Structures de données semi-persistantes

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Structures de données persistantes
Séminaire

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CNRS

30 mars 2023

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Alt-Ergo, un démonstrateur automatique écrit dans un style fonctionnel.

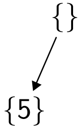
Amène notamment le problème d'une structure persistante pour les classes d'équivalence.

```
let v0 = empty ()
```

$$\{\}$$

```
let v0 = empty ()
```

```
let v1 = add  5 v0
```



pleine persistance

```
let v0 = empty ()
```

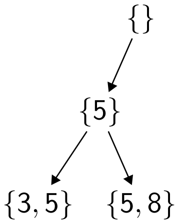
```
let v1 = add  5 v0
```

```
let v2 = add  3 v1
```

$$
\begin{tikzcd}
 & & \{\} \arrow[dl] \\
 & \{5\} \arrow[dl] & \\
\{3, 5\} & &
\end{tikzcd}
$$

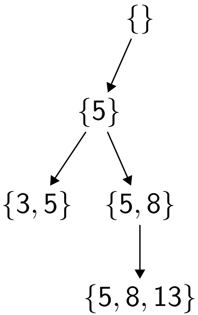
pleine persistance

```
let v0 = empty ()
let v1 = add  5 v0
let v2 = add  3 v1
let v3 = add  8 v1
```



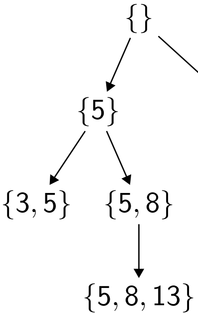
pleine persistance

```
let v0 = empty ()
let v1 = add  5 v0
let v2 = add  3 v1
let v3 = add  8 v1
let v4 = add 13 v3
```



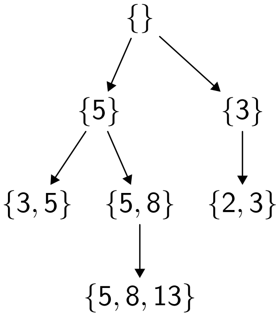
pleine persistance

```
let v0 = empty ()
let v1 = add  5 v0
let v2 = add  3 v1
let v3 = add  8 v1
let v4 = add 13 v3
let v5 = add  3 v0
```

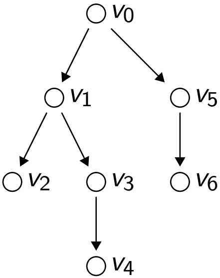


pleine persistance

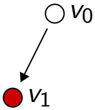
```
let v0 = empty ()
let v1 = add  5 v0
let v2 = add  3 v1
let v3 = add  8 v1
let v4 = add 13 v3
let v5 = add  3 v0
let v6 = add  2 v5
```

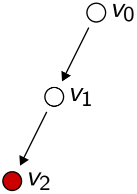


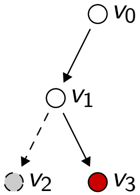
```
let v0 = create ...
let v1 = update v0 ...
let v2 = update v1 ...
let v3 = update v1 ...
let v4 = update v3 ...
let v5 = update v0 ...
let v6 = update v5 ...
```

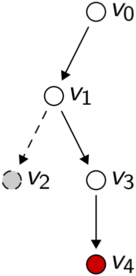


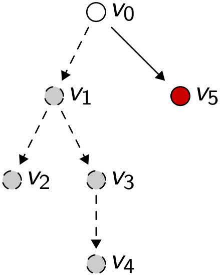

v_0

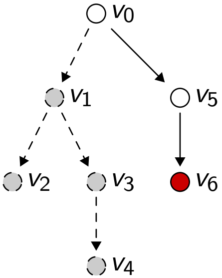

v_0
v_1

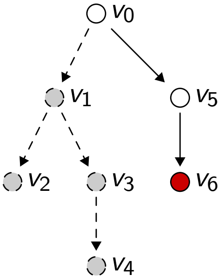

v_0
v_1
v_2


v_0
v_1
v_2
v_3


v_0
v_1
v_2
v_3
v_4


v_0
v_1
v_5
v_2
v_3
v_4


v_0
v_1
v_5
v_2
v_3
v_6
v_4



Une structure de données est dite semi-persistante si elle n'autorise des opérations que sur des ancêtres de la version courante.

