Worst case end-to-end response times for non-preemptive FP/DP* scheduling

Steven Martin, Pascale Minet

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Worst case end-to-end response times for non-preemptive FP/DP scheduling*

Steven MARTIN — Pascale MINET

N° 5418 – version 2

version initiale Décembre 2004 – version révisée Mars 2005

Thème COM

Rapport de recherche

ISSN 0249-6399

Worst case end-to-end response times for non-preemptive FP/DP* scheduling

Steven Martin , Pascale Minet*

Thème COM — Systèmes communicants
Projets Hipercom

Rapport de recherche n° 5418 – version 2† — version initiale Décembre 2004 — version révisée Mars 2005 26 pages

Abstract: In this paper, we are interested in real-time flows requiring quantitative and deterministic Quality of Service (QoS) guarantees. We focus more particularly on two QoS parameters: the worst case end-to-end response time and jitter. We consider a non-preemptive scheduling of flows, called FP/DP*, combining fixed priority and dynamic priority, where the dynamic priority of a flow packet is assigned on the first node visited by the packet in the network. Examples of such a scheduling are FP/FIFO* and FP/EDF*. With any flow is associated a fixed priority denoting the importance of the flow from the user point of view. The arbitration between packets having the same fixed priority is done according to their dynamic priority. A packet can be transmitted only if (i) there is no packet having a higher fixed priority and (ii) there is no packet having a higher dynamic priority. A classical approach used to compute the worst case end-to-end response time is the holistic one, but it leads to pessimistic upper bounds. We propose the trajectory approach to improve the accuracy of the results. Indeed, the trajectory approach only considers worst case scenarios experienced by a flow along its trajectory. It then eliminates scenarios that cannot occur in the network.

Key-words: Fixed priority scheduling, quality of service (QoS), holistic approach, worst case end-to-end response time, trajectory approach, deterministic guarantee.

* INRIA Rocquencourt, 78153 Le Chesnay Cedex, *pascale.minet@inria.fr*
† This version improves the bound given on the worst case end-to-end response time.

Temps de réponse pire cas de bout-en-bout avec un ordonnancement FP/DP* non-préemptif

Résumé : Dans ce rapport, nous nous intéressons aux flux temps-réel nécessitant des garanties quantitatives et déterministes de qualité de service (QoS). Nous étudions plus particulièrement deux paramètres de QoS : le temps de réponse pire cas de bout-en-bout et la gigue pire cas de bout-en-bout. Nous considérons un ordonnancement non-préemptif des flux appelé FP/DP*, combinant priorités fixes et priorités dynamiques. La priorité dynamique d'un paquet est assigné sur son premier noeud visité. Deux exemples d'un tel ordonnancement sont FP/FIFO* et FP/EDF*. A chaque flux est associée une priorité fixe représentant, du point de vue de l'utilisateur, le degré d'importance du flux. Les paquets de même priorité fixe sont départagés selon leur priorité dynamique.Un paquet peut être transmis uniquement si (i) il n'y a aucun paquet de priorité fixe plus élevée et (ii) il n'y a aucun paquet ayant une priorité dynamique plus élevée. Une approche classique utilisée pour calculer le temps de réponse pire cas de bout-en-bout est l'approche holistique, mais celle-ci conduit à des bornes pessimistes. Nous proposons l'approche par trajectoire pour améliorer la précision des résultats. En effet, l'approche par trajectoire ne considère que les scénarios pire cas subis par un flux sur sa trajectoire. Cette approche élimine donc les scénarios impossibles.

Mots-clés : Ordonnancement par priorités fixes, qualité de service (QoS), approche holistique, temps de réponse pire cas de bout-en-bout, approche par trajectoire, garantie déterministe.

Contents

1 Introduction

1.1 Context and motivation

In this paper, we are interested in applications that require deterministic and quantitative guarantees of Quality of Service (QoS). Indeed, such applications need bounds on the worst case end-to-end response times to have a behavior compliant with their specifications. Voice over IP (VoIP) and control-command applications are examples of such applications. That is why we focus on deterministic guarantees of end-to-end response times in a packet network. We will show how to determine these times depending on the flow scheduling used in the network.

With regard to flow scheduling, the assumption generally admitted is that packet transmission is not preemptive. Moreover, *Fixed Priority* (FP) scheduling has been extensively studied in the last years [1, 2]. It exhibits interesting properties:

- The impact of a new flow is limited to flows having equal or lower fixed priorities;
- It is easy to implement;
- It is well adapted for service differentiation: flows with high priorities have smaller response times.

In a network, several packets may have to share the same priority. For example when:

- The number of fixed priorities available on a processor is less than the flow number;
- The priority of a flow is determined by external constraints and cannot be chosen arbitrarily;
- Flows are processed by class of service and the flow priority is this of its class.

In this paper, we consider that such packets are scheduled according to their dynamic priorities (DP) and unlike the state of the art, we account for this scheduling to compute the worst case end-to-end response times. More precisely, we assume that packets are scheduled according to the non-preemptive FP/DP* scheduling.

Definition 1 *With FP/DP*, packets are first scheduled according to their fixed priority. Packets with the same fixed priority are scheduled according to their dynamic priorities, computed on the first node visited (expressed by the star in DP*).*

Moreover, we assume that the dynamic priority of a packet, once assigned on the first node visited, does not evolve with time. For instance, this priority can be based on the packet arrival time or the packet delivery deadline, but not on the remaining laxity of the packet.

The use of the dynamic priority as a secondary scheduling parameter provides a better service differentiation, accounting for the QoS requirements expressed by the applications.

Moreover, DP* does not require to synchronize all clocks in the network but only those of ingress nodes, where dynamic priorities are assigned to flow packets.

Most famous FP/DP* scheduling algorithms are FP/FIFO* and FP/EDF*. It is important to notice that with FP/DP*, the order of packet priority does not depend on the node considered: it is fixed.

In this paper, we show how to compute a bound on the end-to-end response time of any flow. Our results on the worst case end-to-end response time obtained with FP/DP* scheduling can be used in various configurations:

- In a *Differentiated Services* architecture [3], several classes are defined, each class having its own fixed priority. The highest priority class, that is the *Expedited Forwarding* (EF) class, is scheduled Fixed Priority with the other classes. Moreover, if packets belonging to the EF class need to be differentiated, dynamic priorities can be assigned to these packets. Hence, FP/DP* scheduling can be used to provide the requested differentiation.
- In an *Integrated Services* architecture [4], a fixed priority and a deadline are assigned to each flow. FP/EDF* scheduling can be used to provide shorter response times to high priority flows and to flows with the same priority but with short deadlines.
- In an *hybrid* architecture, some flows are managed per class, whereas others are managed individually.

In the three configurations mentionned, several flows can share the same fixed priority. Hence the interest of FP/DP*.

1.2 Paper organization

The rest of the paper is organized as follows. Section 2 presents the problematic we are interested in and defines the assumptions and models adopted in this paper. Section 3 briefly discusses related work in the computation of worst case end-to-end response time. Section 4 shows how to compute an upper bound on the end-to-end response time of any flow, based on a worst case analysis using the trajectory approach. The algorithm to compute the worst case end-to-end response time of any sporadic flow is given. An example gives results obtained with this trajectory approach. Finally, we conclude the paper in Section 5.

2 Problematic

2.1 Assumptions and models

We investigate the problem of providing a deterministic end-to-end response time guarantee to any flow in a distributed system. The end-to-end response time of a flow is defined between its ingress node and its egress node. We want to provide an upper bound on the end-to-end response time of any flow. As we make no particular assumption concerning the arrival times of packets in the distributed system, the feasibility of a set of flows is equivalent to meet the requirement, whatever the arrival times of the packets in the distributed system.

Throughout this paper, we assume that time is discrete. Reference [5] shows that results obtained with a discrete scheduling are as general as those obtained with a continuous scheduling when all flow parameters are multiples of the node clock tick. In such conditions, any set of flows is feasible with a discrete scheduling if and only if it is feasible with a continuous scheduling.

Moreover, we assume the following models.

Scheduling model

We consider that all nodes in the distributed system schedule packets according to the FP/DP* algorithm. Moreover, we assume that packet scheduling is non-preemptive. Therefore, the node scheduler waits for the completion of the current packet transmission (if any) before selecting the next packet.

Network model

We consider a distributed system where links interconnecting nodes are supposed to be FIFO and the network delay between two nodes has known lower and upper bounds: L_{min} and L_{max}. Moreover, we consider neither network failures nor packet losses.

Traffic model

We consider a set $\tau = \{\tau_1, ..., \tau_n\}$ of n sporadic flows. Each flow τ_i follows a path $\mathcal{P}_i$ that is an ordered sequence of nodes whose first node is the ingress node of the flow. This path is assumed to be fixed. This can be obtained by source routing or MPLS. Moreover, a sporadic flow τ_i is defined by:

- T_i, the minimum interarrival time between two successive packets of τ_i;
- C_i^h, the maximum processing time on node $h \in \mathcal{P}_i$ of a packet of τ_i;
- J_i, the maximum release jitter of packets of τ_i at its ingress node. A packet is subject to a release jitter if there exists a non-null delay between its generation time and the time where it is taken into account by the scheduler;
- D_i, the end-to-end deadline of τ_i, its maximum end-to-end response time acceptable. A packet of τ_i generated at time t must be delivered at $t + D_i$.

2.2 Notations and definitions

We consider any flow τ_i, $i \in [1, n]$, following a path $\mathcal{P}_i$. We focus on the packet m of τ_i generated at time t. Then, we denote:

- P_i, the fixed priority of flow τ_i;
- $P_i(t)$, the dynamic priority of packet m.

