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To cite this version:

Dinesh Kumar, Arzad A. Kherani, Eitan Altman. Route Lifetime based Interactive Routing in Intervehicle Mobile Ad Hoc Networks. [Research Report] RR-5691, INRIA. 2006, pp.20. inria-00070323

HAL Id: inria-00070323
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Submitted on 19 May 2006

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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

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N° 5691

September 2005

Thème COM

Rapport de recherche

ISSN 0249-6399 ISRN INRIA/RR--5691--FR+ENG

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Thème COM — Systèmes communicants
Projets Maestro

Rapport de recherche n° 5691 — September 2005 — 20 pages

Abstract: The main goal of this paper is to better understand the route lifetime dynamics in Intervehicle communication networks (IVC) or what we call intervehicle mobile ad hoc networks (iv-MANETs) that are a special class of MANETs but exhibit very different behavior from them. We consider the problem of finding an *optimal* multi-hop route between two vehicular nodes in an iv-MANET. For a given choice of the number of hops and distances between intermediate nodes, we seek the characterizing properties of choice of speeds of the intermediate nodes so as to maximize the expected lifetime of the multi-hop route. Our analytical model inherently incorporates the randomly changing speeds of nodes over time and hence the optimal choice depends on the dynamics of the stochastic process corresponding to the speed of the nodes.

We suppose that for establishing a route the locations and speeds of other vehicles are known. Under a markovian assumption on the process of the speed of nodes, we show that the optimal choice of speeds attempts to equalize the lifetimes of adjacent links in a route. A monotone variation property of the speed of the intermediate nodes under the optimal policy is proved. These solution structures have been confirmed with an extensive simulation study. The heuristics and structures developed in this paper can serve in designing a new set of efficient *interactive* routing protocols specifically tailored for high mobility ad hoc networks and iv-MANETs in particular.

Key-words: MANET, routing, optimization, modelling

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Routage Interactif Basé sur la Durée de Vie des Itinéraires dans les Réseaux Ad Hoc Mobiles Intervéhiculaires

Résumé : Le but principal de cet article est de mieux comprendre la dynamique de vie des itinéraires dans les réseaux de transmission intervéhiculaires (IVC) ou ce que nous appelons réseaux ad hoc mobiles intervéhiculaires (iv-MANETs) qui sont une classe spéciale de MANETs mais ont un comportement très différent de ceux-ci. Nous considérons le problème de trouver un itinéraire multi-sauts *optimal* entre deux nœuds véhiculaires dans un iv-MANET. Pour un choix donné du nombre de sauts et de distances entre les nœuds intermédiaires, nous cherchons les propriétés caractéristiques du choix des vitesses des nœuds intermédiaires afin de maximiser la durée de vie prévue de l'itinéraire multi-sauts. Notre modèle analytique prend en compte le fait que les vitesses des nœuds changent aléatoirement avec le temps et que, par conséquent, le choix optimal dépend de la dynamique du processus stochastique correspondant à la vitesse des nœuds.

Nous supposons que pour établir un itinéraire les emplacements et les vitesses d'autres vehicules sont connus. Dans une hypothèse markovienne sur les processus des vitesse, nous prouvons que le choix optimal de la prochaine voiture essaye d'égaliser la durée de vie des liens adjacents dans un itinéraire. Une propriété de variation monotone de la vitesse des nœuds intermédiaires dans le cadre de la politique optimale est prouvée. Ces structures de solution ont été confirmées par des simulations exhaustives. L'heuristique et les structures développées dans cet article peuvent servir à concevoir un nouvel ensemble de protocoles *interactifs* efficaces d'établissement d'itinéraires spécifiquement adaptés aux réseaux ad hoc à mobilité élevée et aux iv-MANETs en particulier.

Mots-clés : MANET, routing, optimization, modelling

1 Introduction

Mobile Ad Hoc Networks (MANETs) are a promising way of establishing communication in places where wired infrastructure is not available and there is a need for rapid deployment of communication networks. Many such places include conference rooms, natural disaster sites, military operation settings and so on. Various issues about MANETs like expected lifetime of routes, TCP performance, mobility models, rate of change of links and routing protocols have received considerable attention in the recent past [1, 2, 3]. However not much focus has gone to Intervehicle communication (IVC) networks or what we call intervehicle Mobile Ad Hoc Networks (iv-MANETs) which are a special class of MANETs. Vehicles equipped with communication devices can form iv-MANETs for tasks such as intervehicle collision avoidance, road-accident notification from remote areas, traffic situation update, coordinated driving systems or simply intervehicle voice communication. Like MANETs, iv-MANETs do not rely on any fixed infrastructure and instead depend on intermediate *relay* nodes for route establishment protocols and data transmission. However, intervehicle ad hoc networks tend to exhibit a drastically different behavior from the usual MANETs [4]. High speeds of vehicles, mobility constraints on a straight road and driver behavior are some factors due to which intervehicle ad hoc networks possess very different characterisitcs from the typical MANET models. Broadly speaking, four such characteristics are rapid topology changes, frequent fragmentation of the network, small effective network diameter and limited temporal and functional redundancy [4].

Proactive routing and Reactive routing are two main routing techniques that have been studied for ad hoc networks hitherto. Proactive routing is a table-driven approach in which each node maintains one or more tables that contain routing information to every other node in the network. Changes in network topology result in propagation of route update packets by nodes so that consistent and up-to-date routing information about the whole network is maintained. Reactive routing on the other hand is an on-demand approach in which network routes are not updated with changing topology and instead route discovery is invoked when a source node wants to send data to a destination node. It is then clear that in a rapidly changing topology environment like iv-MANETs, proactive routing is highly intensive (in computation and signaling terms) due to packet overhead caused by frequent update of routing information of the whole network, even if there are no user data packets to be transmitted. Proactive routing is thus only suitable for small networks with limited mobility. Reactive routing was designed to improve on this by invoking route discovery only when it is needed. But even reactive routing may impede the efficient performance of iv-MANETs due to the transient nature of links. In an iv-MANET, due to high relative speeds of successive nodes, a route may cease to exist by the time the first *ack* of a routing packet reaches back the sender. Reactive routing can also increase packet delay of an ongoing session since in the event of a route path failure, route rediscovery is commenced only when the sender application has the next burst of packets to send and not sufficiently in advance. This might increase the burst transmission delay by the time needed to rediscover a new route in addition to route path failure overheads.

1.1 Motivation and Related Work

Inaptness of both proactive and reactive routing protocols for iv-MANETs thus gives rise to a need for what we call an *interactive* routing protocol for iv-MANETs or for any other ad hoc network in general that has high mobility patterns. The concept of an interactive routing protocol is very simple. In an interactive routing protocol, if a route exists then using any of the reactive routing protocol mechanisms, a *primero* route based on some *primero* optimality criteria is always chosen. We will define this *primero* optimality criteria later in Section 4. When the chosen route breaks due to high mobility and high relative speeds of nodes, route rediscovery is invoked an optimal time *prior* to the event when one of the links of an on-going session breaks, thus avoiding any interruption in the communication. The sender node thus *interactively* keeps track of the state of a route and invokes a route rediscovery some *optimal* time before a link constituting the route fails, instead of waiting till the point of failure of a link as is done in most of the on-demand reactive protocols. In this way one can avoid excessive routing table maintenance or route path failures resulting from frequent table updates or on-demand route discovery, respectively. Now, the big question is that how can a sender detect in advance as to when exactly, any of the links constituting a route to the destination node will break, so as to be able to invoke a route rediscovery sufficiently in advance. An obvious and well known answer to this question is by means of *node mobility prediction* or *route lifetime prediction*.

There are many node mobility prediction based techniques that have been proposed for ad hoc networks in general [6, 7, 8]. However most of these techniques are based on simulation results. Authors in [9] and [10] have derived expressions for the expected lifetime of a MANET route in a general 2-dimensional random-walk model. But, iv-MANETs exhibit fundamentally different behavior from general MANETs as discussed before and thus need special treatment. If it is somehow possible to predict the *lifetime* of a route in iv-MANETs, route rediscovery can be timely instantiated to shift a connection from the soon to be breaking route to the newly discovered route. This timely rediscovery of a new route can be vital for achieving an improved packet delay and throughput performance. The prediction of route lifetime is feasible since the source node can collect information about the location and speeds of intermediate relay nodes by means of extra bits in the route-reply packets sent by the destination node in response to the route-discovery packets. Also, the lifetime of a route depends very much on the mobility pattern of the nodes and results for general mobility models for MANETs cannot be applied to iv-MANETs which posses very different mobility patterns.