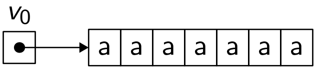
plan

1. Exemples de structures semi-persistantes
    - tableaux
    - classes d'équivalence
    - listes
    - graphes
2. Performance
    - gain en temps
    - gain en mémoire
3. Le bon usage
    - s'assurer qu'on utilise uniquement des versions valides

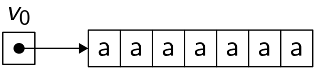
```
type 'a parray =
  'a data ref

and 'a data =
| Base of 'a array
| Diff of int * 'a * 'a parray
```

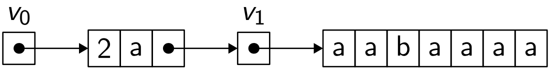
```
let v0 = create 7 a
```



```
let v0 = create 7 a
```

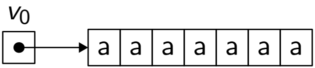



```
let v1 = set v0 2 b
```

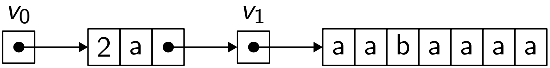


exemple

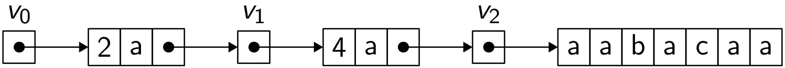
```
let v0 = create 7 a
```

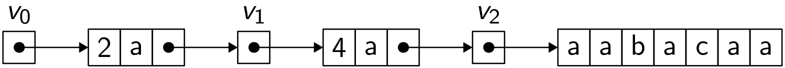



```
let v1 = set v0 2 b
```

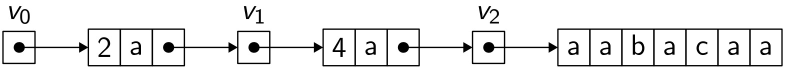



```
let v2 = set v1 4 c
```



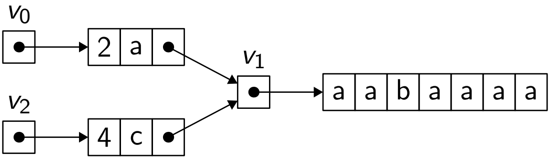


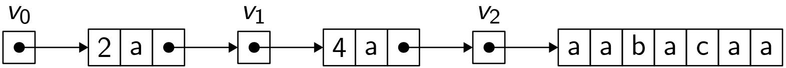

```
let v3 = set v1 5 d
```




```
let v3 = set v1 5 d
```

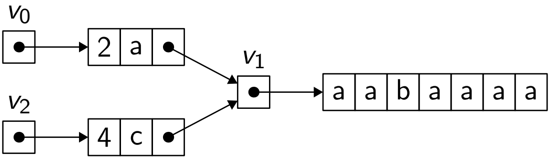
On revient à la version v_1 :



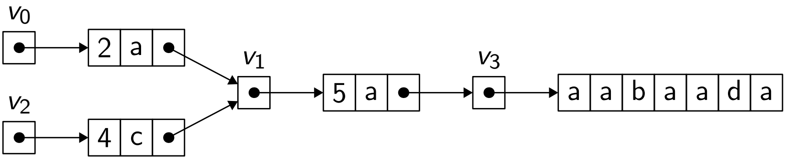


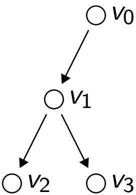

```
let v3 = set v1 5 d
```

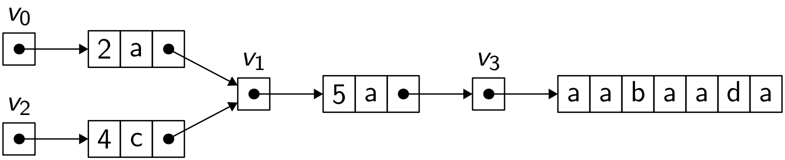
On revient à la version v_1 :



Puis on construit v_3 :



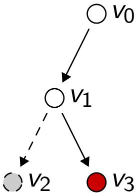

v_0
v_1
v_2
v_3

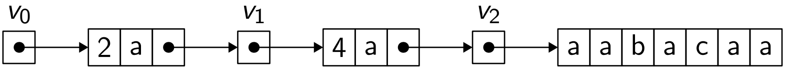

v_0
2 a
v_2
4 c
v_1
5 a
v_3
a a b a a d a

Pour rendre ces tableaux semi-persistants, il suffit de ne faire qu'une partie du travail.

