Positional and conformist effects in public good provision. Strategic interaction and inertia

Francisco Cabo, Alain Jean-Marie, Mabel Tidball

Positional and conformist effects in public good provision. Strategic interaction and inertia

Francisco Cabo, IMUVa, Universidad de Valladolid, Spain

Alain Jean-Marie, INRIA, Sophia Antipolis, France

Mabel Tidball, INRA, CEEM, Montpellier, France

19th International Symposium on Dynamic Games and Applications.
July 25-28 2022.

Context of the paper

In the previous presentation

Positional effects in public good provision.
Strategic interaction and inertia

we considered a game of contribution to a public good, in which players are Positional: concerned with their relative contribution.

i) Positional concerns $\Rightarrow$ positive contributions?

Yes!

ii) Does it necessary lead to social welfare improvements?

No!

iii) If it does, can inertia in adjustments harm welfare?

Yes!

Concluded with possible extension: what if one player is Positional and one is Conformist?

The outline of the paper

A public-good game with subjective effects: one player Positional, one player Conformist.

- The static model.
  - Nash Equilibrium
- The dynamic model with inertia.
  - Some theoretical results.
  - Numerical simulations.
- Conclusions

Progress

The one shot model. The static game

- Two players.
- Each player endowed with w_i and contributes $x_i \in [0, w_i]$ to a public good. Then $w_i - x_i$ privately consumed.
- Preferences:
  intrinsic utility (absolute level of contribution)
  positional payoffs (associated with relative contrib.)

$$U_i(x_i, x_j) = u_i(x_i, X) + V_i(x_i - x_j), \quad X = x_i + x_j$$

  conformist payoffs (associated with distance to contrib.)

$$U_j(x_i, x_j) = u_j(x_j, X) + V_j(x_i - x_j).$$

- Global intrinsic utility: $u = u_i + u_j$
- The global utility or social welfare: $U = U_i + U_j$.

The intrinsic utility. Properties

C1 Individual provision always reduces own welfare:

$$\frac{\partial u_i}{\partial x_i}(x_i, X) < 0, \quad \forall (x_i, x_j) \in [0, w_i] \times [0, w_j].$$

Nash equilibrium is $(0, 0)$.

C2 Individual provision: First unit $\uparrow$ u; last unit $\downarrow$ u.

$$C2a: \ \frac{\partial u}{\partial x_i}(0, 0) > 0, \qquad C2b: \ \frac{\partial u}{\partial x_i}(w_1, w_1 + w_2) < 0.$$

Some contribution to the public good and to the private good are socially desirable.

C3 Agents contributions are substitutes

$$C3: \ \frac{\partial^2 u}{\partial x_i \partial X}(x_i, X) < 0.$$

Intrinsic utility, positional + conformist concerns

Functional specifications:

- The intrinsic utility is an additively separable function:

$$u_i(x_i, X) = w_i - x_i + b_i(X)$$
$$u(X) = w_1 + w_2 - X + b_1(X) + b_2(X)$$
$$b_i(X) = \alpha_i \left(X - \frac{\varepsilon}{2} X^2 \right)$$

  Public Good valuation by i: α_i, satiation ε

- Positional payoff

$$V_i(x_i - x_j) = v_i^{\mathrm{P}} \times (x_i - x_j).$$

  Positional concern of player i: $v_i^{\mathrm{P}} \geq 0$

- Conformist payoff

$$V_j(x_i - x_j) = -\frac{v_j^{\mathrm{C}}}{2} \times (x_i - x_j)^2.$$

  Conformist concern of player j: $v_j^{\mathrm{C}} \geq 0$

Maximization problem

The problem for player $k \in \{i, j\}$ is:

$$\max_{0 \leq x_i \leq w_i} w_i - x_i + b_i(X) + v_i^{\text{P}}(x_i - x_j)$$

$$\max_{0 \leq x_j \leq w_j} w_j - x_j + b_j(X) - \frac{v_j^{\text{C}}}{2}(x_i - x_j)^2$$

The marginal utility of players i and j now reads as:

$$\frac{\partial U_i}{\partial x_i} = -1 + b_i'(X) + v_i^{\text{P}}$$
$$\frac{\partial U_j}{\partial x_j} = -1 + b_j'(X) + v_j^{\text{C}}(x_i - x_j)$$

Positionality raises the marginal benefit of private provision. Conformism does also, as long as j contributes less than i.

