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Abstract: We study a controlled stochastic system whose state is described by a stochastic differential equation where the coefficients are anticipating. This setting is used to interpret markets where insiders have some influence on the dynamics of prices. We give a characterization theorem for the optimal logarithmic portfolio of an investor with a different information flow from that of the insider. As examples, we provide explicit results in the partial information case which we extend in order to incorporate the enlargement of filtration techniques for markets with insiders. Finally, we consider a market with an insider which influences the drift of the asset process. This last example, which does not seem to fit into the enlargement of filtration set-up, gives a situation where it makes a difference for a small agent to acknowledge the existence of the insider in the market.

Key-words: Anticipating systems, Insider market, partial observation control, forward integrals, Malliavin calculus.

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Maximisation d'utilité dans un marché financier influencé par un initié

Résumé : On étudie un système stochastique controllé dont l'état est décrit par une équation différentielle stochastique à coefficients anticipatifs. Ce cadre est utilisé pour modéliser des marchés financiers en présence d'initiés qui influencent les prix des actifs. On donne un théorème de caractérisation de la stratégie d'investissement optimale d'un investisseur qui veut optimiser une fonction d'utilité logarithmique et qui dispose d'une information différente de celle de l'initié. On obtient des résultats explicites dans le cas de l'information partielle que l'on étend pour couvrir également les techniques de grossissement de filtration des marchés avec initiés. Finalement, on considère un exemple qui sort du cadre des techniques de grossissement de filtration et qui concerne un investisseur qui observe des prix dont la dérive est influencée par un initié. Cet exemple permet de mettre en évidence le fait que la prise en compte de la présence de l'initié modifie la stratégie et l'utilité optimale de l'investisseur.

Mots-clés : calcul stochastique anticipatif, contrôle en observation partielle, intégrales forward, calcul de Malliavin, marchés financiers avec initiés.

1 Introduction

In most of the research of modelling of insiders problems (see Karatzas-Pikovsky [KP], Imkeller [Im], Grorud-Pontier [GP], ...) one postulates the asset price dynamics as the one determined by the small investor. The insider has an additional information in the form of a random variable which depends on future events. The financial problem is to evaluate the advantage of the insider in the form of additional utility and optimal portfolio. Mathematically, the problem is to determine the semimartingale decomposition of the Wiener process in the filtration enlarged with the additional information of the insider. Then one can express the dynamics of the prices for the informed agent and compute the optimal investment strategy for this informed agent.

In this article we study this problem from a different point of view. That is, we assume there exists an insider who is also a large trader and therefore can influence the prices of the underlying assets with his/her financial behavior. The small investor is a price taker and the dynamics he assigns to these prices are not necessarily the same as the one observed by the insider due to the information difference. We are interested in analyzing the question of the optimal investment strategy of the small investor in front of such a situation. This point of view was already partly studied in Øksendal-Sulem [ØS] although the financial consequences and modelling possibilities for models of markets with insiders were not exploited there.

Mathematically, the asset price is generated by an anticipating stochastic differential equation, since asset prices have coefficients which are not necessarily adapted to the filtration generated by the Brownian motion. We study a logarithmic utility maximization problem of final wealth in this anticipating market. We suppose that the investor's portfolio is adapted to a filtration which is not necessarily the same as the filtration of the insider or the one generated by the Brownian motion, for example the filtration generated by the underlying asset price.

We give a characterization theorem (Theorem 3.1) of optimal portfolios for the investor which is later applied to various examples to prove optimality of the proposed policies. The optimal portfolios can be interpreted as projection formulas of Merton type solutions plus an extra term (denoted by $a(t)$, see Corollary 3.2) which is interpreted through the examples.

In Section 5 we consider and extend the important example of partial information. We first consider the typical situation of a small investor who does not have the information of the random drift driving the price process (see Example 5.1). That is, the stochastic differential equation is adapted to the filtration generated by the Brownian motion and the filtration of the small investor is smaller than this filtration. This partial observation case is well documented in the filtering theory literature.

In our general set-up, we extend this situation to the case when the random drift can be anticipating (Proposition 5.2). This includes all known models of insiders built with an initial enlargement of filtration technique. In this case, the optimal portfolio of the insider coincides with the optimal portfolio of an investor when the coefficients of the price dynamics are adapted to the enlarged filtration (see Example 5.3).

In this generalized set-up one can also consider the optimal portfolio of the small investor (see Example 5.4). In fact, we will see that in a market where the price dynamics are driven by an insider, using the enlargement of filtrations approach, the small investor with a filtration smaller than the enlarged filtration becomes only a partially informed agent in an anticipating world.

In conclusion the initial enlargement of filtrations approach for insiders modelling becomes a particular case of our generalization of partial information with $a(t) = 0$.