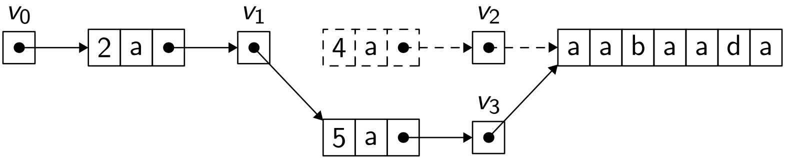
- On suit la chaîne de pointeurs et on écrit les valeurs dans le tableau.
- Mais on ne cherche plus à garantir la cohérence des structures croisées en chemin.

```
let v0 = create 7 a
let v1 = set v0 2 b
let v2 = set v1 4 c
```

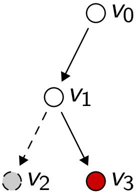




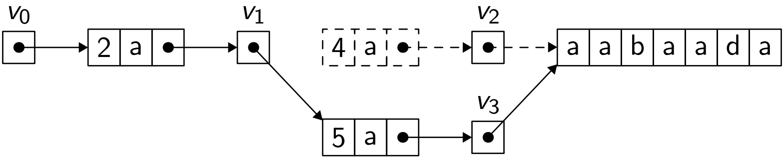

```
let v3 = set v1 5 d
```



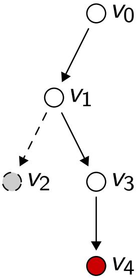
et ainsi de suite

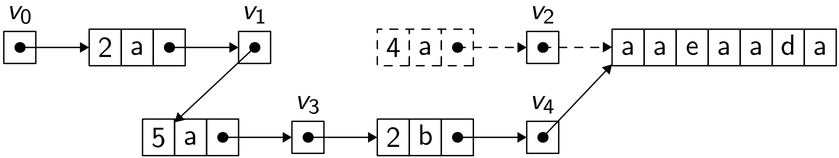


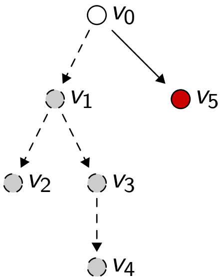

```
...
let v4 = set v3 2 e
let v5 = set v0 6 f
let v6 = set v5 3 g
```



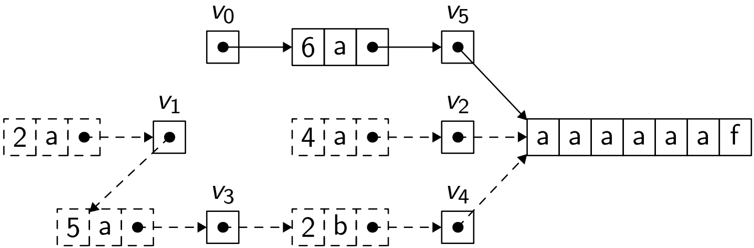
```
...
let v4 = set v3 2 e
let v5 = set v0 6 f
let v6 = set v5 3 g
```



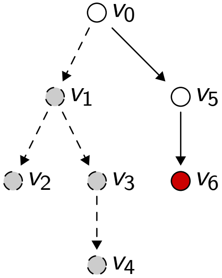


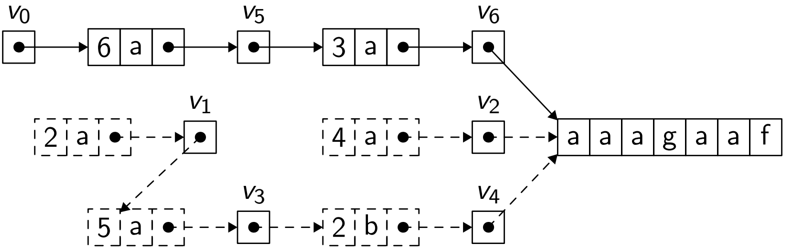



```
...
let v4 = set v3 2 e
let v5 = set v0 6 f
let v6 = set v5 3 g
```



```
...
let v4 = set v3 2 e
let v5 = set v0 6 f
let v6 = set v5 3 g
```





On a économisé quelques opérations (un accès dans un tableau, une allocation et une affectation).

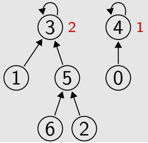
On a caché les informations de retour sur trace dans la structure de données, que l'on continue à utiliser comme une structure persistante.

