A panorama of methods for dealing with sagging cables in cable-driven parallel robots

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Abstract We are considering cable-driven parallel robot (CDPR), where the legs of the robot are constituted of cables that can be independently coiled/uncoiled. We show that whatever the size of the CDPR is we may have slack cables so that using a sagging cable model that takes into account both the mass and elasticity of the cables will improve the positioning accuracy.Being able to solve the inverse and direct kinematics (IK/DK) with sagging cables is crucial for kinematic analysis while being quite complex as both IK/DK may have multiple solutions. We present a panorama of solving methods for the IK/DK with their advantages and drawbacks.

Key words: cable-driven parallel robot, cable sagging, kinematics.

1 Cable-driven parallel robots and sagging cables

Cable-driven parallel robots (CDPR) are now quite well known as a variant of classical parallel robots that use cables instead of rigid legs with the advantage of a large workspace. We will assume that the winch output point of cable j is A_j while the other extremity of this cable is connected to the platform at point B_j. Cables have the drawback of being only able to have a pulling action on the platform. Therefore managing cable tensions is of importance and two classes have to be considered:

- suspended CDPR where B_j is lower than A_j for all cables
- fully constrained CDPR for which some cable(s), denoted the *pulling cables*, have a A_j that is lower than the B_j, the other cables being called *lifting cables*

In the first case only the action of gravity, that cannot be controlled, may lead to have cables under tension while in the second case both gravity and the downward action of the pulling cables influence the tension in the lifting cables, possibly allowing for optimizing the cable tension distribution. Fully constrained CDPRs have the drawback that the pulling cables are obstacles below the platform (which may be troublesome for some tasks) and do not contribute to supporting the platform weight.

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Furthermore it may be conjectured that for a given geometry the workspace of a suspended CDPR will be larger than the one of a fully constrained CDPR with the same number of cables.

Another important issue for the kinematic analysis of CDPR is the cable model i.e. the relationship between cable shape, tension and length. Many works on CDPR use the *ideal cable* model assuming no elasticity, no cable mass and a straight line shape of the cable as soon it is under tension. Some works use different variants of cable model with the purpose of taking elasticity and/or sagging into account. Sagging is important for suspended CDPR even for ideal cables as it has been shown that in general the number of taught cables cannot be larger than the number of d.o.f. of the platform as all cables may be under tension only temporarily.In that case an efficient control requires to identify which cable(s) are slack and to evaluate their slackness using a sagging model is more efficient than trying to measure reliably and accurately the cable tensions (and only them). Furthermore at the actuation level slack cable may leads to winding problems, possibly even leading to reverting the normal coiling process [20].

Simple sagging model may not be sufficient: for example approximating the shape of the slack cable by a parabola is valid only if the cable mass is negligible compared to the tension in the cable which is, by essence, not true for slack cable. Hence a more accurate model must be used and the Irvine textbook model [11], see equations (1,2), is appropriate as it has been experimentally proven to be valid for usual CDPR [23] (more complex model are available [9, 25]).

For improving the accuracy of CDPR taking into account the sagging in the inverse kinematics (IK) and direct kinematics (DK) problems is of high importance. As soon as the CDPR velocities are small and the initial state of the CDPR has been determined real-time kinematics based on a certified Newton scheme to determine the current state of the CDPR is efficient and very fast [18, 21]. In our latest prototype, that has be run continuously for several months, we use redundant measurements [20] for determining the pose of the platform:

- a real-time DK solver that uses a full cable sagging model and a pulley model
- two horizontal and one vertical lidars that measure distance to the surrounding wall and to the ground for calculating very accurately the platform pose at a sampling frequency of 1/2 Hz (horizontal) and 1 Hz (vertical)
- an accelerometer on each cable located close to the platform that measure the direction of the cable tangent. Combined with a sagging model these measurements allows one to estimate the slackness of each cable [17] with the safety advantage of allowing to detect cable failures
- an adaptive coiling model that uses the other measurements to estimate the current drum radius of the multi-layer winch

If managing sagging cables is not a problem with these measurements a difficulty will appear in the design phase where a kinematic analysis has to be performed for evaluating the performances of a given CDPR over its workspace.