On the other hand, it remains to be seen if the general result given in Theorem 3.1 always corresponds to a initial enlargement of filtration setup. A partial negative answer to this question is given in Section 6. It seems that the fact $a(t) \neq 0$ is related to the relationship between three filtrations: (i) the natural filtration of the Brownian motion ($\mathcal{F}$), (ii) the filtration to which the coefficients of the SDE are adapted ($\mathcal{G}$), (iii) the information of the investor ($\mathcal{H}$).

We thus address the issue: Is there a situation where $a(t) \neq 0$ and what is the interpretation of $a(t)$? To answer this question we consider stock dynamics where the drift is influenced by the insider through a smooth (in the sense of stochastic derivatives) random variable and the noise is given by the original Brownian motion (see Section 6). We suppose that a small investor observes the price of the underlying asset and computes his/her optimal portfolio using a logarithmic utility. The results lead to the following conclusion: If the small agent decides that there is no insider in the market, he/she estimates the drift of the underlying with the best estimator (the conditional expectation) with respect to his information, builds a geometric Brownian motion as his/her model to maximize the logarithmic utility. This calculation gives a suboptimal portfolio.

The difference between this suboptimal portfolio and the optimal one assuming an anticipating model for the market with insiders is proportional to $a(t)$. Furthermore the difference in utilities is given by a quantity depending on $a(t)$ which appears due to the anticipating nature of the modelling (see Remark 6.5.4.

In particular, the case considered in Section 6 does not seem to be related with any enlargement of filtration. Nevertheless, this model is still a reasonable one to study markets where the insider has an influence on the drift of the underlying. Therefore this gives a new approach to insider modelling where instead of modelling the information itself one models the effect of the insider on the drift of the underlying.

2 Formulation of the problem

Let $B(t)$ be a Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$. Let $\{\mathcal{G}_t\}_{t\geq 0}$ be a filtration such that

$$\mathcal{F}_t \subseteq \mathcal{G}_t \subseteq \mathcal{F} \qquad \text{for all } \ t \geq 0. \tag{1}$$

Consider a financial market with one risky investment possibility, whose price $S_1(t)$ at time t is described by a stochastic differential equation of the form

$$dS(t) = S(t)\Big[\mu(t)dt + \sigma(t)d^- B(t)\Big], \ S(0) > 0 \tag{2}$$

where $\mu(t) = \mu(t, \omega)$ and $\sigma(t) = \sigma(t, \omega) \geq 0$ are $\mathcal{G}_t$-adapted. Moreover we assume that σ is forward integrable and caglad. Since $B(t)$ need not be a semimartingale with respect to $\{\mathcal{G}_t\}_{t\geq 0}$, the last integral in (2) is an *anticipating* stochastic integral that we choose to interpret as a *forward* integral denoted by $d^- B(t)$. We assume that $\mu(t)$ and $\sigma(t)$ are such that Equation (2) has a unique solution. Suppose the market also has a risk free investment possibility, where the price $S_0(t)$ at time t is described by

$$dS_0(t) = \rho(t)S_0(t)dt; \qquad S_0(0) = 1$$

where $\rho(t) = \rho(t, \omega)$ is a $\mathcal{G}_t$-adapted process which satisfies $\mathbb{E}\int_0^t |\rho(s)|ds < +\infty$ for all t.

Moreover we consider another filtration $\{\mathcal{H}_t\}_{t\geq 0}$ for modelling the information of the investor but no assumption is made on the relation between $\{\mathcal{F}_t\}_{t\geq 0}, \{\mathcal{G}_t\}_{t\geq 0}$ and $\{\mathcal{H}_t\}_{t\geq 0}$.

Let us first address the arbitrage issue. We define the set of admissible strategies defined as $\mathcal{H}_t$-adapted processes $p(t) = (p_0(t), p_1(t))$ giving the numbers of shares hold in each asset, such that p_1 is forward integrable w.r.t. S and caglad. Let $p = (p_0, p_1)$ be an admissible strategy. The associated wealth process is given by

$$W^{(p)}(t) = p_0(t)S_0(t) + p_1(t)S(t).$$

The portfolio p is said to be self-financing if

$$dW^{(p)}(t) = p_0(t)dS_0(t) + p_1(t)d^-S(t).$$

Note that this definition of "self-financing strategy" with forward integrals corresponds to the usual one: this comes from the definition of the forward integral as the limit of Riemann sums.