1.2 Synopsis

In this paper, we first identify the most important optimization parameters to be considered in order to chose a *maximum* lifetime route (this is the primero optimality criteria) among various candidate routes in an iv-MANET. We then discuss about the motion dynamics of a set of nodes constituting a route in an iv-MANET and define the model that is used to track the dynamics of the changing inter-node distances and node connectivity in Section 3. Model definition is followed by the formulation of an objective function consisting of the lifetime of links constituting a route (Section 4). Our goal is to optimize this objective function so as to obtain an optimal choice of intermediate relay nodes between a source and destination such that the route lifetime, which is defined as the least of the expected link lifetimes, is maximized. We further define a parameterized objective function and prove that the solution to the optimization problem with the parameterized objective function, coincides with the solution of the optimization of the original objective function. This gives us the convenience to use any of the two objective functions depending on the characteristics of different model scenario. In Section 5 an attempt to analyticaly determine the link lifetimes is made using a Markov chain model of the randomly travelling vehicles on a highway. Explicit expressions for the link lifetimes and either the optimal intermediate node selection policies or some structural intuitions about them are obtained for certain scenarios that are of most concern in the real life highway traffic situation. In Section 6 we compare the obtained results with the simulation results obtained from an iv-MANET simulator and it is observed that the analyticaly obtained optimal intermediate node policies and other structural results are consistent with the simulation results.

The contributions of this paper are twofold. Firstly, the heuristics and structural characteristics of the optimal selection of intermediate relay nodes developed in this paper can assist in better understanding the dynamics of route lifetime in iv-MANETs, which demonstrate very different performance characteristics when compared to other MANETs. Secondly, the results can serve as a basis for the design of a new set of *interactive* routing protocols which will perform with greater efficiency for high mobility ad hoc networks and iv-MANETs in particular.

2 Optimization Parameters

Consider vehicles (nodes) on an infinitely long straight highway with L lanes, moving in the same direction on either side of the highway. Each lane i has an associated speed limit s_i. Assume that in a given lane, the nodes travel with a speed corresponding to the speed limit of that lane. In other words, it is assumed that all nodes move on the highway with a discrete set of speeds which consists of the speed limits of each lane. We follow the convention that $s_1 < s_2 < \ldots < s_L$.

Now consider 2 tagged nodes, a source and a destination moving in any two (possibly same) lanes, travelling in the same direction. At time 0, these nodes are assumed to be distance D apart. If D is large enough then these nodes may not be able to communicate with each other directly. Intermediate relay nodes are required for these two tagged nodes to form an iv-MANET. In this paper we address the problem of coming up with an *optimal* choice of these intermediate relay nodes such that the primero optimality criteria is satisfied. The constraints under which this decision should be made are mentioned in detail in

Section 3 but here we emphasize on the fact that making such a decision may not be as simple as it seems at first. An evident reason being that the underlying state space over which the route lifetime has to be optimized is composed of different parameters, each representing as a component parameter of the overall optimization problem. Following are the possible optimization parameters that should be considered and the motivation behind their choice is discussed in the remaining part of this section.

1. *Optimization over Number of Intermediate Relay Nodes:* Owing to a constraint on the range over which a node's transmission can be successfully decoded, the number of intermediate relay nodes must be more than a certain minimum that depends on the initial distance D between the two tagged nodes. However, one can not choose an arbitrarily large number of intermediate nodes since with increasing number of intermediate nodes more frequent link failures may occur which degrades the overall end-to-end delay. An optimal choice on the number of intermediate relay nodes is thus necessary.

2. *Optimization over Inter-node Distances:* For a given number of intermediate nodes, an optimal choice of the inter-node distances is essential so that the route satisfies the primero optimality criteria (see Section 4).

3. *Optimization over Speeds of the Intermediate Nodes:* Once the number of intermediate nodes and the inter-node distances are fixed, we need to decide from which lanes the intermediate nodes are to be chosen. In other words, we need to decide what speeds the intermediate nodes should possess for obtaining a route that satisfies the primero optimality criteria (see Section 4).

Most of the existing proactive and reactive routing protocols for ad hoc networks rely on the shortest path (in terms of number of hops between source and destination) approach. The distance covered by a hop is bounded above by the transmission range R which is assumed to be constant. In a dense network (see Section 3.2), minimizing the number of hops would result (with a high probability) in a route path in which the distance covered by each hop is around R meters. This would indeed guarantee a minimum number of hops and this approach though works well in wired networks, it has the following drawbacks for iv-MANETs:

1. Due to high mobility and large speeds of nodes in an iv-MANET, the lifetime of a route path obtained on the basis of shortest path routing (in terms of number of hops) may be very small.

2. As discussed in the following text, smaller number of hops may also result in the use of high transmit energy for nodes.

We assume that a source node transmits to a destination node using a minimum possible amount of transmission power. Let $P_T(d)$ denote the minimum power required for successful transmission between two nodes that are d distance apart. Since the transmit power of a node is bounded, say by P, the maximum distance to which a node can transmit successfully is also bounded by $R \triangleq P_T^{-1}(P)$. This kind of power control clearly aims at reducing interference in the network. Also, since the function $P_T(d)$ is of the form ad^β with $\beta > 1$, $P_T(d)$ increases at a faster-than-linear rate with d. Specifically, if two nodes are distance $d < R$ apart then with no intermediate nodes present, a transmit power of ad^β is required whereas with one intermediate node at a distance $\frac{d}{2}$ from the source a *total* transmit power of $2a\frac{d^\beta}{2^\beta} < ad^\beta$ is required. This illustrates that having a large number of intermediate nodes reduces the overall transmission power used. However, an arbitrarily large number of intermediate nodes can not be chosen since with increasing number of intermediate nodes, frequent link failures may occur which may result in very poor network performance.

The above discussion makes it clear that there is a trade-off between the transmit power, route lifetime and network performance with increasing number of intermediate nodes and an optimal choice of the number of intermediate relay nodes is needed. It is also clear that for a given set of inter-node distances, the route lifetime critically depends on the speeds of the intermediate nodes. In the present work, we assume that the number of intermediate nodes, the inter-node distances and the speeds of the source and destination nodes are somehow known in advance. Given this information, we are interested in obtaining the optimal speeds of intermediate nodes that satisfy the primero optimality criteria. Thus we only attempt to solve the last of the three optimization problems mentioned before. The issue of coming up with an optimal choice of the number of intermediate nodes and the distances between these intermediate nodes is under way as our current research problem and uses the solution of the problem considered in this paper.

3 System Dynamics and Model

3.1 Dynamics of Individual Nodes

The process of changing speed of any individual node (or, vehicle) due to lane change on the highway is assumed to be an independent stationary ergodic stochastic process. We are thus also implicitly assuming that the vehicles do not leave the highway. It is assumed as well that the vehicles do not change their direction of motion. In this paper, we restrict ourselves to the case where the changing speed of any node can be modeled as an irreducible aperiodic Markov process, taking a finite set of constant values $\{s_1, s_2, \ldots, s_L\}$.

We assume that a node continues to move in lane i with an associated speed $s_i,\ 1 \leq i \leq L$ for an exponential amount of time before changing its lane, or its speed equivalently. This time is exponentially distributed with rate μ_i and we denote that a node in lane i transits to another lane j with probability $P_{i,j}$ with $P_{i,i} = 0$. Even though our analysis holds good for generic transition probabilities $P_{i,j}$, we assume the following natural structure on node transitions in our highway scenario: from state (or, lane) i, a node can transit only to the states $(i-1) \vee 1$ or $(i+1) \wedge L$. Clearly, from state 1 a node can transit only to state 2 and from state L the only possible transition is to state $L-1$.

3.2 Placement of Nodes

We assume that node spreadout along the highway is dense in the sense that in a sufficiently small neighbourhood of any point on a lane we can always find atleast one node on the *same lane*. This is like assuming that the transmission range R of a node is significantly large as compared to the distances between two successive nodes in any lane. Most of the results in this paper can be extended to the case where we assume that the existence of a node at any point on a lane is itself a stochastic process. However, since we are more interested in the structural results of optimal speed selections, we will assume that this stochastic process is a constant process, i.e., there is always a node at any given point on any lane.

It is also assumed that the width of the lanes on an highway is negligible when compared to the transmission range of mobile nodes. We call this assumption as the *straight line communication* assumption.

3.3 Evolution of Inter-node Distances and Node Connectivity

Consider any two nodes i and j moving in any two lanes with both the nodes moving in the same direction. Assume that the two nodes have speeds $v_i(t)$ and $v_j(t)$ respectively at time t. Since the two nodes are moving and also have their speeds changing with time due to lane change, the distance between these nodes will also vary with time. Let us denote the distance of node j from node i (measured in the direction of motion) at time t as $d_{ij}(t)$. Assume that node i is the source of transmissions meant for node j. We say that a direct link or single hop route exists between nodes i and j as long as $0 \leq d_{ij}(t) \leq R$, where R is the maximum possible transmission range of a node i.e. a node can successfully transmit at any range $\leq R$.