We recall that the dynamic priority of a packet, once assigned on the first node visited, does not evolve with time. Moreover, we adopt the following assumptions.

Assumption 1 *For any flow τ_i, for any times $t_2 \geq t_1 \geq -J_i$ such that $P_i(t_2) \leq P_i(t_1)$, then for any $z \in [P_i(t_2), P_i(t_1)]$, $\exists\ t_3 \in [t_1, t_2]$ such that $P_i(t_3) = z$.*

Assumption 2 *For any flow τ_i, for any packets m and m' of τ_i generated respectively at time t and t' such that $t' > t$, we have $P_i(t) > P_i(t')$.*

We then define the three following sets:

- $hp_i = \{j \in [1, n]\,,\ P_j > P_i\}$;
- $sp_i = \{j \in [1, n]\,,\ j \neq i\,,\ P_j = P_i\}$;
- $lp_i = \{j \in [1, n]\,,\ P_j < P_i\}$.

Therefore, if $j \in hp_i$ (respectively lp_i), the fixed priority of flow τ_j is higher (respectively lower) than the fixed priority of flow τ_i. If $j \in sp_i$, flows τ_j and τ_i share the same fixed priority. In this case, packets are scheduled according to their dynamic priorities. We then have to distinguish two kinds of flows: (i) flows that are able to generate packets with a dynamic priority higher than this of packet m and (ii) flows that are not able to generate such packets. Then, for any time $t \geq -J_i$, we denote:

- $sp_i(t) = \{j \in sp_i\,,\ P_j(-J_j) \geq P_i(t)\}$;
- $\overline{sp}_i(t) = \{j \in sp_i\,,\ P_j(-J_j) < P_i(t)\}$.

Definition 2 *Let m be the packet of any flow τ_i generated at time t. For any flow τ_j, if $j \in sp_i(t)$, then $G_{j,i}(t)$ denotes the time beyond which τ_j can no longer generate packets with a dynamic priority higher than this of m, that is:*

$$\forall\, t' \in [-J_j, G_{j,i}(t)],\ P_j(t') \geq P_i(t).$$

Assumption 3 *For any flows τ_i and τ_j, $j \in sp_i$, for any time $t \geq -J_i$, for any $\Delta \in \mathbb{N}$, $G_{j,i}(t + \Delta) \leq G_{j,i}(t) + \Delta$.*

For example, we get for any flow τ_j, $j \in sp_i(t)$:

- $G_{j,i}(t) = t$ if the scheduling algorithm is FP/FIFO*;
- $G_{j,i}(t) = t + D_i - D_j$ if the scheduling algorithm is FP/EDF*.

Both scheduling algorithms meet Assumption 3.

Hence, priority of packet m is higher than or equal to this of packet m' belonging to any flow τ_j and generated at time t' if and only if:

$$\begin{cases} P_i > P_j \\ or \\ P_i = P_j \quad and \quad P_i(t) \geq P_j(t'). \end{cases}$$

We also denote:

- $\mathcal{P}_i = [first_i, last_i]$, the path followed by flow τ_i, with $first_i$ (resp. $last_i$) the ingress node (resp. the egress node) of the flow in the network;
- $\mathcal{P}_i^h = [first_i, h] \subseteq \mathcal{P}_i$, the path followed by flow τ_i until node h. Notice that $\mathcal{P}_i^{last_i} = \mathcal{P}_i$;
- $|\mathcal{P}_i|$, the cardinal of path $\mathcal{P}_i$, that is the number of nodes visited by flow τ_i;
- $slow_i$, the slowest node visited by flow τ_i on $\mathcal{P}_i$, that is: $\forall\, h \in \mathcal{P}_i$, $C_i^{slow_i} \geq C_i^h$;
- $slow_i^h$, the slowest node visited by flow τ_i on $\mathcal{P}_i^h$, that is: $\forall\, k \in \mathcal{P}_i^h$, $C_i^{slow_i^h} \geq C_i^k$;
- $first_{j,i}$, the first node on path $\mathcal{P}_i$ visited by flow τ_j;
- $first_{j,i}^h$, the first node on path $\mathcal{P}_i^h$ visited by flow τ_j;
- $last_{j,i}$, the last node on path $\mathcal{P}_i$ visited by flow τ_j;
- $last_{j,i}^h$, the last node on path $\mathcal{P}_i^h$ visited by flow τ_j;
- $slow_{j,i}$, the slowest node of $\mathcal{P}_i$ visited by flow τ_j, that is: for all node $h \in \mathcal{P}_i$ visited by τ_j, $C_j^{slow_{j,i}} \geq C_j^h$;
- $slow_{j,i}^h$, the slowest node of $\mathcal{P}_i^h$ visited by flow τ_j, that is: for all node $k \in \mathcal{P}_i^h$ visited by τ_j, $C_j^{slow_{j,i}^h} \geq C_j^k$;
- $\delta_i(t)$, the maximum delay incurred by the packet of flow τ_i requested at time t while following path $\mathcal{P}_i$ directly due to the non-preemptive effect;
- $\delta_i^h(t)$, the maximum delay incurred by the packet of flow τ_i requested at time t while following path $\mathcal{P}_i^h$ directly due to the non-preemptive effect.

Figure 1 illustrates the notations of $first_{i,j}$, $first_{j,i}$, $last_{i,j}$ and $last_{j,i}$ (1) when flows τ_i and τ_j are in the same direction and (2) when flows τ_i and τ_j are in reverse directions.

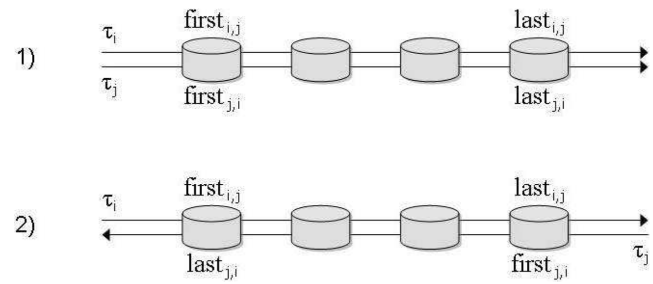

Figure 1: Illustration of $first_{i,j}$, $first_{j,i}$, $last_{i,j}$ and $last_{j,i}$

Finally, we adopt the following notations:

- R_i, the worst case end-to-end response time of flow τ_i;
- R_i^h, the maximum sojourn time of flow τ_i on node h;
- Jin_i^h, the jitter of flow τ_i when entering node h. Notice that $Jin_i^{first_i} = J_i$;
- $Jout_i^h$, the jitter of flow τ_i when leaving node h;
- $Smin_i^h$, the minimum time taken by a packet of τ_i to go from its source node to node h;
- $Smax_i^h$, the maximum time taken by a packet of τ_i to go from its source node to node h;
- $W_i^h(t)$, the latest starting time on node h of the packet of τ_i generated at time t;
- $M_i^h(t) = \sum_{h'=first_i}^{pre_i(h)} (\min_{\substack{j \in hp_i \cup sp_i(t) \cup \{i\} \\ first_{j,i}=first_{i,j}}} \{C_j^{h'}\} + Lmin)$, where $pre_i(h)$ denotes the node visited before node h by flow τ_i.

We now recall the definition of the processor utilization factor for a set of flows.

Definition 3 *For any node h, for any flow τ_i visiting h, the processor utilization factor for the flows belonging to any set $\mathcal{S}$ is denoted $U_{\mathcal{S}}^h$. It is the fraction of processor time spent by node h to process packets belonging to $\mathcal{S}$. It is equal to $\sum_{j\in\mathcal{S}}(C_j^h/T_j)$.*

3 State of the art

To determine the maximum end-to-end response time, several approaches can be used: a stochastic or a deterministic one. A *stochastic approach* consists in determining the mean behavior of the considered network, leading to mean, statistical or probabilistic end-to-end response times [6, 7]. A *deterministic approach* is based on a worst case analysis of the network behavior, leading to worst case end-to-end response times [8, 9]. In this paper, we are interested in the deterministic approach as we want to provide a deterministic guarantee of worst case end-to-end response times for any flow in the network. In this context, two different approaches can be used to determine the worst case end-to-end delay: the holistic approach and the trajectory approach.

• The *holistic approach* [10] considers the worst case scenario on each node visited by a flow, accounting for the maximum possible jitter introduced by the previous visited nodes. If no jitter control is done, the maximum jitter will increase throughout the visited nodes. In this case, the minimum and maximum response times on a node h induce a maximum jitter on the next visited node $h+1$ that leads to a worst case response time and then a maximum jitter on the following node and so on. Otherwise, the jitter can be either cancelled or constrained.

- The *Jitter Cancellation technique* consists in cancelling, on each node, the jitter of a flow before it is considered by the node scheduler [9]: a flow packet is held until its latest possible reception time. Hence a flow packet arrives at node $h+1$ with a jitter depending only on the jitter introduced by the previous node h and the link between them. As soon as this jitter is cancelled, this packet is seen by the scheduler of node $h+1$. The worst case end-to-end response time is obtained by adding the worst case response time, without jitter (as cancelled) on every node;
- The *Constrained Jitter technique* consists in checking that the jitter of a flow remains bounded by a maximum acceptable value before the flow is considered by the node scheduler. If not, the jitter is reduced to the maximum acceptable value by means of traffic shaping.

As a conclusion, the holistic approach can be pessimistic as it considers worst case scenarios on every node possibly leading to impossible scenarios.