Let U be a utility function such that $\lim_{x\to+\infty} U(x) = +\infty$. Assume that for some $x > 0$

$$\max_{p\in A(x)} \mathbb{E}(U(X^{(p)}(T))) < \infty$$

where $X^{(p)}(t) = \exp(-\int_0^t \rho(s)ds)W^{(p)}(t)$ is the discounted wealth process, $A(x) = \{p; p$ is $\mathcal{H}$-adapted, p is self-financing and $W^{(p)}(0) = x\}$. Suppose that $p^* = (p_0^*, p_1^*)$ is an arbitrage, that is, $W^{(p^*)}(0) = 0$ and $W^{(p^*)}(T) \geq 0$ with $P(W^{(p^*)}(T) > 0) > 0$. If one considers the strategy np^* then $W^{(np^*)}(t) = nW^{(p^*)}(t)$ for all $t \in [0, T]$. The strategy $p'(t) = (np_0^*(t) + x, np_1^*(t))$ is also self-financing, $W^{(p')}(0) = x$, $W^{(p')}(T) \geq xS_0(T)$ and $W^{(p')}(T) = xS_0(T) + nW^{(p^*)}(T)$ and on the set $\{W^{(p^*)}(T) > 0\}$ we have that

$$\lim_{n\to+\infty} U(X^{(p')}(T)) = +\infty.$$

Therefore

$$\lim_{n\to+\infty} \mathbb{E}U(X^{(p')}(T)) = +\infty$$

which contradicts the hypothesis of finite utility. Consequently there is no arbitrage in this model as long as the utility is finite which will be the case in all the examples we consider. Furthermore we restrict our portfolios to tame portfolios. That is, we say that a portfolio p is tame if $W^{(p)}(t) > 0$ for all $t \in [0, T]$. With this restriction we can now parametrize our problem using the fraction of wealth invested in the risky asset $\pi(t) = \pi(t,\omega) = p_1(t)S(t)/W^{(p)}(t)$ for all $t \in [0, T]$. We define an admissible portfolio as an $\mathcal{H}_t$-adapted process π giving the *fraction* of the total wealth $W(t)$ of an agent invested in the risky asset at time t, and satisfying some additional conditions that will be precised later (see the definition of $\mathcal{A}_{\mathcal{H}}$). The case when $\mathcal{H}_t \subseteq \mathcal{F}_t \subseteq \mathcal{G}_t \subseteq \mathcal{F}$ is studied in [ØS].

The dynamics of the discounted wealth process

$$X(t) = X^{\pi}(t) = \exp(-\int_0^t \rho(s)ds)W^{(\pi)}(t)$$

corresponding to the portfolio π is then:

$$dX(t) = X(t)\Big[(\mu(t) - \rho(t))\pi(t)dt + \pi(t)\sigma(t)d^-B(t)\Big] \qquad X(0) = x > 0. \quad (3)$$

This equation is justified by using as before the definition of forward integrals as limit of Riemann sums. Using the Itô formula for forward integrals [ØS], we obtain the solution of (3)

$$X^{(\pi)}(T) = x\exp\Big\{\int_0^T ((\mu(t) - \rho(t))\pi(t) - \tfrac{1}{2}\pi^2(t)\sigma^2(t))dt + \int_0^T \pi(t)\sigma(t)d^-B(t)\Big\}.$$

We restrict to logarithmic utility functions and consider the following performance criterion:

$$J(\pi) \equiv \mathbb{E}[\ln X^{(\pi)}(T)] - \ln x = \\ = \mathbb{E}\Big[\int_0^T ((\mu(t)-\rho(t))\pi(t) - \tfrac{1}{2}\pi^2(t)\sigma^2(t))dt + \int_0^T \pi(t)\sigma(t)d^-B(t)\Big]. \tag{4}$$

Definition 2.1 *The space $\mathcal{A}_{\mathcal{H}}$ consists of all processes $\pi(t)$ satisfying the following conditions:*

$$\begin{gathered} \pi(t) \quad \textit{is caglad and } \mathcal{H}_t\textit{-adapted} \\ \pi(t) \quad \textit{is } \text{forward integrable } \textit{w.r.t. } Y(t) := \int_0^t \sigma(s)d^-B(s) \\ \mathbb{E}\Big[\Big|\int_0^T \pi(t)d^-Y(t)\Big|\Big] + \mathbb{E}\Big[\int_0^T (|\mu(t)-\rho(t)|.|\pi(t)| + \sigma^2(t)\pi^2(t))dt\Big] < \infty. \end{gathered} \tag{5}$$

The goal is to find the optimal portfolio $\pi^* \in \mathcal{A}_{\mathcal{H}}$ for the logarithmic utility portfolio problem:

$$\sup_{\pi \in \mathcal{A}_{\mathcal{H}}} \mathbb{E}^x[\ln(X^{(\pi)}(T))] = \mathbb{E}^x[\ln(X^{(\pi^*)}(T))]. \tag{6}$$

3 Characterisation of the optimal portfolio

In this section we give a theorem that characterizes optimal portfolios. This theorem will be used in all examples to determine that the proposed portfolio is optimal.

