Delaunay Triangulation Based Surface Reconstruction : a short survey

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Delaunay Triangulation Based Surface Reconstruction : a short survey

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Abstract: Surface reconstruction consists in computing a model that approximates a surface which is known only through a finite sampling. This document is a short survey on surface reconstruction methods based on Delaunay triangulation. It has been rewritten from a longer survey written by Frédéric Cazals and Joachim Giesen [CG04]. This short survey is meant to be be one of the chapter of the State of the Art Report *Survey on Acquisition and Reconstruction* which is one of the deliverables of the European network of excellence AIM@Shape.

Key-words: Surface reconstruction, shape approximation, reverse ingeneering

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Reconstruction de surface par triangulation de Delaunay : un bref survol

Résumé : La reconstruction de surface consiste à calculer un modèle qui approxime une surface connue seulement par un échantillon de points. Ce document donne un aperçu de différentes méthodes de reconstruction de surfaces basées sur la triangulation de Delaunay. Ce document a été rédigé à partir d'un état de l'art plus approfondi écrit par Frédéric Cazals et Joachim Giesen [CG04]. Cet aperçu est destiné à constituer un chapitre du rapport *State of the Art Report, Survey on Acquisition and Reconstruction*, produit par le réseau d'excellence européen AIM@Shape.

Mots-clés : reconstruction de surface, approximation de forme, ingénierie inverse

1 Introduction

In the surface reconstruction problem we are given only a finite sampling $\mathcal{S}_\mathcal{P}$ of an unknown surface $\mathcal{S}$. The task is to compute a surface $\hat{\mathcal{S}}$ interpolating the sampling $\mathcal{S}_\mathcal{P}$. The reconstructed surface $\hat{\mathcal{S}}$ is generally represented as a triangulated surface that can be directly used by downstream computer programs for further processing. The reconstruction should match the original surface in terms of geometric and topological properties, but in general surface reconstruction is an ill-posed problem since there are several triangulated surfaces that might fulfill these criteria. Meeting geometric or topological criteria depends on the specificities of the sampling and on the properties of the sampled surface. In particular, sparsity, redundancy, noisiness of the sampling or non-smoothness and boundaries of the original surface make surface reconstruction a challenging problem.

Because reconstruction boils down to establishing connections between samples which are neighbors on the surface, any geometric construction defining a simplicial complex on these samples is a candidate auxiliary data structure for reconstruction. Because of its ability to code the neighboring relations in any direction, the Delaunay triangulation is obviously a first class candidate. Reconstruction methods based on Delaunay triangulation flourished over the past four years. In fact they are the only ones for which one can prove theoretical results on the quality of the reconstructed surface. These proofs hold under sufficient conditions —rather restrictive, and do not qualify the behaviour of the algorithms if the conditions are not met.

After, a short presentation of the main concepts used in the field, this section reviews some of the main Delaunay based reconstruction methods. This review is by no mean exhaustive, the reader is referred to [CGar] for a much more comprehensive survey.

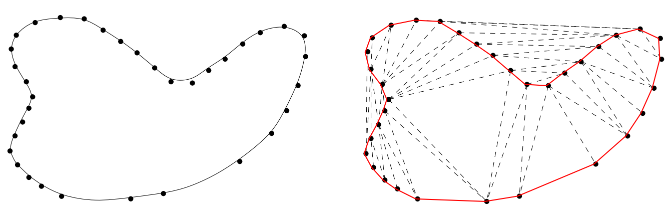

Figure 1: Left: a sampled curve. Right: Delaunay contains a piece-wise linear approximation of the curve

2 Pre-requisites

Voronoi diagrams

The Voronoi diagram $V(\mathcal{S}_\mathcal{P})$ of a sampling $\mathcal{S}_\mathcal{P}$ is a cellular complex covering $\mathbb{R}^3$, with one cell for each point in $\mathcal{S}_\mathcal{P}$. The cell $V(p)$ of the point p is the locus of points which are closer to p than to any other point of $\mathcal{S}_\mathcal{P}$.

$$V(p) = \{x \in \mathbb{R}^3 : \forall q \in \mathcal{S}_\mathcal{P} \quad \|x - p\| \leq \|x - q\|\}.$$

The cells of $V(\mathcal{S}_\mathcal{P})$ are convex polytopes. The faces of $V(\mathcal{S}_\mathcal{P})$ are called respectively *Voronoi cells, facets, edges and vertices.*