• The *trajectory approach* [11] consists in examining the scheduling produced by all the visited nodes of a flow. In this approach, only possible scenarios are examined. For instance, the fluid model (see [12] for GPS) is relevant to the trajectory approach. This approach produces the best results as no impossible scenario is considered but is somewhat more complex to use. This approach can also be used in conjunction with a jitter control (see [13] for EDF, and [12] for GPS). In this paper, we adopt the trajectory approach without jitter control in a distributed system to determine the maximum end-to-end response time of a flow.

We can also distinguish two main traffic models: the sporadic model and the token bucket model. The sporadic model has been used in the holistic approach and in the trajectory approach, while the token bucket model has been used only in the trajectory approach.

• The *sporadic model* is classically defined by three parameters: the maximum processing time, the minimum interarrival time and the maximum release jitter, (see the traffic model in subsection 2.1). This model is natural and well adapted for real-time applications.

• The *token bucket* [8, 12, 13] is defined by two parameters: σ, the bucket size and ρ, the token throughput. The token bucket can model a flow or a flow aggregate. In the first case, it requires to maintain per flow information on every visited node. This solution is not scalable. In the second case, the choice of good values for the token bucket parameters is complex when flows have different characteristics. A bad choice can lead to bad response times, as the end-to-end response times strongly depend on the choice of the token bucket parameters [13, 14]. Furthermore, the token bucket parameters can be optimized for a given configuration, only valid at a given time. If the configuration evolves, the parameters of the token bucket should be recomputed on every node to remain optimal. This is not generally done.

In this paper, we establish new results by adopting the trajectory approach with the sporadic traffic model.

4 Worst case analysis

In this section, we determine the worst case end-to-end response time of any flow τ_i by applying the trajectory approach. For the sake of simplicity, we first assume that, with regard to flow τ_i following path $\mathcal{P}_i$, any flow τ_j, $j \in hp_i \cup sp_i$, crosses path $\mathcal{P}_i$ at most once. We will see in subsection 4.7 how to remove this assumption.

Assumption 4 *For any flow τ_i following path $\mathcal{P}_i$, for any flow τ_j, $j \in hp_i \cup sp_i$, following path $\mathcal{P}_j$ such that $\mathcal{P}_i \cap \mathcal{P}_j \neq \emptyset$, we have:*

- *either $[first_{j,i}, last_{j,i}] \subseteq \mathcal{P}_i$*
- *or $[last_{j,i}, first_{j,i}] \subseteq \mathcal{P}_i$.*

In other words, Assumption 4 means that with regard to a flow τ_i, if any flow τ_j, $j \in hp_i \cup sp_i$, after having visited a node belonging to $\mathcal{P}_i$, visits a node not belonging to $\mathcal{P}_i$, then τ_j will never visit a node belonging to $\mathcal{P}_i$.

Remark 1 *With regard to any flow τ_i and any flow $\tau_j \in hp_i \cup sp_i$, if Assumption 4 is met and $first_{j,i} = first_{i,j}$, then flows τ_i and τ_j visit their single common sequence of nodes in the same order. Otherwise, flows τ_i and τ_j visit their single common sequence of nodes in the reverse order.*

On Figure 2, if flow τ_2 belongs to $hp_1 \cup sp_1$, Assumption 4 is not met. We will see in Section 4.7 how to remove this assumption and extend our results to the general case.

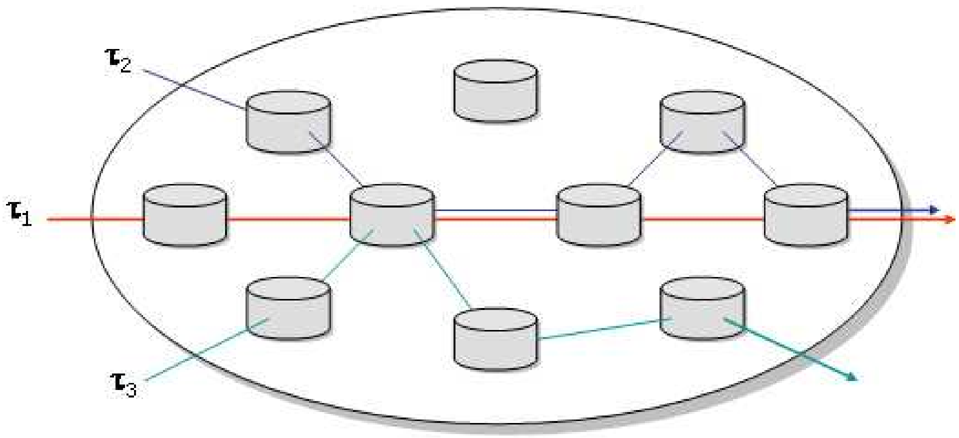

Figure 2: Illustration of Assumption 4

In this section, we show how to compute a bound on the worst case end-to-end response time of any flow τ_i. As said in the previous section, we adopt the trajectory approach [11], that consists in examining the scheduling produced by all the visited nodes of a flow. In this approach, only possible scenarios are examined. For instance, the fluid model (see [12] for GPS) is relevant to the trajectory approach. This approach produces the best results as no impossible scenario is considered but is somewhat more complex to use. This approach can also be used in conjunction with a jitter control (see [13] for EDF, and [12] for GPS).

In the following, m denotes the packet of flow τ_i generates at time t. We first evaluate in Subsection 4.1 the impact of the non-preemption, that is the maximum delay incurred by m while following its path due to the non-preemptive effect. Next, we compute in Subsection 4.2 the latest starting time of packet m on its last node visited and then analyze the mathematical expression of this latest starting time, that is an iterative expression. From these results, we determine in Subsection 4.3 the kind of packets belonging to flow τ_i that lead to the worst case end-to-end response time. Indeed, we show that only packets generated at specific times have to be considered. Finally, we establish the worst case end-to-end response time of flow τ_i in Subsection 4.4 and give the worst case response time computation algorithm in Subsection 4.5.

4.1 Non-preemptive effect

We recall that packet scheduling is non-preemptive. Hence, despite the high priority of any packet m, released at time t, a packet with a lower priority can delay m processing due to non-preemption. Indeed, if a packet m of any flow τ_i arrives on node h while a packet m' belonging to $lp_i \cup \overline{sp}_i(t)$ is being processed, m has to wait until m' completion. It is important to notice that the non-preemptive effect is not limited to this waiting time. The delay incurred by packet m on node h directly due to m' may lead to consider packets belonging to $hp_i \cup sp_i(t)$, arrived after m on the node but before m starts its execution. Then, we denote: $\delta_i(t)$, the maximum delay incurred by packet m while following its path directly due to the non-preemptive effect.

Property 1 *Let τ_i, $i \in [1,n]$, be a flow following path $\mathcal{P}_i = [first_i, ..., last_i]$. When flows are scheduled FP/DP*, the maximum delay incurred by the packet of τ_i generated at time t directly due to flows belonging to $lp_i \cup \overline{sp}_i(t)$ meets:*

$$\delta_i(t) \leq \max\left(0\,;\max_{\substack{j\in lp_i\cup\overline{sp}_i(t)\\ first_{j,i}=first_i}}\left\{C_j^{first_i}\right\}-1\right)$$
$$+\sum_{\substack{h\in\mathcal{P}_i\\ h\neq first_i}}\max\left(0\,;\max_{\substack{j\in lp_i\cup\overline{sp}_i(t)\\ first_{j,i}=h}}\left\{C_j^h\right\}-1\,;\max_{\substack{j\in lp_i\cup\overline{sp}_i(t)\\ h\in(first_{j,i},last_{j,i}]\\ first_{j,i}\neq first_{i,j}}}\left\{C_j^h\right\}-1\,;\right.$$
$$\left.1_\alpha\cdot\left(\max_{\substack{j\in lp_i\cup\overline{sp}_i(t)\\ h\in(first_{j,i},last_{j,i}]\\ first_{j,i}=first_{i,j}}}\left\{C_j^h\right\}-C_i^{pre_i(h)}+L_{max}-L_{min}\right)\right),$$

where $\max\limits_{j\in lp_i\cup\overline{sp}_i(t)}\{C_j^h\}=0$ *if* $lp_i\cup\overline{sp}_i(t)=\emptyset$ *and* $1_\alpha=1$ *if* $lp_i\cup\overline{sp}_i(t)\neq\emptyset$ *and* 0 *otherwise.*

Proof: By recurrence on the number of nodes visited. On the first node visited, Property 1 is true. Assuming that Property 1 is true at rank h. We prove it at rank $h+1$. Let us consider packet m of flow τ_i generated at time t. Due to the non-preemption, on any node $h \in (first_i, last_i]$, a packet m' belonging to a flow τ_j, $j \in lp_i \cup \overline{sp}_i(t)$, can delay the execution of packet m if m arrives on node h while m' is being processed. Then, we have to distinguish three cases:

- Node h is the first node of $\mathcal{P}_i$ visited by τ_j ($first_{j,i} = h$). Hence, the maximum delay incurred by m directly due to flow τ_j meets: $C_j^h - 1$;
- Node h is not the first node of $\mathcal{P}_i$ visited by τ_j ($h \in (first_{j,i}, last_{j,i}]$) and $first_{j,i} \neq first_{i,j}$. Hence, the maximum delay incurred by m directly due to flow τ_j meets: $C_j^h - 1$;
- Node h is not the first node of $\mathcal{P}_i$ visited by τ_j ($h \in (first_{j,i}, last_{j,i}]$) and $first_{j,i} = first_{i,j}$. Packet m' leaves node $pre_i(h)$ at the latest at time $W_i^{pre_i(h)}(t)$. Then, packet m' ends its processing on node h at the latest at time $W_i^{pre_i(h)}(t) + L_{max} + C_j^h$. As packet m arrives on node h at the earliest at time $W_i^{pre_i(h)}(t) + C_i^{pre_i(h)} + L_{min}$, the maximum delay incurred by m directly due to flow τ_j meets: $\max\left(0\,; C_j^h - C_i^{pre_i(h)} + L_{max} - L_{min}\right)$.

