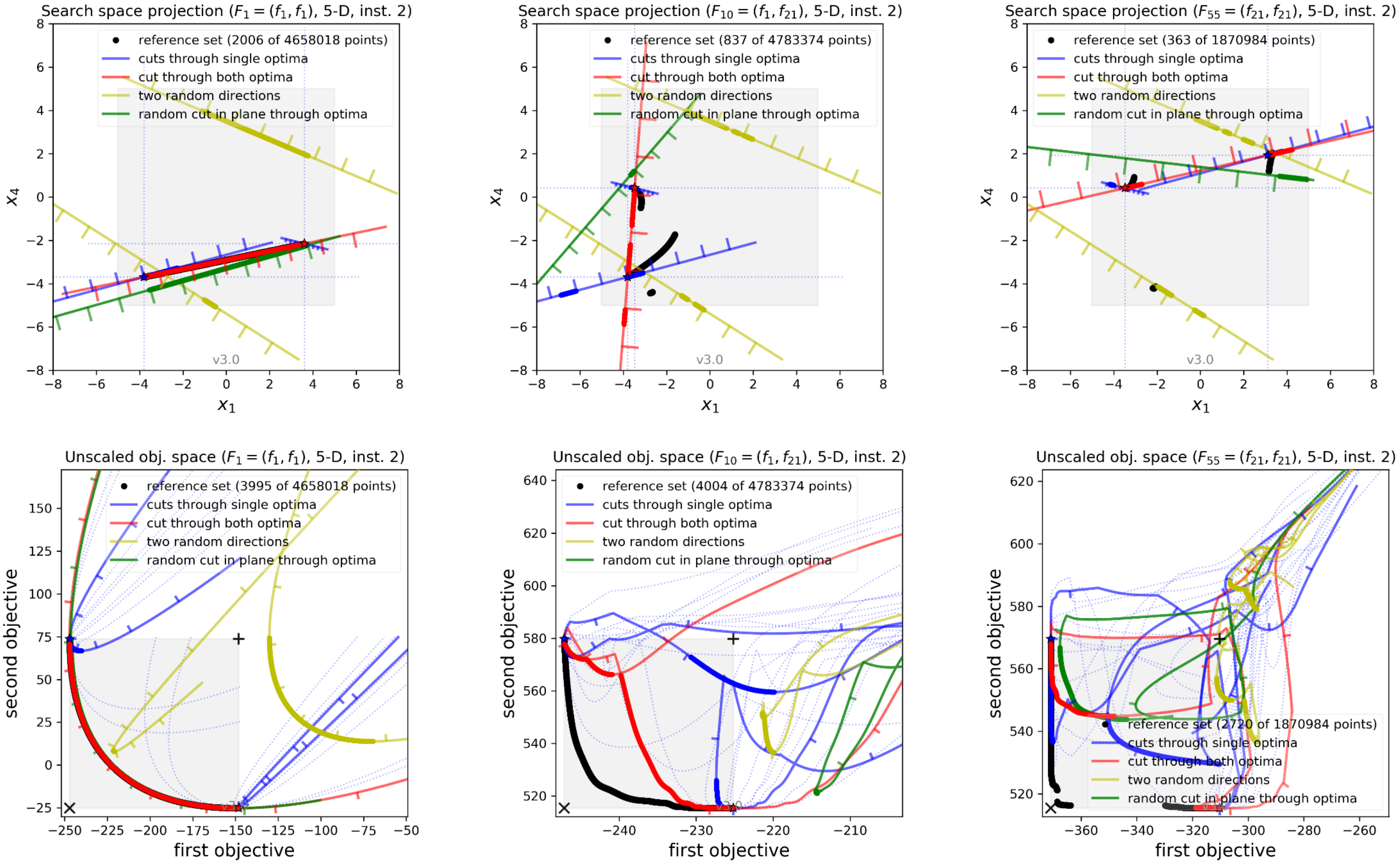
# Local Dominance

[Fieldsend et al. 2019]

- Green: Dominance-neutral local optima regions
  - Points that are mutually nondominated with all their 8 neighbors (not equal to Pareto sets)
- Pink: Basins of attraction
  - Points that are dominated by at least one neighbor and whose dominating paths lead to the same green region
- White: Boundary regions
  - Points whose dominating paths lead to different green regions



# Line Cuts


Search space projection ($F_1 = (f_1, f_1)$, 5-D, inst. 2)

Search space projection ($F_{10} = (f_1, f_{21})$, 5-D, inst. 2)

Search space projection ($F_{55} = (f_{21}, f_{21})$, 5-D, inst. 2)

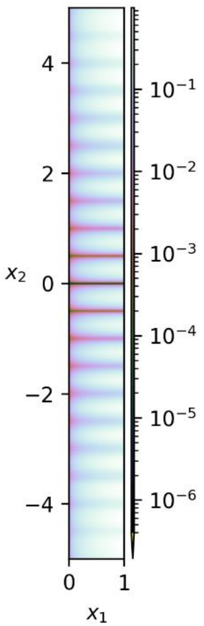
Unscaled obj. space ($F_1 = (f_1, f_1)$, 5-D, inst. 2)

