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Abstract: A model describing the discovery by means of traceroute of the topology an Internet access network with a tree structure is proposed in this paper. This model allows us to assess the efficiency of traceroute procedures to determine the complete set of routers of the network. Under some stochastic assumptions, explicit analytical expressions are obtained for the mean number of routers discovered when a subset of the stations is used in the traceroute procedure. Several tree architectures are then discussed when the total number of routers gets large, and asymptotic expansions are derived. The results are compared with real data obtained from measurements.

Key-words: Internet, topology, traceroute, trees.

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Modèles probabilistes pour l'inférence de topolgie dans les réseaux en arbre

Résumé : Un modèle probabiliste de l'algorithme de découverte des réseaux par traceroute est étudié dans cet article

Mots-clés :

1 Introduction

Over the past few years, a huge research effort has been devoted to the study of Internet topology. The absence of hierarchical structure, the lack of agreement between wide area network operators, local operators (e.g., metropolitan area networks), and service providers has led to a highly disordered expansion of the Internet. When considering the global topology of the Internet, different lines of investigations can be followed in order to describe the underlying structure of the network. One popular approach consists of using the theory of complex networks and random graph models (e.g. Erdös and Rényi, small worlds, etc.), see Albert and Barabási [1] and Newman and Watts [2]. This line of investigation has all the more been supported by the fact that it has empirically been observed that the degree of routers obeys a power law distribution as reported in the celebrated paper by Faloutsos *et al.* [3]. This approach consists of representing the Internet by means of a random graph satisfying some local properties observed by measurements. It is nevertheless worth noting that a model based on incomplete data may lead to erroneous interpretation of the reality, as argued in Radoslavov *et al.* [4], Dall'Astra *et al* [5].

Many projects have been initiated in order to discover the global topology of the Internet (by CAIDA, AT&T, etc.). Most of them rely on the use of the traceroute capability offered by routers. By using traceroute, one can discover the routers along the path between a source and a destination. Even though traceroute may yield unreliable data, since paths may change within the network and all routers do not respond to a traceroute request, it gives an indication on the global structure of the network (see for instance [6]). In particular, it allows the embedding of the Internet graph into a non Euclidean multi-dimensional space in order to evaluate the distance between two hosts Shavitt and Tankel [7] and Lakhina *et al.* [8].

The inference of network topology is highly relevant for studying and possibly anticipating the propagation of attacks through the whole of the Internet (worms, DDoS, etc.). The magnitude of an attack greatly depends on how the network is structured and it is of prime importance to be able to locate the weak points of the network in order to implement tools to block the propagation of attacks.

In all the above cited studies, the network appears as black box whose structure is inferred by injecting probes and by reconstructing the global topology by different methods (tomography). The situation is however quite different for an network operator, who is able to know the topology of his own AS. By listening the routing messages exchanged between the different routers, it is possible to reconstruct the physical as well as the routing graph of the network (IGP information). Listening BGP routing messages can in addition be used to know, to some extent, what happens outside a network operator's AS. IGP routing information also gives the state of the different links within the network. The instability of links is an important factor to account for when inferring the topology of an AS since the up or down states of interfaces or links impacts the routing graph and then the information obtained by a traceroute procedure.

In addition, for inferring the topology of the Internet, it is important to take into account the different components of the network. Indeed, the global Internet is composed of customer premises networks (LANs, home networks), access networks (GigaBit Ethernet or ATM networks possibly enhanced with level 3 functions), collect networks (including PoPs) and transit networks. The last ones, owned by tiers one operators, are composed of very high speed links (e.g., OC 12 or OC 48 links) connecting different collect networks, which are most of the time concentration networks in charge of collecting and distributing data among different users. Furthermore, at a macroscopic level, users connected to a collect network are geographically close one to each other (for instance in a same country). The topology of a collect network is, in first approximation, a tree, with possibly cross links at intermediate levels. Note that the existence of collect network naturally provides the global Internet with a small world structure. Collect networks are dense networks connected one to each other via high speed links between high capacity routers.

In this paper, we focus on the topology discovery of collect networks. While transit networks are composed of a relatively small number of routers, the structure of collect networks is much more intricate and involves a large number of routers in a rather small geographical area. The topology of such a network is not exactly a tree but this model can be used as a first approximation. Indeed, when inferring the topology of a network, we have to take into account different factors, in particular the instability of links, the possibility that some routers do not respond to traceroute requests, etc. Thus, we are led to make some worst case assumptions on the behavior of the network. Assuming a tree structure amounts to supposing that the cross links at intermediate levels cannot be seen by a traceroute exploration. Thus, any traceroute message has to go up to the root in order to discover the links along the path between a source and a destination located in areas connected to the Internet by two different sons of the root node.

We consider a non-homogeneous tree network in which the degree of a node may depend upon the depth and place different traceroute capable hosts randomly on the leaves of the tree. We suppose that these hosts exchange traceroute messages and we compute the number of links discovered by the traceroute procedure. We specifically show that the number of links discovered rapidly increases for moderate values of the number of hosts but slowly increases when the number of hosts becomes large. This indicates that the discovery of the complete topology of a network requires a massive deployment of hosts exchanging traceroute messages.

The organization of this paper is as follows: In Section 2, the main variables and the topologies of trees are introduced. Section 3 gives an explicit expression for the mean number of nodes discovered by a traceroute procedure used by a set of stations. A connection is established with classical coupon collector problems. In order to have some insight into the impact of the topology of networks, Section 4 deals with asymptotic results, when the size of the network tends to infinity, for the formulas obtained in Section 3. In particular the rate of growth of the number of discovered routers is analyzed in great detail. In Section 5, we investigate the degree of a node, which is discovered by the traceroute procedure. The results obtained in the previous sections are compared in Section 6 against some experiments on real data from the Scan+Lucent map obtained from traceroutes collected by the Internet Mapping project at Lucent Bell Laboratories. Concluding remarks are presented in Section 7

2 Notation and definitions

2.1 Assumptions and notation

Throughout this paper, we assume that the graph of the network is a partially homogeneous tree with height $n \geq 2$. For $1 \leq j < n$, a node of the jth level has d_j sons. The total number of nodes at level j is denoted by l_j and N is the total number of routers in the network. We clearly have

$$l_1 = 1, \quad j > 1,\ l_j = \prod_{k=1}^{j-1} d_k \quad \text{and} \quad N = \sum_{j=1}^{n} l_j.$$

For $p \geq 1$, the set S of traceroute capable hosts is composed of p elements, taken at random on the last (nth) level of the tree; the nodes of this last level are referred to as the leaves of the tree network. Several traceroute capable hosts may be attached to a single leaf. In practice, when considering an ADSL collect network, the leaves of the tree network are the access routers to the IP backbone network. We assume that there is no restriction on the number of hosts, which can be attached to a router.

The traceroute capable hosts are terminals of customers, who performed traceroute procedures. In the following, each of these users knows the IP addresses of the others in order to perform a traceroute. This implicitly assumes that there exists a server which knows the IP address of each user willing to perform a traceroute; this server is required since IP addresses for ADSL customers are dynamic.