The distance between any two adjacent nodes i and $i+1$ of a route denoted simply by $d_i(t)$, forms a stochastic process that begins with an initial value of $d_i(0) = d_i$ and whose evolution over time, $d_i(t)$, depends on the initial speeds of the two nodes. We assume that two successive nodes i and $i+1$ of a route remain connected *only* until when $d_i(t)$ takes a value outside the interval $[0, R]$ for the first time (see Figure 1). The convention followed is that the link between two successive nodes i and $i+1$ of a route, breaks, if

1. node $i+1$ is ahead of node i in the direction of motion and the distance between node i and node $i+1$ exceeds R so that node $i+1$ is outside the maximum transmission range of node i, or
2. node i moves ahead of node $i+1$ in the direction of motion. This convention can be easily relaxed to incorporate the case where the link between node i and $i+1$ breaks only when node i moves ahead of node $i+1$, in the direction of motion, by a distance R. The results of our analysis will still hold good with this relaxed convention.

In brief, we consider nodes i and j to be connected if node j lies within the maximum transmission range of node i *only* in the direction of motion and not otherwise.

Note that since the communication devices mounted in the vehicles operate on car battery which is recharged by the vehicle engine, battery-life of nodes is not an issue in our model.

Assume M intermediate relay nodes between the source and destination in a route with the source being the 0^{th} node and the destination as the $(M+1)^{th}$ node. Let v_0 and v_{M+1} be the velocities of the source and destination nodes and let D be the distance between them. For a given value of M, let $d_i, 0 \leq i \leq M$ be the distance between node i and node $i+1$. We impose that $\sum_{i=0}^{M} d_i = D$ so that the last hop distance $d_M = D - \sum_{i=0}^{M-1} d_i$. For a non-broken route formed by nodes $0, 1, 2, \ldots, M+1$, we require that $0 \leq d_i \leq R$ and let $v_i, 0 \leq i \leq M+1$ be the velocity of the i^{th} node with v_0 and v_{M+1} known in advance. Note that v_is may take any one of the set of constant values $\{s_1, \ldots, s_L\}$ and there are L^M different possible values that the vector $\underline{v} = (v_1, \ldots, v_M)$ can take.

4 The Problem Formulation

As discussed before, in an interactive routing protocol a source node chooses a primero route according to an associated primero optimality criteria. In our model described in the previous section, we assume a dense vehicle traffic scenario on the highway. Due to this assumption multiple candidate routes may exist for choosing a primero route. But, once a route has been chosen and established, it is desirable to use this route for as long as possible in order to sustain increased network stability and avoid signaling and computation overhead costs. Thus if multiple candidate routes are available then the *primero* optimality criteria can be to choose the route with the *maximum* lifetime.

We are given that there are $M+2$ nodes, indexed $0, 1, \ldots, M+1$, constituting a route. Node 0 is the source node and node $M+1$ is the destination node. Now consider any two successive nodes i and $i+1$ in the route, that are distance d apart at time zero. Assume also that at time zero, node i is in lane k and node $i+1$ is in lane l such that $d_i(0) = d$, $v_i(0) = s_k$ and $v_{i+1}(0) = s_l$. Let $T(d, v_i, v_{i+1})$ be the expected time after which the link between these two nodes breaks (see Section 3.3). We refer to the quantity $T(d, v_i, v_{i+1})$ as the *link lifetime* of the link between the successive nodes i and $i+1$ in a route.

For a route comprised of $M+1$ links with initial distances between successive nodes in the route denoted by $\underline{d} = (d_0, \ldots, d_{M-1})$, our problem is to find an optimal speed assignment, denoted by $\underline{v} = (v_1, \ldots, v_M)$, to the M intermediate nodes such that the primero optimality criteria of maximum route lifetime is satisfied. We thus seek the optimal speed vector $\underline{v}$ such that the *least* of the link lifetimes of the route is maximized. Our optimization problem is therefore the following,

$$\underset{\underline{v}}{\text{Maximize}} \ \underset{i=0..M}{\text{Minimum}} \ T(d_i, v_i, v_{i+1}) \ . \tag{1}$$

Instead of solving the above problem directly, we can also attempt to optimize a different, parameterized, objective function. This objective function will coincide with the original one in Equation 1 when the parameter takes a special value. To define the parameterized objective function, we first state the following simple lemma (which we prove here for the sake of completeness).

Lemma 1 *For any* finite dimensional *vector $\underline{x}$ with positive elements $x_i, \ 1 \leq i \leq n$, if $||\underline{x}||_\alpha$ denotes the $l_\alpha-$norm, i.e.,*

$$||\underline{x}||_\alpha = \left[\sum_{1 \leq i \leq n} x_i^\alpha \right]^{\frac{1}{\alpha}} ,$$

and $||\underline{x}||_\infty$ denotes its $l_\infty-$norm, i.e.,

$$||\underline{x}||_\infty = \max_{1 \leq i \leq n} \{x_i\},$$

then

$$\lim_{\alpha \to \infty} ||\underline{x}||_\alpha = ||\underline{x}||_\infty.$$

Proof: Let, for the given vector $\underline{x}$,

$$i^* \stackrel{\Delta}{=} \arg \max_{1 \le i \le n} x_i.$$

Then, for any α,

$$||\underline{x}||_\infty = x_{i^*} \le ||\underline{x}||_\alpha,$$

and also,

$$||\underline{x}||_\alpha \le n^{\frac{1}{\alpha}} ||\underline{x}||_\infty.$$

This completes the proof. •

Theorem 4.1 *The solution of the optimization problem in Equation 1 is identical to that of the optimization problem*

$$\underset{\underline{v}}{\textit{Minimize}} \left[\sum_{j=0}^{M} (T(d_j, v_j, v_{j+1}))^{-\alpha} \right]^{\frac{1}{\alpha}}, \tag{2}$$

as $\alpha \to \infty$.

Proof: The optimization problem of Equation 1 clearly has the same solution as that of the problem

$$\underset{\underline{v}}{\text{Minimize}} \ \underset{i=0..M}{\text{Maximum}} \ \frac{1}{T(d_i, v_i, v_{i+1})}. \tag{3}$$

Now, for any integer $\alpha > 0$, we can compute the l_α−norm of an M-dimensional vector whose i^{th} element is $1/T(d_i, v_i, v_{i+1})$. The l_α−norm of this vector, for any given values of v_i's is

$$\left[\sum_{j=0}^{M} (T(d_j, v_j, v_{j+1}))^{-\alpha} \right]^{\frac{1}{\alpha}}.$$

Since $1/T(d_i, v_i, v_{i+1})$'s are strictly positive and bounded quantities, we can invoke Lemma 1 to conclude with the statement of the present theorem. •

In fact, we can say something more about the relation between the two optimization problems of Equation 1 and 2.

Theorem 4.2 *There exists a* finite α^* *such that the maximizer of optimization problem of Equation 1 is identical to that of Equation 2* for all values of $\alpha > \alpha^*$.

Proof: Fix a vector $\underline{x}$ with elements $x_i, 1 \le i \le n$. Then, from Lemma 1 we know that $\lim_{\alpha \to \infty} ||\underline{x}||_\alpha = \max_{1 \le i \le n} x_i$. Now, form a vector $\underline{y}$ whose i^{th} element y_i is the i^{th} maximum among the elements of $\underline{x}$ (so that $y_1 = \max_{1 \le i \le n} x_i = ||\underline{x}||_\infty$). Since the number of elements in $\underline{x}$ is n, which is finite, the difference $y_1 - y_2 > 0$ (assuming that no two elements of $\underline{x}$ are equal; the case where some of the elements of $\underline{x}$ are equal can also be easily considered.). Since $\lim_{\alpha \to \infty} ||\underline{x}||_\alpha = \lim_{\alpha \to \infty} ||\underline{y}||_\alpha \to y_1$, for any $\epsilon > 0$ there exists a finite $\alpha^*_\epsilon(\underline{x})$ such that , $y_1 - ||\underline{x}||_\alpha < \epsilon$ for all $\alpha > \alpha^*_\epsilon(\underline{x})$.

Now, since the set of possible values of the speed vector $\underline{v}$ over which optimization is carried out is finite (of cardinality L^M), then for a given combination of inter-node distances $\underline{d} = (d_0, \ldots, d_M)$ we can define,

$$\delta \stackrel{\Delta}{=} \min_{s_1 \le v_i, v_j \le s_L,} \text{POS}(|T(d_i, v_i, v_{i+1}) - T(d_j, v_j, v_{j+1})|),$$

where

$$\text{POS}(|x-y|) = \begin{cases} |x-y| & \text{if } |x-y| > 0, \\ |x| & \text{otherwise} \end{cases}$$

Then, $\alpha^* \stackrel{\Delta}{=} \max_{\underline{x}} \alpha^*_\delta(\underline{x}) < \infty$ is the finite quantity that we were seeking. •

Theorem 4.2 ensures that there is no discontinuity in the solution of the optimization problem of Equation 2 with respect to the solution of Equation 1, as $\alpha \to \infty$. Working with the objective function of Equation 2 in fact has an advantage that we can optimize it for some *finite* value of $\alpha > \alpha^*$ and elegantly obtain the solution to the optimization problem of Equation 1.