Theorem 3.1 *The following assertions are equivalent:*
(i) *There exists an optimal portfolio $\pi^* \in \mathcal{A}_{\mathcal{H}}$ for Problem* (6)*.*
(ii) *There exists $\pi^* \in \mathcal{A}_{\mathcal{H}}$ such that the process*

$$M_{\pi^*}(t) := \mathbb{E}\Big[\int_0^t (\mu(s) - \rho(s) - \sigma^2(s)\pi^*(s))ds + \int_0^t \sigma(s)d^-B(s)|\mathcal{H}_t\Big] \tag{7}$$

is an $\mathcal{H}_t$-martingale (w.r.t. P)
(iii) *There exists $\pi^* \in \mathcal{A}_{\mathcal{H}}$ such that for a.a. t, ω, the function*

$$s \mapsto \mathbb{E}[\int_0^s \sigma(u)d^- B(u)|\mathcal{H}_t] \; ; s > t$$

is absolutely continuous and

$$\frac{d}{ds}\mathbb{E}[\int_0^s \sigma(u)d^- B(u)|\mathcal{H}_t] = -\mathbb{E}\Big[\mu(s) - \rho(s) - \sigma^2(s)\pi^*(s)|\mathcal{H}_t] \; ; \; a.a. \; s > t \quad (8)$$

Proof. (i) $\Rightarrow$ (ii): Suppose (i) holds. Since $\pi^* \in \mathcal{A}_{\mathcal{H}}$ is optimal, we have

$$J(\pi^*) \geq J(\pi^* + r\beta)$$

for all $\beta \in \mathcal{A}_{\mathcal{H}}$ and $r \in \mathbb{R}$. Therefore

$$\frac{d}{dr}J(\pi^* + r\beta)\Big|_{r=0} = 0.$$

This gives

$$\mathbb{E}\Big[\int_0^T \{\mu(t) - \rho(t) - \sigma^2(t)\pi^*(t)\}\beta(t)dt + \int_0^T \beta(t)(\sigma(t)d^- B(t))\Big] = 0 \qquad (9)$$

for all $\beta \in \mathcal{A}_{\mathcal{H}}$. In particular, applying this to

$$\beta(u) = \beta_0(t)1_{[t,s]}(u)$$

for $0 \leq t < s \leq T$, $u \in [t,s]$, where $\beta_0(t)$ is $\mathcal{H}_t$-measurable, bounded, smooth and satisfies $D_{s^+}\beta(t) = D_s\beta(t)$ a.a. $s \in [0,T]$, we obtain

$$\mathbb{E}\Big[(\int_t^s \{\mu(u) - \rho(u) - \sigma^2(u)\pi^*(u)\}du + \int_t^s \sigma(u)d^- B(u))\beta_0(t)\Big] = 0. \qquad (10)$$

Since this holds for all such $\beta_0(t)$ we conclude that

$$\mathbb{E}\Big[(\int_t^s \{\mu(u) - \rho(u) - \sigma^2(u)\pi^*(u)\}du + \int_t^s \sigma(u)d^- B(u))|\mathcal{H}_t\Big] = 0. \qquad (11)$$

This is equivalent to saying that the process

$$K_{\pi^*}(t) := \int_0^t \{\mu(u) - \rho(u) - \sigma^2(u)\pi^*(u)\}du + \int_0^t \sigma(u)d^-B(u)$$

satisfies

$$\mathbb{E}[K_{\pi^*}(s)|\mathcal{H}_t] = \mathbb{E}[K_{\pi^*}(t)|\mathcal{H}_t] \quad \text{for all } s \geq t. \tag{12}$$

From this we get, for $s \geq t$

$$\mathbb{E}[M_{\pi^*}(s)|\mathcal{H}_t] = \mathbb{E}[\mathbb{E}[K_{\pi^*}(s)|\mathcal{H}_s]|\mathcal{H}_t] = \mathbb{E}[K_{\pi^*}(s)|\mathcal{H}_t] = \mathbb{E}[K_{\pi^*}(t)|\mathcal{H}_t] = M_{\pi^*}(t),$$

which is (ii).

(ii) $\Rightarrow$ (iii): Suppose (ii) holds. Then, for $s \geq t$,

$$\mathbb{E}[K_{\pi^*}(s)|\mathcal{H}_t] = \mathbb{E}[\mathbb{E}[K_{\pi^*}(s)|\mathcal{H}_s]|\mathcal{H}_t] = \mathbb{E}[M_{\pi^*}(s)|\mathcal{H}_t] = M_{\pi^*}(t) = \mathbb{E}[K_{\pi^*}(t)|\mathcal{H}_t].$$

Hence (12) - and then also (11) - holds. And (11) clearly implies (iii).