Note it is also possible that any hosts may send traceroute messages to hosts it does not know explicitly but by guessing the IP address. This may however generate a large number of test packets and very unreliable information. This why we assume the existence of a server in charge of collecting the IP addresses of hosts performing traceroute.

With the above assumptions, any pair of hosts in S exchanges packets to determine the nodes between them. The routers discovered can embedded into a minimal subtree of the total graph, referred to as the spanning tree of the discovered routers. Its height is denoted by $H(N, p)$. Clearly, for $j \leq n$, one has $H(N, p) < n - j$ when none of the routers at level j has been discovered.

One denotes by $D(N, p)$ the total number of nodes between level 1 and level n discovered by the traceroute procedure.

2.2 A dynamical picture for the placement of hosts

The random placement of the traceroute capable hosts at the leaves of the tree network can be seen as a dynamical stochastic process as follows: initially the p points are at the root of the tree, then they are thrown randomly on the d_1 sons; at the next step the subset of points at level 2 are thrown randomly among the d_2 sons, and so on ... until they reach the leaves of the tree.

With this description, when the points are at level j, let $(X_{1j}, \dots, X_{l_j j})$ denotes the vector describing the number of points at each of the l_j nodes at level j. This vector

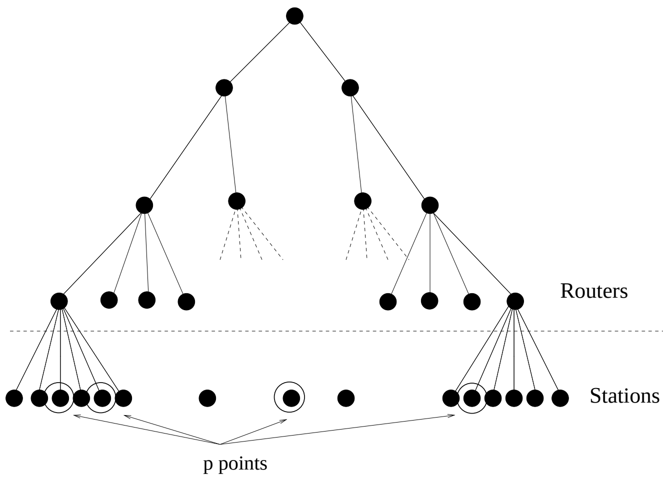

Figure 1: Partially Homogeneous Tree with d_1=d_2=2, d_3=4, d_4=6

has a multinomial distribution with parameter p and $(1/l_j, \ldots, 1/l_j)$. This means that for $(n_1, \ldots, n_{l_j}) \in \mathbb{N}^{l_j}$ such that $n_1 + \cdots + n_{l_j} = p$,

$$\mathbb{P}\left((X_{1j}, \ldots, X_{l_j j}) = (n_1, \ldots, n_{l_j})\right) = \frac{p!}{n_1! \ldots n_{l_j}!} \frac{1}{l_j^p}.$$

In the following sections, we study the properties of the random variable $D(N, p)$ describing the number of nodes discovered by the traceroute procedure, in particular its mean value as well as its asymptotic behavior when the height n of the tree tends to infinity while the ratio of the number of stations p to the total number of routers N in the network is of the order of some fixed $\lambda > 0$.

2.3 The shape of the network

The above tree model offers some flexibility with regard to the structure of the network, in particular via the choice of the number of sons of a node at level j, denoted by d_j. Ideally, this number should be a random variable and the tree would describe a the dynamics of a Galton-Watson branching process. In this paper, we restrict the analysis to the case when d_j is deterministic.

In spite of the above restriction, one may consider different tree structures for the graph of the network. We shall specifically analyze the different cases defined as follows.

Definition 1. *For a tree with height* n*,*

1. *A regular tree (REG*$_d$ *tree) with degree* d *is such that* $d_j = d \geq 2$*,* $\forall j \in \{1, \ldots, n-1\}$*.*

2. Power law tree with index $\alpha > 0$, $1 \leq j < n$,

— *with increasing growth rate (PLI$_\alpha$ trees) if*
$d_j = \lfloor j^\alpha \rfloor$,

— *with decreasing growth rate (PLD$_\alpha$ trees) if*
$d_j = \lfloor (n-j)^\alpha \rfloor$,

with $\lfloor y \rfloor$, the integer part of $y \in \mathbb{R}$.

As it will be seen in the following, even by restricting to the case of deterministic trees, the impact of the topology is crucial in the speed at which nodes of the network are discovered.

The tree structure with nodes having a degree growing as a power law is motivated by measurements of the Internet. Various studies have already shown that the underlying graphs of the Internet and of the web exhibit the power law property for the degree of nodes. See Faloutsos *et al.* [3].

3 Average number of discovered nodes

3.1 Distribution of the number of nodes discovered by two stations

In a first step, we examine the case of two stations placed at random at the leaves of the tree network and performing a traceroute procedure. We specifically compute the distribution of the number of nodes which are discovered by the two stations. The stations are randomly attached to the leaves of the tree, which are numbered from 1 to l_n. Station 1 sends traceroute packets to the other station. The number of routers discovered with this procedure is denoted by D. From the root of the tree, one denotes by H the maximal level such that the two stations under consideration are contained in the same subtree (see Figure 2).

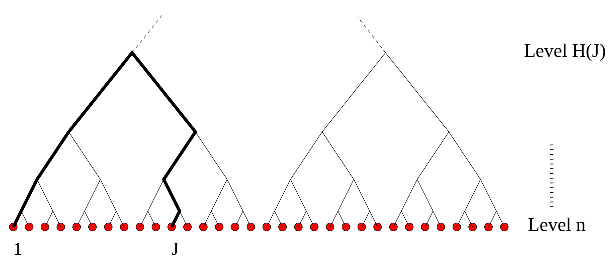

Figure 2: Routers discovered by two stations.

When the two stations are contained in a subtree, the root of this subtree is located at level H. It is obvious that $\mathbb{P}(H \geq 1) = 1$. We have $H \geq 2$ if and only if the two stations are contained in the same subtree issued from one of the d_1 sons of the root. The probability that the two stations are in such a subtree is equal to $1/d_1^2$. Since there are d_1 possibilities, we have $\mathbb{P}(H \geq 2) = 1/d_1$. More generally, we have $H \geq k$ if the two stations are contained in a subtree issued from one of the node at level k. Simple arguments show that we then have

$$\mathbb{P}(H \geq k) = \frac{1}{l_k} = \frac{1}{d_1 ... d_{k-1}}.$$

Since we deal with a tree, the number of routers discovered by the two stations can take the values $2k+1$ for $k = 0, ..., n-1$ and we clearly have by definition $D = 2(n-H)+1$. By combining the different above arguments, we easily come up with the following result.