2 Sagging cables model

The non-algebraic Irvine model is a planar one with a planar frame in which A,B have respectively (0,0), (x_b, z_b) as coordinates. The model relates x_b, z_b to the cable length at rest L_0 and to the horizontal and vertical forces $F_x > 0, F_z$ applied on the cable at B. They are different formulations to describe this model and one of them is:

$$x_b = F_x\left(\frac{L_0}{EA_0} + \frac{sinh^{-1}(\frac{F_z}{F_x}) - sinh^{-1}(\frac{F_z-\mu g L_0}{F_x})}{\mu g}\right) \quad (1)$$

$$z_b = \frac{F_z L_0}{EA_0} - \mu g L_0^2/2 + \frac{\sqrt{F_x^2+F_z^2} - \sqrt{F_x^2+(F_z-\mu g L_0)^2}}{\mu g} \quad (2)$$

where E is the Young modulus of the cable material, μ its linear density and A_0 the area of a cross-section of the cable. Let us consider the IK/DK for a spatial CDPR with n cables based on the Irvine model. Regarding equations we have $2n$ equations coming from the cable model and 6 equations that characterize the mechanical equilibrium of the platform so that we have a total of $2n+6$ equations. For the IK we have $3n$ unknowns, namely the L_0, F_x, F_z for each cable, leading to a square system only if $n = 6$. For the DK the unknowns are the $2n$ F_x, F_z for each cable and the 6 pose parameters, namely 3 coordinates in the reference frame of a specific point on the platform and 3 rotation angles that will characterize its orientation (the coordinates x_b, z_b of cable model can be derived from the pose parameters). Hence we have $2n+6$ unknowns and the DK system is always square. The important point is that with this sagging model both the IK and DK may have multiple solutions and we will address in section 4 the possible methods that may be used for finding these solutions. But we will first emphasize the importance of being able to solve the IK/DK for kinematics analysis in the design phase.

3 Sagging and kinematics analysis

Kinematic analysis is used in the design phase to check the performances of a given CDPR design. For example it is of interest to

1. being able to calculate cross-sections of the workspace assuming for example an upper limit L_0^M for the L_0
2. calculate the maximal cable tensions over a specified workspace for a given load

In many works the workspace problem is approximately solved by using a discretisation of the workspace: a regular grid is used and at each node of the grid the IK is solved for determining if the node is in or out of the workspace. A faster and more accurate method consists in determining only the workspace border. Being given the non-algebraic nature of the Irvine equations it seems to be difficult to find an analytic description of the border curve so that we have to rely on a numerical method. For that purpose an efficient approach [13] is to start from a pose $\mathbf{X_a}$, that may be arbitrarily chosen as soon as it lies in the workspace, and from one of its IK solutions S_a^{ik}. We then moves incrementally the platform along an arbitrary unit vector $\mathbf{N}$ by setting the pose $\mathbf{X}$ as $\mathbf{X}] = \mathbf{X_a} + \lambda\mathbf{N}$ and solving its IK for a given λ by using the previous IK solution as guess for the Newton method, until we obtain a pose $\mathbf{X_s}$ such that its current IK solution S_s^{ik} has a L_0^j very close to L_0^M so that $\mathbf{X_s}$ is close to the border. We then find a pose $\mathbf{X_c}(x_c, y_c)$ that is arbitrarily close to the border by solving with Newton a new square IK system $\mathscr{S}$ obtained by setting $L_0^j = L_0^M$ and λ as new unknown. Note that by using classical mathematical tools we are able to calculate an upper bound on the smallest distance between $\mathbf{X_b}$ and the border and adjust the accuracy of the Newton scheme so that this bound is lower than a fixed threshold. Then we follow the border by setting the pose parameters to $x_c + \varepsilon, y_c$ where ε is a small increment and solving with Newton the IK system with $L_0^j = L_0^M$ and ε as new unknown. By repeating this process successive poses on the border are obtained, leading to an approximation of the border by a polygonal line. Note that this border is specific to one of the IK solutions at $\mathbf{X_a}$ so that we obtain a border curve for each IK solutions at $\mathbf{X_a}$. Finding the pose $\mathbf{X_s}$ may fail if a singularity is encountered when moving from $\mathbf{X_a}$ so that several initial poses $\mathbf{X_a}$ and/or $\mathbf{N}$ may have to be used for obtaining a closed region and to find all closed components of the workspace.

The workspace example illustrates the necessity of being able to find all the IK solutions for a finite set of poses. Now if we are willing to determine the maximum of the cable tensions over the CDPR articular workspace we have to consider all solutions of the DK as we cannot predict the history of the robot motion.