Delaunay triangulation and Gabriel simplices

The Delaunay complex of the sampling $\mathcal{S}_\mathcal{P}$ is the dual of the Voronoi diagram $V(\mathcal{S}_\mathcal{P})$. The Delaunay complex includes a face for each subset $T \in \mathcal{S}_\mathcal{P}$, such that the intersection of the Voronoi cells $\cap_{p \in T} V(p)$ is non empty. This face is the convex hull $conv(T)$ of T. The Delaunay complex covers the convex hull of $\mathcal{S}_\mathcal{P}$. If the sampling $\mathcal{S}_\mathcal{P}$ is non degenerate, i. e. includes no subset of five cosperical points, the Delaunay complex is a simplicial complex called the Delaunay triangulation of $\mathcal{S}_\mathcal{P}$ and noted $D(\mathcal{S}_\mathcal{P})$. If degeneracies occur in $\mathcal{S}_\mathcal{P}$, the Delaunay complex may have cells which are not simplicial. A Delaunay triangulation $D(\mathcal{S}_\mathcal{P})$ can still be obtained by triangulating in any way the non simplicial cells, but it is no longer uniquely defined.

It follows from the definition that the Delaunay faces (cells, facets, edges and vertices) are characterized by the *empty ball property* saying that there is a ball empty of sample points having the vertices of the face on its boundary. Delaunay facets and edges that have the stronger property that their smallest circumscribing ball is empty of sample points, are called respectively Gabriel facets and Gabriel edges.

The combinatorial and algorithmic worst case complexity of the Delaunay triangulation of n points in $\mathbb{R}^3$ is $\Theta(n^2)$. However this worst case complexity is subquadratic for samplings of polyhedral or smooth surfaces [AB02, ABL03] and the complexity is in fact linear for most of the surface samplings.

Restricted Voronoi diagram and restricted Delaunay triangulation.

Let $\mathcal{C}$ be a subset of $\mathbb{R}^3$. For any face F of $V(\mathcal{S}_\mathcal{P})$, the intersection $F \cap \mathcal{C}$ is called the restriction of F to $\mathcal{C}$. The set of non empty restriction $F \cap \mathcal{C}$ of faces F of $V(\mathcal{S}_\mathcal{P})$ forms the Voronoi diagram restricted to $\mathcal{C}$, noted $V_\mathcal{C}(\mathcal{S}_\mathcal{P})$.

The dual of the restricted Voronoi diagram $V_\mathcal{C}(\mathcal{S}_\mathcal{P})$ is the subcomplex of $D(\mathcal{S}_\mathcal{P})$ formed by the faces of $D(\mathcal{S}_\mathcal{P})$ whose dual Voronoi faces have a non empty intersection with $\mathcal{S}$. This subcomplex is called the Delaunay triangulation restricted to $\mathcal{C}$ an noted $D_\mathcal{C}(\mathcal{S}_\mathcal{P})$.

The Voronoi diagram and the Delaunay triangulation restricted to $\mathcal{S}$, $V_\mathcal{S}(\mathcal{S}_\mathcal{P})$ and $D_\mathcal{S}(\mathcal{S}_\mathcal{P})$, are important concepts for the reconstruction of the surface $\mathcal{S}$ from the sampling $\mathcal{S}_\mathcal{P}$. See Fig. 2 for an illustration.

Power diagram and regular triangulation.

The concepts of Voronoi diagrams and Delaunay triangulations can be generalized to sets of weighted points. A *weighted point* p in $\mathbb{R}^3$ is a pair (z, r) where $z \in \mathbb{R}^3$ denotes the point

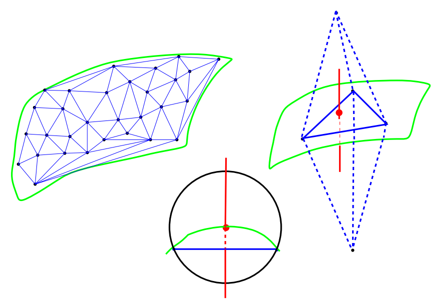

Figure 2: Restricted Delaunay triangles for a surface in $3D$

itself and $r \in \mathbb{R}$ its weight. Every weighted point gives rise to a distance function, namely the *power distance function*,

$$\pi_p : \mathbb{R}^3 \to \mathbb{R}, x \mapsto \|x - z\|^2 - r^2.$$

Replacing the Euclidean distance by the power distance respectively yields the power diagram and the regular triangulation instead of the Voronoi diagram and the Delaunay triangulation.

