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Abstract: In this research report, we study the algorithmic complexity of different broadcast and multicast ad hoc routing problems given a wireless medium.

Key-words: ad hoc networks, multicast routing, complexity, NP-completeness

NP-Complétude du routage multicast ad hoc

Résumé : Dans ce rapport de recherche, nous étudions la complexité algorithmique de plusieurs problèmes de routage multicast et broadcast ad hoc étant donné un médium radio.

Mots-clés : réseaux ad hoc, routage multicast, complexité, NP-complétude

1 Introduction

In wired networks, the search for a partial broadcast scheme (multicast scheme) is a hard task. Generally, depending on the evaluation criteria, the problem is equivalent to the search for a Steiner tree [3,6,7,8] in the network connectivity graph. A Steiner tree is a minimum weighted tree covering a designated subset of vertices, the multicast group members, in a weighted graph. Computation of a Steiner tree is a NP-hard problem. However, some subproblems such as the search for a broadcast scheme with a constraint on the number of forwarders may be done in polynomial time.

Intuitively, partial broadcast seems both simpler and harder in an ad hoc environment. Simpler because it is possible to reach several hosts at once, thanks to the diffusion nature of the radio medium also known as the *Wireless Multicast Advantage* (WMA), but harder because this same property makes it hard to compare the cost of two different paths without taking into account the whole set of nodes reached by the different forwarding operations.

To theoretically study the ad hoc multicast problem, we abstract an ad hoc network into its connectivity graph: the networks is modeled by a graph $G = (V, E)$ where V is the set of ad hoc nodes and E is all possible communication links, *i.e.* $(u, v) \in E$ if and only if u is in the coverage area of v, *i.e.* $u \in \Gamma_1(v)$, and reciprocally. Moreover, we apply the following communication rule: (1) while a vertex emits, the packet is sent through all its edges (Δ-port emission $[1, 4]$); (2) if two packets transit simultaneously through the same edge, they can not be understood by any of the adjacent vertices (half-duplex communication $[1, 4]$); (3) finally, if a vertex receives two packets simultaneously, it can not understand any of them (1-port reception $[1, 4]$).

Remark. Broadcasting being a subproblem of multicasting, we limit ourselves to the search for broadcast schemes in the complexity analysis section. Indeed, if the search for a broadcast scheme under a given constraint (*e.g.* number of steps, number of forwarding operations...) is NP-complete, then the search for a multicast scheme under the same constraint is also NP-complete.

In this report, we will study the existence of broadcast schemes under three different constraints: number of steps, number of forwarders, number of forwarding operations. We will prove that the problem is NP-complete under the three constraints.

2 Broadcast in minimum time (number of steps)

We first study the existence of broadcast schemes in less than k steps. A k step broadcast scheme is a succession of k subsets of V, $\mathcal{F}_k = V_1, V_2, ..., V_k$, describing the forwarders for each step, *i.e.*, V_i is the set of forwarders for step i and V_1 is limited to the source vertex s. In our ad hoc communication model, a broadcast scheme rooted at s is valid if and only if:

- $V_1 = \{s\}$
- $\forall i \geq 2, \forall u \in V_i, \exists 1 \leq j < i, \exists v \in V_j$ such as:
  - $\forall k \leq j, u \notin V_k$

  - $u \in \Gamma_1(v)$
  - $\forall w \in V_j$ such as $w \neq v, u \notin \Gamma_1(w)$

- $\forall u \notin \bigcup_{i=1}^{i=k} V_i, \exists 1 \leq j < i, \exists v \in V_j$ such as:
  - $u \in \Gamma_1(v)$
  - $\forall w \in V_j$ such as $w \neq v, u \notin \Gamma_1(w)$

where $\Gamma_1(u)$ is the 1-neighborhood of u, *i.e.*, $v \in \Gamma_1(u) \Leftrightarrow (u,v) \in E$.

Problem 1. [B_{adhoc}] Given $G = (V, E)$, $s \in V$ et $k \in \mathbb{N}$, given the communication rules described previously, is there a valid broadcast scheme rooted at s with at most k steps?

This problem belongs to NP. Given a broadcast scheme rooted at s, the validity of the scheme and its depth can be computed in polynomial time. To show that B_{adhoc} is NP-complete, we propose a reduction from the 3DM decision problem.