Bien entendu, on s’oblige à utiliser la structure correctement, c’est-à-dire en effectuant uniquement des retours en arrière.

Nous verrons plus loin comment l’assurer.

Avec deux tableaux semi-persistants, on construit une structure semi-persistante pour les classes d'équivalence de $\{0, 1, \ldots, N-1\}$.

```
type spuf = {
  link: int sparray;
  rank: int sparray;
}
```



Elle a l'interface d'une structure persistante :

```
type spuf
val create: int -> spuf
val find: spuf -> int -> spuf * int
val union: spuf -> int -> int -> spuf
```

```
type 'a splist

val  nil: unit -> 'a splist
val cons: 'a -> 'a splist -> 'a splist

val is_nil: 'a splist -> bool
val   head: 'a splist -> 'a
val   tail: 'a splist -> 'a splist
```

Constatation : une liste valide est un suffixe de la version courante.

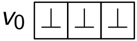
D'où l'idée de réutiliser les cellules d'anciens préfixes.

```
type 'a splist = {
  mutable head: 'a;
          tail: 'a splist;
  mutable prev: 'a splist;
}
```

illustration

```
type 'a splist = {
  mutable head: 'a;
          tail: 'a splist;
  mutable prev: 'a splist;
}
```

```
let v0 = nil ()
let v1 = cons a v0
let v2 = cons b v1
let v3 = cons c v1
let v4 = cons d v3
let v5 = cons e v0
let v6 = cons f v5
```



```
type 'a splist = {
  mutable head: 'a;
          tail: 'a splist;
  mutable prev: 'a splist;
}
```

```
let v0 = nil ()
let v1 = cons a v0
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let v4 = cons d v3
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let v6 = cons f v5
```
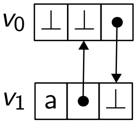
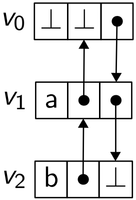



```
type 'a splist = {
  mutable head: 'a;
          tail: 'a splist;
  mutable prev: 'a splist;
}
```

```
let v0 = nil ()
let v1 = cons a v0
let v2 = cons b v1
let v3 = cons c v1
let v4 = cons d v3
let v5 = cons e v0
let v6 = cons f v5
```



```
type 'a splist = {
  mutable head: 'a;
          tail: 'a splist;
  mutable prev: 'a splist;
}
```

```
let v0 = nil ()
let v1 = cons a v0
let v2 = cons b v1
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let v4 = cons d v3
let v5 = cons e v0
let v6 = cons f v5
```
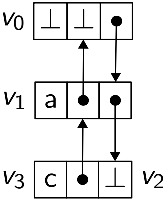
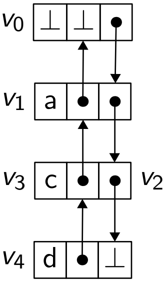



```
type 'a splist = {
  mutable head: 'a;
          tail: 'a splist;
  mutable prev: 'a splist;
}
```

```
let v0 = nil ()
let v1 = cons a v0
let v2 = cons b v1
let v3 = cons c v1
let v4 = cons d v3
let v5 = cons e v0
let v6 = cons f v5
```



```
type 'a splist = {
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          tail: 'a splist;
  mutable prev: 'a splist;
}
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let v0 = nil ()
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```

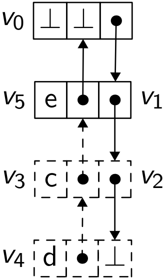

```
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  mutable head: 'a;
          tail: 'a splist;
  mutable prev: 'a splist;
}
```

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let v0 = nil ()
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let v4 = cons d v3
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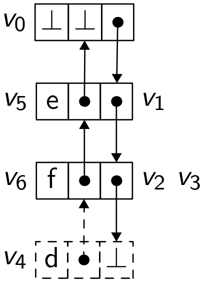
```



```
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}
```