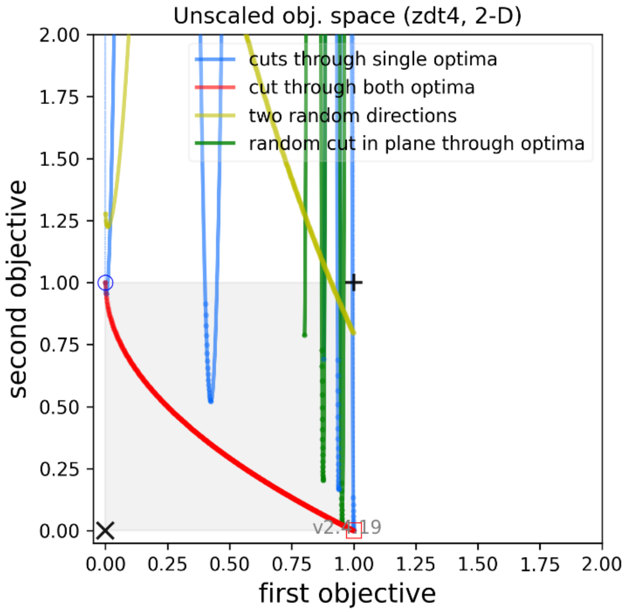
Unscaled obj. space ($F_{10} = (f_1, f_{21})$, 5-D, inst. 2)

Unscaled obj. space ($F_{55} = (f_{21}, f_{21})$, 5-D, inst. 2)

# Comparison of Problem Landscapes

ZDT4



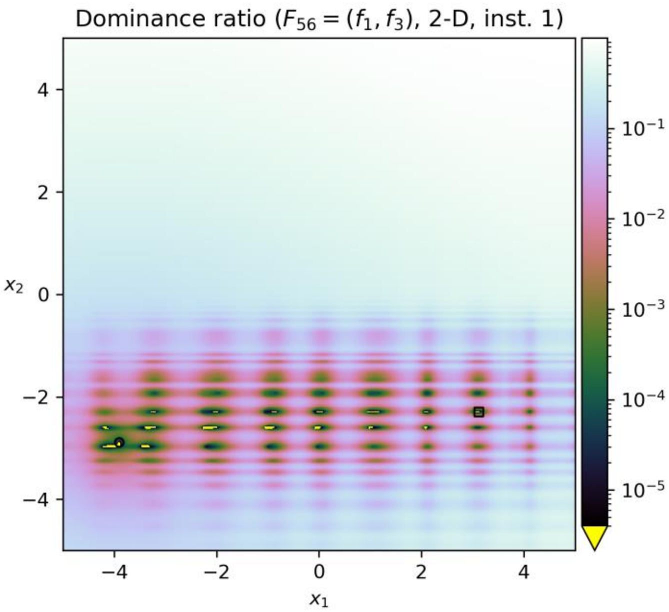


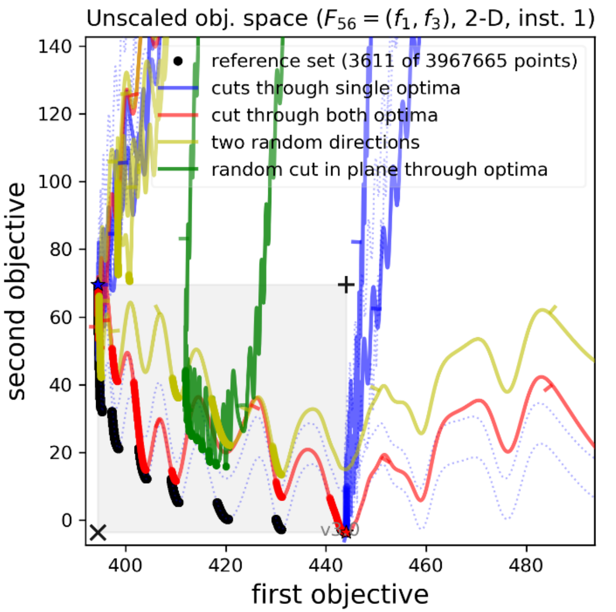
Two problems where both objectives are separable, first is unimodal and second is multimodal