Problem 2. [3DM] Given $q \in \mathbb{N}$, X, Y, Z three distinct sets of cardinality q and $M \subset X \times Y \times Z$, is there a maximum matching M' in M ? In other terms, is there M' such as:

- $|M'| = q$
- $M' \subset M$
- $\forall (c_1 = (x_1, y_1, z_1), c_2 = (x_2, y_2, z_2)) \in M' \Rightarrow x_1 \neq x_2, y_1 \neq y_2, z_1 \neq z_2$

Theorem 1 [NP-completeness of 3DM [2]]. 3DM is NP-complete.

Given (q, X, Y, Z, M), an instance of 3DM, we construct $(G = (V, E), s, k)$ an instance of B_{adhoc} using the following function. Each element of $M = \{m_1, ..., m_{|M|}\}$, $X = \{x_1, ..., x_q\}$, $Y = \{y_1, ..., y_q\}$ and $Z = \{z_1, ..., z_q\}$ is associated to a vertex (called m_i, x_i, y_i or z_i). We consider

- $V = \{m_i | 1 \leq i \leq |M|\} \cup \{x_i | 1 \leq i \leq q\} \cup \{y_i | 1 \leq i \leq q\} \cup \{z_i | 1 \leq i \leq q\} \cup \{s\}$
- $E = \{(s, m_i), (m_i, x_j), (m_i, y_{j'}), (m_i, z_{j''}) \mid m_i = (x_j, y_{j'}, z_{j''}) \in M, 1 \leq i \leq |M|\}$

To conclude this instance of B_{adhoc}, we set $k = 2$. Let us call $\mathcal{T}$ this transformation. $\mathcal{T}$ can be computed in polynomial time.

Lemma 1. Let (q, X, Y, Z, M) an instance of 3DM and $F(q, X, Y, Z, M) = (G = (V, E), s, k)$ the corresponding instance of B_{adhoc} built using $\mathcal{T}$. There is a maximum matching M' in M if and only if there is a broadcast scheme from s in G with at least $k = 2$ steps.

Proof. Suppose there is a maximum matching $M' = \{m'_1, ..., m'_q\} \subset M$. The 2-step broadcast scheme $V_1 = \{s\}, V_2 = \{m'_1, ..., m'_q\}$ is valid in the ad hoc communication model. All vertices of G are reached: the first forwarding step covers thee vertices of M and the second one covers the vertices of $X \cup Y \cup Z$ (M' is maximum). The half-duplex property is respected: s is the only forwarder at step one and for the second step, elements of M are not adjacent. Finally, there is no multiple reception by a single vertex at step one, s being the only node to emit, and the only multiple reception at step 2 is for s (M' is a matching) which already has the information.

Consider a valid broadcast scheme B in $k \leq 2$ steps in G. Given the diameter of G, 2, the broadcast scheme has necessarily two steps. At step one, the only node to emit is s. At step two, only a subset of $M \cup \{s\}$ may forward the data. Given the fact that s has already emitted, we can derivate a broadcast B' from B in two steps for which only a subset M' of M forwards the data at step two. As B' is a broadcast, any node of $X \cup Y \cup Z$ is adjacent to M'. Hence $|M'| = q$ and M' is maximal. Given the communication rules and given the fact that B' is a broadcast scheme, we can deduce that M' is a matching. □

Theorem 2 [NP-completeness of B_{adhoc}]. B_{adhoc} is NP-complete.

Remark. This problem is also NP-complete [2] for a wired communication model (1-port).

3 Broadcast with a minimum number of forwarders

We are now interested in the search for broadcast schemes with a minimum number of forwarders. As for the number of steps, we show that the associated decision problem is NP-complete.

Problem 3. [B^2_{adhoc}] Given $G = (V, E)$, $s \in V$ and $k \in \mathbb{N}$, given the communication rules described previously, is there a broadcast scheme from s in G with at most k forwarders?

This problem belongs to NP. Given a broadcast scheme from s, the number of forwarders can be computed in polynomial time. To show that B^2_{adhoc} is NP-complete, we propose a derivation from the MPR [5] decision problem.

Problem 4. [MPR] Given $G = (V, E)$, $s \in V$ and $k \in \mathbb{N}$, is there a MPR set of s with size less or equal to k?