Medial axis, skeleton and medial axis transform

The medial axis $MA(\mathcal{S})$ of a closed subset $\mathcal{S}$ of $\mathbb{R}^3$ is the subset of $\mathbb{R}^3 \backslash S$ that consists of all points in $\mathbb{R}^3 \backslash S$ having two or more nearest points in $\mathcal{S}$. The medial axis of a finite point set of $\mathbb{R}^3$ is simply the union the Voronoi faces of dimension 2 and less.

The medial axis of a surface $\mathcal{S}$ is closely related to the skeleton of $\mathbb{R}^3 \backslash S$, which consists of the centers of maximal spheres included in $\mathbb{R}^3 \backslash S$. Here maximal is meant with respect to inclusion among spheres. For a smooth surface S the closure of the medial axis is actually equal to the skeleton of $\mathbb{R}^3 \backslash S$.

Any smooth surface $\mathcal{S}$ can be recovered as the envelope of the maximal spheres included in one of the two component of $\mathbb{R}^3 \backslash S$. This transformation is called the medial axis transform.

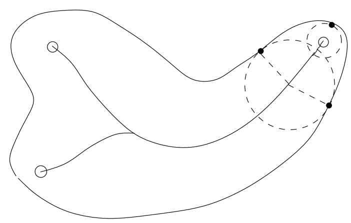

Figure 3: The three points marked with a bullet belong to the skeleton but not the medial axis

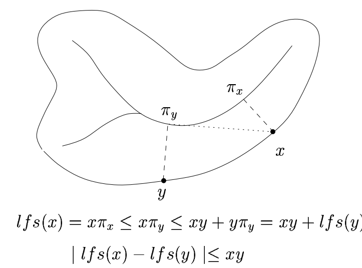

Figure 4: The feature size is 1-Lipschitz

Poles, local feature size and ε-sampling

A vertex of the Voronoi cell $V(p_i)$ of a sample point is called a pole if :
- either it is the vertex v_i of $V(p_i)$ that is the farthest from p_i
- or it is the vertex w_i of $V(p_i)$ that is the farthest from p_i in the halfspace H_i^-, set of points x such that $(v_i - p_i) \cdot (x - pi) \leq 0$.

The local feature size is a function $lfs : S \rightarrow \mathbb{R}$ that assigns to each point in S its distance to the medial axis of S. An immediate consequence of the triangle inequality is that the local feature size of smooth surface is Lipschitz continuous with Lipschitz constant 1, see Figure 4 for an illustration.

The function lfs can be seen as a measure of the local thickness of an object. Ambiguities arise in reconstruction processes as soons as the samples are not dense enough with respect to the local feature size of the shape. The following definition, due to Amenta and Bern [AB99], captures exactly this intuition and provides a way to specify whether samples are localy dense enough:

Definition 1 *For $\varepsilon > 0$ a sampling $\mathcal{S}_\mathcal{P}$ of a surface $\mathcal{S}$ is called an ε-sampling of $\mathcal{S}$ if every point x on $\mathcal{S}$ is at a distance less than $\varepsilon\ lfs(x)$ from a sample point.*

The followings results [AB99] are corner stones for Delaunay based reconstruction method. Let $\mathcal{S}_\mathcal{P}$ be an ε-sample of a smooth surface $\mathcal{S}$

- When ε goes to zero, the poles of the Voronoi diagram $V(\mathcal{S}_\mathcal{P})$ converge to the medial axis $MA(\mathcal{S})$ of $\mathcal{S}$. Furthermore, if v_i is a pole of sample point p_i, the vector $v_i - p_i$ is close to the normal of $\mathcal{S}$ at p_i.
- For sufficiently small ε, the restricted Delaunay triangulation $D_S(\mathcal{S}_\mathcal{P})$ forms a piecewise linear surface homeomorphic and even ambient isotopic [APR03] to $\mathcal{S}$.