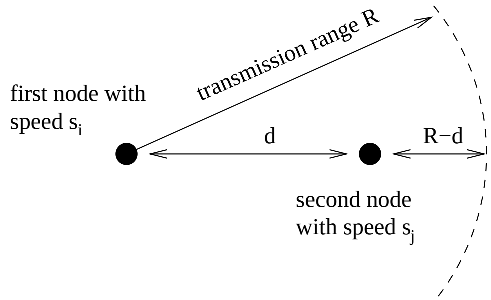

Figure 1: Two successive nodes constituting a route path

5 Determining the expected lifetimes and optimal solution

Having done with the problem formulation, here we seek to obtain explicit expressions for the link lifetimes, to be able to explicitly define the objective function of either Equation 1 or Equation 2. We study the expected lifetime of the connection between two nodes that are d distance apart at time 0 and have speeds s_i and s_j respectively. We use the notation that a pair of nodes k and l is in state s_{ij} when node k is in lane i with associated speed s_i and node l is in lane j with associated speed s_j. Here onwards we will also use the notation $T(d, s_{ij})$ for the link lifetimes of any two nodes alongwith $T(d, v_k, v_l)$, interchangeably. With some abuse of notation we use the same notation for the state s_{ij} and the relative speeds between the two nodes $s_{ij} \triangleq s_j - s_i$, interchangeably. Consider a pair of successive nodes forming a link in a route as shown in Figure 1. If the second node is within the range R of the first node then using the *straight line communication* assumption mentioned before in Section 3.2, the expected remaining link lifetime is given by $T(d, s_{ij})$ and we state the following theorem.

Theorem 5.1 $T(d, s_{ij})$ *satisfies the following renewal-type recursions*

$$s_{ij} > 0 \quad T(d,s_{ij}) = e^{-(\mu_i+\mu_j)\frac{R-d}{s_{ij}}}\frac{R-d}{s_{ij}} + \int_0^{\frac{R-d}{s_{ij}}} (\mu_i+\mu_j)e^{-(\mu_i+\mu_j)u}\Big[u+ \sum_l P_{i,l}\frac{\mu_i}{\mu_i+\mu_j}T(d+s_{ij}u, s_{lj}) + \sum_l P_{j,l}\frac{\mu_j}{\mu_i+\mu_j}T(d+s_{ij}u, s_{il})\Big]du, \tag{4}$$

$$s_{ij} < 0 \quad T(d,s_{ij}) = e^{-(\mu_i+\mu_j)\frac{d}{|s_{ij}|}}\frac{d}{|s_{ij}|} + \int_0^{\frac{d}{|s_{ij}|}} (\mu_i+\mu_j)e^{-(\mu_i+\mu_j)u}\Big[u+ \sum_l P_{i,l}\frac{\mu_i}{\mu_i+\mu_j}T(d-|s_{ij}|u, s_{lj}) + \sum_l P_{j,l}\frac{\mu_j}{\mu_i+\mu_j}T(d-|s_{ij}|u, s_{il})\Big]du, \tag{5}$$

$$s_{ij} = 0 \quad T(d,s_{ij}) = \int_0^{\infty} (\mu_i+\mu_j)e^{-(\mu_i+\mu_j)u}\Big[u + \sum_l P_{i,l}\frac{\mu_i}{\mu_i+\mu_j}T(d, s_{lj}) + \sum_l P_{j,l}\frac{\mu_j}{\mu_i+\mu_j}T(d, s_{il})\Big]du. \tag{6}$$

Proof: Since a node continues to stay in lane i for an exponentially distributed amount of time with mean $\frac{1}{\mu_i}$, it follows that a pair of nodes i and j, remains in state s_{ij} for an exponential amount of time with mean $\frac{1}{\mu_i+\mu_j}$ before one of them transits to any of the adjacent lanes. Then for $s_{ij} > 0$, the probability that the second node escapes from the range of the first node before any of them changes lane is given by $e^{-(\mu_i+\mu_j)\frac{R-d}{s_{ij}}}$ and $e^{-(\mu_i+\mu_j)\frac{R-d}{s_{ij}}}\frac{R-d}{s_{ij}}$ is the corresponding expected time to escape. If any one of the nodes makes a transition to another lane before the second node escapes from first node's range, then $P_{i,l}\frac{\mu_i}{\mu_i+\mu_j}$ is the probability that the first node made a transition to lane l and $P_{j,l}\frac{\mu_j}{\mu_i+\mu_j}$ is the probability that the second node made a transition to lane l. Hence with $(\mu_i + \mu_j)e^{-(\mu_i+\mu_j)u}$ as the exponential distribution function and $T(d+s_{ij}u, s_{lj})$ and $T(d+s_{ij}u, s_{il})$ being the remaining expected lifetimes of the first and second nodes respectively, after time u has elapsed, we get Equation 4 above. Now, note again that we consider two nodes to be connected if the first node lies within the maximum transmission range of the second node i *only* in the direction of motion and not otherwise. Then for $s_{ij} < 0$, the probability that the second node escapes from the range of the first node before any of them makes a transition is given by $e^{-(\mu_i+\mu_j)\frac{d}{|s_{ij}|}}$ and $e^{-(\mu_i+\mu_j)\frac{d}{|s_{ij}|}}\frac{d}{|s_{ij}|}$ is the corresponding expected time to escape. The rest of the terms in Equation 5 are obtained with similar arguments as for Equation 4 except that the remaining expected lifetimes after the event that one of the nodes transited at time u is given by $T(d-|s_{ij}|u, s_{lj})$ and $T(d-|s_{ij}|u, s_{il})$ for the first and second nodes respectively. •

Instead of solving the system of Equations 4, 5, and 6 explicitly in its most general form, we solve it only for some special cases. The main reason for considering only these special cases is that these are the only cases which are of relevance in a real life highway scenario and solutions for cases other than these cannot be applied to real life traffic movement on highways. Another interesting aspect of considering these special cases is that the results that we obtain for these cases constitute a simple form and provide important insights into the structure of the corresponding optimal speed policies. Later with the help of simulations we attempt to validate the obtained structure for any general case.

In the following sub-sections we attempt to solve the link lifetime recursion equations for particular cases of $L = 2$ and $L \geq 3$. The case $L = 1$ is trivial because there is no breakdown of routes, since all nodes are always travelling with the same speed s_1. Firstly, we consider the case $L = 2$ and, assuming $\mu_1 = \mu_2$, we obtain explicit expressions for the quantities $T(d, s_{ij})$'s. We then solve the optimization problem of Equation 1 directly for $M = 1$ and $\frac{R}{s_{12}} < \frac{1}{2\mu}$. Second, we consider the case with general values of $L \geq 3$ and $\frac{1}{\mu_i} >> \frac{R}{s_i}$ so that a node remains in lane i for a very long period as compared to the lifetime of a link. For this case we derive some interesting properties of the solution to the optimization problem of Equation 1 and develop some structural heuristics about the solution to the optimization problem of Equation 2. Both these cases provide important guidelines on choosing the speed of the intermediate nodes.

5.1 $L = 2$

Consider the case where the number of lanes is $L = 2$. There are only two possible speeds s_1 and s_2 in this case with $s_2 > s_1$. At any time t, let the source have speed $v_0(t)$ and destination have speed $v_{M+1}(t)$. Recall that the processes $\{v_0(t)\}$ and $\{v_{M+1}(t)\}$ are assumed to be independent Markov processes over the state space $\{s_1, s_2\}$. The infinitesimal generator matrix is then given by:

	s_1	s_2
s_1	$-\mu_1$	μ_1
s_2	μ_2	$-\mu_2$

Here μ_i is the rate of the exponentially distributed sojourn time when the process $\{v_0(t)\}$ (or, $\{v_{M+1}(t)\}$) is in state s_i. We state the following lemma without proof.

Lemma 2 *If* $\mu_1 = \mu_2 = \mu$ *then,*

1. *The process of the speed of destination node with respect to the source node, i.e.,* $\{v_0(t) - v_{M+1}(t)\}$ *forms an irreducible periodic Markov process over (finite) state space* $\{0, s_{12}, s_{21}\}$ *with the mean sojourn time in any state being exponentially distributed with rate* 2μ.

2. *The state transition probability matrix is of the form*

	s_{12}	0	s_{21}
s_{12}	0	1	0
0	0.5	0	0.5
s_{21}	0	1	0

In words, from the states with non-zero relative speed, transition is always to the one with a relative speed of 0 *and from the state with relative speed* 0*, the transition is to either of the other two states, each with probability* 0.5.