bbob-biobj-ext $F_{56}$

$f_1$ Sphere function

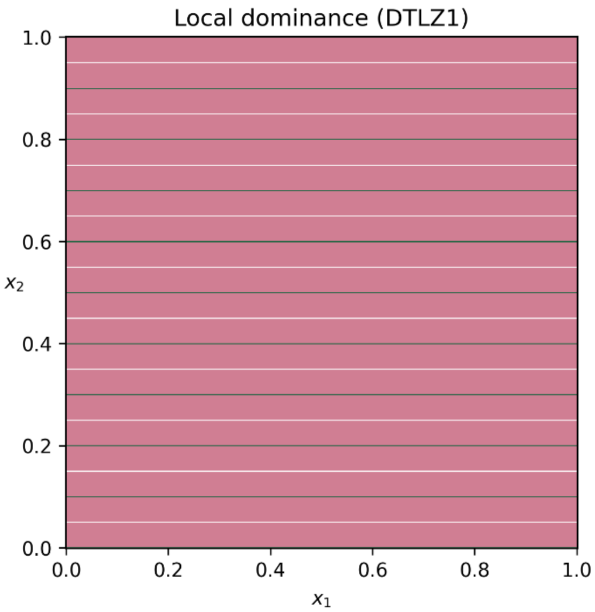
$f_3$ Rastrigin function

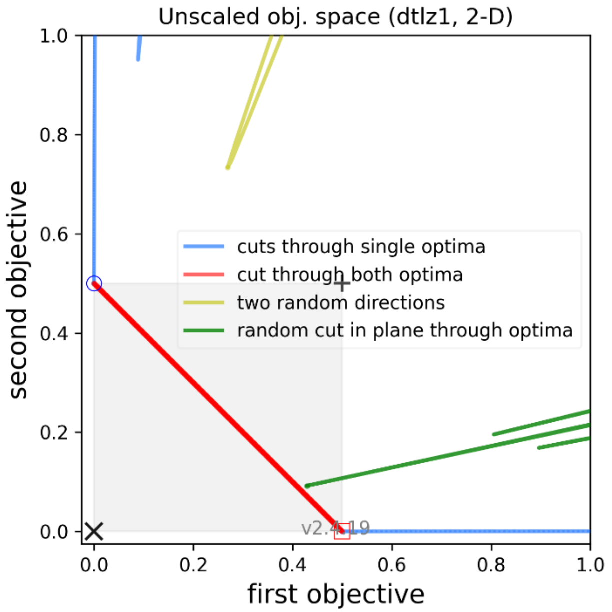




# Comparison of Problem Landscapes

DTLZ1



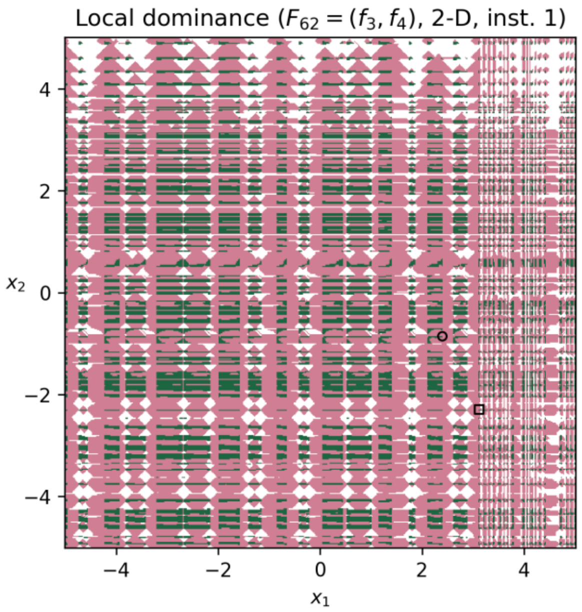


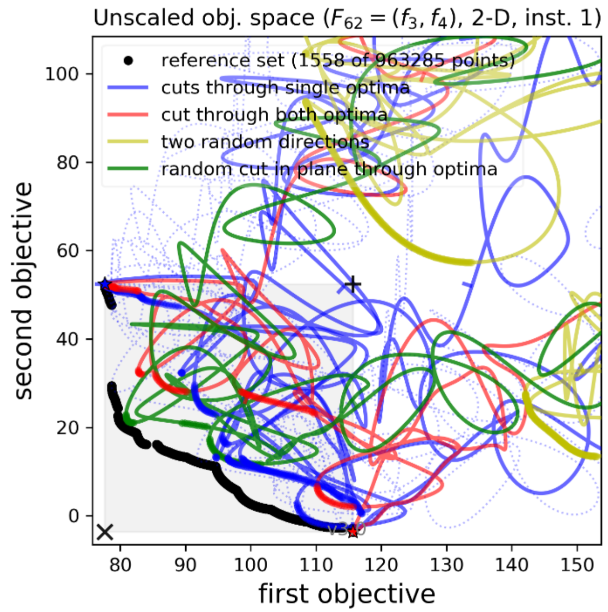
Two problems where both objectives are separable and multimodal