Proposition 1. *The number of routers discovered by two stations placed at random at the leaves at the tree network has the probability distribution*

$$k = 0, ..., n-1, \quad \mathbb{P}(D \leq 2k+1) = \frac{1}{d_1 \dots d_{n-1-k}}.$$

From the above result, we can easily deduce the following formula for the mean value of D.

Corollary 1. *The mean value of the random variable D is given by*

$$\mathbb{E}(D) = 2n + 1 - 2\sum_{k=1}^{n-1} \frac{1}{d_1 \dots d_k} \tag{1}$$

For a regular tree, we have $d_j = d > 1$ for all j and simple computations show that the mean number of routers discovered by two stations exchanging traceroute messages is given by

$$\mathbb{E}(D) = 2n + \frac{d+1}{d-1} - \frac{2}{(d-1)d^{n-1}}.$$

When n is large, we have $\mathbb{E}(D) \sim 2n$. In addition, under the same condition, the variance of the random variable D satisfies

$$\text{Var}(D) \sim 4n\frac{d+1}{d-1}.$$

The above simple computations show that when n is large, the mean length of the path (in terms of discovered routers) is closed to the longest path through the network, equal to $2n-1$. Hence, even if the ratio of the number of discovered routers to the total number of routers in the network is small:

$$\frac{\mathbb{E}(D)}{N} \sim \frac{2n(d-1)}{d^n},$$

two stations placed at random can derive a fair estimate of the diameter of the network via a traceroute procedure.

The same property is satisfied for power law trees since

$$\sum_{k=1}^{\infty} \frac{1}{d_1 \dots d_k} < \infty.$$

3.2 Mean number of discovered nodes with p hosts

While in the previous section, we have investigated what we can learn from a traceroute procedure in a tree network by using only two hosts, we consider the case when there are p traceroute capable hosts placed at random at the leaves of the tree network. We assume that p stations are exchanging traceroute messages and we are interested in the total number of routers discovered by means of traceroute. We specifically have the following result.

Proposition 2. *The average number $D(N,p)$ of routers discovered with p stations is given by*

$$\mathbb{E}(D(N,p)) = \sum_{j=1}^{n} l_j \left(1 - \left(1 - \frac{1}{l_j}\right)^p\right) - \sum_{j=2}^{n} \frac{1}{l_j^{p-1}}. \quad (2)$$

Proof. For $1 \leq j \leq n$, a node A at the jth level has not been discovered if either

— with the dynamic picture, when the points are at level j, none of them is at node A. The probability of this event is given by

$$\left(1 - \frac{1}{l_j}\right)^p$$

— at level $j < n$, all the p points are at node A and all of them chose the same subtree below A. See Figure 3.

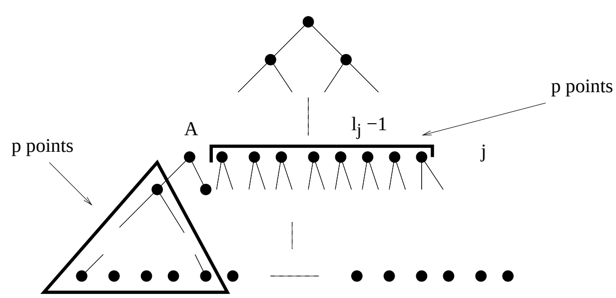

Figure 3: The Two Situations where Node A is not Discovered

Since there are d_j such trees, the probability of this event is given by

$$d_j \frac{1}{l_j^p} \frac{1}{d_j^p} = d_j \frac{1}{l_{j+1}^p}$$

The average number of nodes discovered is thus given by

$$\sum_{j=1}^{n} l_j \left[1-\left(1-\frac{1}{l_j}\right)^p - d_j \frac{1}{l_{j+1}^p} \mathbb{1}_{\{j<n\}}\right] = \sum_{j=1}^{n} l_j \left[1-\left(1-\frac{1}{l_j}\right)^p\right] - \sum_{j=2}^{n} \frac{1}{l_j^{p-1}},$$

and Equation (2) follows. □

When p is large, it is not difficult to see that the second term on the right hand side of Equation (2) is negligible (since $l_j \geq 2^{j-1}$). It is moreover convenient to write the first term as

$$\sum_{j=1}^{n} l_j \mathbb{P}\left(m_p \leq \frac{1}{l_j}\right) \tag{3}$$

where $m_p = \inf(U_i, 1 \leq i \leq p)$ and (U_i) are independent random variables uniformly distributed on $[0,1]$. This expression is in fact an equivalent of $\mathbb{E}(D(N,p))$ when p is large. Under this condition, the variable pm_p converges in distribution to the random variable X, which is an exponentially distributed random variable with parameter 1:

$$\mathbb{P}\left(pm_p \geq x\right) = \left(1-\frac{x}{p}\right)^p \sim e^{-x}.$$

when p tends to infinity. Concerning the spanning tree of discovered routers, we have the following result.

Corollary 2. *For $0 \leq j < n$ and $p > 1$, the distribution of the height of the spanning tree of discovered routers is given by*

$$\mathbb{P}(H(N,p) \leq n-j) = \frac{1}{l_{j+1}^{p-1}},$$

and the proportion of nodes of level j discovered is given by

$$1-\left(1-\frac{1}{l_j}\right)^p - d_j \frac{1}{l_{j+1}^p} \mathbb{1}_{\{j<n\}}.$$

Proof. The second identity is clear from the above proof. To prove the first one, it is sufficient to remark that, in order to have no node of level j discovered, then all the points must be in some subtree whose root is at the $(j+1)$th level. □

3.3 A Coupon Collector Analogy

3.3.1 The case of symmetric networks

Let $D_j(N,p)$ denote the number of routers at level j which have been discovered when there are p traceroute capable hosts. The random variable $D_j(N,p)$ is equal to the number of different nodes seen when p nodes are independently drawn among l_j nodes. This random variable can be written as

$$D_j(N,p) \stackrel{\text{dist.}}{=} \sum_{i=1}^{l_j} \mathbb{1}_{\{i \in \{A_1, A_2, \ldots, A_p\}\}}, \tag{4}$$

where (A_i) are i.i.d. uniformly distributed random variables on $\{1, \ldots, l_{n-1}\}$. This is precisely the classical coupon collector variable. (See Comtet [9] for a general presentation of the coupon collector problem.)

Indeed, assume that we have l_j different types of coupons, which are drawn independently and uniformly. The random variable $D_j(N,p)$ represents the total number of different types of coupons after p trials. From the coupon collector's problem, it is known that

$$\mathbb{P}(D_j(N,p) = l_j) = \frac{l_j! \left\{ \begin{array}{c} p-1 \\ l_j - 1 \end{array} \right\}}{l_j^p} \tag{5}$$

where

$$\left\{ \begin{array}{c} n \\ k \end{array} \right\} = \sum_{\ell=1}^{k} \frac{(-1)^{k-\ell} \ell^{n-1}}{(k-\ell)!(\ell-1)!} \tag{6}$$

is a Stirling number of the second kind. These numbers satisfy the recursion

$$\left\{ \begin{array}{c} n \\ k \end{array} \right\} = \left\{ \begin{array}{c} n-1 \\ k-1 \end{array} \right\} + k \left\{ \begin{array}{c} n-1 \\ k \end{array} \right\}$$

and their generating function, for fixed k, is given by

$$\sum_{n=0}^{\infty} \left\{ \begin{array}{c} n \\ k \end{array} \right\} x^n = \frac{x^k}{(1-x)(1-2x)\ldots(1-kx)}.$$

(See Wilf [10] for details).