(iii) $\Rightarrow$ (i): Suppose (iii) holds.

Then integrating (8), we get (11), which again implies (10). By taking linear combination of (10), we obtain that (9) holds for all $\beta \in \mathcal{A}_{\mathcal{H}}$ of the form

$$\beta(u) = \Sigma_{i=1}^N \beta_i(t_i) 1_{(t_i,t_{i+1}]}(u)$$

where $0 = t_0 < t_1 < \ldots < t_{N+1} = T$, $\Delta_i = t_{i+1} - t_i$, and $\beta_i(t_i)$ is $\mathcal{H}_{t_i}$-measurable, bounded, smooth and satisfies $D_{s^+}\beta_i(t) = D_s\beta_i(t)$ a.a. $s \in [0,T]$. By (5), we have for all $\beta \in \mathcal{A}_{\mathcal{H}}$

$$\int_0^T \beta(t)(\sigma(t)d^-B(t)) = \int_0^T \beta(t)d^-Y(t) = \lim_{\Delta_i \to 0} \Sigma_{i=1}^N \beta(t_i) \int_{t_i}^{t_{i+1}} \sigma(s)d^-B(s),$$

and hence (9) holds for all $\beta \in \mathcal{A}_{\mathcal{H}}$.

This means that the directional derivative of J at π^* with respect to the direction β, denoted by $D_\beta J(\pi^*)$ is 0, i.e.

$$D_\beta J(\pi^*) := \lim_{r \to 0} \frac{J(\pi^* + r\beta) - J(\pi^*)}{r} = 0 \quad ; \beta \in \mathcal{A}_{\mathcal{H}}. \tag{13}$$

Note that $J : \mathcal{A}_{\mathcal{H}} \to \mathbb{R}$ is concave, in the sense that

$$J(\lambda\alpha + (1-\lambda)\beta) \geq \lambda J(\alpha) + (1-\lambda)J(\beta); \quad \lambda \in [0,1], \alpha, \beta \in \mathcal{A}_{\mathcal{H}}.$$

Therefore, for all $\alpha, \beta \in \mathcal{A}_{\mathcal{H}}$ and $\varepsilon \in (0,1)$, we have

$$\begin{aligned} J(\alpha + \varepsilon\beta) - J(\alpha) &= J((1-\varepsilon)\frac{\alpha}{1-\varepsilon} + \varepsilon\beta) - J(\alpha) \\ &\geq (1-\varepsilon)J(\frac{\alpha}{1-\varepsilon}) + \varepsilon J(\beta) - J(\alpha) \\ &= J(\frac{\alpha}{1-\varepsilon}) - J(\alpha) + \varepsilon(J(\beta) - J(\frac{\alpha}{1-\varepsilon})). \end{aligned} \tag{14}$$

Now, with $\frac{1}{1-\varepsilon} = 1 + \eta$ we have

$$\lim_{\varepsilon \to 0} \frac{1}{\varepsilon}(J(\frac{\alpha}{1-\varepsilon}) - J(\alpha)) = \lim_{\eta \to 0} \frac{1+\eta}{\eta}(J(\alpha + \eta\alpha) - J(\alpha)) = D_\alpha J(\alpha).$$

Combining this with (14) we get

$$D_\beta J(\alpha) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon}(J(\alpha + \varepsilon\beta) - J(\alpha)) \geq D_\alpha J(\alpha) + J(\beta) - J(\alpha).$$

We conclude that

$$J(\beta) - J(\alpha) \leq D_\beta J(\alpha) - D_\alpha J(\alpha) \ ; \alpha, \beta \in \mathcal{A}_{\mathcal{H}}.$$

In particular, applying this to $\alpha = \pi^*$ and using that $D_\beta J(\pi^*) = 0$ by (13), we get

$$J(\beta) - J(\pi^*) \leq 0 \quad \text{for all } \beta \in \mathcal{A}_{\mathcal{H}},$$

which proves that π^* is optimal. □

Using this characterization theorem in (iii) one can also give a closed formula for the optimal strategy π^*.

Corollary 3.2 *Suppose that an optimal portfolio $\pi^* \in \mathcal{A}_{\mathcal{H}}$ for Problem* (6) *exists. Then it must satisfy*

$$\pi^*(t)\mathbb{E}[\sigma^2(t)|\mathcal{H}_t] = \mathbb{E}[(\mu(t) - \rho(t))|\mathcal{H}_t] + a(t). \tag{15}$$

where

$$a(t) \equiv \lim_{h \to 0^+} \frac{1}{h}\mathbb{E}[\int_t^{t+h} \sigma(s) d^- B(s)|\mathcal{H}_t]. \tag{16}$$

Note that the optimal portfolio has a similar form as the solution of the Merton problem. Here the rate of appreciation and volatility are replaced by their best estimators, the conditional expectations. There is an extra term $a(t)$ which appears due to the anticipative nature of the original equation. An interpretation of this term is given in Section 6.