4 Sagging and solving the IK/DK

As finding all solutions of the IK/DK is crucial for the kinematic analysis we will now present possible solving methods that may provide all solutions for the IK/DK. Numerous works have addressed the DK problem with ideal cables [1,4,6,10] but much less works have considered Irvine-based sagging cables [7,22,24]. To the best of the author knowledge they are only 3 available methods for dealing with solving the IK/DK having sagging cables and aiming at finding **all** solutions. We present them for the Irvine model but they may be used with any cable model as soon as it is analytic:

Interval analysis (IA): this method looks for solutions within a search space defined by intervals for the unknowns that have to be carefully selected. For the IK with 6 sagging cables the unknowns are the $L_0 > 0, F_x > 0, F_z$. As IK solutions with very large L_0, F_x can be found we have to choose a large upper bound $\overline{L_0}, \overline{F_x}$ for the L_0, F_x intervals. If M is the platform mass, then a lower bound for the F_z interval may be $-Mg - (n-1)\mu g\overline{L_0}$ while its upper bound may be set to $\mu g\overline{L_0}$. For the DK the unknowns are the pose parameters and the $F_x > 0, F_z$. The translational part of the pose parameters may easily be bounded being given the geometry of the CDPR while the rotation angles intervals may be set to $[0, 2\pi]$. Intervals for F_z may be chosen in the same way than for the IK. As for the upper bound of the F_x intervals we choose a very large value as a DK solution may have cables close to the horizontal. As we have unknowns with very large intervals the IA branch-and-bound process may lead to a large computation time (hours for the IK and days for the DK for a 6-cables CDPR [16]). For both the IK and DK we may miss solution(s) because of the unknown upper bound limit on the F_x.

Continuation [2]: if $E \to \infty$ and $\mu \to 0$, then Irvine equations reflect the behavior of an ideal cable. For the DK we first assume ideal cables and calculate all DK solutions $\{S_k^i\}$ for all cable configurations having from 6 to 1 cables under tension, the other cables being slack and therefore disregarded. Then we set E to a large value E_l and μ to a small value μ_l and use the Newton scheme with each S_k^i as guess for deriving the DK solutions C_k of the CDPR with sagging cables. If E_0, μ_0 are the real E, μ values we define a single parameter *continuation path* as

$$E = E_l + \lambda(E_0 - E_l) \qquad \mu = \mu_l + \lambda(\mu_0 - \mu_l) \tag{3}$$

The continuation parameter λ is such that C_k is a DK solution for $\lambda = 0$ while the sought DK solutions will be obtained for $\lambda = 1$. Starting from $\lambda = 0$ we increase λ by an increment ε that is automatically calculated at each step and solve the DK problem for the current λ using the previous solution as guess for Newton. We then repeat the process until $\lambda = 1$. Continuation is in general much faster than IA but the continuation path (3) may end-up in a singularity while another path (e.g. adjusting first E then μ) may avoid any singularity and leads to a DK solution. Therefore we may also miss DK solution(s) with the continuation approach [14]. For the IK the process is similar: we consider all configurations of IK with $m = 6$ to $m = 1$ ideal cables under tension (therefore having m fixed pose parameters among the desired pose). Then we use first a continuation on E, μ and if $m < 6$ a second continuation on the $6 - m$ free pose parameters so that they reach their assigned values [19]. Here again the continuation may end up in a singularity aand therefore miss an IK solution.

Neural networks: a *multi-layer perceptron (MLP)* [8] is an universal estimator that is theoretically able to provide an estimation $\mathbf{V}$ of any set of functions $\mathbf{F}(\mathbf{U})$ with input $\mathbf{U}$, even if $\mathbf{F}$ is not known. For constructing a MLP we need a large training set of pairs $(\mathbf{U_i}, \mathbf{F}(\mathbf{U_i}))$. In our case $\mathbf{V}$ will be the solutions of the IK/DK problems while $\mathbf{U}$ will be the pose parameters for the IK and the L_0s for the DK.

A major problems with a MLP is that it produces a single estimation $\mathbf{V}$ of the IK/DK while we may have several solutions whose number cannot be predicted. Furthermore the estimation error should be acceptable for being used in the kinematic analysis. A MLP has been used for the IK of a redundant planar CDPR with 4 cables by fixing the minimal cable tension and therefore having a single IK solution [5]. The prediction error on the L_0 was acceptable (less than 0.1) for the kinematic analysis but not for control while the errors on the F_x, F_z where not acceptable (more than 10 N for a load of 1 kg).