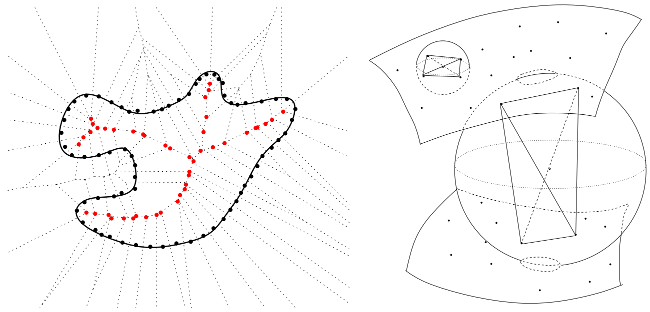

Figure 5: In $2D$, all Voronoi vertices converge to the medial axis. In $3D$, some Voronoi vertices may be far from the medial axis but poles are guaranteed to converge to the medial axis.

3 Overview of Delaunay based reconstruction methods

Using Delaunay triangulation or Voronoi diagrams still leaves room for quite different approaches to solve the reconstruction problem. The classification proposed below is based on what we consider as the dominant idea behind an algorithm, although such a classification is difficult because many algorithm combine different approaches.

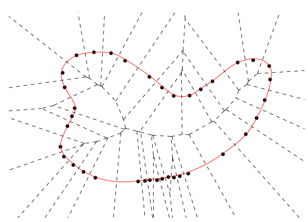

Figure 6: The Voronoi cells of good sampling are elongated in the normal direction.

Tangent plane methods

If one considers a smooth surface $\mathcal{S}$ with a sufficiently dense sampling $\mathcal{S}_{\mathcal{P}}$, the *neighbors* of a point in $\mathcal{S}_{\mathcal{P}}$ should not deviate too much from the tangent plane of the surface at that point. It turns out that this tangent plane can be well approximated by exploiting the fact that the Voronoi cell of the sample point is elongated in the direction of the surface normal at the sample point. This normal or tangent plane information, respectively, can be used to derive a local triangulation around each point.

The algorithm proposed by Gopi, Krihsnan and Silva ([GKS00]), has three major steps. First, it computes for each sample point, an approximation of the tangent plane and a selection of sample points which are neighbooring points. Second, in each approximate tangent plane, a 2-dimensional Delaunay triangulation including the sample point and the projection of its neighbors is computed. The third step is a stitching step in which pqr is selected as a triangle of the reconstructed surface iff points $p, q, r \in \mathcal{S}_\mathcal{P}$ correspond to mutual neighbors in the local 2d triangulations. The triangles output by the algorithm form a surface homeomorphic to $\mathcal{S}$ provided a that some sampling condition holds.

Also related to tangent planes, the greedy algorithm [CSD04] grows an oriented manifold $\hat{\mathcal{S}}$ by selecting triangles out of the Delaunay triangulation $D(\mathcal{S}_\mathcal{P})$. The selection starts with a seed triangle and is guided by the circumradii of the triangles and the dihedral angles they form with already selected adjacent triangles. By construction the output is an oriented triangulated surface but it may fail to interpolate all the sample points or to provide a close surface essentially due to the possible presence of slivers. Heuristic are provided to overcome this difficulty and also to handle boundaries or sharp features.

Restricted Delaunay triangulation methods

The restricted Delaunay triangulation $D_\mathcal{S}(\mathcal{S}_\mathcal{P})$ is known to be homeophorm to the $\mathcal{S}$ if the sampling is dense enough. Of course, because the surface $\mathcal{S}$ is unknown, it is not so easy to extract $D_\mathcal{S}(\mathcal{S}_\mathcal{P})$ from $D(\mathcal{S}_\mathcal{P})$. The main idea behind algorithms in this category is to extract from $D(\mathcal{S}_\mathcal{P})$ a subcomplex which the restriction of $D(\mathcal{S}_\mathcal{P})$ to a subset of $\mathbb{R}^3$ that is a good approximation $\mathcal{S}$.

Crust. The crust algorithm proposed by Bern and Amenta [AB99] was the first to yield guarantees for the reconstruction provided some ε-sampling condition is fulfilled. The crust algorithm computes the Delaunay triangulation $D(\mathcal{S}_\mathcal{P})$ of the sample, the set $\mathcal{Q}$ of poles in the Voronoi diagram $V(\mathcal{S}_\mathcal{P})$ and finally the Delaunay triangulation $D(\mathcal{S}_\mathcal{P} \cup \mathcal{Q})$ of the set $\mathcal{S}_\mathcal{P} \cup \mathcal{Q}$. The algorithm retains as candidate for the reconstructed surface, the facets of $D(\mathcal{S}_\mathcal{P} \cup \mathcal{Q})$ whose vertices are sample points. This amounts to restrict the Delaunay triangulation of $\mathcal{S}_\mathcal{P}$ by the subset $\mathcal{C}$ of $\mathbb{R}^3$ which is the union of the cells of the sample points in the Voronoi diagram $V(\mathcal{S}_\mathcal{P} \cup \mathcal{Q})$.