Remark 3.3 *If $\mathcal{G}_{t+\delta} \subseteq \mathcal{H}_t$, $\delta > 0$, then in most cases $a(t)$ does not exist, because in such a case*

$$\frac{1}{h}\mathbb{E}[\int_t^{t+h} \sigma(s)d^- B(s)|\mathcal{H}_t] = \frac{1}{h}\int_t^{t+h} \sigma(s)d^- B(s) \quad \text{for } h \le \delta.$$

Similarly, if $\mathcal{H}_t = \mathcal{F}_{t+\delta}$, $a(t)$ does not exist. This is also related to the fact that such insiders obtain an infinite amount of wealth and that the market admits arbitrage by the insider.

4 Optimal utility

In this section we compute the value function when the optimal portfolio exists. We start by a lemma (see (Corollary 2.10) in [ØS]) resulting from the relation between forward and Skorohod integrals. From now on, δ denotes the Skorohod integral and D denotes the stochastic derivative operator.

Lemma 4.1 *Suppose $f : [0,T] \times \Omega \to \mathbb{R}$ is Skorohod integrable and caglad. Moreover, assume that*

$$D_{t^+} f(t) := \lim_{s \to t^+} D_s f(t)$$

exists for a.a. $t \in [0,T]$ and

$$\int_0^T |D_{t^+} f(t)| dt < \infty.$$

Then

$$\mathbb{E}\Big[\int_0^T f(t) d^- B(t)\Big] = \mathbb{E}\Big[\int_0^T D_{t^+} f(t) dt\Big],$$

provided that the expectations exist.

Theorem 4.2 *Suppose that* $\sigma(t) \neq 0$ *for a.a.* (t,ω). *Suppose there exists an optimal portfolio* $\pi^* \in \mathcal{A}_\mathcal{H}$ *for Problem* (6). *Suppose that the function* $f : s \mapsto \pi^*(s)\sigma(s)$ *satisfies the conditions of Lemma 4.1. The optimal utility is then given by*

$$J(\pi^*) = \mathbb{E}\Big[\int_0^T \Big\{\frac{1}{2}\frac{\mathbb{E}[\mu(s)-\rho(s)|\mathcal{H}_s]^2}{\mathbb{E}[\sigma^2(s)|\mathcal{H}_s]} - \frac{1}{2}\frac{a(s)^2}{\mathbb{E}[\sigma^2(s)|\mathcal{H}_s]} + D_{s^+}\left(\sigma(s)\frac{\mathbb{E}[\mu(s)-\rho(s)|\mathcal{H}_s]+a(s)}{\mathbb{E}[\sigma^2(s)|\mathcal{H}_s]}\right)\Big\}ds\Big]. \tag{17}$$

Proof. From (15) we have

$$\pi^*(t) = \frac{\mathbb{E}[\nu(t)|\mathcal{H}_t]+a(t)}{\mathbb{E}[\sigma^2(t)|\mathcal{H}_t]} \tag{18}$$

where we have set $\nu(s) = \mu(s) - \rho(s)$. Plugging (18) into (4) we obtain

$$J(\pi^*) = \mathbb{E}\Big[\int_0^T \left\{\nu(s)(\frac{\mathbb{E}[\nu(s)|\mathcal{H}_s]+a(s)}{\mathbb{E}[\sigma^2(s)|\mathcal{H}_s]}) - \frac{\sigma^2(s)}{2}\left[\frac{\mathbb{E}[\nu(s)|\mathcal{H}_s]+a(s)}{\mathbb{E}[\sigma^2(s)|\mathcal{H}_s]}\right]^2\right\} ds + \int_0^T \sigma(s)\left[\frac{\mathbb{E}[\nu(s)|\mathcal{H}_s]+a(s)}{\mathbb{E}[\sigma^2(s)|\mathcal{H}_s]}\right] d^-B(s)\Big].$$

Now we use that

$$\mathbb{E}\left[\nu(s)\mathbb{E}[\nu(s)|\mathcal{H}_s]\right] = \mathbb{E}[\mathbb{E}[\nu(s)|\mathcal{H}_s]^2],$$

and $a(s)$ is $\mathcal{H}_s$-measurable $0 \le s \le T$, so that

$$\mathbb{E}\left[\nu(s)a(s)\right] = \mathbb{E}\left[\nu(s)\mathbb{E}[a(s)|\mathcal{H}_s]\right] = \mathbb{E}\left[\mathbb{E}[\nu(s)|\mathcal{H}_s]a(s)\right].$$