bbob-biobj-ext $F_{62}$

$f_3$ Rastrigin function

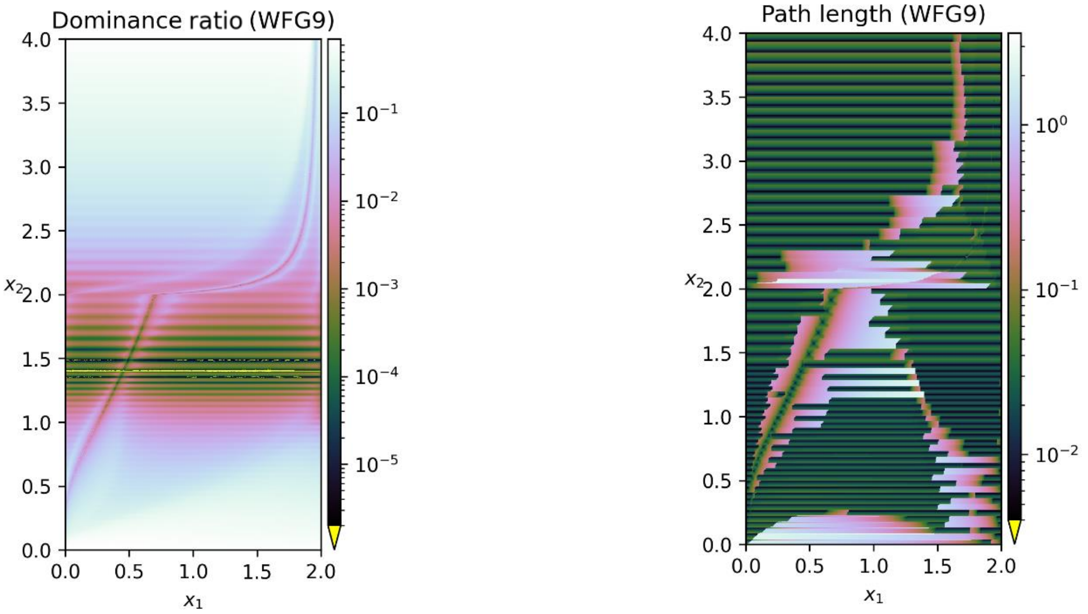
$f_4$ Skew Rastrigin-Bueche





# Comparison of Problem Landscapes

WFG9

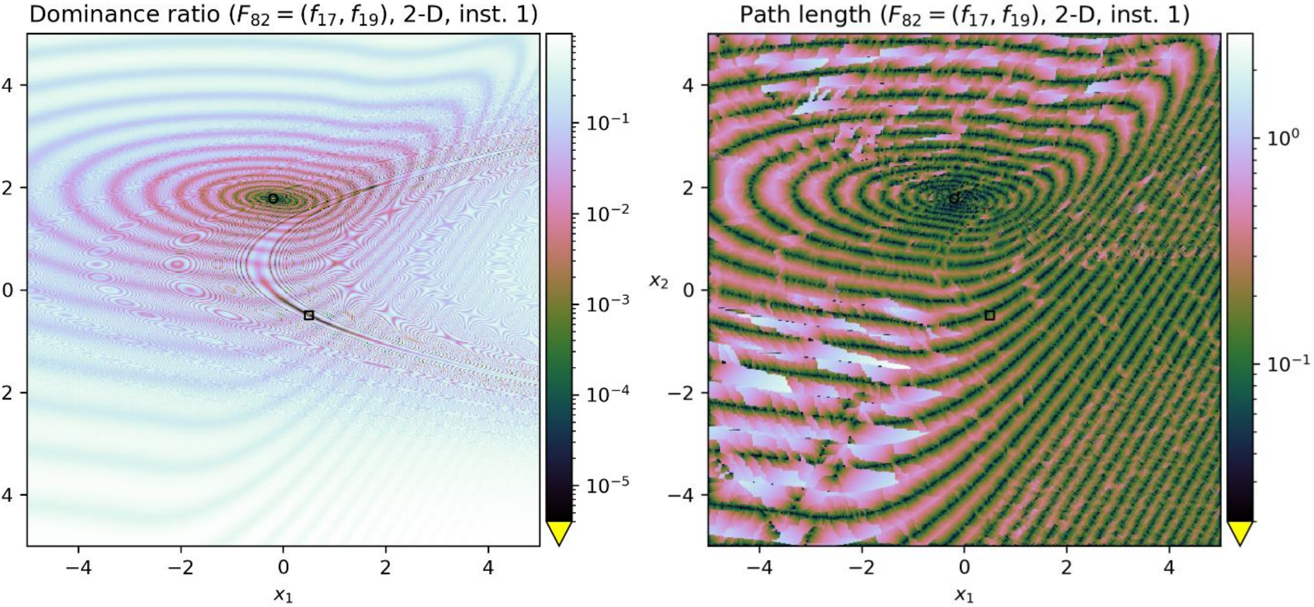


Two problems where both objectives are nonseparable and multimodal

bbob-biobj-ext $F_{82}$

$f_{17}$ Schaffer F7

$f_{19}$ Griewank-Rosenbrock

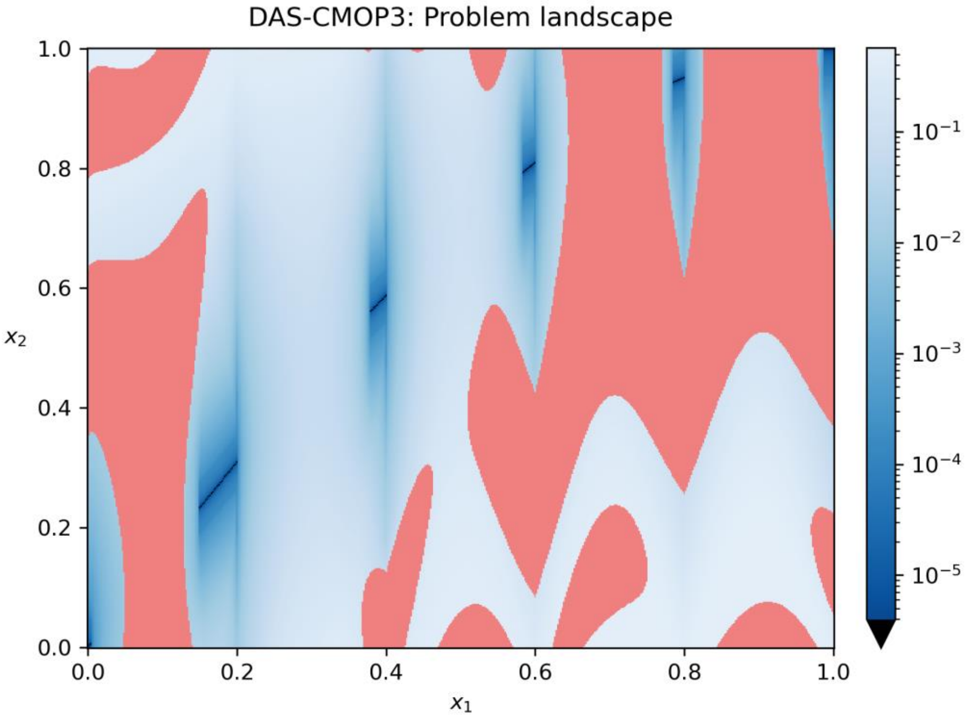


# Other artificial problems

# Other Artificial Problems

## Suites of multiobjective problems with constraints

- CTP [Deb et al. 2001]
- CF [Zhang et al. 2009]
- C-DTLZ [Jain and Deb 2014]
- NCTP [Li et al. 2016]
- DC-DTLZ [Li et al. 2019]
- LIR-CMOP [Fan et al. 2019a]
- DAS-CMOP [Fan et al. 2019b]
- MW [Ma and Wang 2019]