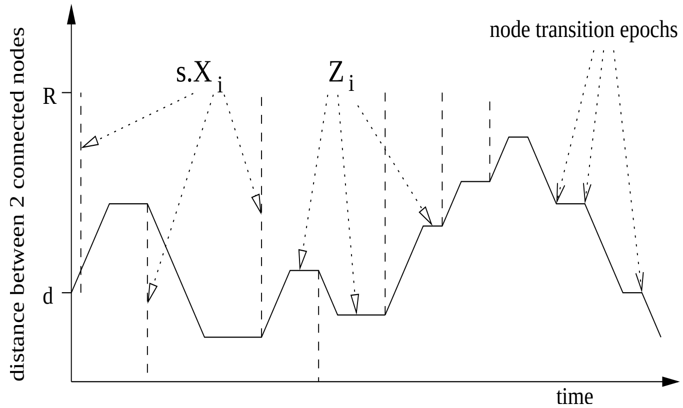

Figure 2: Random walk model for 2 successive nodes in a route

An important consequence of the observation of Lemma 2 is that the function $T(d, v_i, v_j)$ depends on v_i and v_j only via $v_i - v_j$ with $v_i - v_j \in \{0, s_{12}, s_{21}\}$. We will see later that the observation of Lemma 2 also helps us to compute the function $T(d, 0)$ directly via a simple application of Wald's lemma without solving any integral equation for $T(d, 0)$. We have the following recursions for $T(d, s_{12})$ and $T(d, s_{21})$ from Equations 4 and 5:

$$T(d, s_{12}) = e^{-2\mu\frac{(R-d)}{s}}\frac{R-d}{s} + \int_{u=0}^{\frac{R-d}{s}} (u + T(d + su, 0))2\mu e^{-2\mu u} du, \qquad \text{for} \quad s_{12} > 0, s = s_2 - s_1 \tag{7}$$

$$T(d, s_{21}) = e^{-2\mu\frac{d}{s}}\frac{d}{s} + \int_{u=0}^{\frac{d}{s}} (u + T(d - su, 0))2\mu e^{-2\mu u} du, \qquad \text{for} \quad s_{21} < 0, s = s_2 - s_1 \tag{8}$$

For obtaining $T(d, 0)$ we follow a different approach. Recall that $T(d, 0)$ is the expected time for which the distance between the two nodes remains in the interval $[0, R]$, starting with distance d apart and 0 relative velocity. Clearly, the distance between the nodes can change only when the relative velocity between the two nodes is non-zero. The periods of zero and non-zero relative velocity alternate and the instants of the beginning of zero relative speed form renewal instants for the relative speed process.

Consider a particle starting at point d. In each time unit, the particle moves to left or right (each with probability $\frac{1}{2}$) and moves by an exponentially distributed amount. The mean of the jump size is $\frac{1}{m}$ where $m = 2\mu$. Let $S_n, n \geq 1$ be the position of particle just after n^{th} jump. It is then seen that

$$S_n = d + \sum_{i=1}^{n} X_i$$

where $|X_i|$s are exponentially distributed random variables (with rate m) corresponding to the jump sizes (see Figure 2). X_i takes negative and positive values with probability $\frac{1}{2}$ each. Let N be the random variable corresponding to the number of jumps required by the particle to exit the interval $[0, R]$ with $R > d$. Let q be the probability that the particle exits via R. The treatment of [5, Chapter 7] can then be used to show that, since $|X_i|$s are independent and identically distributed,

$$\begin{aligned} E\sum_{i=1}^{N} |X_i| &= E[N]E[|X_1|] \\ E[(S_N - d)^2] &= E[N]E[|X_1|^2]. \end{aligned}$$

To compute $E\sum_{i=1}^{N} |X_i|$, we need $E[N]$ which is derived from the second relation above as follows. Since $|X_i|$ are exponentially distributed, we can invoke the memoryless property of exponential distribution to see that

$$S_N - d = \begin{cases} R - d + Y & w.p.\ q \\ -d - Y & w.p.\ 1-q \end{cases}, \tag{9}$$

where Y is an exponentially distributed random variable with rate m. Hence,

$$\begin{aligned} E[(S_N - d)^2] &= E[N]E[|X_1|^2] \\ &= qE[(R-d+Y)^2] + (1-q)E[(d+Y)^2] \\ &= (d^2 + E[Y^2] + 2dE[Y]) + q(R-2d)[R+2E[Y]]. \end{aligned}$$

From the above expression, since $E[Y] = E[X_1] = \frac{1}{m}$, we can obtain $E[N]$ if we know q. We now obtain q using the fact that [5]

$$E[S_N - d] = E\sum_{i=1}^{N} X_i = E[N]E[X_1] = 0.$$

Now, using the possible values of $S_N - d$ mentioned in Equation 9,

$$E[S_N - d] = 0 = q(R - d + E[Y]) + (1-q)(-d - E[Y])$$

hence

$$q = \frac{d + E[Y]}{R + 2E[Y]} = \frac{md+1}{mR+2}$$

where we have used the fact that $E[Y] = \frac{1}{m}$. From this value of q, we get (using the fact that $E[Y] = \frac{1}{m}$ and $E[X_1^2] = \frac{2}{m^2}$)

$$E[N] = ((R-2d)(d + \frac{1}{m}) + d^2 + \frac{2}{m^2} + 2\frac{d}{m})\frac{m^2}{2}.$$

It is then seen that, assuming $s = 1$ with out loss of generality,

$$T(d,0) = E[\sum_{i=1}^{N}(Z_i + |X_i|) - (\sum_{i=1}^{N} X_i - (R-d))I_{\{R-d<\sum_{i=1}^{N} X_i\}} - (-d - \sum_{i=1}^{N} X_i)I_{\{-d>\sum_{i=1}^{N} X_i\}}]$$

where Z_is are also exponentially distributed random variables with rate m and they correspond to the time when the distance between the two nodes does not change because of zero relative speed (see Figure 2). Using the memoryless property of exponential distribution, we see that if

$$I_{\{R-d<\sum_{i=1}^{N} X_i\}} = 1$$

then $\sum_{i=1}^{N} X_i - (R-d)$ is (independent and) exponentially distributed with rate m. Similarly, if

$$I_{\{-d>\sum_{i=1}^{N} X_i\}} = 1$$

then $(-d - \sum_{i=1}^{N} X_i)$ is exponentially distributed with rate m. Also,

$$E[I_{\{R-d<\sum_{i=1}^{N} X_i\}}] = q = 1 - E[I_{\{-d>\sum_{i=1}^{N} X_i\}}].$$

Hence,

$$T(d,0) = \frac{2E[N]}{m} - \frac{1}{m} = (R-d)md + R + \frac{1}{m}.$$

It then follows from the recursion Equations 7 and 8 that

$$T(d,1) = md(R-d) + 2(R-d),$$

and

$$T(d,-1) = md(R-d) + 2d.$$

We have thus derived explicit expressions for the link lifetimes above.

5.1.1 Direct Solution to the optimization problem of Equation 1 for the case of $\frac{R}{s} < \frac{1}{m}$

We consider the case where $\frac{R}{s} < \frac{1}{m}$. This scenario is of relevance since in normal real life highway traffic, a node remains in its lane for an average time greater than the lifetime of the link formed by this node and its next hop. Assuming $s = 1$ with out loss of generality, it is easy to see that for this case

$$T(d, 1) \leq T(d, 0), \quad d \leq R,$$

and

$$T(d, -1) \leq T(d, 0), \quad d \leq R.$$

Now, let the distance between the source and destination be D such that $R < D < 2R$. Thus one needs at least two hops or equivalently one intermediate relay node for communication. Let the number of intermediate relay nodes be $M = 1$. Also, let the speed of destination with respect to the source be $s = 1$ (i.e. $s_{ij} > 0$). Then for a given distance d between the source and the intermediate node, the decision is to be made on the speed v of this intermediate node. Let the expected lifetime of the link between source and intermediate node be $L_1(v)$ and that of the link between intermediate node and destination be $L_2(v)$. The value of these quantities then are

v	$L_1(v)$	$L_2(v)$
s_1	$T(d, 0)$	$T(D - d, 1)$
s_2	$T(d, 1)$	$T(D - d, 0)$

Now,

$$T(D - d, 0) - T(d, 0) = m(D - R)(2d - D),$$

and,

$$T(D - d, 1) - T(d, 1) = (m(D - R) + 2)(2d - D).$$

Hence, for $d > \frac{D}{2}$,

$$\max_{v \in \{s_1, s_2\}} (L_1(v) \wedge L_2(v)) = s_1,$$

and for $d < \frac{D}{2}$,

$$\max_{v \in \{s_1, s_2\}} (L_1(v) \wedge L_2(v)) = s_2.$$

Thus, we see that by the solution to the optimization problem of Equation 1, for $s_{ij} > 0$ *the speed of the intermediate node should be the same as the speed of the farther node.* Similarly, it is easy to derive that when the source node has speed s_2 and destination node has speed s_1 (i.e. $s_{ij} < 0$) *the speed of the intermediate node should be the same as the speed of the nearer node.*

5.2 $L \geq 3$

5.2.1 Some properties of the solution to the optimization problem of Equation 2 with $\frac{R}{s_i} << \frac{1}{\mu_i}$

Here we derive some structural properties of the solution to the optimization problem of Equation 2 for the particular case of interest when $\frac{R}{s_i} << \frac{1}{\mu_i}$ so that a node stays in its lane for a time much greater than its link lifetimes. Assume any value of $L \geq 3$ and consider the link lifetime dynamics of two nodes in lanes i and j that are seperated by an initial distance $d < R$. It can be easily seen that for $i \neq j$ and $\frac{R}{s_i} << \frac{1}{\mu_i}$, Equations 4 and 5 can be rewritten as

$$\begin{aligned} T(d, s_{ij}) &= \frac{R - d}{s_{ij}} \quad \forall s_{ij} > 0, \\ T(d, s_{ij}) &= \frac{d}{s_{ij}} \quad \forall s_{ij} < 0. \end{aligned}$$