It can be shown that for a closed smooth surface $\mathcal{S}$ and a sufficiently dense ε-sampling $\mathcal{S}_\mathcal{P}$ of $\mathcal{S}$ the set of candidate triangles contains all the triangles of the restricted Delaunay triangulation $D_\mathcal{S}(\mathcal{S}_\mathcal{P})$. The algorithm includes a post processing to eliminate dangling triangles and extract a manifold from the set of candidate triangles. Extensions of the algorithm are provided to deal with boundaries and a certain amount of sharp features.

Cocone. The cocone algorithm was designed by Amenta et al. [ACDL00] as a successor and improvement of the crust algorithm. The cocone algorithm builds as the crust algorithm on the idea of computing from $\mathcal{S}_\mathcal{P}$ a subset $\mathcal{C} \subset \mathbb{R}^3$ which is a thickened version of $\mathcal{S}$ such that the Delaunay triangulation $D_\mathcal{C}(\mathcal{S}_\mathcal{P})$ restricted to $\mathcal{C}$ can be computed. In cocone, the subset $\mathcal{C}$ is defined as follows. For every sample point $p \in \mathcal{S}_\mathcal{P}$, the normal of $\mathcal{S}_\mathcal{P}$ at p is approximated using one of the poles of p. The cocone at p is now defined as the intersection of $V(p)$ with the

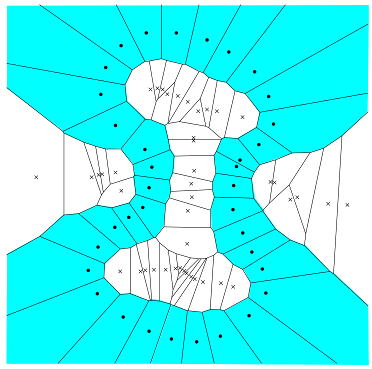

Figure 7: The subset $\mathcal{C}$ of the crust algorithm is a thickening of the surface S.

complement of a double cone with apex p, axis parallel to the approximate normal at p and a fixed opening angle. The set $\mathcal{C}$ is the union of all such cocones. The triangles in $D_{\mathcal{C}}(\mathcal{S}_{\mathcal{P}})$ form a set of candidates which is known to include $D_{\mathcal{S}}(\mathcal{S}_{\mathcal{P}})$ for sufficiently dense sample. In the last step the algorithm extracts a manifold surface out of these candidates. Cocone provides the same guarantees as the crust in the same conditions. Dey and Goswami [DG03, DG04] designed an extension called *tight cocone* to provide a watertight reconstructed surface and another one called *robust cocone* to deal with noisy samplings.

Implicit function reconstruction It is well known (this is Whitney's theorem) that any smooth surface occurs as the zero level set $f^{-1}(0)$ of some smooth function $f : \mathbb{R}^3 \mapsto \mathbb{R}$. The main idea here is to define from the sampling $\mathcal{S}_{\mathcal{P}}$ a smooth function f whose zero level set approximate $\mathcal{S}$. Then the reconstruction is just the restriction $D_{(f^{-1}(0))}(\mathcal{S}_{\mathcal{P}})$ of the Delaunay triangulation to the zero level set of f. In the natural neighbor interpolation method [BC00, BC01], the tangent plane at the sample points are estimated together with oriented normals. The function f is defined from the distance funtions to those normals plane through natural neighbor interpolation, that is $f(x) = \Sigma_i \lambda_i(x) f_i(x)$ where $f_i(x)$ is the signed distance to the estimated tangent plane at point $p_i \in \mathcal{S}_{\mathcal{P}}$ and $lambda_i(x)$ are the natural coordinates of x in $\mathcal{S}_{\mathcal{P}}$. In this setting, it can be shown that the Hausdorff distance between $f^{-1}(0)$ and $\mathcal{S}$ goes to zero when the sampling density increases.