Theorem 3 [NP-completeness of MPR [5]]. MPR is NP-complete.

Given $(G = (V, E), s, k)$ an instance of MPR, we build $(G' = (V', E'), s, k + 1)$ an instance of B^2_{adhoc} with:

- $V' = \{s\} \cup \Gamma_1(s) \cup \Gamma_2(s)$

- $E' = \{(s,u)|u \in \Gamma_1(s), (s,u) \in E\} \cup \{(u,v)|(u,v) \in E, u \in \Gamma_1(s), v \in \Gamma_2(s)\}$

Let us call $\mathcal{T}_2$ this transformation. $\mathcal{T}_2$ can be computed in polynomial time.

Lemma 2. Given $(G = (V,E), s, k)$ an instance of MPR and $F_2(G = (V,E), s, k) = (G' = (V', E'), s, k' = k+1)$ the corresponding instance of B^2_{adhoc} built using $\mathcal{T}_2$. There is a MPR set of s with at most k MPRs if and only if there is a broadcast scheme from s in G' with at most $k+1$ forwarders.

Proof. Suppose there is a MPR set $M = \{x_i | 1 \le i \le q, x_i \in \Gamma_1(s)\} \subset V$ of s. Consider the broadcast scheme B rooted at s in G' such as at first step, only s emits and at step i, only x_i emits. By construction, B has $q+1 \le k+1$ forwarders. Moreover, as M is a MPR set of s, B reaches all nodes of G'.

Suppose there is a broadcast scheme B rooted at s in G' with at most $k+1$ forwarders. From B, we can derivate a broadcast scheme B' which forwarders belong to $\{s\} \cup \Gamma_1(s)$. Indeed, by construction of G', the remaining nodes belong to $\Gamma_2(s)$ and are not adjacent. B' has at most $k+1$ forwarders. Let us call M the forwarders of B'. We have $M = \{s\} \cup M'$ with $M' \subset \Gamma_1(s)$ and $|M'| \le k$. As B' is a broadcast, we know that M' is adjacent to all nodes of V'. As $\Gamma_2(s) \subset V'$, M' is a MPR set of s in G. Moreover, $|M'| \le k$. $\square$

Theorem 4 [NP-completeness of B^2_{adhoc}]. B^2_{adhoc} is NP-complete.

Remark. In the case of a wired communication model, this problem is also NP-complete. An identical proof can be applied.

4 Broadcast with a minimum number of forwarding operations

We are now interested in the number of forwarding operations required to perform a broadcast. As opposed to the wire case, this problem is NP-complete.

Problem 5. [B^3_{adhoc}] Given a graph $G = (V,E)$, $s \in V$ and $k \in \mathbb{N}$, given the communication rules described previously, is there a broadcast scheme from s in G with at most k forwarding operations?

It is interesting to notice that this problem is similar to B^2_{adhoc}. Given our communication model, Δ-port in emission, any broadcast scheme with q forwarders and $l \ge q$ forwarding operations can be trivially derivated in a broadcast scheme using the same forwarders but with only q forwarding operations. Thanks to the *Wireless Multicast Advantage* (WMA), one emission is enough for a forwarder to cover all its neighbors.

Theorem 5 [NP-completeness of B^3_{adhoc}]. B^3_{adhoc} is NP-complete.

Proof. Trivial given the preceding remark.

Remark. With a 1-port emission communication model, this problem is polynomial. Indeed, any graph with n vertices admit a minimal broadcast scheme with $n-1$ forwarding operations. Such a scheme may be computed using a classical spanning tree algorithm. However, the question of a multicast scheme in limited number of forwarding operations is NP-complete [2].

5 Conclusion

To conclude, table 1 summarizes the complexities of multicast and broadcast problems in function of the communication model and constraint.

Communication type	$\leq k$ steps	$\leq k$ forwarding op.	$\leq k$ forwarders
Broadcast wire	NP-Complete	P	NP-Complete
Multicast wire	NP-Complete	NP-Complete	NP-Complete
Broadcast ad hoc	NP-Complete	NP-Complete	NP-Complete
Multicast ad hoc	NP-Complete	NP-Complete	NP-Complete

Table 1: Complexity of several routing problems.

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