If both the nodes are initially in the same lane, then the distance between these two nodes remains constant till the instant when any one of them changes lanes, so that $\forall s_{ii} = 0$,

$$T(d, s_{ii}) = \frac{1}{2\mu_i} + \sum_{j \neq i} \frac{P_{i,j}}{2}(T(d, s_{ij}) + T(d, s_{ji})).$$

Now consider a route consisting of M intermediate nodes so that the source and destination nodes have speeds v_0 and v_{M+1} respectively, and the distance vector $\underline{d} = (d_0, \ldots, d_M)$ is known in advance. For obtaining the speed vector $\underline{v} = (v_1, \ldots, v_M)$ that maximizes the route lifetime, we can consider minimizing the objective function of Equation 2. Let us make a simplifying assumption here that $T(d, 0) = \infty$ so that $\frac{1}{T(d,0)} = 0$. Though this assumption is not necessary for the analysis that follows, it is well justified here for the case under consideration. We see that the objective function of Equation 2 for any given value of α is given by,

$$\left[\sum_{j=0}^{M}\left[\frac{1}{T(d_j, v_j, v_{j+1})}\right]^{\alpha}\right]^{\frac{1}{\alpha}}.$$

Define $f_i(x, y) = \frac{1}{T(d_i, v_i, v_j)}$ such that $x = v_i$ and $y = v_j$. Clearly, if it is allowed to chose an intermediate node i with any arbitrary continuum speed x (thus not restricting to the discrete set of speeds $s_i, 1 \leq i \leq L$), the following condition should be satisfied for an optimal speed assignment to node i,

$$\frac{d}{dx}\left[(f_{i-1}(v_{i-1}, x))^{\alpha} + (f_i(x, v_{i+1}))^{\alpha}\right]^{\frac{1}{\alpha}} = 0.$$

This implies, in particular, that

$$\frac{f_{i-1}(v_{i-1}, x)}{f_i(x, v_{i+1})} = \left[-\frac{df_i(x, v_{i+1})}{df_{i-1}(v_{i-1}, x)}\right]^{\frac{1}{\alpha-1}}.$$

Now it is easy to show that $\frac{df_i(x,v_{i+1})}{df_{i-1}(v_{i-1},x)} < 0$. Taking $\alpha \to \infty$, we see that we need

$$\frac{f_{i-1}(v_{i-1}, x)}{f_i(x, v_{i+1})} = 1,$$

implying that the lifetimes of adjacent links should be *equalized* in order to optimize the objective function of Equation 2. Note that this is only a necessary condition and not a sufficient one, i.e., not all configurations that result in equal lifetimes of adjacent links will be the solution of the optimization problem under consideration. However, *any solution of the optimization problem will satisfy the above mentioned property.*

The above structure also holds good for the case where the speeds of the intermediate nodes are restricted to a finite discrete set. However, it is obvious that exact equalization of the lifetimes of adjacent links is not achieved due to the lack of the choice of being able to freely choose the speeds for the intermediate nodes. We discuss this issue next.

Let $\underline{v}^* = (v_1^*, \ldots, v_M^*)$ be an optimal speed assignment of intermediate nodes for the problem of Equation 2, for given values of M, v_0, v_{M+1} and $\underline{d}$. Assume that $\underline{v}^*$ is the optimal speed assignment for all values of $\alpha > \alpha^*$ (so that the optimal policy remains unchanged, as proved in Theorem 4.2). If the speed of a j^{th} intermediate node of an optimal route is changed from v_j^* to some fixed value v_j then, for all values of $\alpha > \alpha^*$, we obtain the following relation after some simple algebra,

$$f_{j-1}(v_{j-1}^*, v_j^*) \leq \left[f_{j-1}(v_{j-1}^*, v_j)^{\alpha} + f_j(v_j, v_{j+1}^*)^{\alpha} - f_j(v_j^*, v_{j+1}^*)^{\alpha}\right]^{\frac{1}{\alpha}}.$$

Restricting ourselves to only odd values of $\alpha > \alpha^*$, we note that the right hand side of the above relation is the l_α norm of the three-dimensional vector $(f_{j-1}(v_{j-1}^*, v_j), f_j(v_j, v_{j+1}^*), -f_j(v_j^*, v_{j+1}^*))$. Since the third element of this vector is non-positive and the relation is valid for all values of $\alpha > \alpha^*$, the right hand side of the relation above converges to $\max(f_{j-1}(v_{j-1}^*, v_j), f_j(v_j, v_{j+1}^*))$. This is valid since Lemma 1 is independent of whether the limit is achieved for only odd values of α. Hence we get, for any allowed choice of v_j

$$f_{j-1}(v_{j-1}^*, v_j^*) \leq \max(f_{j-1}(v_{j-1}^*, v_j), f_j(v_j, v_{j+1}^*)).$$

Similarly, we get

$$f_j(v_j^*, v_{j+1}^*) \leq \max(f_{j-1}(v_{j-1}^*, v_j), f_j(v_j, v_{j+1}^*)).$$

In particular,

$$f_{j-1}(v_{j-1}^*, v_j^*) \leq \min_{v_j} \max(f_{j-1}(v_{j-1}^*, v_j), f_j(v_j, v_{j+1}^*)).$$

Since $f_{j-1}(\cdot,\cdot)$ and $f_j(\cdot,\cdot)$ are convex functions of v_j, their pointwise maximum is also a convex function of v_j. Hence, the minimum of the pointwise maximum is attained at one of the intersection points of $f_{j-1}(\cdot,\cdot)$ and $f_j(\cdot,\cdot)$ where they attain equal values. (This is indeed the case in our situation because the two functions under consideration are actually piecewise linear and intersect at at least one point so that one of these points of intersection forms the minimum).

Moreover it is easy to show in the present case that, the minimizer under consideration will lie between v^*_{j-1} and v^*_{j+1}. If there is no other allowed speed in between these two speeds, then the minimizer is one of these two itself. This observation further simplifies the search for an optimizer since now the speeds of the intermediate nodes are known to be lieing between the speeds of the source and destination nodes. This *monotone* transition of the speeds of intermediate nodes in an optimal policy is confirmed by our simulation study which we discuss next.

6 Simulation Study of an iv-MANET

In order to validate our structural heuristics on the solutions to the optimization problems of Equations 1 and 2 developed in the previous sections, we have developed a simulator for an iv-MANET. The simulator is based on the model and assumptions proposed in Section 3 and is implemented such that the nodes move in their lanes in a discrete time space. A node in lane i transits to any of the adjacent lanes at the beginning of a time slot of length 0.1 seconds and the transition takes place with probability $1-p_i$. Given that a node in lane i transits, the transition is to lane j with the same lane transition probability $P_{i,j}$ as discussed before. For our simulations we consider the probabilities $p_1 = \cdots = p_L = p$ to be identical for all the lanes. The probability p is related to μ_i by the relation $\frac{1}{1-p} = \frac{0.1}{\mu_i}$ and for $\frac{R}{s_i} << \frac{1}{\mu_i}$, it is equivalently said that $p \rightarrow 1$.

The simulator computes the expected link lifetimes of all possible links by exhaustively simulating over all possible speed assignments $\underline{v}$ of the intermediate nodes for a given scenario of M intermediate nodes, L lanes, the inter-node distance vector $\underline{d}$, speeds of source and destination v_0 and v_{M+1}, transmission range R, source and destination seperation D and the probability p. Once an exhaustive set of link lifetimes for all possible values of $\underline{v}$ is obtained by employing this brutforce method, either of the objective functions of Equation 1 or 2 is applied over this set to obtain an optimal speed assignment policy.

6.1 Simulation scenarios

A car battery operated mobile device has a typical transmission range of around 200 meters. We therefore consider inter-node distances of an iv-MANET to vary from 140 to 200 meters and transmission range of 200 meters is considered for all the simulation scenarios. It has been shown in a previous work [11] that large number of hops in an ad hoc network can significantly degrade the TCP throughput performance. Based on this result, we consider the number of hops $(M+1)$ to vary from 2 to 7 only and the distance between the source and destination nodes is varied from 800 to 1200 meters. We perform simulations for the number of lanes L varying from 2 to 6 and unless explicitly stated in the discussion on the simulation results, the associated speeds are taken as shown in the table that follows,

l	s_l (m/s)	$\approx s_l$ (km/hr)
1	14	50
2	17	60
3	22	80
4	30	110
5	42	150
6	55	200

Simulations were carried out for a large set of real life scenarios for which heuristics has been developed in the previous sections using analytical models. In the following part of this section we discuss the various scenarios that were simulated and compare their results with the structural results and heuristics obtained analytically.