The quantity $\mathbb{P}(D_j(N,p) = l_j)$ is the probability that with p traceroute capable hosts, all the nodes at level j are discovered. Conversely, one may consider the problem of determining the number of traceroute hosts one needs to discover all the nodes at level j. Let ν_j denote this random variable.

From the coupon collector problem, the probability $\mathbb{P}(\nu_j = p)$ is given by Formula (5). The mean value of the random variable ν_j is given by

$$\mathbb{E}(\nu_j) = 1 + \frac{l_j}{l_j - 1} + \frac{l_j}{l_j - 2} + \cdots + \frac{l_j}{2} + l_j$$

and its standard deviation by

$$\sigma_j^2 = l_j^2 \sum_{i=1}^{l_j} \frac{1}{i^2} - \left(1 + \frac{l_j}{l_j - 1} + \frac{l_j}{l_j - 2} + \cdots + \frac{l_j}{2} + l_j\right). \quad (7)$$

It is worth noting that, as $l_j$becomes large,

$$\mathbb{E}(\nu_j) = l_j \left(\log l_j + \gamma + O\left(\frac{1}{l_j}\right)\right), \quad (8)$$

where γ is the Euler constant. In addition, we have the Chernoff bound like formula

$$\lim_{l_j \to \infty} \mathbb{P}(\nu_j > l_j \log(l_j) + x l_j) = 1 - e^{-e^{-x}}.$$

The above limit gives the deviation of the random variable ν_j with respect to its mean value when $l_j \to \infty$.

From the above results on the coupon collector problem, we observe that for discovering the l_j routers at level j, we approximately need $l_j \log l_j$ traceroute capable hosts. In addition, for discovering the half of the nodes, we have

$$\mathbb{E}(\#\text{hosts needed for discovering } l_j/2 \text{ nodes}) = l_j \log 2 + O(1).$$

This indicates that in order to discover all the nodes at level j, we need a number of traceroute capable hosts, which is much larger than the number of nodes. Take for instance a constant degree d, then the number of nodes in the network is $N = (d^n - 1)/(d - 1)$ and $l_n = d^{n-1}$. We then have $l_n = ((d-1)N + 1)/d$ and all the nodes in the network are discovered if all the nodes at level n are discovered. This requires a number ν_n of hosts such that

$$\mathbb{E}(\nu_n) \sim \frac{(d-1)N}{d} \log N$$

when N is large. The number of hosts required to discover all the nodes in the network is hence much greater than the number of nodes in the whole network.

As a consequence, a complete topology discovery requires a massive deployment of hosts exchanging traceroute messages. This will be further illustrated in the next section when considering all the nodes in the network.

3.3.2 The case of asymmetric networks

So far, we have assumed that the network is completely symmetric. In reality, however, this is rarely the case. In fact, collect networks are highly asymmetrical. For instance, the number of users in a city (dense area) is much larger than the number of users on the country side (sparse area). Hence, when considering the discovery of concentration routers, located at intermediate levels, say, level $j < n$, the probability that a user belongs to a dense area is larger than the probability the user is in a sparse area.

Keeping in mind the analogy with the coupon collector's problem, we now assume that the probability of drawing a coupon of type r is denoted by p_r. Without loss of generality, we assume that $p_1 \le p_2 \le \ldots \le p_{l_j}$.

If traceroute capable hosts are placed at random at the leaves of the network, the number of hosts needed to discover all the routers at level j is such that as $l_j \to \infty$,

$$\log \mathbb{P}(p_1 \nu_j - b_j \le z) \sim - \sum_{r}^{l_j} \exp\left(-\frac{p_r}{p_1}(z + b_j)\right),$$

where b_j is chosen so that

$$\sum_{r=1}^{l_j} \exp\left(-p_r \frac{b_j}{p_1}\right) < \infty$$

as $l_j \to \infty$. (See Klaassen [11] for details.)

The above result shows that the growth rate of ν_j is determined by the less dense area since $\nu_j \sim b_j/p_1$ when $l_j \to \infty$. This indicates that the discovery of routers connecting sparse area is quite expensive if hosts are placed at random. It follows that the discovery of an asymmetrical network with sparse areas requires a coordination between traceroute capable hosts. Those hosts cannot be drawn at random but their placement should take into account the density of the areas connected by collect routers.

To quantitatively illustrate the above phenomenon, assume that the population attached to routers is proportional to α^j for some $\alpha > 1$. In this case, we have $p_r = p_1 \alpha^{r-1}$ and $1/p_1 = (\alpha^{l_j} - 1)/(\alpha - 1)$. We can take $b_j = 1$ so that $\nu_j \sim \alpha^{l_j}/(\alpha - 1)$ when $l_j \to \infty$. We hence see that the number of hosts needed to discover routers collecting sparse areas is explosive.

3.3.3 The versatility of the coupon collector analogy

So far, we have considered tree networks. However, the coupon collector analogy still pertains for more complex topologies as long as the network presents a hierarchical structure. Indeed, if the network is organized in layers (or levels) and if a node at a given level connects a certain population of users, we easily see that we can still use the coupon collector analogy for computing the number of hosts needed to discover the nodes at a given level. Moreover, in view of the preceding section, we see that the populations attached to the different routers can be asymmetric. The application of this observation to more complex network topologies will be addressed in further studies.

4 Asymptotic results

To understand more closely the basic properties of topology discovery by means of traceroute, it is assumed in this section that the ratio of the number p of traceroute capable hosts to the total number N of routers in the network is fixed and equal to a constant λ and we suppose that N goes to infinity. We are specifically interested in the behavior of $\mathbb{E}(D(N,p))/N$ when N tends to infinity. This quantity gives information about the ratio of the number of routers discovered to the total number of nodes in the network.

4.1 Regular trees

In a first step, we investigate the case when the degree of nodes is constant, equal to $d \geq 2$.