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let v0 = nil ()
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let v6 = cons f v5
```
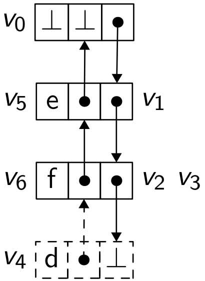
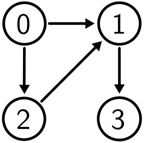
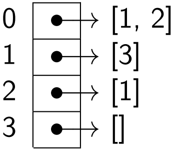



(Peut conduire à des fuites mémoire.)

Avec des tableaux semi-persistants et des listes semi-persistantes, on peut construire une structure semi-persistante pour des graphes dont les sommets sont $0, 1, \ldots, N-1$, avec des listes d'adjacence.

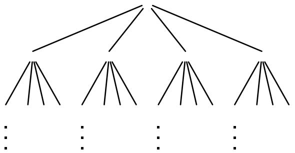
```
type spgraph =
  int splist sparray
```





performance

On explore un arbre de hauteur 6 et de branchement 4.



Entre deux nœuds, on fait N opérations sur la structure de données.

Au total, on effectue donc

- $4^6 N$ opérations sur la structure ;
- $3(4^6 - 1)$ retours en arrière.

N	1000	5000	10 000
tableaux persistants	0,26	2,06	5,71
semi-persistants	0,13	0,90	2,27
listes persistantes	0,03	0,24	0,53
semi-persistantes	0,03	0,12	0,24
graphes persistants	0,34	3,09	8,84
semi-persistants	0,23	1,65	3,93

On mesure le nombre total de Mo alloués par le GC.

N	1000	5000	10 000
tableaux persistants	750	3750	7490
semi-persistants	250	1250	2500
listes persistantes	125	625	1250
semi-persistantes	0,22	1,08	2,15
graphes persistants	875	4370	8740
semi-persistants	375	1880	3750

le bon usage

On manipule différentes versions d'une structure de données globale d'un type `semi`, avec deux opérations :

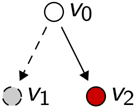
- construire une nouvelle version de la structure

```
val update: semi -> semi
```

- utiliser une version de la structure

```
val access: semi -> elt
```

Ces deux opérations exigent une version valide.



Correct :

```
let good (v0: semi) =
  let v1 = update v0 in
  let v2 = update v0 in
  access v2
```

Incorrect :

```
let bad (v0: semi) =
  let v1 = update v0 in
  let v2 = update v0 in
  access v1
```

Correct :

```
let rec backtracking (v: semi) =
  if ... access v ... then (
    ...
    backtracking (update v);
    ...
    backtracking (update v);
    ...
  )
```

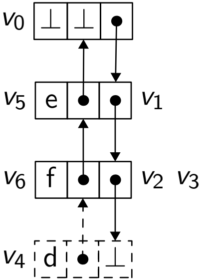
Il serait facile d'invalider un tableau semi-persistant, en ajoutant un troisième constructeur.

```
type 'a parray =
  'a data ref

and 'a data =
| Base of 'a array
| Diff of int * 'a * 'a parray
| Invalid
```

C'est à peine plus de travail.

C'est en revanche moins immédiat pour les listes.

Sur l'exemple suivant,



il est facile d'invalider v_4, mais moins évident d'invalider v_1, v_2, v_3.

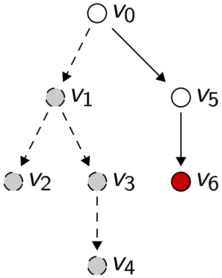
Montrons le bon usage de la structure semi-persistante en utilisant la vérification déductive.

1. Annoter les programmes avec un langage logique.
2. Calculer les conditions de vérification.
3. Montrer leur validité.

Notons

- *cur* la version courante de la structure,
- $\texttt{prev}(x)$ l'ancêtre immédiat de la version x,
- $\texttt{path}(x, y)$ la relation « x est un ancêtre de y »,
  i.e., $\texttt{path}$ est la clôture réflexive transitive de $\texttt{prev}^{-1}$.

En particulier, la validité de la version x s'écrit $\texttt{path}(x, cur)$.



Donnons à nos fonctions un contrat formel de la forme

$$f : (x : \tau_1) \to^{\varepsilon} (r : \tau_2)$$
$$\texttt{requires}\ P$$
$$\texttt{ensures}\ \ Q$$

où

- P est une précondition
- Q est une postcondition
- ε est un effet latent
    - $\top$: la version courante *cur* a (possiblement) changé
    - $\bot$: la version courante *cur* n'a pas changé

$$\begin{array}{l}
\mathtt{update} : (x : \mathtt{semi}) \rightarrow^{\top} (r : \mathtt{semi}) \\
\quad\quad \mathtt{requires}\ \mathtt{path}(x, cur) \\
\quad\quad \mathtt{ensures}\quad r = cur \wedge \mathtt{prev}(cur) = x
\end{array}$$

$$\begin{array}{l}
\mathtt{access} : (x : \mathtt{semi}) \rightarrow^{\top} \mathtt{elt} \\
\quad\quad \mathtt{requires}\ \mathtt{path}(x, cur) \\
\quad\quad \mathtt{ensures}\quad cur = x
\end{array}$$