Similarities and differences: Pos+Conf vs Pos+Pos

For the positional player, a similar situation:
Best-reply function:

$$x_i^b(x_j) = \begin{cases} 0 & A_i \leq x_j \\ A_i - x_j & A_i - w_i \leq x_j \leq A_i \\ w_i & x_j \leq A_i - w_i. \end{cases}$$

Definition (Wished amount)

$$A_i = \begin{cases} 0 & \text{if } v_i^{\mathrm{P}} < 1 - \alpha_i, \\ \dfrac{v_i^{\mathrm{P}} - (1 - \alpha_i)}{\alpha_i \varepsilon} & \text{otherwise} \\ w_i + w_j & \text{if } v_i^{\mathrm{P}} > 1 - \alpha_i + \alpha_i \varepsilon (w_1 + w_2). \end{cases}$$

Positive contribution $\boxed{v_i^{\mathrm{P}} > 1 - \alpha_i}$

Public Good Assumption $\boxed{v_i^{\mathrm{P}} < 1}$

Similarities and differences (continued)

For the conformist player, different situations

Referring to condition C3:

$$\frac{\partial^2 U_j}{\partial x_i \partial x_j} = -\alpha_j \varepsilon + v_j^c$$

1. $v_j^c < \alpha_j \varepsilon$: contributions are still substitutes
2. $v_j^c > \alpha_j \varepsilon$: conformism is so strong that contributions from the other agent increases the willingness to contribute: contributions become complements for Agent j.

Best-reply function (complementarity case):

$$x_j^b(x_i) = \begin{cases} 0 & x_i \leq B_j := \dfrac{1-\alpha_j}{v_j^c - \alpha_j \varepsilon} \\ \dfrac{v_j^c - \alpha_j \varepsilon}{v_j^c + \alpha_j \varepsilon}(x_i - B_j) & \text{otherwise} \\ w_j & w_j(v_j^c + \alpha_j \varepsilon) \leq (x_i - B_j)(v_j^c - \alpha_j \varepsilon) \end{cases}$$

Nash equilibrium

Unique Nash equilibrium in all cases.

c) If $v_i^{\mathrm{P}} \leq 1 - \alpha_i$, $A_i = 0$: no contribution at all

$$(x_i^{\mathrm{N}}, x_j^{\mathrm{N}}) = (0, 0)$$

a) If $v_i^{\mathrm{P}} > 1 - \alpha_i$, $A_i > 0$, and

$$v_j^{\mathrm{C}} \ \leq \ \underline{v}_j^{\mathrm{C}} \ := \ \alpha_j \varepsilon + \frac{1 - \alpha_j}{\min\{A_i, w_i\}}$$

then

$$(x_i^{\mathrm{N}}, x_j^{\mathrm{N}}) = (\min\{A_i, w_i\}, 0)$$

Agent j is not incentivized enough to contribute. This includes the substituability case $v_j^{\mathrm{C}} \leq \alpha_j \varepsilon$, but also the weak complementarity case $v_j^{\mathrm{C}} \in (\alpha_j \varepsilon, \underline{v}_j^{\mathrm{C}}]$.