Proposition 3. *For a regular tree with degree $d \geq 2$, when $N \to +\infty$ and $p/N \to \lambda$,*

$$\frac{\mathbb{E}(D(N,p))}{N} \sim T_{REG_d}(\lambda) \stackrel{def.}{=} \sum_{j=1}^{+\infty} \frac{d-1}{d^j}\left(1-\exp\left(-\frac{\lambda d^j}{d-1}\right)\right). \quad (9)$$

Proof. Using that $N \sim d^n/(d-1)$ and the equivalent (3) as N goes to infinity, one gets that

$$\frac{1}{N}\mathbb{E}(D(N,p)) \sim (d-1)\sum_{j=1}^{n} \frac{d^{j-1}}{d^n}\mathbb{P}\left(pm_p \leq \frac{\lfloor \lambda \frac{d^n}{d-1} \rfloor}{d^{j-1}}\right)$$

and then,

$$\frac{1}{N}\mathbb{E}(D(N,p)) \sim (d-1)\sum_{j=1}^{n} \frac{1}{d^j}\mathbb{P}\left(X \leq \frac{\lambda}{(d-1)}d^j\right)$$

where X is an exponentially distributed random variable with unit mean, since as mentioned in Section 3, the random variable pm_p converges in distribution to X. □

We now take benefit the closed form of the ratio of discovered nodes given by equation (9) in order to illustrate the speed of the exploration process. For this purpose, we study the dependence of the number of discovered nodes on the ratio $\lambda \sim p/N$ when N is large while λ is small. It describes in some sense the initial speed of the exploration process through the network. Indeed, the analysis gives the ratio of discovered nodes with a small number of traceroute capable hosts.

Proposition 4. *For a regular tree with degree $d \geq 2$, when λ tends to 0 the asymptotic proportion of discovered nodes satisfies the following equivalence*

$$T_{REG_d}(\lambda) \sim -\lambda \log_d(\lambda). \tag{10}$$

Proof. As it can easily be seen, the asymptotic behavior of Equation (10) when λ tends to 0 is quite delicate since the series is divergent. The technique used to get an expansion relies on a convenient rewriting of the equation together with the use of Fubini's Theorem for non-negative functions. (See Robert [12] for a general description of the method.)

We have

$$\begin{aligned}\sum_{j=1}^{+\infty} \frac{1}{d^j}&\left(1-\exp\left(-\frac{\lambda d^j}{d-1}\right)\right)\\ &= \sum_{j=1}^{+\infty} \frac{1}{d^j} \int_0^{+\infty} \mathbb{1}_{\left\{u \leq \frac{\lambda d^j}{d-1}\right\}} e^{-u}\, du\\ &= \int_0^{+\infty} \sum_{j=1}^{+\infty} \frac{1}{d^j} \mathbb{1}_{\left\{u \leq \frac{\lambda d^j}{d-1}\right\}} e^{-u}\, du,\end{aligned}$$

where we have used Fubini's theorem to exchange the sum and the integral signs. This entails that

$$\sum_{j=1}^{+\infty} \frac{1}{d^j}\left(1-\exp\left(-\frac{\lambda d^j}{d-1}\right)\right) = \frac{d}{d-1}\int_0^{\lambda a} e^{-u} du + \frac{1}{d-1}\int_{\lambda a}^{+\infty} \frac{1}{d^{\lfloor \log_d(u(d-1)/\lambda)\rfloor}} e^{-u}\, du, \tag{11}$$

with $a = d/(d-1)$.

The first term on the right hand side of Equation (11) vanishes when λ tends to 0. For the second term, we note that with regard to the asymptotic behavior for λ close to 0, the integration interval $[\lambda a, \infty]$ can be replaced by any interval $[\lambda a, b]$ with $b > \lambda a$. To simplify, we take $b = a$ so that

$$\int_a^{+\infty} \frac{1}{d^{\lfloor \log_d(u(d-1)/\lambda)\rfloor}} e^{-u}\, du \leq \lambda(1-e^{-a}),$$

since for $u \geq a$,

$$\lfloor \log_d(u(d-1)/\lambda)\rfloor \geq -\log_d \lambda.$$

It follows that the function $T_{\mathrm{REG}_d}(\lambda)$ defined by Equation (9) has the following equivalent in the neighborhood of 0

$$\begin{aligned} T_{\mathrm{REG}_d}(\lambda) &\sim \int_{\lambda a}^{a} d^{\{\log_d(u(d-1)/\lambda)\}} \frac{\lambda e^{-u}}{u(d-1)}\, du \\ &= \frac{\lambda}{d-1}\left(\int_{\lambda}^{1} d^{\{\log_d u/\lambda\}} \frac{e^{-au}-1}{u}\, du \right. \\ &\qquad \left. + \int_1^{1/\lambda} \frac{d^{\{\log_d u\}}}{u}\, du\right), \end{aligned} \tag{12}$$

with $\lfloor y \rfloor$ is the integer part of $y \in \mathbb{R}$ and $\{y\} = y - \lfloor y \rfloor$ its fractional part.

The asymptotic behavior of the function $T_{\mathrm{REG}_d}(\lambda)$ when λ tends to 0 is determined by the second term on the right hand side of Equation (12). Hence, as $\lambda \sim 0$, we have the equivalence

$$T_{\mathrm{REG}_d}(\lambda) \sim \frac{\lambda}{d-1} \int_1^{1/\lambda} \frac{d^{\{\log_d u\}}}{u}\, du.$$

If $\lambda = 1/(xd^m)$ with fixed $1 < x \leq d$ and $m \in \mathbb{N}$, then as $\lambda \to 0$

$$\begin{aligned} \int_1^{1/\lambda} \frac{d^{\{\log_d u\}}}{u}\, du &\sim \int_x^{1/\lambda} \frac{d^{\{\log_d u\}}}{u}\, du \\ &= \sum_{k=0}^{m-1} \int_{xd^k}^{xd^{k+1}} \frac{d^{\{\log_d u\}}}{u}\, du \\ &= \sum_{k=0}^{m-1} \int_1^d \frac{d^{\{\log_d (xu)\}}}{u}\, du \\ &= -\log_d(\lambda x) \int_1^d \frac{d^{\{\log_d (xu)\}}}{u}\, du. \end{aligned}$$

The last integral can be expressed as follows, recall that $\lambda = 1/(xd^m)$ with $x \in [1, d]$,

$$\begin{aligned} \int_1^d \frac{d^{\{\log_d (xu)\}}}{u}\, du & \\ &= \int_1^{d/x} \frac{d^{\{\log_d (xu)\}}}{u}\, du + \int_{d/x}^{d} \frac{d^{\{\log_d (xu)\}}}{u}\, du \\ &= \int_1^{d/x} \frac{d^{\log_d (xu)}}{u}\, du + \int_{d/x}^{d} \frac{d^{\log_d (xu)-1}}{u}\, du \\ &= x\left(\frac{d}{x} - 1\right) + \frac{x}{d}\left(d - \frac{d}{x}\right) = d - 1. \end{aligned}$$

Equivalence (10) is therefore established. □

From equivalence (10), it is worth noting that the ratio $T_{\text{REG}_d}(\lambda)/\lambda$ is not constant but is equal to $-\log_d(\lambda)$. Thus, when the number p of traceroute capable hosts is small when compared to the number N of nodes in a tree network, the number of nodes discovered is more than linear in the ratio $\lambda = p/N$. This indicates that the speed of the exploration process is quite fast when the number of hosts is small. This encouraging observation is however to be counterbalanced by the fact that, by Equation (9), the speed of the exploration process is nevertheless decreasing exponentially fast with respect to λ. In addition, the speed depends on the degree of nodes via $\log_d \lambda$. This indicates that the greater the degree of nodes, the smaller is the speed of the learning process.