```
let good (v0: semi) =
  (* requires path(v0, cur) *)
  let v1 = update v0 in
  let v2 = update v0 in
  access v2
```

$$
\begin{array}{l}
\forall v_0.\, \forall cur.\, \mathtt{path}(v_0, cur) \Rightarrow \\
\quad \mathtt{path}(v_0, cur) \wedge \\
\quad \forall v_1.\, \forall cur_1.\, (v_1 = cur_1 \wedge \mathtt{prev}(v_1) = v_0) \Rightarrow \\
\qquad \mathtt{path}(v_0, cur_1) \wedge \\
\qquad \forall v_2.\, \forall cur_2.\, (v_2 = cur_2 \wedge \mathtt{prev}(v_2) = v_0) \Rightarrow \\
\qquad\quad \mathtt{path}(v_2, cur_2)
\end{array}
$$

```
let bad (v0: semi) =
  (* requires path(v0, cur) *)
  let v1 = update v0 in
  let v2 = update v0 in
  access v1
```

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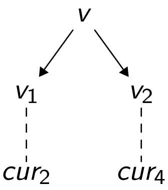
\end{array}
$$

```
let rec backtracking (v: semi) =
  (* requires path(v, cur) *)
  (* ensures  path(v, cur) *)
  if ... access v ... then (  ...
    backtracking (update v);  ...
    backtracking (update v);  ... )
```

$\forall v. \forall cur. \mathtt{path}(v, cur) \Rightarrow$



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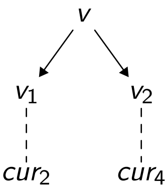
$$\forall v. \forall cur. \mathtt{path}(v, cur) \Rightarrow$$
$$\quad \mathtt{path}(v, cur) \land$$



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&\forall v.\,\forall cur.\,\mathtt{path}(v, cur) \Rightarrow \\
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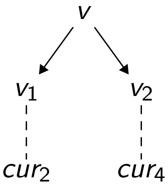

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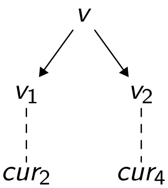

conditions de vérification

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let rec backtracking (v: semi) =
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&\qquad \mathtt{path}(v_1, cur_1) \wedge
\end{aligned}
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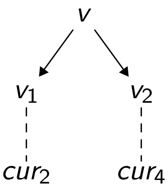

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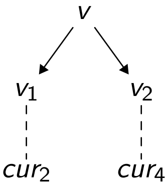

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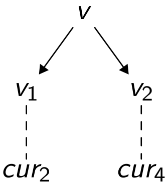

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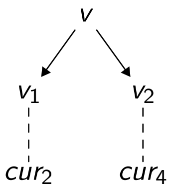

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\quad\quad\quad \forall v_2.\,\forall cur_3.\,(v_2 = cur_3 \wedge \mathtt{prev}(v_2) = v) \Rightarrow \\
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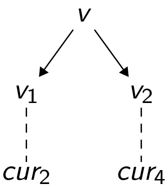

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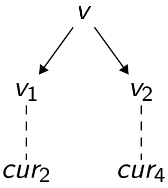
\qquad\quad \mathtt{path}(v, cur_2) \wedge \\
\qquad\quad \forall v_2. \forall cur_3. (v_2 = cur_3 \wedge \mathtt{prev}(v_2) = v) \Rightarrow \\
\qquad\qquad \mathtt{path}(v_2, cur_3) \wedge \\
\qquad\qquad \forall cur_4. \mathtt{path}(v_2, cur_4) \Rightarrow \mathtt{path}(v, cur_4)
\end{array}
$$



Nos conditions de vérification ϕ ont une forme simple :

$$\begin{array}{rrcl}
\text{termes} & t & ::= & x \mid \texttt{prev}(t) \\
\text{atomes} & a & ::= & t = t \mid \texttt{path}(t, t) \\
\text{cond. de vérif.} & \phi & ::= & a \mid \phi \wedge \phi \mid a \Rightarrow \phi \mid \forall x.\, \phi
\end{array}$$

Elles doivent être valides dans la théorie $\mathcal{T}$ qui combine l'égalité et les trois règles suivantes :

$$\frac{}{\texttt{path}(x, x)} \qquad \frac{}{\texttt{path}(\texttt{prev}(x), x)} \qquad \frac{\texttt{path}(x, y) \quad \texttt{path}(y, z)}{\texttt{path}(x, z)}$$

Il se trouve que déterminer

$$\mathcal{T} \models a_1 \wedge a_2 \wedge \cdots \wedge a_m \Rightarrow a$$

est décidable.

On veut décider la formule $\phi \stackrel{\text{def}}{=} a_1 \wedge a_2 \wedge \cdots \wedge a_m \Rightarrow a$.

1. On construit l'ensemble $H = \{a_1, \ldots, a_m\}$.
2. On clôt H par congruence :

$$\frac{t \in \phi}{t = t \in H} \qquad \frac{t_1 = t_2 \in H}{t_2 = t_1 \in H} \qquad \frac{t_1 = t_2 \in H \quad t_2 = t_3 \in H}{t_1 = t_3 \in H}$$

$$\frac{\mathtt{prev}(t_1), \mathtt{prev}(t_2) \in \phi \quad t_1 = t_2 \in H}{\mathtt{prev}(t_1) = \mathtt{prev}(t_2) \in H}$$

$$\frac{\mathtt{path}(t_1, t_2) \in \phi \quad t_1 = t_3 \in H \quad t_2 = t_4 \in H}{\mathtt{path}(t_3, t_4) \in H}$$

On fait cela avec une structure de classes d'équivalence.

On veut décider la formule $\phi \stackrel{\text{def}}{=} a_1 \wedge a_2 \wedge \cdots \wedge a_m \Rightarrow a$.

3. Si a est de la forme $t_1 = t_2$,
   on teste $t_1 = t_2 \in H$.

4. Si a est de la forme $\mathtt{path}(t_1, t_2)$,
   4.1 on construit un graphe orienté G dont les sommets sont les termes de H et
      - si $\mathtt{prev}(t) \in H$, on a un arc $\mathtt{prev}(t) \to t$,
      - si $\mathtt{path}(t, t') \in H$, on a un arc $t \to t'$,
      - si $t = t' \in H$, on a deux arcs $t \to t'$ et $t' \to t$;
   4.2 on teste l'existence d'un chemin de t_1 à t_2 dans G.

On peut le faire incrémentalement, en parcourant la condition de vérification,