Moreover, $C_j^h \leq \max_{j\in lp_i\cup\overline{sp}_i(t)}\{C_j^h\}$. Hence the property. ■

4.2 Latest starting time computation

We consider any flow τ_i, $i \in [1, n]$, and focus on the packet m of τ_i generated at time t. We assume that flow τ_i follows path $\mathcal{P}_i$ consisting of q nodes numbered from 1 to q. As we consider a non-preemptive scheduling, we are interested in determining the latest starting time of packet m processing on its last node visited. Unlike the holistic approach, the trajectory approach is based on the analysis of the worst case scenario experienced by packet m on its trajectory and not on any node visited. The strategy is to move backwards through the sequence of nodes m traverses, each time identifying preceding packets and busy periods that ultimately affect the delay of m. We proceed in that way till finally at node 1, the very first packet $f(1)$ to affect the delay of m is identified. Thereafter, the worst case delay is estimated from the time that $f(1)$ enters the domain to the time that m exits the domain. This, when subtracted by the difference between the arrival times of m and $f(1)$ to the domain, yields the worst case delay for m.

To compute the latest starting time on node q of packet m, we focus on the busy period of level corresponding to m priority[1] in which m is processed on node q. Let bp^q this busy period. We define $f(q)$ as the first packet processed in bp^q with a priority higher than or equal to this of packet m. Due to the non-preemption, this packet can be delayed by at most one packet with a priority less than this of packet m.

As flows do not necessarily follow the same path in the network considered, it is possible that packet $f(q)$ does not come from node $q-1$. We then define $p(q-1)$ as the first packet processed between $f(q)$ and m such that $p(q-1)$ comes from node $q-1$. Packet $p(q-1)$ has been processed in a busy period of level corresponding to m priority on node $q-1$. Let bp^{q-1} this busy period.

We then define $f(q-1)$ as the first packet processed in bp^{q-1} with a priority higher than or equal to this of packet m. And so on until the busy period of level corresponding to m priority on node 1 in which the packet $f(1)$ is processed (see figure 3).

[1] A busy period of level $\mathcal{L}$ is defined by an interval $[t, t')$ such that t and t' are both idle times of level $\mathcal{L}$ and there is no idle time of level $\mathcal{L}$ in (t, t'). An idle time t of level $\mathcal{L}$ is a time such that all packets with a priority greater than or equal to $\mathcal{L}$ generated before t have been processed at time t.

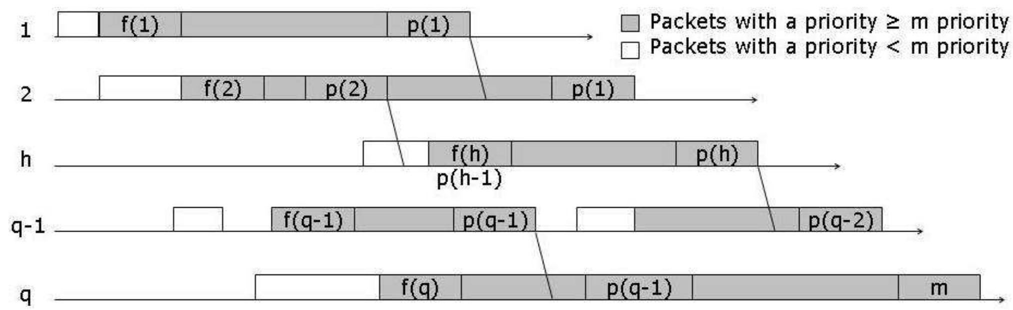

Figure 3: Response time of packet m

For the sake of simplicity, on a node h, we number consecutively the packets processed after $f(h)$ and before $p(h)$ (with $p(q) = m$). Hence, on node h, we denote $m' - 1$ (resp. $m' + 1$) the packet preceding (resp. succeeding to) m'. Moreover, we denote $a^h_{m'}$ the arrival time of packet m' on node h and consider the arrival time of packet $f(1)$ in node 1 as the time origin ($a^1_{f(1)} = 0$). By adding parts of the busy periods considered, we can express the latest starting time of packet m in node q, that is:

	the processing time on node 1 of packets $f(1)$ to $p(1)$ $+$ L_{max}
$+$	the processing time on node 2 of packets $f(2)$ to $p(2)$ $+$ L_{max} $-$ $(a^2_{p(1)} - a^2_{f(2)})$
$+$	...
$+$	the processing time on node q of packets $f(q)$ to $(m-1)$ $-$ $(a^q_{p(q-1)} - a^q_{f(q)})$
$+$	$\delta_i(t)$, the maximum delay incurred by m directly due to the non-preemptive effect.

We can notice that on any node $h \in \mathcal{P}_i$, if there exists no flow τ_j such that $h = first_{j,i}$, then $p(h-1) = f(h)$ and so $a^h_{p(h-1)} - a^h_{f(h)} = 0$. In other words, if $p(h-1) \neq f(h)$, then there exists a flow τ_j such that $h = first_{j,i}$. In such a case, by definition of $p(h)$, all the packets in $[f(h), p(h-1))$ cross path $\mathcal{P}_i$ for the first time at node h. We can then act on their arrival times. Postponing the arrivals of these packets in the busy period where $p(h-1)$ is processed, would increase the departure time of m from node q. Hence, in the worst case, $p(h) = f(h+1)$ on any node $h \in \mathcal{P}_i$. Thus, we get:

$$W^q_i(t) = \sum_{h=1}^{q} \left(\sum_{g=f(h)}^{f(h+1)} C^h_{\tau(g)} \right) - C^q_i + \delta_i(t) + (q-1) \cdot L_{max}.$$

Notice that $\sum_{h=1}^{q}(\sum_{g=f(h)}^{f(h+1)} C^h_{\tau(g)}) - C^q_i$ represents the maximum delay incurred by m due to packets with a priority higher than or equal to this of m. We now evaluate the term: $X = \sum_{h=1}^{q}(\sum_{g=f(h)}^{f(h+1)} C^h_{\tau(g)})$. By definition, for any node $h \in [1, slow_i)$, $f(h+1)$ is the first packet with a priority higher than or equal to this of m, processed in bp^{h+1} and coming from node h. Moreover, $f(h+1)$ is the last packet considered in bp^h. Hence, if we count packets processed in bp^h and bp^{h+1}, only $f(h+1)$ is counted twice. In the same way, for any node $h \in (slow_i, q]$, $f(h)$ is the first packet with a priority higher than or equal to this of m, processed in bp^h and coming from node $h-1$. Moreover, $f(h)$ is the last packet considered in bp^{h-1}. Thus, $f(h)$ is the only packet counted twice when counting packets processed in bp^{h-1} and bp^h. Hence, X is equal to:

$$\underbrace{\sum_{h=1}^{slow_i-1}\left(\sum_{g=f(h)}^{f(h+1)-1} C^h_{\tau(g)} + C^h_{\tau(f(h+1))}\right)}_{\text{nodes visited before } slow_i} + \underbrace{\sum_{g=f(slow_i)}^{f(slow_i+1)} C^{slow_i}_{\tau(g)}}_{\text{node } slow_i} + \underbrace{\sum_{h=slow_i+1}^{q}\left(\sum_{g=f(h)+1}^{f(h+1)} C^h_{\tau(g)} + C^h_{\tau(f(h))}\right)}_{\text{nodes visited after } slow_i}.$$

Moreover, for any node $h \in [1, q]$, for any packet g visiting h, the processing time of g on node h is less than or equal to $C^{slow_{\tau(g),i}}_{\tau(g)}$. Then, as packets are numbered consecutively from $f(1)$ to $f(q+1) = m$, we get inequation (1). In addition, by considering that on any node $h \in [1, slow_i)$ (resp. $h \in (slow_i, q]$), the processing time of packet $f(h+1)$ (resp. $f(h)$) on node h is less than or equal to $\max_{j \in hp_i \cup sp_i(t) \cup \{i\}}\{C^h_j\}$ and in the worst case, $f(h+1)$ is a packet coming from node h, we get inequation (2).

$$\sum_{h=1}^{slow_i-1}\left(\sum_{g=f(h)}^{f(h+1)-1} C^h_{\tau(g)}\right) + \sum_{g=f(slow_i)}^{f(slow_i+1)} C^{slow_i}_{\tau(g)} + \sum_{h=slow_i+1}^{q}\left(\sum_{g=f(h)+1}^{f(h+1)} C^h_{\tau(g)}\right) \leq \sum_{g=f(1)}^{m} C^{slow_i}_{\tau(g)} \quad (1)$$

$$\sum_{h=1}^{slow_i-1} C^h_{\tau(f(h+1))} + \sum_{h=slow_i+1}^{q} C^h_{\tau(f(h))} \leq \sum_{\substack{h=1\\ h \neq slow_i}}^{q} \max_{\substack{j \in hp_i \cup sp_i(t) \cup \{i\}\\ first_{j,i} = first_{i,j}}} \{C^h_j\} \quad (2)$$

By (1) and (2), we get:

$$X \leq \sum_{g=f(1)}^{m} C^{slow_{\tau(g),i}}_{\tau(g)} + \sum_{\substack{h=1\\ h \neq slow_i}}^{q} \max_{\substack{j \in hp_i \cup sp_i(t) \cup \{i\}\\ first_{j,i} = first_{i,j}}} \{C^h_j\}.$$

The term X corresponds to the maximum delay incurred by m due to packets belonging to flows having a priority higher than or equal to this of m. The term $\sum_{g=f(1)}^{m} C^{slow_{\tau(g),i}}_{\tau(g)}$ is maximized when the workload generated by such flows is maximum. An upper bound on X is given in Lemma 1.