Analysis and visualization of multiobjective problems with constraints: https://vodopijaaljosa.github.io/cmop-web/

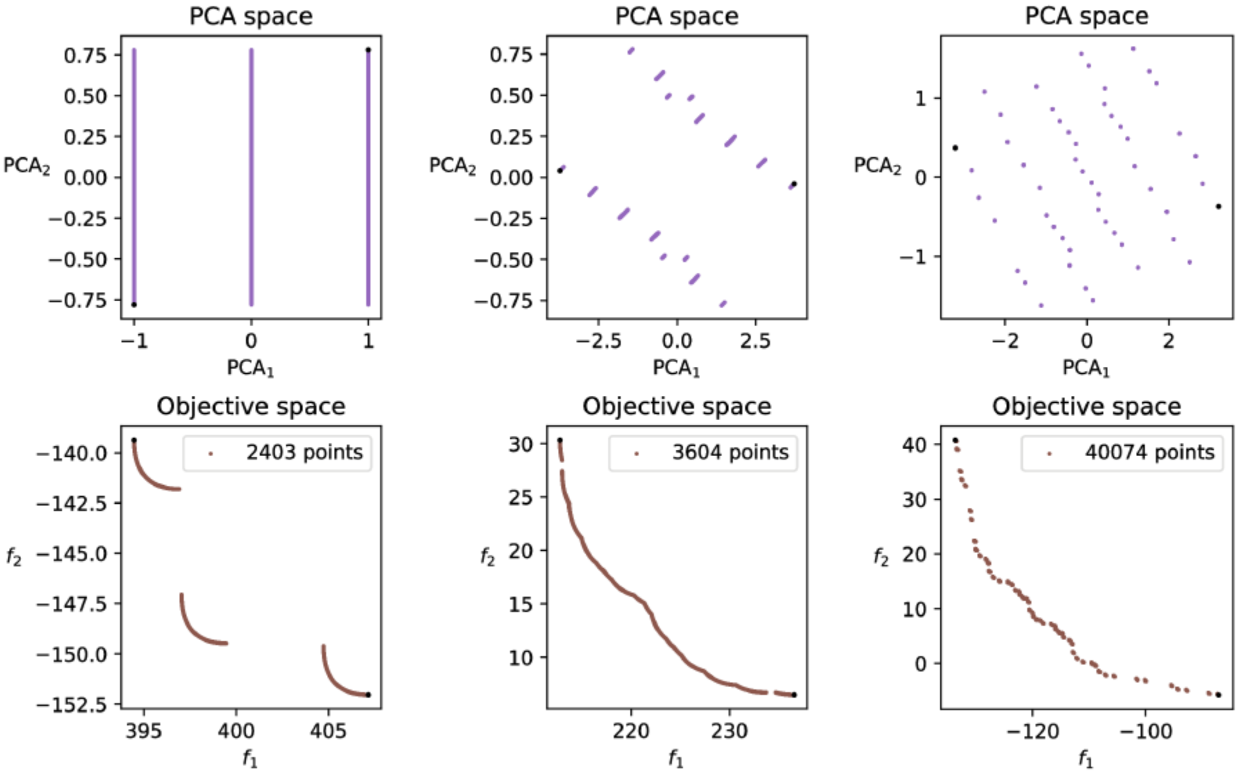
Tutorial on Multiobjective optimization in the presence of constraints: https://dis.ijs.si/filipic/cec2021tutorial/

# Other Artificial Problems

## Suites of multiobjective mixed-integer problems

- Exeter suite of 6 problems constructed with the bottom-up approach [McClymont and Keedwell 2011]
- bbob-biobj-mixint suite of 92 bi-objective problems [Tušar et al. 2019]

Pareto set and front approximations for three different instances of the double sphere function



# Real-world problems

# v0.1

# Real-World Problems

## Individual problems

- Radar waveform design problem with a varying number of variables and 9 objectives [Hughes 2007]
- HBV problem of calibrating the HBV rainfall-runoff model with 14 variables and 4 objectives [Reed et al. 2013]
- MAZDA car structure design problem with 222 integer variables, 2 objectives and 54 constraints [Kohira et al. 2018]

# v0.2

# Suites of Real-World Problems

## Suites of unscalable problems

[Tanabe and Ishibuchi 2020]

- RE suite of 16 test problems with a different number of variables (2–7) and objectives (2–9)
  - 11 continuous problems
  - 1 integer problem
  - 4 mixed-integer problems
- CRE suite of 8 test problems with constraints a different number of variables (3–7) and objectives (2–5)
  - 6 continuous problems
  - 1 integer problem
  - 1 mixed-integer problem
- Both suites consist of previously published problems

# v0.5

# Suites of Real-World Problems

## Suites of scalable problems

- Heat exchanger design problem with scalable variables and 1 or 2 objectives [Daniels et al. 2018]
- Suite of 3 bi-objective TopTrumps problems in multiple dimensions and instances [Volz et al. 2019]
- Suite of 26 bi-objective MarioGAN problems in multiple dimensions and instances [Volz et al. 2019]