```
let rec decide ctx = function
  | Fatom a        -> decide_atom ctx a
  | Fand (f1, f2) -> decide ctx f1 && decide ctx f2
  | Fimp (a, f)    -> decide (assume ctx a) f
  | Fforall f      -> decide ctx f
```

où `ctx` désigne l'état de la procédure de décision, c'est-à-dire

- une structure de classes d'équivalence ;
- un graphe.

La fonction `decide` fait un usage semi-persistant du contexte `ctx`.

On peut donc le réaliser ainsi :

```
type ctx = {
  uf: spuf;
  gr: spgraph;
}
```

En particulier, on peut vérifier le bon usage de ce contexte par la fonction `decide`... avec la fonction `decide` elle-même !

- En pratique, on veut également permettre
  - la création dynamique de structures ;
  - l'utilisation simultanée de plusieurs ensembles disjoints de structures semi-persistantes.

- On a vérifié le bon usage d'une structure semi-persistante, mais il faut également vérifier son implémentation.

Une structure semi-persistante, c'est à la fois

- moins de possibilités qu'avec une structure persistante, mais on gagne en performances ;
- plus de possibilités qu'avec une structure éphémère, car on peut faire des retours en arrière.

On peut vérifier mécaniquement la bonne utilisation d'une structure semi-persistante.

- Sylvain Conchon & Jean-Christophe Filliâtre
  Semi-Persistent Data Structures
  ESOP 2008, LNCS vol. 4960, pp 322–336
  `https://hal.inria.fr/hal-04045849`

- Code :
  `https://github.com/backtracking/spds`

- Gabriel Scherer
  Backtracking reference stores
  JFLA 2023
  `https://hal.inria.fr/hal-03936704/`