Moreover

$$\mathbb{E}\left[\frac{\sigma^2(s)}{\mathbb{E}[\sigma^2(s)|\mathcal{H}_s]}\right] = 1$$

and by Lemma 4.1

$$\mathbb{E}\left[\int_0^T \sigma(s)\frac{\mathbb{E}[\nu(s)|\mathcal{H}_s]+a(s)}{E[\sigma(s)^2|\mathcal{H}_s]}d^-B(s)\right] = \mathbb{E}\left[\int_0^T D_{s^+}\left(\sigma(s)\frac{\mathbb{E}[\nu(s)|\mathcal{H}_s]+a(s)}{E[\sigma(s)^2|\mathcal{H}_s]}\right)ds\right].$$

The conclusion follows. □

5 An extension of the partial information framework

In this section we consider a generalization of the partial observation control problem which will include most known cases of utility maximization for markets with insiders where enlargement of filtration techniques are used.

Example 5.1 *[Partial observation case.] Suppose $\mathcal{H}_t \subseteq \mathcal{F}_t$ and $\mathcal{F}_t = \mathcal{G}_t$. Then, we have*

$$\frac{d}{ds}\mathbb{E}[\int_0^s \sigma(u)d^- B(u)|\mathcal{H}_t] = 0, \quad s > t.$$

That is, $a(t) = 0$ and the optimal portfolio π^* is thus given by

$$\pi^*(t) = \frac{\mathbb{E}[\mu(t) - \rho(t)|\mathcal{H}_t]}{\mathbb{E}[\sigma^2(t)|\mathcal{H}_t]},$$

if the right hand side is well defined as an element in $\mathcal{A}_\mathcal{H}$. Furthermore the optimal utility is

$$J(\pi^*) = \frac{1}{2}\mathbb{E}\Big[\int_0^T \frac{\mathbb{E}[\mu(s) - \rho(s)|\mathcal{H}_s]^2}{\mathbb{E}[\sigma^2(s)|\mathcal{H}_s]} ds\Big].$$

This result follows directly from Theorem 3.1 (iii). One set of conditions that assures that $\pi^* \in \mathcal{A}_\mathcal{H}$ is that μ and ρ are uniformly bounded and $|\sigma(t)| \geq c > 0$ for all (t, ω). Similar existence conditions can also be found for the following examples.

We consider now a more general situation:

Proposition 5.2 *[Partial observation in an anticipative market]. Suppose $\mathcal{H}_t \subseteq \mathcal{F}_t \subseteq \mathcal{G}_t$. Moreover suppose that σ is Skorohod integrable. Then*

$$\pi^*(t) = \frac{\mathbb{E}[\mu(t) - \rho(t) + D_{t^+}\sigma(t)|\mathcal{H}_t]}{\mathbb{E}[\sigma^2(t)|\mathcal{H}_t]}$$

provided that the right hand side is a well defined element of $\mathcal{A}_\mathcal{H}$, $D_{t^+}\sigma(t)$ exists and belongs to $L^1(P)$. Furthermore if the conditions of Theorem 4.2 are *satisfied then*

$$J(\pi^*) = \frac{1}{2}\mathbb{E}\Big[\int_0^T \Big\{\frac{\mathbb{E}[\mu(s)-\rho(s)|\mathcal{H}_s]^2}{\mathbb{E}[\sigma^2(s)|\mathcal{H}_s]} - \frac{1}{2}\frac{a(s)^2}{\mathbb{E}[\sigma^2(s)|\mathcal{H}_s]}\Big\}ds\Big]$$

where $a(s) = \mathbb{E}[D_{s^+}\sigma(s)|\mathcal{H}_s]$.

PROOF. Let M be a smooth $\mathcal{H}_t$-measurable random variable. Then

$$\begin{aligned}
\mathbb{E}\Big[M.\int_t^{t+h}\sigma(s)d^-B(s)\Big] &= \mathbb{E}\Big[\int_t^{t+h} M\sigma(s)d^-B(s)\Big] \\
&= \mathbb{E}\Big[\int_t^{t+h} D_{s^+}(M\sigma(s))ds\Big] \\
&= \mathbb{E}\Big[\int_t^{t+h} MD_{s^+}\sigma(s)ds\Big] \\
&= \mathbb{E}\Big[M\int_t^{t+h} D_{s^+}\sigma(s)ds\Big].
\end{aligned}$$

This proves that

$$\mathbb{E}\Big[\int_t^{t+h}\sigma(s)d^-B(s)|\mathcal{H}_t\Big] = \mathbb{E}\Big[\int_t^{t+h} D_{s^+}\sigma(s)ds|\mathcal{H}_t\Big].$$

Hence

$$a(t) \equiv \lim_{h\to 0^+}\frac{1}{h}\mathbb{E}\Big[\int_t^{t+h}\sigma(s)d^-B(s)|\mathcal{H}_t\Big] = \mathbb{E}[D_{t^+}\sigma(t)|\mathcal{H}_t]. \quad \square$$

We conclude by using Theorems 3.1 and 4.2.