# Conclusions

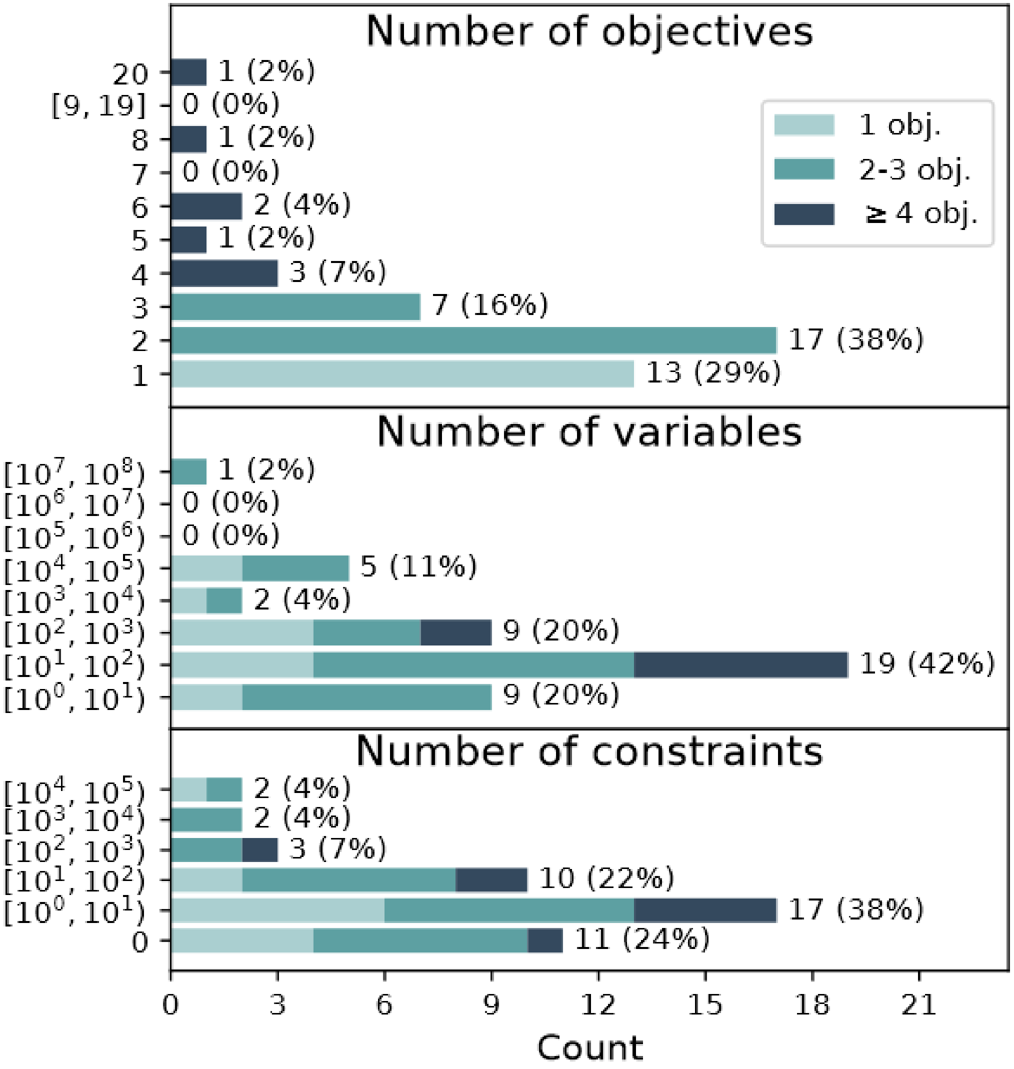
We should think about the usefulness of our research

Results of a questionnaire on the properties of real-world problems has shown their diversity [van der Blom et al. 2020]

https://sites.google.com/view/macoda-rwp/home

Most research is done on continuous unconstrained problems

Although the test problems are scalable, most studies use a fixed number of variables