Lemma 1 *Let m be the packet of flow τ_i generated at time t. When flows are scheduled FP/DP*, the maximum delay incurred by m due to packets having a priority higher than or equal to this of m is bounded by:*

$$\sum_{j\in hp_i}\left(1+\left\lfloor\frac{W_i^{last_{i,j}}(t)-Smin_j^{last_{i,j}}-M_i^{first_{i,j}}(t)+Smax_j^{first_{i,j}}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{slow_{j,i}}$$
$$+\sum_{j\in sp_i(t)}\left(1+\left\lfloor\frac{\min\left(G_{j,i}(t)\,;\,W_i^{last_{i,j}}(t)-Smin_j^{last_{i,j}}\right)-M_i^{first_{i,j}}(t)+Smax_j^{first_{i,j}}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{slow_{j,i}}$$
$$+\left(1+\left\lfloor\frac{t+J_i}{T_i}\right\rfloor\right)\cdot C_i^{slow_i}+\sum_{\substack{h\in\mathcal{P}_i\\ h\neq slow_i}}\max_{\substack{j\in hp_i\cup sp_i(t)\cup\{i\}\\ first_{j,i}=first_{i,j}}}\{C_j^h\}.$$

Proof: Considering a packet m of flow τ_i generated at time t:

- Packets of flow τ_j, $j\in hp_i$, can delay m if they are generated at the earliest at time $a_{f(first_{i,j})}^{first_{i,j}}-Smax_j^{first_{i,j}}-J_j$ and at the latest at time $W_i^{last_{i,j}}(t)-Smin_j^{last_{i,j}}$;
- Packets of flow τ_j, $j\in sp_i(t)$, can delay m if they are generated at the earliest at time $a_{f(first_{i,j})}^{first_{i,j}}-Smax_j^{first_{i,j}}-J_j$ and at the latest at time $\min(G_{j,i}(t),W_i^{last_{i,j}}(t)-Smin_j^{last_{i,j}})$;
- Packets of flow τ_i can delay m if they are generated at the earliest at time $-J_i$ and at the latest at time t.

As (i) the maximum workload generated by any flow τ_j in the interval $[a,b]$ on node h is equal to $(1+\lfloor(b-a)/T_j\rfloor)\cdot C_j^h$ and (ii) $a_{f(first_{i,j})}^{first_{i,j}}\geq M_i^{first_{i,j}}(t)$, we get the lemma. ■

Property 2 *Let m be the packet of flow τ_i generated at time t. When flows are scheduled FP/DP*, then $W_i^{last_i}(t)$, the latest starting time of packet m on its last node visited, is bounded by:*

$$\sum_{j\in hp_i}\left(1+\left\lfloor\frac{W_i^{last_{i,j}}(t)-Smin_j^{last_{i,j}}-M_i^{first_{i,j}}(t)+Smax_j^{first_{i,j}}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{slow_{j,i}}$$
$$+\sum_{j\in sp_i(t)}\left(1+\left\lfloor\frac{\min\left(G_{j,i}(t)\,;\,W_i^{last_{i,j}}(t)-Smin_j^{last_{i,j}}\right)-M_i^{first_{i,j}}(t)+Smax_j^{first_{i,j}}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{slow_{j,i}}$$
$$+\left(1+\left\lfloor\frac{t+J_i}{T_i}\right\rfloor\right)\cdot C_i^{slow_i}-C_i^{last_i}+\sum_{\substack{h\in\mathcal{P}_i\\ h\neq slow_i}}\max_{\substack{j\in hp_i\cup sp_i(t)\cup\{i\}\\ first_{j,i}=first_{i,j}}}\{C_j^h\}+\delta_i(t)+(|\mathcal{P}_i|-1)\cdot Lmax.$$

Proof: By lemma 1 and Property 1. ∎

The expression of time $W_i^{last_i}(t)$ is recursive. Let us consider the following series for any node $h \in \mathcal{P}_i$:

$$\begin{cases} \mathcal{W}_i^{h\,(0)}(t) = \sum_{j \in hp_i \cup sp_i(t)} C_j^{slow_{j,i}^h} + \left(1 + \left\lfloor \frac{t+J_i}{T_i} \right\rfloor\right) \cdot C_i^{slow_i^h} \\ \quad + \sum_{\substack{k \in \mathcal{P}_i^h \\ k \neq slow_i^h}} \max_{\substack{j \in hp_i \cup sp_i(t) \cup \{i\} \\ first_{j,i} = first_{i,j}}} \{C_j^k\} - C_i^h + \delta_i^h(t) + (|\mathcal{P}_i^h| - 1) \cdot Lmax \\ \\ \mathcal{W}_i^{h\,(q+1)}(t) = \sum_{j \in hp_i} \left(1 + \left\lfloor \frac{\mathcal{W}_i^{last_{i,j}^h\,(q)}(t) - Smin_j^{last_{i,j}^h} - M_i^{first_{i,j}^h}(t) + Smax_j^{first_{i,j}^h\,(q)} + J_j}{T_j} \right\rfloor^+\right) \cdot C_j^{slow_{j,i}^h} \\ \quad + \sum_{j \in sp_i(t)} \left(1 + \left\lfloor \frac{\min\left(G_{i,j}(t)\,;\ \mathcal{W}_i^{last_{i,j}^h\,(q)}(t) - Smin_j^{last_{i,j}^h}\right) - M_i^{first_{i,j}^h}(t) + Smax_j^{first_{i,j}^h} + J_j}{T_j} \right\rfloor^+\right) \cdot C_j^{slow_{j,i}^h} \\ \quad + \left(1 + \left\lfloor \frac{t+J_i}{T_i} \right\rfloor\right) \cdot C_i^{slow_i^h} \\ \quad + \sum_{\substack{k \in \mathcal{P}_i^h \\ k \neq slow_i^h}} \max_{\substack{j \in hp_i \cup sp_i(t) \cup \{i\} \\ first_{j,i} = first_{i,j}}} \{C_j^k\} - C_i^h + \delta_i^h(t) + (|\mathcal{P}_i^h| - 1) \cdot Lmax. \end{cases}$$

To prove the convergence of $\mathcal{W}_i^h(t)$, established in Condition 1, we have to consider the set $\mathcal{S}_i$, that is the set of flows τ_j, $j \in hp_i \cup sp_i \cup \{i\}$, crossing directly or indirectly flow τ_i. More formally, we proceed as follows to determine the set $\mathcal{S}_i$:

- $\mathcal{S}_i = \{\tau_i\}$;
- $\mathcal{S}_i = \mathcal{S}_i \cup \{\tau_j, j \in hp_i \cup sp_i, \tau_j$ crosses directly $\tau_i\}$;
- $\mathcal{S}_i = \mathcal{S}_i \cup \{\tau_k, k \in hp_i \cup sp_i, \exists \tau_j \in \mathcal{S}_i$ such that τ_k crosses directly $\tau_j\}$.

Condition 1 *Let m be the packet of flow τ_i generated at time t. Let $\mathcal{S}_i$ be the set of flows τ_j, $j \in hp_i \cup sp_i \cup \{i\}$, crossing directly or indirectly flow τ_i. When flows are scheduled FP/DP* and $\max_{j \in \mathcal{S}_i}\{\sum_{k \in hp_j \cup sp_j} C_k^{slow_{k,j}}/T_k\} < 1/2$, the series $\mathcal{W}_i^{last_i}(t)$ is convergent. The solution is equal to $W_i^{last_i}(t)$.*

Proof: We prove by recurrence the following property:
if $\max_{j\in\mathcal{S}_i}\{\sum_{k\in hp_j\cup sp_j} C_k^{slow_{k,j}}/T_k\} < 1/2$, we have for any flow $\tau_j \in \mathcal{S}_i$ and for any time t:

- $\mathcal{W}_j^{last_j}(t) \leq X/(1-2\cdot U)$
- $Smax_j^{last_j} \leq X/(1-2\cdot U) + J_j$

where $X = \max_{j\in\mathcal{S}_i}\left\{X_j = \sum_{k\in hp_j\cup sp_j(t)}(1+\frac{2\cdot J_k}{T_k})\cdot C_k^{slow_{k,j}} + \Delta_j(t)\right\}$,

$$\Delta_j(t) = \left(1+\left\lfloor\frac{t+J_j}{T_j}\right\rfloor\right)\cdot C_j^{slow_j} + \sum_{\substack{h\in\mathcal{P}_j\\ h\neq slow_j}} \max_{\substack{k\in hp_j\cup sp_j(t)\cup\{j\}\\ first_{k,j}=first_{j,k}}}\{C_k^h\} - C_j^{last_j} + \delta_j(t) + (|\mathcal{P}_j|-1)\cdot Lmax$$

and $U = \max_{j\in\mathcal{S}_i}\{\sum_{k\in hp_j\cup sp_j} C_k^{slow_{k,j}}/T_k)\}$.