Inside/Outside labelling

The main idea of this class of methods is to build from the sampling $\mathcal{S}_{\mathcal{P}}$ a spatial subdivision and to classify the regions of this subdivision as inside or outside the domain bounded by $\mathcal{S}$. The reconstructed surface $\hat{S}$ is then the boundary of the union of inside regions.

Sculpting the Delaunay triangulation In a seminal paper [Boi84], Boissonnat applied this idea on the Delaunay triangulation. The algorithm starts from the Delaunay triangulation $D(\mathcal{S}_\mathcal{P})$ and removes in turn tetrahedra that are likely to be outside so that every sample point appears on the boundary. Care is taken that no singularity arise on the boundary surface from the removal of a tetrahedra. In [AS00], Attene and Spagnuolo further constrained the removal process so that Gabriel facets and EMST edges (edges of the euclidean minimum spanning trees) appear on the boundary. They also used EMST edges to guide the creation of holes and be able to reconstruct surfaces with a non 0 genus. The convection algorithm proposed by Chaine [Cha03] may also be view as a sculpting process. This algorithm removes in turn tetrahedra having a facet on the current boundary that does not have the oriented Gabriel property. The oriented Gabriel property considers the intersection of the smallest circumscribing sphere of a facet with the half-space bounded by the facet and on the inner side and requires that this instersection is empty of sample points.

Power crust The power crust algorithm was introduced by Amenta et al. in [ACK01b, ACK01a]. The main idea is that the medial axis transform of $\mathcal{S}$ can be approximated by the union of the Delaunay ball of $D(\mathcal{S}_\mathcal{P})$ centered on the poles that are internal to $\mathcal{S}$. Briefly, the algorithm computes, the Delaunay triangulation $D(\mathcal{S}_\mathcal{P})$ and the poles of each sample point. The poles are tagged as either inside or outside through a greedy algorithm starting from poles of sample points on the bounding box and using the fact that balls of neighboring poles intersect deeply if the two poles are on the same side of $\mathcal{S}$ and shallowly if they are on opposite sides. The algorithm compute the power diagram of the poles weighted by the circumradii of the corresponding Delaunay ball. The surface $\hat{\mathcal{S}}$ is obtained as the boundary of the union of the cells of inner poles in the power diagram.

The reconstructed surface is watertight but need not be manifold. The method is provably good for dense sample and can be extended to detect sharp edges and accomodate boundaries.

Flow methods The flow methods consider the flow diagram induced by the distance to the sampling $\mathcal{S}_\mathcal{P}$. The local maxima of the function distance to $\mathcal{S}_\mathcal{P}$ are some of the Voronoi vertices of $V(\mathcal{S}_\mathcal{P})$. The flow methods reconstruct the approximated surface $\hat{\mathcal{S}}$ as the boundary of the union of the stable manifolds of some of those maxima. The wrap algorithm proposed by Edelsbrunner [Ede04] and then marketed by his company belongs to this category, as well as the flow complex algorithm of Giesen and John [GJ02]. In either case, no guarantee is given except that the reconstructed surface always bound a solid.

Methods based on radii of empty balls

The prototype of these methods are the alpha shape reconstruction methods [EM94]. These methods can be viewed are inside/outside methods basing the inside/outside labelling of Delaunay tetrahedra on the radii of their circumscribing spheres. To account for sampling with non uniform density, the weighted alpha shape methods affects weights to the sample points, build the regular triangulation of these weighted samples and label the tetrahedra according to the radii of their orthogonal spheres. Bernardini et al [BMR+99] proposed a ball pivoting algorithm to construct efficiently the boundary of an alpha shape.

Petitjean and Boyer [PB01] introduce the notions of interpolants and regular interpolants of a sampling $\mathcal{S}_\mathcal{P}$ and propose a reconstruction method based on interpolants. The method amount to build $\hat{\mathcal{S}}$ from a selected subset of Gabriel facets in $D(\mathcal{S}_\mathcal{P})$ where the selection is based on the radius of their smallest circumcribing spheres. The corresponding algorithm builds the surface incrementally starting from a seed triangle of $D(\mathcal{S}_\mathcal{P})$. At each step, a boundary edge of the current reconstructed surface is considered and the surface is extended by adding the incident Gabriel facet with the minimum circumscribing radius.

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