The above remarks show that we rapidly learn about the topology of a tree network with a small initial number of hosts but the speed of learning decreases as the number of hosts increases.

4.2 Power law trees

4.2.1 Power Law trees with increasing degree

For power law trees with increasing degree, we have the following result.

Proposition 5. *For a power law tree with increasing degree, the proportion of discovered nodes is such that*

$$\lim_{\substack{N\to+\infty,\\ p/N\to\lambda}} \frac{D(N,p)}{N} = T_{PLI_\alpha}(\lambda) \stackrel{def.}{=} 1 - e^{-\lambda}. \tag{13}$$

Proof. For power trees with increasing degree, we have $l_{n+1} \sim (n!)^\alpha$. It is then quite easy to see that $N \sim l_n$ and if $p \sim \lfloor \lambda N \rfloor$, then

$$\frac{D(N,p)}{N} \sim \sum_{j=1}^{n} \frac{l_j}{l_n} \mathbb{P}\left(pm_p \leq \lambda \frac{l_n}{l_j} \right).$$

When $n \to +\infty$, all the terms of the series vanish except the last one with index n. One consequently obtains, by recalling that pm_p converges in distribution to an exponentially distributed random variable with parameter 1 when $p \to +\infty$,

$$\lim_{\substack{N\to+\infty,\\ p/N\to\lambda}} \frac{D(N,p)}{N} = 1 - e^{-\lambda},$$

and the result follows. □

Equation (13) is not really informative since it does not depend on the growth parameter α. This is not the case for regular trees, see Equation (9) which gives the growth rate of the number of discovered routers with respect to d (via the $\log_d$ term). An asymptotic analysis of this equation will give further insight in this case.

4.2.2 Power Law trees with decreasing degree

We can state the analogue of Proposition 5 for power law trees with decreasing degree. We specifically have:

Proposition 6. *For a PLD_α tree, when $N \to +\infty$ and $p/N \to \lambda$, then*

$$\frac{D(N,p)}{N} \to T_{PLD_\alpha}(\lambda)$$

with

$$T_{PLD_\alpha}(\lambda) = \sum_{j=1}^{+\infty} \frac{1}{H(\alpha)(j!)^\alpha}\left(1 - e^{-\lambda H(\alpha)(j!)^\alpha}\right) \tag{14}$$

and

$$H(\alpha) = \sum_{j=1}^{+\infty} \frac{1}{(j!)^\alpha}.$$

Proof. For a PLD_α tree, we have for large n, $l_j \sim ((n-1)!/(n-j)!)^\alpha$

$$\frac{N}{((n-1)!)^\alpha} \sim \sum_{j=1}^{+\infty} \frac{1}{(j!)^\alpha} = H(\alpha).$$

If Equivalence (3) is used again, one gets that, when N goes to infinity, the ratio $D(N,p)/N$ is equivalent to

$$\sum_{j=1}^{n} \frac{1}{H(\alpha)(n-j)!} \mathbb{P}\left(pm_p \leq \lambda H(\alpha)((n-j)!)^\alpha\right).$$

and Equation (14) follows. □

4.3 The growth rate of the exploration process

As before, the behavior of $T_{\mathrm{PLI}_\alpha}(\lambda)$ and $T_{\mathrm{PLD}_\alpha}(\lambda)$ when λ is small is investigated. From Equation (13), one directly concludes that $T_{\mathrm{PLD}_\alpha}(\lambda) \sim \lambda$ when λ gets small.

For a PLD_α tree, we have the following result.

Proposition 7. *For a PLD_α tree, when λ tends to 0 the asymptotic proportion of discovered nodes satisfies the following equivalence*

$$T_{PLD_\alpha}(\lambda) \sim \frac{\lambda \log(1/\lambda)}{\alpha \log\log(1/\lambda)}. \tag{15}$$

Proof. Equation (14) gives

$$\frac{1}{\lambda} T_{\mathrm{PLD}_\alpha}(\lambda) = \sum_{j=1}^{+\infty} \frac{1}{(j!)^\alpha \tilde{\lambda}}\left(1 - e^{-(j!)^\alpha \tilde{\lambda}}\right),$$

where $\tilde{\lambda} = \lambda H(\alpha)$. If h is the function defined by

$$h(x) = -\frac{1 - e^{-x}}{x},$$

Equation (14) shows that the quantity $T_{\mathrm{PLD}_\alpha}(\lambda)/\lambda$ is given by

$$\sum_{j=1}^{+\infty} \int_0^{+\infty} \mathbb{1}_{\{\tilde{\lambda}(j!)^\alpha \leq u\}} h'(u)\, du. \tag{16}$$

Let us introduce the classical Euler's Gamma function Γ defined by

$$\Gamma(x) = \int_0^{+\infty} u^{x-1} e^{-u}\, du$$

on $(0, +\infty)$ (recall that $\Gamma(n+1) = n!$ when $n \in \mathbb{N}$). The function Γ is a bijection from $[2, \infty) \to [1, \infty)$; its inverse is denoted by Γ^{-1}.

By Fubini's Theorem, Equation (16) can be rewritten as

$$\begin{aligned} &\int_{\tilde{\lambda}}^{+\infty} \sum_{j=1}^{+\infty} \mathbb{1}_{\left\{j+1 \leq \Gamma^{-1}\left(\left(\frac{u}{\tilde{\lambda}}\right)^{1/\alpha}\right)\right\}} h'(u)\, du \\ &= \int_{\tilde{\lambda}}^{+\infty} \left\lfloor \Gamma^{-1}\left(\left(\frac{u}{\tilde{\lambda}}\right)^{1/\alpha}\right) - 1 \right\rfloor h'(u)\, du. \end{aligned} \tag{17}$$

Stirling's Formula for the Gamma function gives that $\Gamma(x) \sim x^{x-1/2} e^{-x} \sqrt{2\pi x}$ for a large x. Therefore, if $y = \Gamma(x)$, one gets the relation

$$\log y \sim x \log x.$$

If we write $x = \phi(y) \log y$, then

$$\frac{1}{\log x} \sim \phi(y)$$

hence $\phi(y) \to 0$ as $y \to +\infty$. Therefore, one gets

$$1 \sim \phi(y) \log \phi(y) + \phi(y) \log \log y,$$

as y gets large, so $\phi(y) \log \log y \sim 1$. Thus, the inverse function satisfies the equivalence

$$\Gamma^{-1}(y) \sim \frac{\log(y)}{\log(\log(y))}$$

when y tends to infinity.