We have investigated the use of MLP for CDPR with 2 and 3 cables for both the IK and DK that have a single solution in this case [3]. The training set is obtained by solving with IA or continuation the IK/DK for a limited number of inputs P_j and using continuation to calculate the IK/DK solutions for $P_j + \lambda \mathbf{N_{jk}}$ where $\mathbf{N_{jk}}$ are randomly chosen unit vectors (the verification set is obtained in the same manner but with different $\mathbf{N_{jk}}$) Beside the training set a MLP requires to fix several parameters: number of layers, number of neurons per layer, type of activation functions and the cost function. As the learning time is small (mean value: 7 minutes) we have used a systematic brute force approach for creating MLPs with between 1 and 6 layers, 10 to 200 neurons per layer and various combinations of activation functions. We have then systematically calculated statistics (mean, max, variance, ...) of the estimation errors on a large verification set. It appears that none of the MLPs were producing estimation whose accuracy was sufficient for kinematic analysis. However in some cases we have got an exact solution by using the MLP estimation as guess for the Newton scheme. Furthermore the statistical analysis have shown that some MLPs were able to provide a reasonable prediction for part of the unknowns and poor one for the others. Hence we have devised a solving strategy that is based on various MLPs whose estimation is used as guess for Newton and if necessary we combine the good MLPs estimations to create new estimations that are fed to Newton. We have then tested this strategy on another large verification set with 100% of success. The interest of this approach is that the computation time is very low (less than 5ms while the IA/continuation require about one minute)). Still we cannot guarantee that we will always find the solution for the IK/DK.

It remains to determine if MLPs can manage the case of multiple IK/DK solutions. We are currently investigating an approach for that purpose. A first step is to establish **all** IK/DK solutions for m inputs. A training set will be established for each solution by using a continuation process. This training set will be used to build 12 different MLPs with different parameters. If we get n_j solutions for input j and define $k = \sum_{j=1}^{j=m} n_j$, then $12k$ MLPs will be built. The number m of inputs must be chosen large enough so that k is large as we cannot get more than k IK/DK solutions. Being given the MLPs we use the same solving strategy than in the previous example.

Building the MLPs is computer intensive but has to be done only once for possibly getting a very fast IK/DK solver so that the pre-processing time will be small compared to the time necessary for performing the kinematic analysis. Still to reduce the pre-processing time we are currently investigating the use of specific processors devoted to neural networks learning with a large number of GPUs. In parallel we plan to use also these processors for speeding up IA solving as this approach is intrinsically appropriate for a distributed implementation.

In **summary** IA is guaranteed to find all solutions within a search domain in a large computation time but will miss non realistic solutions (e.g. for the IK the one having one or several very large cable length) while continuation, that is usually faster than IA, may miss even realistic solution(s) because of the singularity problem. Regarding MLP it is currently unclear if all solutions may be determined but possibly may determine all realistic solutions in a very short computation time.

Another possible solving approach is to use a *cable lumped model* that is a much simpler alternate to Irvine cable model. It has been tested for a 3-cables CDPR with satisfactory experimental results [12] but this CDPR is relatively simple. A large number of issues has still to be solved for this approach to determine if this model will allow for faster IA/continuation solvers such as how to choose the number of elements, their elasticity and mass in order to get the best compromise between computation time of the solver and errors on the kinematic parameters (considering that the model estimation may be used as Newton guess for getting the exact result).

5 Conclusion

In this paper we have addressed different issues for taking sagging into account for CDPR in the design phase and during control. We have shown that the ideal cable model limits the possibility of accurate control and is not sufficient for kinematic analysis. We then have emphasized that using a sagging model is not an issue for real-time kinematics and that sagging may also be measured with additional sensors for improving control and safety. For kinematic analysis it is crucial to have efficient solving methods for both the IK and DK (that usually have multiple solutions) and we have presented a panorama of existing methods with their advantages and drawback and the alternate lumped mass model that seems to be worth investigating.

A kinematic issue is not addressed in this paper: singularity. Indeed CDPR with sagging cables may exhibit singularities beside classical parallel robots one [15] but this issue will be addressed in a devoted paper.

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