Indeed, for any flow $\tau_j \in \mathcal{S}_i$ and for any time $t \geq -J_j$, the series $\mathcal{W}_j^{last_j}(t)$ is non-decreasing as the floor function ($\lfloor\ \rfloor$) is non-decreasing. Moreover, at rank 0, we have: $\mathcal{W}_j^{last_j\ (0)}(t) = \sum_{k\in hp_j\cup sp_j(t)} C_k^{slow_{k,j}} + \Delta_j \leq X_j$. As $X_j \leq X$ and $U < 1/2$, $\mathcal{W}_j^{last_j\ (0)}(t) \leq X/(1-2\cdot U)$. In the same way, $Smax_j^{last_j\ (0)} = \sum_{h=first_j}^{pre_j(last_j)}(C_j^h + Lmax)$, that is less than or equal to $X/(1-2\cdot U) + J_j + C_j^{last_j}$.

We now assume that the property is true until rank q and prove it at rank $q+1$.

$$\mathcal{W}_j^{last_j\ (q+1)}(t) = \sum_{k\in hp_j}\left(1+\left\lfloor\frac{\mathcal{W}_j^{last_{j,k}\ (q)}(t) - Smin_k^{last_{j,k}} - M_j^{first_{j,k}}(t) + Smax_k^{first_{j,k}\ (q)} + J_k}{T_k}\right\rfloor^+\right)\cdot C_k^{slow_{k,j}}$$

$$+\sum_{k\in sp_j(t)}\left(1+\left\lfloor\frac{\min\left(G_{j,k}(t)\,;\,\mathcal{W}_j^{last_{j,k}\ (q)}(t) - Smin_k^{last_{j,k}}\right) - M_j^{first_{j,k}}(t) + Smax_k^{first_{j,k}\ (q)} + J_k}{T_k}\right\rfloor^+\right)C_k^{slow_{k,j}}$$

$+\Delta_j(t)$.
As for any $x \in \mathbb{R}^+$, $\lfloor x\rfloor \leq x$ and for any $(a,b)\in\mathbb{N}^2$, $\min(a;b)\leq b$, we get:

$$\mathcal{W}_j^{last_j\ (q+1)}(t) \leq \sum_{k\in hp_j\cup sp_i(t)}\left(1+\frac{\mathcal{W}_j^{last_{j,k}\ (q)}(t) + Smax_k^{first_{j,k}\ (q)} + J_k}{T_k}\right)\cdot C_k^{slow_{k,j}} + \Delta_j(t).$$

As the property is true at rank q, we have:

$$\mathcal{W}_j^{last_j\ (q+1)}(t) \leq \frac{2\cdot X}{1-2\cdot U}\cdot\sum_{k\in hp_j\cup sp_i(t)}\frac{C_k^{last_k}}{T_k} + \sum_{k\in hp_j\cup sp_i(t)}\left(1+\frac{2\cdot J_k}{T_k}\right)\cdot C_k^{slow_{k,j}} + \Delta_j(t).$$

$$\leq \frac{2\cdot X}{1-2\cdot U}\cdot U + X = X/(1-2\cdot U).$$

In the same way, we have:

$$S_{max_j}^{last_j\,(q+1)} = \max_t\{W_j^{pre_j(last_j)\,(q)}(t) - t + C_j^{pre_j(last_j)}\}$$

$$\leq \max_t\{W_j^{last_j\,(q)}(t) - t\} \quad \leq \quad X/(1-2\cdot U) + J_j.$$

Hence, for any flow $\tau_j \in \mathcal{S}_i$ and for any time $t \geq -J_j$, the series $\mathcal{W}_j^{last_j}(t)$ is non-decreasing and upper bounded, that is convergent. ■

4.3 Set of times to be tested

The worst case end-to-end response time of packet m of flow τ_i, generated at time t, is equal to: $W_i^{last_i}(t)+C_i^{last_i}-t$. The worst case end-to-end response time of flow τ_i is then equal to: $R_i = \max_{t\geq -J_i}\{W_i^{last_i}(t) + C_i^{last_i} - t\}$. In order not to test all times $t \geq -J_i$, we establish the following lemma.

Lemma 2 *When flows are scheduled FP/DP*, for any flow τ_i following a path $\mathcal{P}_i$, for any time t such that $\overline{sp}_i(t) = \emptyset$, we get: $W_i^{last_i}(t+\mathcal{B}_i^{slow}) \leq W_i^{last_i}(t) + \mathcal{B}_i^{slow}$, with $\mathcal{B}_i^{slow} = \sum_{j\in hp_i\cup sp_i\cup\{i\}}\lceil \mathcal{B}_i^{slow}/T_j\rceil \cdot C_j^{slow_{j,i}}$.*

Proof: We consider the series $\mathcal{W}_i^{last_i}(t)$ and prove this lemma by recurrence. For any time t such that $\overline{sp}_i(t) = \emptyset$, we have: $\forall\, \alpha \in \mathbb{N}$, $sp_i(t+\alpha) = sp_i(t)$ and $\overline{sp}_i(t+\alpha) = \emptyset$. Then, we get: $\forall\, t$ such that $\overline{sp}_i(t) = \emptyset$, $\forall\, \alpha \in \mathbb{N}$, $\delta_i(t+\alpha) = \delta_i(t)$. Moreover, $\forall\,(a,b) \in {\mathbb{R}^+}^2$, $\lfloor a+b\rfloor \leq \lfloor a\rfloor + \lceil b\rceil$. Hence, on node $first_i$, we have for any time t such that $\overline{sp}_i(t) = \emptyset$:

$$\mathcal{W}_i^{first_i\,(0)}(t+\mathcal{B}_i^{slow}) = \sum_{j\in hp_i\cup sp_i(t+\mathcal{B}_i^{slow})} C_j^{first_i} + \left(1+\left\lfloor \frac{t+\mathcal{B}_i^{slow}+J_i}{T_i}\right\rfloor\right)\cdot C_i^{first_i} - C_i^{first_i} + \delta_i(t+\mathcal{B}_i^{slow})$$

$$\leq \sum_{j\in hp_i\cup sp_i(t)} C_j^{first_i} + \left(1+\left\lfloor\frac{t+J_i}{T_i}\right\rfloor\right)\cdot C_i^{first_i} + \left\lceil \frac{\mathcal{B}_i^{slow}}{T_i}\right\rceil \cdot C_i^{first_i} - C_i^{first_i} + \delta_i(t).$$

As $\lceil \mathcal{B}_i^{slow}/T_i\rceil \cdot C_i^{first_i} \leq \mathcal{B}_i^{slow} = \sum_{j\in hp_i\cup sp_i(t)\cup\{i\}}\lceil \mathcal{B}_i^{slow}/T_j\rceil\cdot C_j^{slow_{j,i}}$, we get:
$\mathcal{W}_i^{first_i\,(0)}(t+\mathcal{B}_i^{slow}) \leq \mathcal{W}_i^{first_i\,(0)}(t) + \mathcal{B}_i^{slow}$.

We now show that if the recurrence if true at rank q, then it is true at rank $q+1$. Indeed, for any time t such that $\overline{sp}_i(t) = \emptyset$, $\mathcal{W}_i^{first_i\,(q+1)}(t+\mathcal{B}_i^{slow})$ is equal to:

$$\sum_{j\in hp_i}\left(1+\left\lfloor\frac{\mathcal{W}_i^{first_i\,(q)}(t+\mathcal{B}_i^{slow})-Smin_j^{first_i}+Smax_j^{first_i}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{first_i}$$

$$+\sum_{j\in sp_i(t+\mathcal{B}_i^{slow})}\left(1+\left\lfloor\frac{\min\left(G_{j,i}(t+\mathcal{B}_i^{slow});\ \mathcal{W}_i^{first_i\,(q)}(t+\mathcal{B}_i^{slow})-Smin_j^{first_i}\right)+Smax_j^{first_i}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{first_i}$$

$$+\quad\left(1+\left\lfloor\frac{t+\mathcal{B}_i^{slow}+J_i}{T_i}\right\rfloor\right)\cdot C_i^{fisrt_i}-C_i^{first_i}+\delta_i(t+\mathcal{B}_i^{slow}).$$

$$\leq\sum_{j\in hp_i}\left(1+\left\lfloor\frac{\mathcal{W}_i^{first_i\,(q)}(t)-Smin_j^{first_i}+Smax_j^{first_i}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{first_i}+\sum_{j\in hp_i}\left\lceil\frac{\mathcal{B}_i^{slow}}{T_j}\right\rceil\cdot C_j^{first_i}$$

$$+\sum_{j\in sp_i(t)}\left(1+\left\lfloor\frac{\min\left(G_{j,i}(t);\ \mathcal{W}_i^{first_i\,(q)}(t)-Smin_j^{first_i}\right)+Smax_j^{first_i}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{first_i}$$

$$+\sum_{j\in sp_i(t)}\left\lceil\frac{\mathcal{B}_i^{slow}}{T_j}\right\rceil\cdot C_j^{first_i}+\left(1+\left\lfloor\frac{t+J_i}{T_i}\right\rfloor\right)\cdot C_i^{first_i}+\left\lceil\frac{\mathcal{B}_i^{slow}}{T_i}\right\rceil\cdot C_i^{first_i}-C_i^{first_i}+\delta_i(t).$$

$$\leq\quad\mathcal{W}_i^{first_i\,(q)}(t)+\sum_{j\in hp_i\cup sp_i(t)\cup\{i\}}\left\lceil\frac{\mathcal{B}_i^{slow}}{T_j}\right\rceil\cdot C_j^{first_i}.$$

$$\leq\quad\mathcal{W}_i^{first_i\,(q)}(t)+\mathcal{B}_i^{slow}.$$

Hence, $\mathcal{W}_i^{first_i}(t+\mathcal{B}_i^{slow})\leq\mathcal{W}_i^{first_i}(t)+\mathcal{B}_i^{slow}$. We now assume that property is true on any node visited between node $first_i$ and $pre_i(h)$, with $h\in\mathcal{P}_i$, and prove that it is true on node h. Indeed, $\mathcal{W}_i^{h\,(0)}(t)$ is equal to:

$$\sum_{j\in hp_i\cup sp_i(t+\mathcal{B}_i^{slow})}C_j^{slow_{j,i}^h}+\left(1+\left\lfloor\frac{t+\mathcal{B}_i^{slow}+J_i}{T_i}\right\rfloor\right)\cdot C_i^{slow_i^h}$$

$$+\sum_{\substack{k\in\mathcal{P}_i^h\\ k\neq slow_i^h}}\max_{\substack{j\in hp_i\cup sp_i(t+\mathcal{B}_i^{slow})\cup\{i\}\\ first_{j,i}=first_{i,j}}}\left\{C_j^k\right\}-C_i^{last_i^h}+\delta_i^h(t)+(|\mathcal{P}_i^h|-1)\cdot Lmax$$

$$\leq\sum_{j\in hp_i\cup sp_i(t)}C_j^{slow_{j,i}^h}+\left(1+\left\lfloor\frac{t+J_i}{T_i}\right\rfloor\right)\cdot C_i^{slow_i^h}+\left\lceil\frac{\mathcal{B}_i^{slow}}{T_i}\right\rceil\cdot C_i^{slow_i^h}$$

$$+\sum_{\substack{k\in\mathcal{P}_i^h\\ k\neq slow_i^h}}\max_{\substack{j\in hp_i\cup sp_i(t)\cup\{i\}\\ first_{j,i}=first_{i,j}}}\left\{C_j^k\right\}-C_i^{last_i^h}+\delta_i^h(t)+(|\mathcal{P}_i^h|-1)\cdot Lmax.$$

As $\lceil\mathcal{B}_i^{slow}/T_i\rceil\cdot C_i^{slow_i^h}\leq\mathcal{B}_i^{slow}=\sum_{j\in hp_i\cup sp_i(t)\cup\{i\}}\lceil\mathcal{B}_i^{slow}/T_j\rceil\cdot C_j^{slow_{j,i}}$, we get: $\mathcal{W}_i^{h\,(0)}(t+\mathcal{B}_i^{slow})\leq\mathcal{W}_i^{h\,(0)}(t)+\mathcal{B}_i^{slow}$.

We now show that if the recurrence if true at rank q, then it is true at rank $q+1$. Indeed, for any time t such that $\overline{sp}_i(t)=\emptyset$, $\mathcal{W}_i^{h\,(q+1)}(t+\mathcal{B}_i^{slow})$ is equal to:

$$\sum_{j\in hp_i}\left(1+\left\lfloor\frac{\mathcal{W}_i^{last^h_{i,j}\,(q)}(t+\mathcal{B}_i^{slow})-Smin_j^{last^h_{i,j}}-M_i^{first^h_{i,j}}(t+\mathcal{B}_i^{slow})+Smax_j^{first^h_{i,j}}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{slow^h_{j,i}}$$

$$+\sum_{j\in sp_i(t+\mathcal{B}_i^{slow})}\left(1+\left\lfloor\frac{\min\left(G_{j,i}(t+\mathcal{B}_i^{slow})\,;\,\mathcal{W}_i^{last^h_{i,j}\,(q)}(t+\mathcal{B}_i^{slow})-Smin_j^{last^h_{i,j}}\right)-M_i^{first^h_{i,j}}(t+\mathcal{B}_i^{slow})+Smax_j^{first^h_{i,j}}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{slow^h_{j,i}}$$

$$+\left(1+\left\lfloor\frac{t+\mathcal{B}_i^{slow}+J_i}{T_i}\right\rfloor\right)\cdot C_i^{slow^h_i}$$

$$+\sum_{\substack{k\in\mathcal{P}_i^h\\ k\neq slow_i^h}}\max_{\substack{j\in hp_i\cup sp_i(t+\mathcal{B}_i^{slow})\cup\{i\}\\ first_{j,i}=first_{i,j}}}\left\{C_j^k\right\}-C_i^{last_i^h}+\delta_i^h(t+\mathcal{B}_i^{slow})+(|\mathcal{P}_i^h|-1)\cdot Lmax.$$

From Assumption 3, $G_{j,i}(t+\mathcal{B}_i^{slow})\leq G_{j,i}(t)+\mathcal{B}_i^{slow}$. Moreover, $\forall\,(a,b)\in\mathbb{R}^{+^2}$, $\lfloor a+b\rfloor\leq\lfloor a\rfloor+\lceil b\rceil$. Hence, $\mathcal{W}_i^{h\,(q+1)}(t+\mathcal{B}_i^{slow})$ is less than or equal to:

$$\sum_{j\in hp_i}\left(1+\left\lfloor\frac{\mathcal{W}_i^{last^h_{i,j}\,(q)}(t)-Smin_j^{last^h_{i,j}}-M_i^{first^h_{i,j}}(t)+Smax_j^{first^h_{i,j}}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{slow^h_{j,i}}+\sum_{j\in hp_i}\left\lceil\frac{\mathcal{B}_i^{slow}}{T_j}\right\rceil\cdot C_j^{slow^h_{j,i}}$$

$$+\sum_{j\in sp_i(t)}\left(1+\left\lfloor\frac{\min\left(G_{j,i}(t)\,;\,\mathcal{W}_i^{last^h_{i,j}\,(q)}(t)-Smin_j^{last^h_{i,j}}\right)-M_i^{first^h_{i,j}}(t)+Smax_j^{first^h_{i,j}}+J_j}{T_j}\right\rfloor^+\right)\cdot C_j^{slow^h_{j,i}}$$

$$+\sum_{j\in sp_i(t)}\left\lceil\frac{\mathcal{B}_i^{slow}}{T_j}\right\rceil\cdot C_j^{slow^h_{j,i}}+\left(1+\left\lfloor\frac{t+J_i}{T_i}\right\rfloor\right)\cdot C_i^{slow^h_i}+\left\lceil\frac{\mathcal{B}_i^{slow}}{T_i}\right\rceil\cdot C_i^{slow^h_i}$$

$$+\sum_{\substack{k\in\mathcal{P}_i^h\\ k\neq slow_i^h}}\max_{\substack{j\in hp_i\cup sp_i(t)\cup\{i\}\\ first_{j,i}=first_{i,j}}}\left\{C_j^k\right\}-C_i^{last_i^h}+\delta_i^h(t)+(|\mathcal{P}_i^h|-1)\cdot Lmax.$$

$$\leq\ \mathcal{W}_i^{h\,(q)}(t)+\sum_{j\in hp_i\cup sp_i(t)\cup\{i\}}\left\lceil\frac{\mathcal{B}_i^{slow}}{T_j}\right\rceil\cdot C_j^{slow^h_{j,i}}\quad=\quad\mathcal{W}_i^{h\,(q)}(t)+\mathcal{B}_i^{slow}.$$

Hence, for any time t such that $\overline{sp}_i(t)=\emptyset$, $W_i^{last_i}(t+\mathcal{B}_i^{slow})\leq W_i^{last_i}(t)+\mathcal{B}_i^{slow}$. ■

4.4 Worst case end-to-end response time

Property 3 *When flows are scheduled FP/DP*, if* $\max_{j \in \mathcal{S}_i} \{\sum_{k \in hp_j \cup sp_j} C_k^{slow_{k,j}} / T_k\} < \frac{1}{2}$*, then the worst case end-to-end response time of any flow* τ_i *is bounded by:*

$R_i = \max_{t \in [-J_i, \bar{t}_i^{\emptyset} + \mathcal{B}_i^{slow})} \left\{ W_i^{last_i}(t) - t \right\} + C_i^{last_i}$, *with:*

$$W_i^{last_i}(t) = \sum_{j \in hp_i} \left(1 + \left\lfloor \frac{W_i^{last_{i,j}}(t) - Smin_j^{last_{i,j}} - M_i^{first_{i,j}}(t) + Smax_j^{first_{i,j}} + J_j}{T_j} \right\rfloor^+ \right) \cdot C_j^{slow_{j,i}}$$

$$+ \sum_{j \in sp_i(t)} \left(1 + \left\lfloor \frac{\min\left(G_{j,i}(t)\,;\; W_i^{last_{i,j}}(t) - Smin_j^{last_{i,j}}\right) - M_i^{first_{i,j}}(t) + Smax_j^{first_{i,j}} + J_j}{T_j} \right\rfloor^+ \right) \cdot C_j^{slow_{j,i}}$$

$$+ \left(1 + \left\lfloor \frac{t + J_i}{T_i} \right\rfloor \right) \cdot C_i^{slow_i} - C_i^{last_i} + \sum_{\substack{h \in \mathcal{P}_i \\ h \neq slow_i}} \max_{\substack{j \in hp_i \cup sp_i(t) \cup \{i\} \\ first_{j,i} = first_{i,j}}} \left\{ C_j^h \right\} + \delta_i(t) + (|\mathcal{P}_i| - 1) \cdot L_{max},$$

$\bar{t}_i^{\emptyset}$ *the first time* t *such that* $\overline{sp}_i(t) = \emptyset$ *and* $\mathcal{B}_i^{slow} = \sum_{j \in hp_i \cup sp_i \cup \{i\}} \lceil \mathcal{B}_i^{slow} / T_j \rceil \cdot C_j^{slow_{j,i}}$.