Nash Equilibrium (continued)

b) If $v_i^{\mathrm{P}} > 1 - \alpha_i$, $A_i > 0$, and $v_j^{\mathrm{C}} > \underline{v}_j^{\mathrm{C}}$ (strong complementarity)

b$_{\text{int}}$) unique interior equilibrium

$$x_{i\ int}^{\mathrm{N,PC}} = \frac{1}{2}(A_i + \Delta_x) \quad x_{j\ int}^{\mathrm{N,PC}} = \frac{1}{2}(A_i - \Delta_x)$$

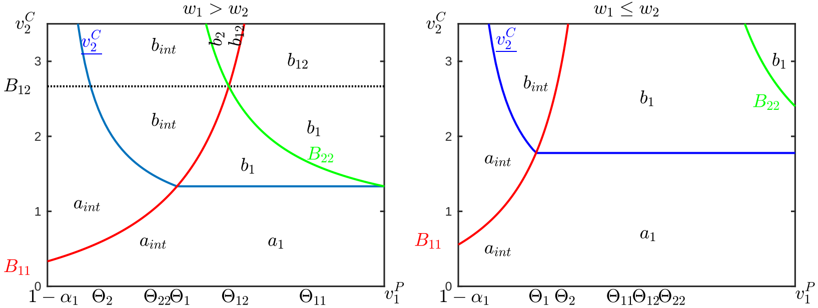
$$\Delta_x := \frac{\alpha_i - \alpha_j(1 - v_i^{\mathrm{P}})}{\alpha_i v_j^{\mathrm{C}}}$$

b$_1$) $(w_i, x_j^b(w_i))$

b$_{12}$) (w_i, w_j)

b$_2$) $(A_i - w_j, w_j)$

Nash Equilibrium (end)



Nash equilibria with positive contribution
for $w_1 > w_2$ (left); $w_1 \leq w_2$ (right)

Progress

A simple dynamic model with inertia

We consider the intrinsic utility function of the static model...

1. At time t the **positional** agent gets joy from contributions above the other player at time $t-1$: $+v_i^{\mathrm{P}}(x_{it} - x_{jt-1})$.
2. The **conformist** agent gets joy from contributions close to the other player at time $t-1$: $-v_j^{\mathrm{C}}(x_{jt} - x_{it-1})^2/2$.
3. Agents have **Inertia** from previous actions. Disutility from deviations from the previous action: $-v_k^{\mathrm{I}}(x_{kt} - x_{kt-1})^2/2$, $k \in \{i, j\}$.

Simple ynamic model with inertia (continued)

Utilities at time t:

$$U_i(\bullet) = w_i - x_{it} + \alpha_i \left[x_{it} + x_{jt} - \frac{\varepsilon}{2}(x_{it} + x_{jt})^2 \right]$$
$$+ v_i^{\mathsf{P}}(x_{it} - x_{jt-1}) - \frac{v_i^{\mathsf{I}}}{2}(x_{it} - x_{it-1})^2$$
$$U_j(\bullet) = w_j - x_{jt} + \alpha_j \left[x_{it} + x_{jt} - \frac{\varepsilon}{2}(x_{it} + x_{jt})^2 \right]$$
$$- \frac{v_j^{\mathsf{C}}}{2}(x_{jt} - x_{it-1})^2 - \frac{v_j^{\mathsf{I}}}{2}(x_{jt} - x_{jt-1})^2$$

Simple dynamic model with inertia (end)

The problem of a myopic agent $k \in \{i, j\}$ is:

$$\max_{x_{kt}} U_k(x_{kt}, x_{-kt}, \hat{x}_{kt}, \hat{x}_{-kt}),$$
$$\text{s.t.: } \hat{x}_{kt} = x_{kt-1}.$$

The first-order conditions for an interior equilibrium are:

$$\begin{pmatrix} \varepsilon\alpha_i + v_i^{\mathrm{I}} & \varepsilon\alpha_i \\ \varepsilon\alpha_j & \varepsilon\alpha_j + v_j^{\mathrm{I}} + v_j^{\mathrm{C}} \end{pmatrix} \begin{pmatrix} x_{it} \\ x_{jt} \end{pmatrix} = \begin{pmatrix} v_i^{\mathrm{I}} & 0 \\ v_j^{\mathrm{C}} & v_j^{\mathrm{I}} \end{pmatrix} \begin{pmatrix} x_{it-1} \\ x_{jt-1} \end{pmatrix} + \begin{pmatrix} -1 + \alpha_i + v_i^{\mathrm{P}} \\ -1 + \alpha_j \end{pmatrix}.$$

$\implies$ dynamic system $x_t = Mx_{t-1} + V_0$.