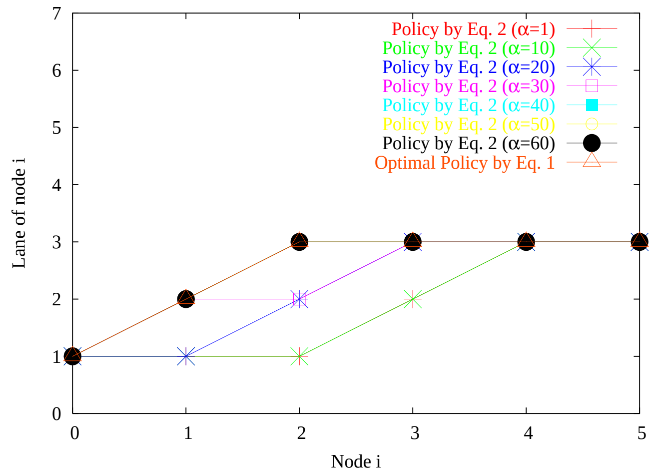

Figure 3: Policies showing existence of α^* of Theorem 4.2

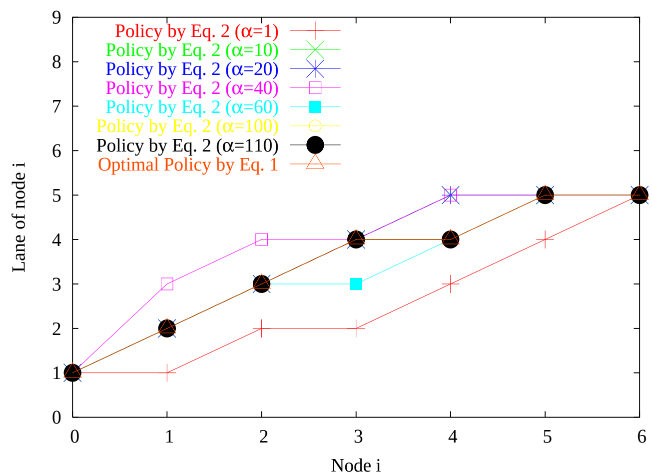

Figure 4: Policies showing existence of α^* of Theorem 4.2

1. *Existence of α^* of Theorem 4.2*: In Figure 3 we consider the scenario $L = 4$, $M = 4$, $v_0 = s_1$, $v_{M+1} = s_3$, $p = 0.9995$, $D = 800m$ and $\underline{d} = (162, 153, 155, 158, 172)$. We plot the various optimal policies obtained using both the objective functions of Equation 1 and 2 and we clearly see that the optimal policies obtained by Equation 2 coincide with that obtained by Equation 1 for $\alpha \geq 40$. Figure 4 shows similar characteristics for another scenario of $L = 5$, $M = 5$, $v_0 = s_1$, $v_{M+1} = s_5$, $p = 0.99$ $D = 1000m$ and $\underline{d} = (168, 162, 167, 166, 163, 174)$ with the optimal policies coinciding for $\alpha \geq 100$. This validates our Theorem 4.2 on the existence of α^*.

2. *Structure of optimal policy for $L = 2$ (Section 5.1)*: Figure 5 shows plots of optimal policies obtained from Equation 1 for $L = 2$, $M = 1$, $p = 0.9995$, $D = 300m$ and $\underline{d} = (158, 142)$. Figure 6 shows a similar plot for $L = 2$, $M = 5$, $p = 0.9995$, $D = 1000m$ and $\underline{d} = (162, 164, 165, 161, 170, 178)$. The former figure clearly illustrates that under optimality, an intermediate node is assigned the speed of the farther node for $s_{ij} > 0$ and that of the nearer node for $s_{ij} < 0$. The latter figure shows that a next hop in an optimal route goes across lanes of different speeds when the expected remaining lifetimes are the maximum possible.

3. *Lifetime Equalization over continuum set of speeds for $L \geq 3$ and $\frac{R}{s_i} << \frac{1}{\mu_i}$ (Section 5.2)*: In Figure 7 we consider the scenario $L = 3$, $M = 1$, $v_0 = s_3 = 30m/s$, $v_2 = s_1 = 14m/s$, $p = 0.99999$, $D = 300m$ and $\underline{d} = (143, 157)$. In order to be able to validate the equalizing structure obtained in Section 5.2 over a continuum set of intermediate node speeds, we vary the speed associated with lane 2 from $14m/s$ to $30m/s$ in small steps of $1m/s$ and plot the link lifetimes for each such speed of lane 2

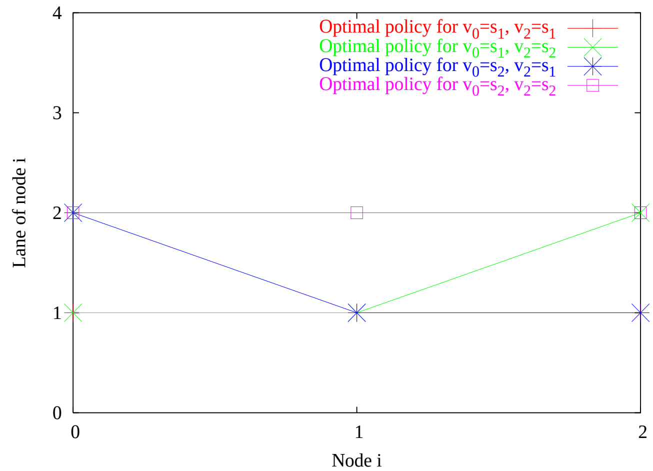

Figure 5: Structure of optimal policy for $L = 2$

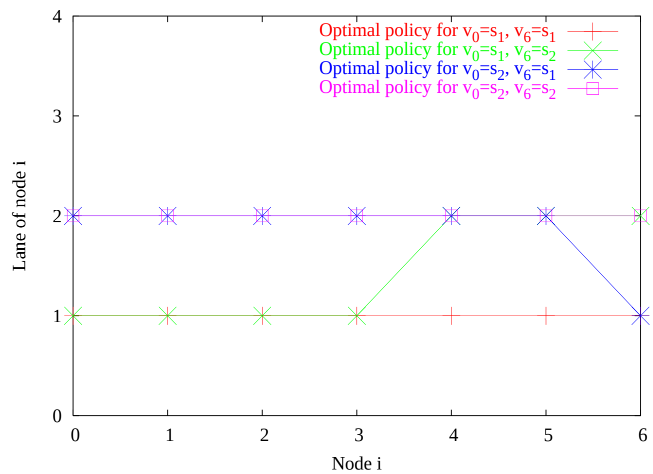

Figure 6: Structure of optimal policy for $L = 2$

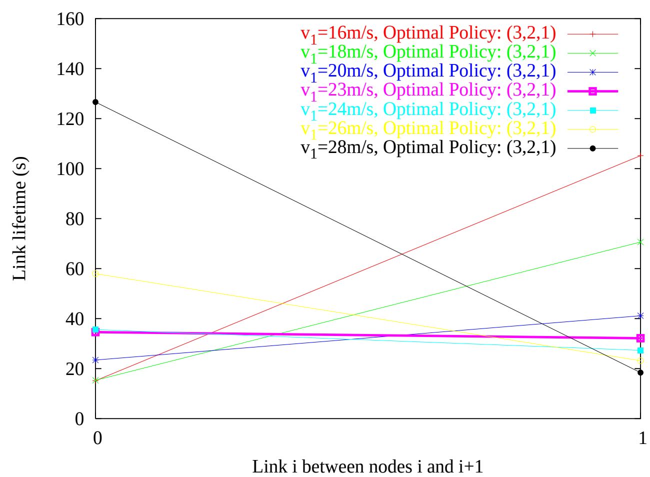

Figure 7: Lifetime Equalization over continuum set of speeds

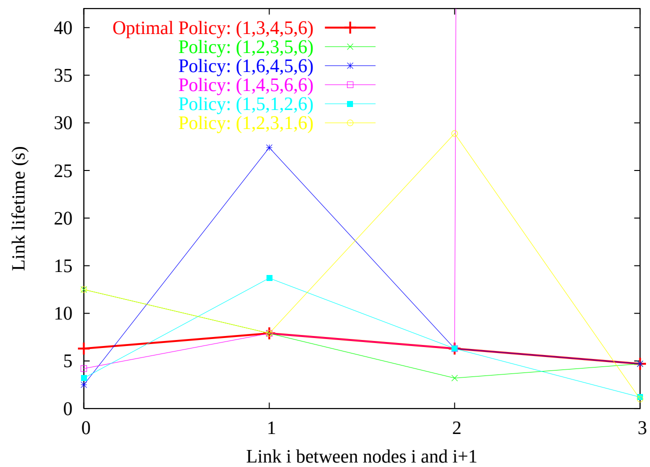

Figure 8: Lifetime Equalization under the optimal policy

seperately. This allows the only intermediate node 1 to be assigned one of the quasi-continuum set of speeds for the optimization problem of Equation 2. It is seen in the figure that under optimality, for varying values of v_1, the optimal lifetimes of the links between node 0 and 1 and node 1 and 2 are different. However at $v_1 = 23m/s$ the optimal lifetimes of the two adjacent links are almost equal thus confirming our result obtained in Section 5.2 that the lifetimes of adjacent links should be equalized in order to optimize the objective function of Equation 2. Infact, it can be observed that we obtain the maximum of the least of the two lifetimes for speed $v_1 = 23m/s$ and the optimal lifetimes obtained for other values of v_1 are not truely optimal because of the unavailability of the choice of speed $23m/s$ in those scenarios.