Proof: From Property 2, Condition 1 and Lemma 2. ■

4.5 Computation algorithm

To compute the worst case end-to-end response time of any flow τ_i when Assumption 4 is met, we apply the following algorithm: (i) we first determine the set $\mathcal{S}_i$ of flows in $hp_i \cup sp_i \cup \{i\}$ crossing directly or indirectly flow τ_i, that is any flow τ_j belongs to $\mathcal{S}_i$ iff $j \in hp_i \cup sp_i \cup \{i\}$ and τ_j directly crosses τ_i or a flow $\tau_k \in \mathcal{S}_i$ (ii) we then initialize for the iteration $q = 1$ the value of $Smax_j^{first_{k,j}}(q)$ for any flow $\tau_k \in \mathcal{S}_i$ and for any flow τ_j crossing τ_k: we have $Smax_j^{first_{k,j}}(1) = \sum_{h=first_j}^{pre_j(first_{k,j})} (C_j^h + L_{max})$, (iii) we proceed iteratively:

```
q = 0
Repeat
  q=q+1
  for any flow τ_k ∈ S_i
    for h = first_k to last_k
      if (h = last_k) or (∃τ_j crossing τ_k such that h = last_{k,j} or h = pre_k(first_{j,k})) then
          compute W_k^h(t) using Smax_k^h(q) for any flow τ_k ∈ S_i
          if ∃j such that h = pre_k(first_{j,k}) then
            Smax_k^{first_{j,k}}(q+1) = max_t(W_k^h(t) − t) + C_k^h + Lmax
          if h = last_k then compute R_k = max_t(W_k^h(t) − t) + C_k^h
Until (∃τ_k ∈ S_i, R_k > D_k)
   or (∀τ_k ∈ S_i, ∀h = pre_k(first_{j,k}), Smax_k^{first_{j,k}}(q+1) = Smax_k^{first_{j,k}}(q))
```

4.6 Example

In this subsection, we give an example of bounds on the end-to-end response times of sporadic flows, when these flows are scheduled according to FP/EDF*. We consider that the network meets: $L_{max} = L_{min} = 1$. Moreover, we assume that: $\tau = \{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5\}$. All these flows have a period equal to 36 and enter the network without jitter. The maximum processing time of any packet of flow τ_i on node $h \in \mathcal{P}_i$ is assumed to be equal to 4. Moreover, for any flow τ_i, we have $D_i^{first_i} = \lfloor D_i / |\mathcal{P}_i| \rfloor$. Table 2 gives the fixed priority and the end-to-end deadline of each flow.

Table 2. Priorities and end-to-end deadlines

	τ_1	τ_2	τ_3	τ_4	τ_5
P_i	10	10	11	11	12
D_i	47	50	44	45	39

The path taken by each flow is defined as follows:

- $\mathcal{P}_1 = \{1, 3, 4, 5\}$;
- $\mathcal{P}_2 = \{9, 10, 7, 6\}$;
- $\mathcal{P}_3 = \{2, 3, 4, 7, 10, 11\}$;
- $\mathcal{P}_4 = \{2, 3, 4, 7, 10, 11\}$;
- $\mathcal{P}_5 = \{2, 3, 4, 7, 8\}$.

Applying Property 3, we obtain Table 3 giving the worst case end-to-end response times.

Table 3. End-to-end response times of sporadic flows

	τ_1	τ_2	τ_3	τ_4	τ_5
R_i	31	39	46	48	33

4.7 Generalization

Property 3, giving the worst case end-to-end response time of any flow τ_i following path $\mathcal{P}_i$, can be extended to the general case (i.e. by removing Assumption 4). To achieve that, the idea is to consider any flow τ_j crossing path $\mathcal{P}_i$ after it left $\mathcal{P}_i$ as a new flow. We proceed by iteration until meeting Assumption 4. We then apply Property 3 considering all these flows.

Decomposition example In Figure 4, to compute the worst case end-to-end response time of flow τ_i, flow τ_j has to be decomposed in three flows to meet Assumption 4, that is:

- τ_{j1}, following path $\mathcal{P}_{j1} = [1, 2, 3]$;
- τ_{j2}, following path $\mathcal{P}_{j2} = [4, 5, 6]$;
- τ_{j3}, following path $\mathcal{P}_{j3} = [7, 8, 9]$.

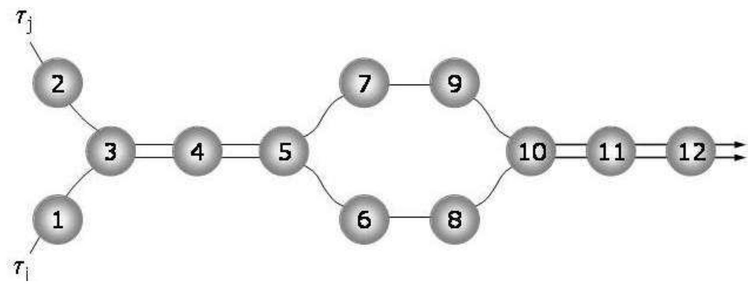

Figure 4: Decomposition of flow τ_j to meet Assumption 4

Then, each of these three flows crosses path $\mathcal{P}_i$ only once. It is important to notice that the release jitter of flow τ_{j2} is equal to the output jitter of flow τ_{j1} on node $last_{j1,i} = 3$ plus $Lmax - Lmin$. In the same way, the release jitter of flow τ_{j3} is equal to the output jitter of flow τ_{j2} on node $last_{j2,i} = 6$ plus $Lmax - Lmin$.

5 Conclusion

In this paper, we have shown how to obtain new results for non-preemptive Fixed Priority scheduling in the distributed case, assuming that flow packets sharing the same fixed priority are scheduled according to their Dynamic Priorities assigned on the first node visited. Such a scheduling is called FP/DP*. Examples are FP/FIFO* and FP/EDF*. We have presented the trajectory approach taking into account the worst case scenario experienced by a flow packet on its trajectory, unlike the holistic approach that considers the worst case scenario experienced by a flow on each visited node. Based on this worst case analysis, deterministic guarantees on the end-to-end response time and jitter can be granted to any sporadic flow in the network.

References

[1] K. Tindell, A. Burns, and A. J. Wellings. Analysis of hard real-time communications. *Real-Time Systems*, Vol. 9, pp. 147-171, 1995.

[2] J. Liu. *Real-time systems.* Prentice Hall, 2000.

[3] S. Blake, D. Black, M. Carlson, et al. An Architecture for Differentiated Services. *RFC 2475*, December 1998.

[4] R. Braden, D. Clark, and S. Shenker. Integrated Services in the Internet Architecture: an Overview. *RFC 1633*, June 1994.

[5] S. Baruah, R. Howell, and L. Rosier. Algorithms and complexity concerning the preemptive scheduling of periodic real-time tasks on one processor. *Real-Time Systems*, Vol. 2, pp. 301-324, 1990.

[6] V. Sivaraman, F. Chiussi, and M. Gerla. End-to-end statistical delay service under GPS and EDF scheduling: a comparaison study. *INFOCOM'01*, Anchorage, USA, April 2001.

[7] M. Vojnovic and J. Y. le Boudec. Stochastic analysis of some expedited forwarding networks. *INFOCOM'02*, New York, USA, June 2002.

[8] F. M. Chiussi and V. Sivaraman. Achieving high utilization in guaranteed services networks using Early-Deadline-First scheduling. *IWQoS'98*, Napo, USA, May 1998.

[9] L. George, D. Marinca, and P. Minet. A solution for a deterministic QoS in multimedia systems. *International Journal on Computer and Information Science*, Vol. 1, No. 3, September 2001.

[10] K. Tindell and J. Clark. Holistic schedulability analysis for distributed hard real-time systems. *Microprocessors and Microprogramming, Euromicro Journal*, Vol. 40, 1994.

[11] J. Y. le Boudec and P. Thiran. A note on time and space methods in network calculus. *Technical Report, No. 97/224*, Ecole Polytechnique Fédérale de Lausanne, Suisse, April 1997.

[12] A. Parekh and R. Gallager. A generalized processor sharing approach to flow control in integrated services networks: the multiple node case. *IEEE ACM Transactions on Networking*, Vol. 2, No. 2, April 1994.

[13] L. Georgiadis, R. Guérin, V. Peris, et al. Efficient network QoS provisioning based on per node traffic shaping. *IEEE/ACM Transactions on Networking*, Vol. 4, No. 4, August 1996.

[14] V. Sivaraman, F. Chiussi, and M. Gerla. Traffic shaping for end-to-end delay guarantees with EDF scheduling. *IWQoS'2000*, Pittsburgh, USA, 2000.

Unité de recherche INRIA Rocquencourt
Domaine de Voluceau - Rocquencourt - BP 105 - 78153 Le Chesnay Cedex (France)

Unité de recherche INRIA Futurs : Parc Club Orsay Université - ZAC des Vignes
4, rue Jacques Monod - 91893 ORSAY Cedex (France)
Unité de recherche INRIA Lorraine : LORIA, Technopôle de Nancy-Brabois - Campus scientifique
615, rue du Jardin Botanique - BP 101 - 54602 Villers-lès-Nancy Cedex (France)
Unité de recherche INRIA Rennes : IRISA, Campus universitaire de Beaulieu - 35042 Rennes Cedex (France)
Unité de recherche INRIA Rhône-Alpes : 655, avenue de l'Europe - 38334 Montbonnot Saint-Ismier (France)
Unité de recherche INRIA Sophia Antipolis : 2004, route des Lucioles - BP 93 - 06902 Sophia Antipolis Cedex (France)

Éditeur
INRIA - Domaine de Voluceau - Rocquencourt, BP 105 - 78153 Le Chesnay Cedex (France)
http://www.inria.fr
ISSN 0249-6399