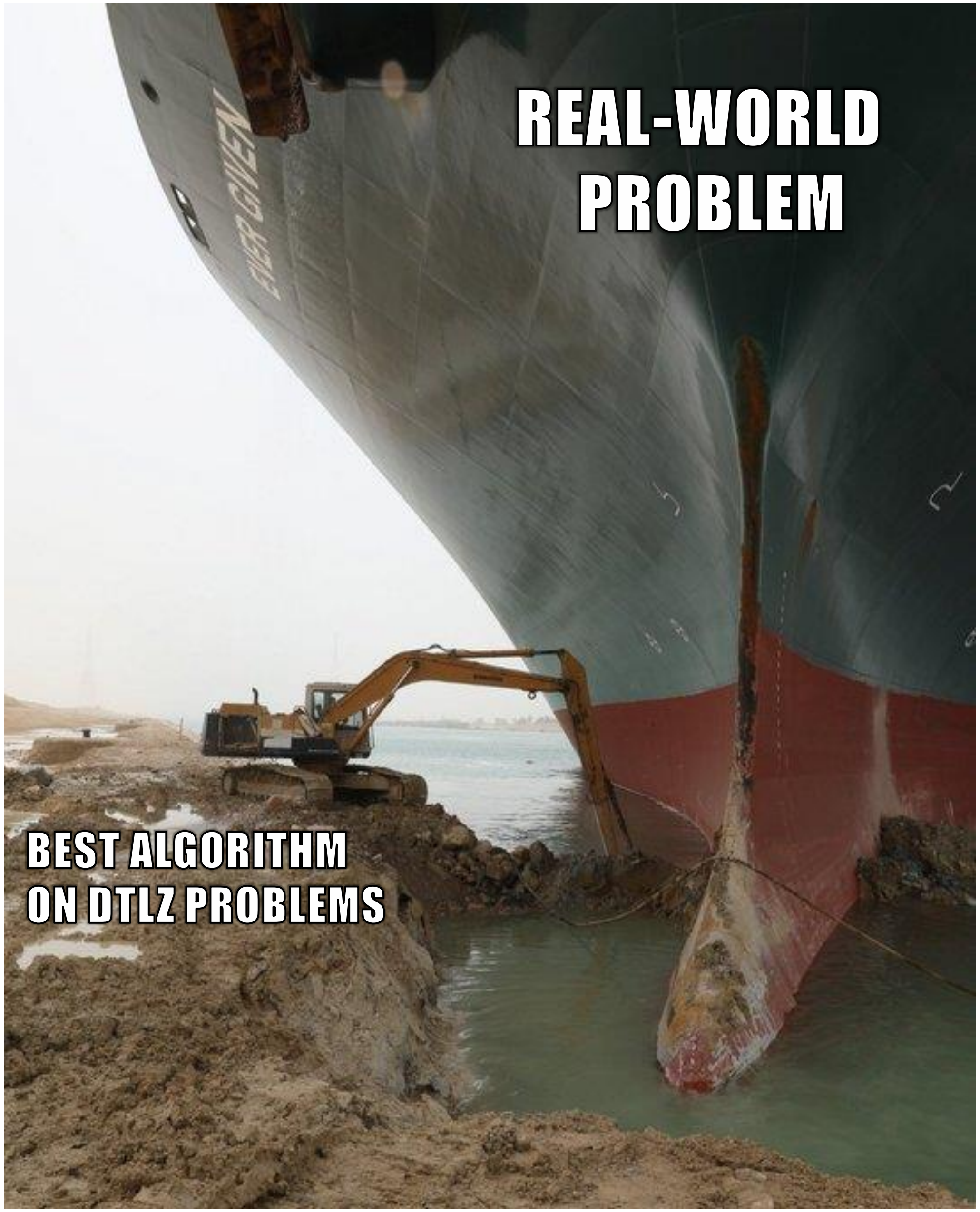


# Conclusions

Problem suites constructed with the bottom-up approach have unrealistic properties

Algorithms are overfitting to these problems (especially the overused DTLZ and WFG) [Ishibuchi et al. 2017]

Using separate functions for the objectives looks like a step in the right direction