4. *Lifetime Equalization under the optimal policy for $L \geq 3$ and $\frac{R}{s_i} << \frac{1}{\mu_i}$*: To strengthen our validation of the lifetime equalization structure discussed in the previous note, in Figure 8, we plot the link lifetimes for a subset of the all possible speed combinations (including the optimal combination) for a given particular scenario. It is clearly seen in these figures that at the optimal policy the link lifetimes are equalized and the maximum of the least lifetimes is obtained at the optimal policy, by definition. Figures 9 and 10 illustrate the same result for two more set of given scenarios.

   Figure 8 uses the scenario $L = 6$, $M = 3$, $v_0 = 14m/s$, $v_4 = 30m/s$, $p = 0.99999$, $D = 700m$ and $\underline{d} = (160, 175, 180, 185)$. Figure 9 uses the same scenario as Figure 8 except for $\underline{d} = (182, 162, 176, 180)$. Figure 10 uses the scenario $L = 55$, $M = 3$, $v_0 = 14m/s$, $v_4 = 30m/s$, $p = 0.99999$, $D = 700m$ and $\underline{d} = (169, 166, 167, 198)$.

5. *Monotone speed transitions for $L \geq 3$ and $\frac{R}{s_i} << \frac{1}{\mu_i}$ of Section 5.2* : The phenomenon of monotone transition of speeds from a given node to any one of the next hops in a route, is observed in all of the figures discussed until now. Specifically, Figure 11 shows the optimal policy assignment for the scenario $L = 5$, $M = 5$, $v_0 = s_1$, $v_6 = s_3$, $p = 0.99999$, $D = 1000m$ and $\underline{d} = (168, 162, 167, 166, 163, 174)$ where all the intermediate nodes have speeds v_i such that $v_0 \leq v_i \leq v_6$. This validates the corresponding analytical result stated in Section 5.2.

7 Conclusion

Designing efficient routing protocols for iv-MANETs is quite a challenging task owing to the fast speed of nodes and mobility constraints on the movement of nodes in a straight line highway. An attempt has been made in this paper to help accomplish this task better.

The problem of optimal multi-hop routing in an iv-MANET has been identified to be composed of three sub-optimization problems. These sub-problems aim at achieving an *optimal* route performance. The three sub-problems consist of finding the number of hops, the distances between the intermediate

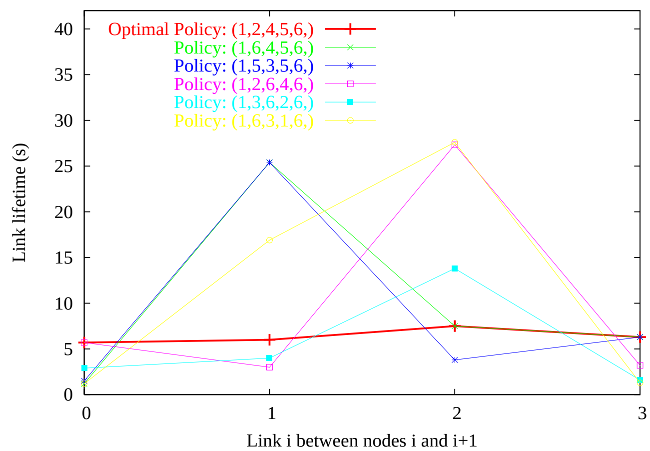

Figure 9: Lifetime Equalization under the optimal policy

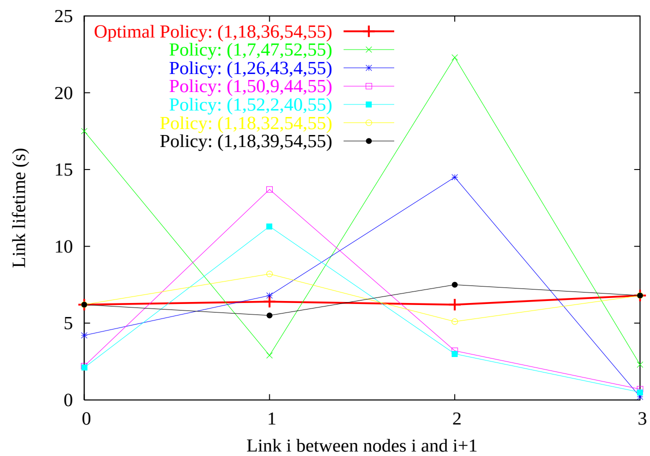

Figure 10: Lifetime Equalization under the optimal policy

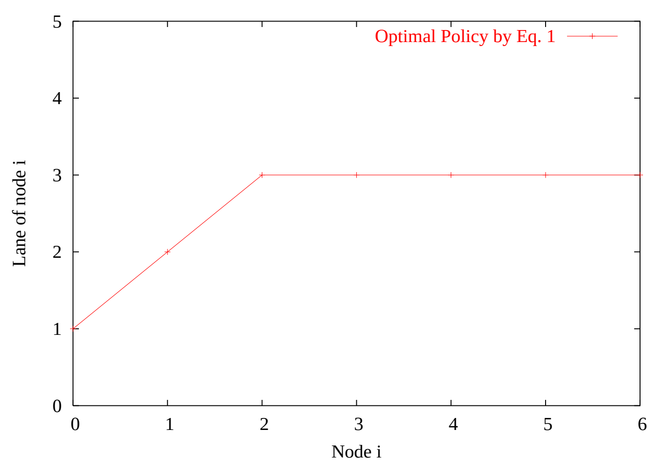

Figure 11: Monotone speed transitions for $L \geq 3$ and $\frac{R}{s_i} << \frac{1}{\mu_i}$

relay nodes and the speeds of intermediate nodes, respectively. Each of these sub-problems have their own optimization objectives. For a given number of hops and inter-node distances, we have considered the problem of *characterizing* the solution of the third sub-problem assuming that the objective is to find the speeds of the intermediate nodes, that achieve maximum route lifetimes.

The analysis of this paper has established that the solution of the optimization problem under consideration tends to equalize the lifetimes of adjacent links in a route. Moreover, there is a monotone variation of the speeds of the intermediate nodes under the optimal policy. These solution structures have also been confirmed with an extensive simulation study. The structures obtained are of considerable practical interest as they reduce the space over which a real-time routing algorithm (for e.g., the Dijkstra's algorithm) would search for the optimal policy.

References

[1] P. Samar and S.B. Wicker. "On the Behavior of Communication Links of a Node in a Multi-Hop Mobile Environment", MobiHoc, May 2004.

[2] P. Gupta and P.R. Kumar. "A System and Traffic Dependent Adaptive Routing Algorithm for Ad Hoc Networks", 36th IEEE Conference on Decision and Control, Dec, 1997.

[3] S. Agarwal, A. Ahuja, J.P. Singh and R. Shorey. "Route-Lifetime Assessment Based Routing (RABR) Protocol for Mobile Ad Hoc Networks", IEEE ICC, June 2000.

[4] J.J. Blum, A. Eskandarian and L.J. Hoffman. "Challenges of Intervehicle Ad Hoc Networks", IEEE Transactions on Intelligent Transportation Systems, Vol. 5, No. 4, Dec 2004.

[5] R. G. Gallager. "Discrete Stochastic Processes", Kluwer, 1998.

[6] R. Dube, C.D. Rais, K-Y. Wang and S.K. Tripathi, "Signal Stability based Adaptive Routing (SSA) for Ad Hoc Mobile Networks", IEEE Personal Communications, pp. 36-45, Feb, 1997.

[7] J. Broch, R. Punnoose, P. Nikitin and D. Stancil. "Optimizing wireless network protocols using real-time predictive propagation modeling", IEEE Radio and Wireless Conference, 1999.

[8] S-J. Lee, W. Su and M. Gerla. "Ad hoc Wireless Multicast with Mobility Prediction", ICCN, pp. 4-9, 1999.

[9] D. Turgut, S.K. Das and M. Chatterjee. "Longevity of Routes in Mobile Ad Hoc Networks", IEEE VTC, pp. 2833-2837, Spring 2001.

[10] Y-C. Tseng, Y-F. Li and Y-C. Chang. "On Route Lifetime in Multihop Mobile Ad Hoc Networks", IEEE Transactions on Mobile Computing, vol. 02, no. 4, pp. 366-376, Oct-Dec, 2003.

[11] M. Gerla, K. Tang and R. Bagrodia. "TCP Performance in Wireless Multihop networks", IEEE WMCSA, Feb, 1999.

[12] J. Luo and J.P. Hubaux. "A Survey of Inter-Vehicle Communication", Technical Report, EPFL, Switzerland, 2004.

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ISSN 0249-6399