Dynamic problem: general properties

Properties of the free dynamical system

- When $v_k^{\mathrm{I}} \neq 0$ at least for one player $k \in \{i, j\}$,
  - both eigenvalues have modulus < 1
  - when they are real, they have the same sign
  - the dynamical system x_t converges to a unique steady state which is the point x_{int}
- When $v_i^{\mathrm{I}} = v_j^{\mathrm{I}} = 0$
  - one eigenvalue is equal to -1, the other one is 0
  - the dynamical system does not converge in general: its trajectories tend asymptotically to a cycle of order 2 which is organized around the point x_{int}

General properties (continued)

Behavior different from the Positional+Positional case

Eigenvalues

A sufficient condition for both eigenvalues to be complex is:

$$v_i^{\mathrm{l}} \; < \; \frac{\alpha_i}{\alpha_j} \left(v_j^{\mathrm{l}} + v_j^{\mathrm{c}}\right)$$

Spiralling behavior:

- small inertia of Positional player
- large inertia or conformism from Conformist player

When taking budget constraints into account:

Property of the constrained dynamical system

If $x_0 = (0, 0)$, then $x_1 = (x_{i1}, 0)$.

Illustration 1: no inertia

Parameters: $\varepsilon = 0.4$, $\alpha_1 = 0.8$, $\alpha_2 = 0.5$, $v_1^P = 0.56$, $v_2^C = 2$.
Wished amount $A_1 = 1.125$, $A_1/2 = 0.5625$.

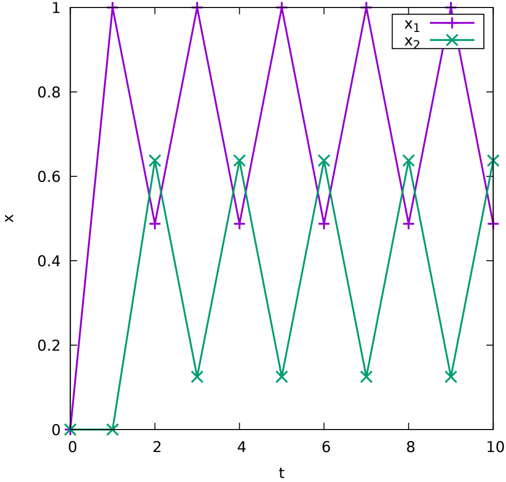
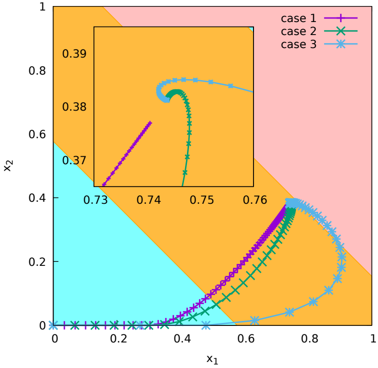


Illustration 2: spiral or direct convergence

Varying inertia parameters:

- case 1: $v_1^l = 10$, $v_2^l = 10$: $\lambda_1 \simeq 0.8543$, $\lambda_2 \simeq 0.9302$
- case 2: $v_1^l = 5$, $v_2^l = 10$: $\lambda_i \simeq 0.8757 \pm 0.0655i$
- case 3: $v_1^l = 20$, $v_2^l = 20$: $\lambda_i \simeq 0.8201 \pm 0.1068i$



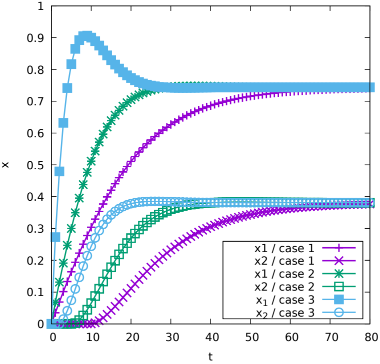


Illustration 2: welfare analysis

Welfare analysis: remember the typology of total contribution X:

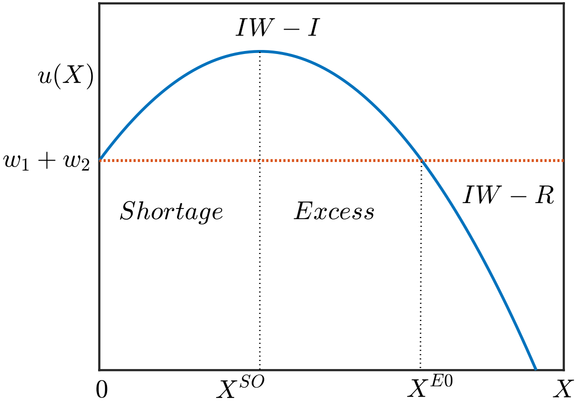


Illustration 2: welfare analysis (continued)

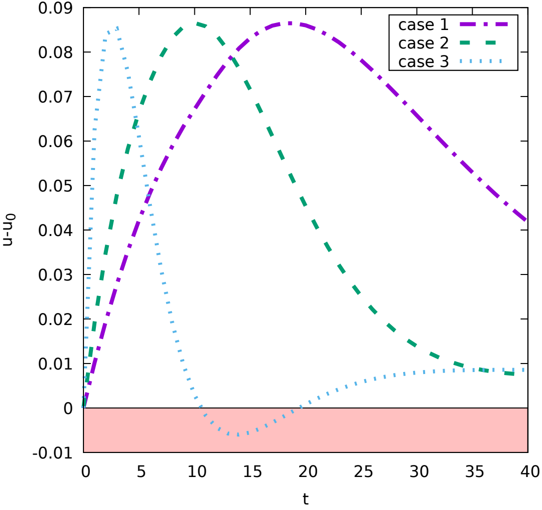
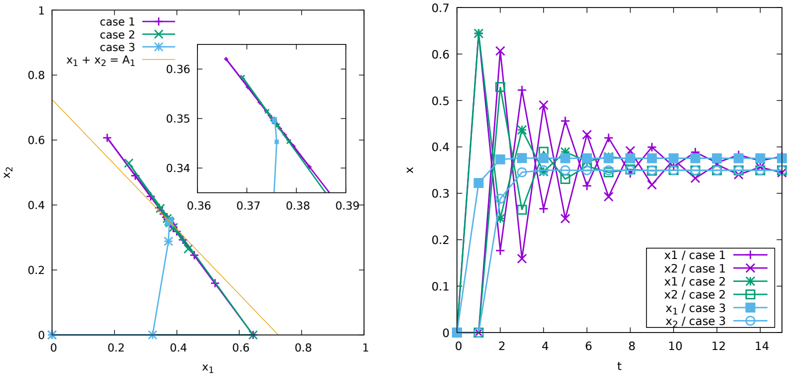


Illustration 3: oscillating convergence

Varying inertia parameters:

- case 1: $v_1^I = 0.1$, $v_2^I = 0.5$: $\lambda_1 \simeq -0.7392$, $\lambda_2 \simeq -0.0025$
- case 2: $v_1^I = 0.1$, $v_2^I = 5$: $\lambda_1 \simeq -0.4705$, $\lambda_2 \simeq -0.0337$
- case 3: $v_1^I = 1$, $v_2^I = 1$: $\lambda_i \simeq 0.0848 \pm 0.1024i$



Progress

Conclusions

With respect to the Positional+Positional case, there are common features and qualitative differences.

In the static framework

- $\neq$ The standard situation with large endowments is an *interior* Nash equilibrium

In the myopic dynamic framework with inertia

- $=$ Convergence to the static equilibrium
- $\neq$ Possibility of spiralling or oscillating convergence
- $=$ Possibility of overshooting
- $=$ Possibility of transient welfare reduction

Extensions

As in the previous talk:

- Consider another dynamic: the stock of public good.
- Consider farsighted agents (true dynamic game).
- More than two players.
- ...

Obrigado!