# Overview

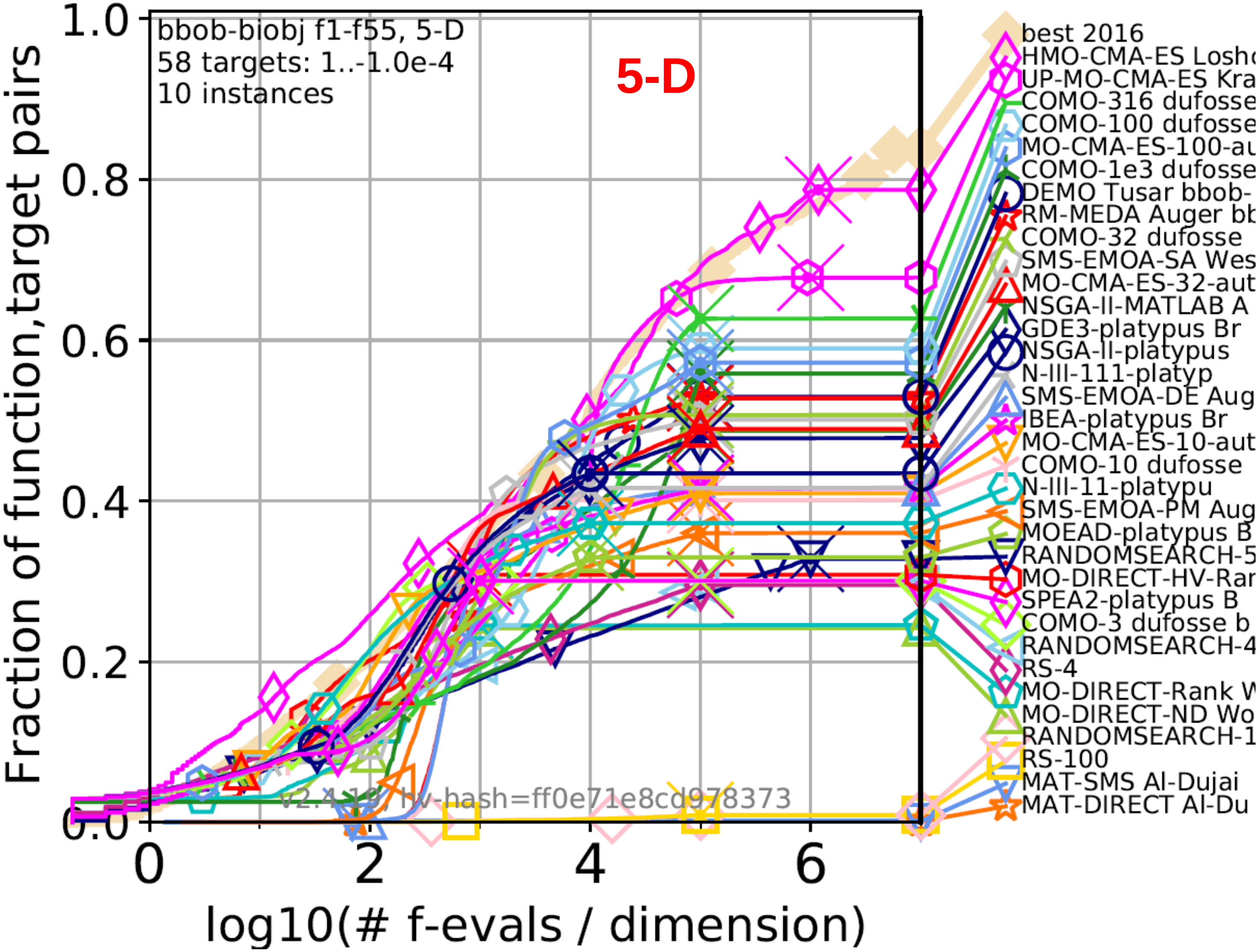
❶ Performance Assessment

❷ Test Problems and Their Visualizations

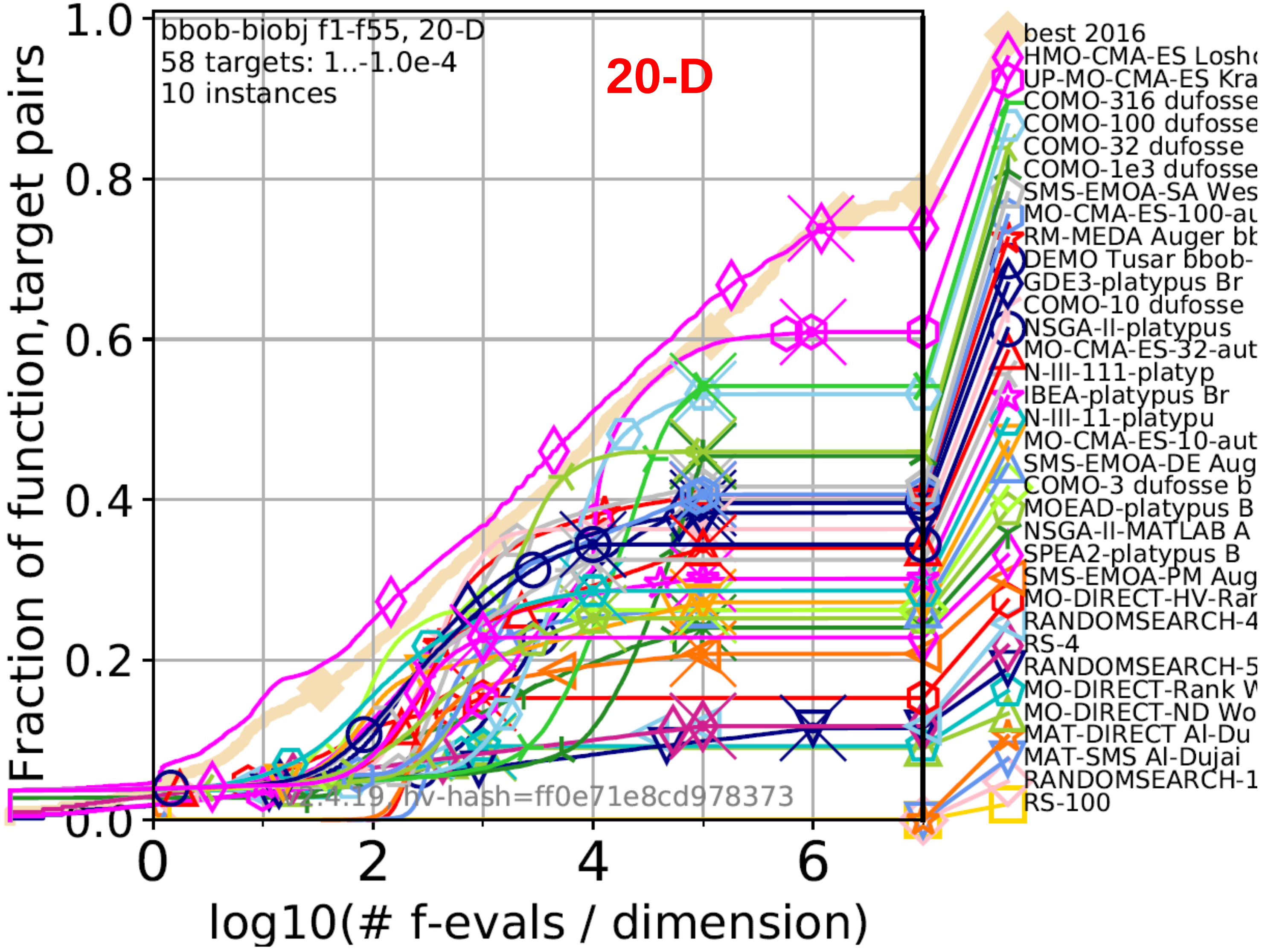
❸ Recommendations from Numerical Results

```
python -m cocopp bbob-biobj*
```

# Aggregated Results Over All 55 Functions



# Aggregated Results Over All 55 Functions



# Multiobjective Benchmarking 3.0?

## a.k.a Challenging Open Research Directions

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  - problems/suites
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## a.k.a Challenging Open Research Directions

- many-objective problems
    - problems/suites
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    - efficient implementations
- constraints, mixed-integer, …
- real-world benchmarking?
    - simulation crashes
    - parallelism
    - dynamic changes
    - interactive decision making
    - …

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## a.k.a Challenging Open Research Directions

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  - interactive decision making
  - …
- benchmarking results from more classical approaches

# Three “New Year” Resolutions

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❶ Show convergence graphs/ECDF

anything else than tables for fixed budget

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**Thank you!**

# Supplementary Material

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