Since

$$h'(x) = \frac{1-(1+x)e^{-x}}{x^2},$$

it is easy to see that the integral (17) is diverging when λ tends to 0. Moreover, it is equivalent to

$$\int_{\tilde{\lambda}a}^{+\infty} \left[\Gamma^{-1}\left(\left(\frac{u}{\tilde{\lambda}}\right)^{1/\alpha}\right) - 1\right] h'(u)\,du,$$

for an arbitrary $a > 1$, since the remaining part of the integral converges as $\lambda \to 0$. By choosing a sufficiently large so that, for $\varepsilon > 0$,

$$1 - \varepsilon \leq \frac{\Gamma^{-1}(x)}{\log(x)/\log(\log(x))} \leq 1 + \varepsilon, \quad x \geq a,$$

it is sufficient to study the expansion of the quantity

$$I(\tilde{\lambda}) = \int_{\tilde{\lambda}a}^{+\infty} \frac{\log(u/\tilde{\lambda})/\alpha}{\log\left(\log(u/\tilde{\lambda})/\alpha\right)} h'(u)\,du. \tag{18}$$

Let us introduce

$$\phi(\tilde{\lambda}) = \log\left(\tilde{\lambda}^{-1}\right) \Big/ \log\log\left(\tilde{\lambda}^{-1}\right)$$

The integral (18) can be rewritten as

$$I(\tilde{\lambda}) = \frac{1}{\alpha} \int_{\tilde{\lambda}a}^{+\infty} \frac{\log(u) + \log(\tilde{\lambda}^{-1})}{D(u)} h'(u)\,du,$$

where

$$D(u) = -\log(\alpha) + \log(\log(\tilde{\lambda}^{-1})) + \log\left(1 + \log(u)/\log(\tilde{\lambda}^{-1})\right).$$

Since $\log(u)h'(u)$ is integrable on $\mathbb{R}_+$, trite inequalities and Lebesgue's Theorem show that $I(\tilde{\lambda})/\phi(\tilde{\lambda})$ converges to

$$\int_0^{+\infty} h'(u)\,du = 1$$

as $\tilde{\lambda} \to 0$ and the proposition follows. □

4.4 Discussion

4.4.1 The asymptotic profile of tree structures

In order to give an intuitive explanation for the initial growth of the exploration process seen in the above section, we first introduce the concept of the profile of a tree.

Definition 2. *The asymptotic profile of a tree is given by the sequence* (p_k) *defined by*

$$p_k = \limsup_{n\to+\infty} \frac{\sum_{j=n-k}^{n-1} l_j}{N} = \limsup_{n\to+\infty} \frac{\sum_{j=n-k}^{n-1} l_j}{\sum_{j=1}^{n-1} l_j}.$$

For $k \geq 1$, the quantity p_k is the proportion of nodes of the tree in the last k levels. In the following, the notation $p_k(X)$ shall be used, where X is one of the classes of trees: REG_d, PLI_α and PLD_α. The following proposition is quite straightforward to prove.

Proposition 8. *For* $k \geq 1$*,*

$$\begin{aligned} p_k(REG_d) &= 1 - \frac{1}{d^k}, \\ p_k(PLD_\alpha) &= \sum_{j=1}^{k} \frac{1}{(j!)^\alpha} \Bigg/ \sum_{j=1}^{+\infty} \frac{1}{(j!)^\alpha}\ , \\ p_k(PLI_\alpha) &= \mathbb{1}_{\{k=1\}}, \end{aligned}$$

The initial growth rate of the exploration process of the three tree topologies proved in Section 4.3 is recalled, when the proportion λ of stations involved in the exploration process is small. We specifically have

$$\begin{aligned} T_{\mathrm{REG}_d}(\lambda) &\sim \lambda \log(1/\lambda), \\ T_{\mathrm{PLD}}(\lambda) &\sim \frac{\lambda \log(1/\lambda)}{\alpha \log\log(1/\lambda)}, \\ T_{\mathrm{PLI}}(\lambda) &\sim \lambda \end{aligned}$$

when λ tends to 0.

For the PLI_α tree, most of the N nodes are at the bottom of the network: $p_1 = 1$ and $p_k = 0$ if $k \neq 1$. So a traceroute procedure initiated by a host discovers essentially the router the hosts is attached to. This explains the low speed of the initial phase of the exploration. See the section on the coupon collector analogy.

On the contrary, see Proposition 8, for regular trees and PLD_α trees, a non-negligible proportion of nodes are in the upper levels of the network. Since a traceroute procedure discovers several routers in these layers, it speeds up the rate of the discovery process. Note that for the proportion of nodes above the k last levels is of the order of $1/d^k$ for REG_d trees and $1/((k+1)!)^\alpha$ for PLD_α trees, which explains that the exploration process is initially slightly faster for regular trees.

4.4.2 Growth rate for large λ

Formula (9) shows that the growth rate of the exploration process decreases exponentially fast with respect to λ for any tree architecture. Figure 4 displays the proportion of nodes discovered as a function of the ratio λ. It clearly appears from this figure that the total discovery of the network requires a large number of hosts. The structure, which requires the smallest number of hosts, is the regular tree while the plower law increasing structure is the most demanding.

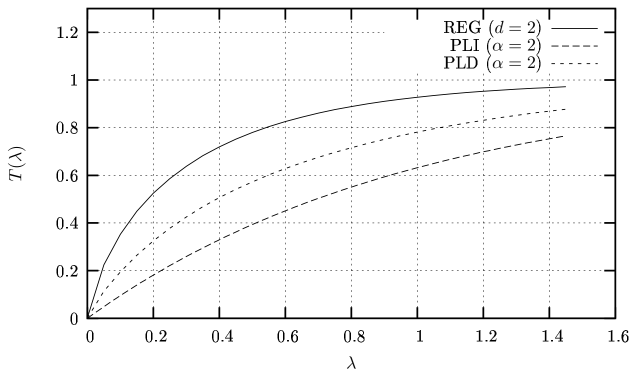

Figure 4: Proportion of discovered nodes by using $p = \lfloor \lambda N \rfloor$ stations for REG_d, PLD_α and PLI_α trees with $d = 2$ and $\alpha = 2$.

5 Degree of discovered nodes

In this section, we investigate the degree of a random node in the network seen by the exploration process with p traceroute capable hosts. That degree is denoted by the random variable L. Before giving the distribution, let us introduce some additional notation: for $a \geq 1$ and $0 < k \leq a$, $(a)_k = a(a-1)\dots(a-k+1)$, $(a)_0 = 1$ and $(a)_k = 0$ for $k > a$. Moreover, let $b(p, n, l_j)$ be the Bernoulli probabilities defined by

$$b(p, n, l_j) = \binom{p}{n} \left(\frac{1}{l_j}\right)^n \left(1 - \frac{1}{l_j}\right)^{p-n}. \tag{19}$$

Proposition 9. *The degree L of a node discovered by the exploration by means of traceroute is given by: for $k \geq 1$,*

$$\mathbb{P}(L = k) = \sum_{j=1}^{n} \frac{l_j}{N} \frac{(d_j)_k}{\delta_j} \sum_{m=k}^{p} \frac{b(p, m, l_j)}{d_j^m} \times \left\{ \begin{array}{c} m \\ k \end{array} \right\} \left(1 - \mathbb{1}_{\{k=1, m=p\}}\right), \tag{20}$$

where $\left\{ \begin{array}{c} m \\ k \end{array} \right\}$ *is the Stirling number of the second kind defined by Equation* (6) *and* δ_j *is the probability of not seing a node at level* j*, given by*

$$\delta_j = 1 - \left(1 - \frac{1}{l_j}\right)^p - \frac{d_j}{l_{j+1}^p}. \tag{21}$$

Proof. Let us pick up a node at random. This node belongs to level j with probability l_j/N. Such a node is not discovered if no traceroute hosts are attached to his node or if all the hosts fall into the same subtree attached to this node. The probability δ_j given by equation (21) then readily follows.

For a discovered node at level j, let M denote the number of hosts in the subtree associated to this node. The probability $\mathbb{P}(M = m)$ is given by the Bernoulli probabilities (19). Assuming $M = m$, the degree seen via traceroute is equal to the number of sons of the node considered associated to the traceroute hosts. Since the degree of a node at level j is d_j, this number, denoted by L_j, is exactly equal to the number of nonempty cells when m objects are distributed into d_j different cells. The distribution of this number is given by (see Riordan [13, p. 100])

$$\mathbb{P}(L_j = k) = \frac{(d_j)_k}{d_j^m} \left\{ \begin{array}{c} m \\ k \end{array} \right\}. \tag{22}$$

By deconditioning, Equation (20) follows. □

The mean value of the degree L of a discovered node is given by the following result.

Corollary 3. *The mean value of the random variable* L *is given by*

$$\mathbb{E}(L) = \sum_{j=1}^{n} \frac{l_j d_j}{N} \frac{1 - \left(1 - \frac{1}{d_j l_j}\right)^p - \frac{1}{l_{j+1}^p}}{1 - \left(1 - \frac{1}{l_j}\right)^p - \frac{d_j}{l_{j+1}^p}}. \tag{23}$$

Proof. The mean value of the random variable L_j with the distribution defined by Equation (22) is given by

$$\mathbb{E}(L_j) = d_j \left(1 - \left(1 - \frac{1}{d_j}\right)^m\right).$$

By deconditioning, Equation (23) follows. □

In the case of a regular graph with constant degree d, we have when $n \to \infty$ and $p = \lambda N$,

$$\mathbb{E}(L) \sim (d-1) \sum_{j=0}^{\infty} \frac{1}{d^j} \frac{1 - e^{\lambda d^j}}{1 - e^{\lambda d^{j+1}}}.$$

When λ tends to 0, we have $\mathbb{E}(L) \sim 1$. This means that at the beginning of the exploration process, the nodes seen in a regular tree network are seen only once.

6 Experimental results

Several experiments were conducted on a set of real graphs. We used the Scan+Lucent map which is a merge of a map from the Internet Mapping project at Lucent Bell Laboratories on November 1999 and a map obtained from the Mercator software. The Scan+Lucent map contains 284804 routers, the degree of the nodes ranges from 1 (150397 stations) to 1978 (1 station) with an average of 3.02 and a variance of 8.624. Figure 6 gives an indication of how far the distribution of the degree is from a power law distribution.

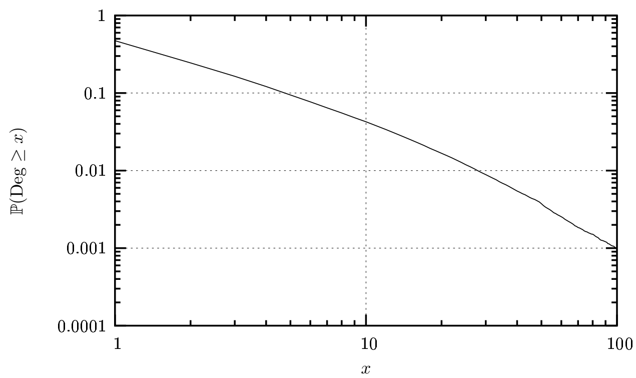

Figure 5: The function $x \to \mathbb{P}(\text{Deg} \geq x)$ on a logarithmic scale of Scan+Lucent graphs

By using the edges of this graph, several spanning trees have been constructed recursively. Root nodes are chosen either at random or are the nodes with the highest degree, the other levels are defined recursively in the following way:

- Bounded degree: if a node is in the tree, only d of its sons are chosen in the next level.
- Unbounded: all its sons are taken in the next level.

With this method, one does not, of course, recover the complete graph. But, as noted in the introduction, this gives an upper bounded on the real discovery process if these spanning trees are used instead of the "real" graph. It must be mentioned that, in our experiment, only a small fraction of edges connects nodes belonging to different subtrees of the top level. The edges which are discarded connect essentially nodes of different layers within the top subtrees.

Figure 6 shows, as expected, a steady growth at the origin when the degree is bounded to 2 or 4. When the degree is not bounded, the initial growth suggests a PLI tree architecture.

It must be noted that these experiments are incomplete since the size of the data sets available is not sufficiently large to investigate with a reasonable accuracy the parameters of the power law features of these architectures.

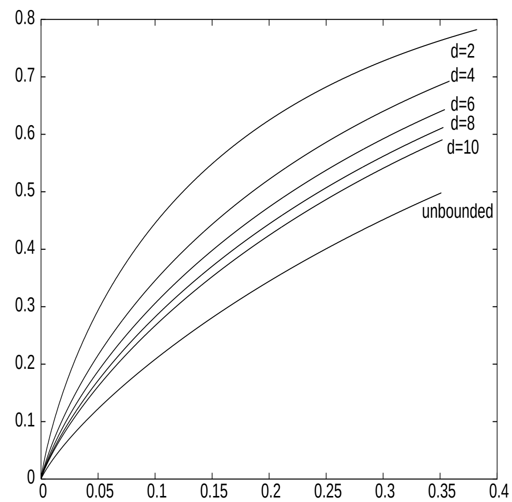

Figure 6: The ratio of discovered nodes for the spanning trees for Scan+Lucent graphs

7 Conclusion

We have investigated in this paper the topology discovery of a collect network, represented by means of a tree network, when using ideal traceroute procedures. We have obtained, for different network structures, closed formulas for the mean number of discovered nodes and for the speed of the exploration process when the height of the tree tends to infinity while the ratio of the number of traceroute capable hosts to the number of nodes in the network remains constant. We have in particular introduced a connection with the coupon collector problem, which seems to appear as a generic tool in the problem of topology discovery.

From the analysis carried out in this paper, we can make the following points:

- the rate of the exploration process is quite large with a small number of hosts but rapidly decreases when the number of hosts increases,
- the exhaustive exploration of a network requires a massive deployment of traceroute capable hosts,
- sparse areas significantly slow down the exploration process unless a coordination exists between the different hosts,
- the degree of nodes is not easy to determine when the number of hosts is small.

It follows that it seems very difficult to explore the topology of a network by means of traceroute procedures. The computations carried out in this paper for a simple network structure (namely a tree) show that the output of such procedures are quite unreliable. Of course, the validity of these results will be investigated in further studies for more complex network structures (e.g., random tree networks and complex networks à la Barábasi).

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