The difference between the optimal utilities of Proposition 5.2 and Example 5.1 can be explained as in Remark 6.5 below.

Next we want to show that Proposition 5.2 which generalizes the partial information framework also includes the case of financial markets with insiders modelled through enlargement of filtrations. To this purpose, let us first recall the classical set-up for models of markets with insiders through enlargement of filtrations in a simple case:

Let $\mathcal{G}_t = \mathcal{F}_t \vee \sigma(B_T)$ and $\mathcal{H}_t = \mathcal{G}_t$. Consider an insider who can influence the asset prices in the following way

$$dS_t = (\mu + \frac{B_T - B_t}{T - t})S_t dt + \sigma S_t d\tilde{B}_t, \quad t \in [0, T'], T' < T$$

where $\tilde{B}_t = B_t - \int_0^t \frac{B_T - B_t}{T-t} dt$ is a $\mathcal{G}_t$-Brownian motion, μ and σ are constants, and B_t is a $\mathcal{F}_t$-Brownian motion.

Note that in this case $\mathcal{H}_t \not\subseteq \mathcal{F}_t$ and $\tilde{B}$ is not a $\mathcal{F}_t$-Brownian motion and thus it may seem that Proposition 5.2 can not be applied here. Therefore, instead of continuing in this way, we now modify the above formulation in order that the enlargement of filtration approach fits into this proposition. Consider the following model:

$$dS_t = (\mu + \frac{B_T - B_t}{T - t})S_t dt + \sigma S_t d\tilde{B}_t, \tag{19}$$

where $\tilde{B}$ is an $\mathcal{F}_t := \mathcal{F}_t^B \vee \sigma(B_T)$-Brownian motion and $\mathcal{F}^B$ stands for the filtration generated by the Brownian motion B. Furthermore, let $\mathcal{G}_t = \mathcal{F}_t^B \vee \sigma(B_T)$. We consider two examples:

Example 5.3 *[The insider case]. Let $\mathcal{H}_t = \mathcal{F}_t^B \vee \sigma(B_T)$ and consider model (19). This fits into Example 5.1 with $\mathcal{F} = \mathcal{G} = \mathcal{H}$. We have*

$$a(t) = \lim_{h \to 0^+} \frac{1}{h} \sigma \mathbb{E}[\tilde{B}_{t+h} - \tilde{B}_t | \mathcal{H}_t] = 0.$$

The optimal policy for the insider is

$$\pi^*(t) = \frac{1}{\sigma^2}(\mu - \rho(t) + \frac{B_T - B_t}{T - t}).$$

We have

$$\mathbb{E} \ln(X^{\pi^*})(T') = \frac{1}{2\sigma^2} \mathbb{E} \int_0^{T'} (\mu - \rho(t) + \frac{B_T - B_t}{T - t})^2 dt \sim \ln \sqrt{\frac{1}{T' - T}} \text{ when } T' \to T.$$

Consequently the optimal utility is infinite:

$$\lim_{T' \to T} \mathbb{E} \ln(X^{\pi^*})(T') = \infty.$$

This is the well-known result of Karatzas-Pikovsky [KP]. The general case $\mathcal{H} \subset \mathcal{G}$ can also be considered similarly.

Example 5.4 *[The case of a small investor]. Let $\mathcal{H}_t = \mathcal{F}_t^B$ and consider model (19). Then*

$$a(t) = \lim_{h \to 0^+} \frac{1}{h} \sigma \mathbb{E}[\tilde{B}_{t+h} - \tilde{B}_t | \mathcal{F}_t] = 0.$$

Consequently, if $\rho(t) \equiv \rho$,

$$\pi^*(t) \quad = \frac{\mu - \rho}{\sigma^2}$$

and the optimal utility is

$$J(\pi^*) = \frac{(\mu - \rho)^2 T}{2\sigma^2} \qquad \text{(Merton problem).}$$

One can generalize this example to a general situation as follows.

Corollary 5.5 *Let S be described as the unique solution of*

$$dS_t = (\mu + X_t) S_t dt + \sigma S_t d^- B_t,$$

where $(X_t, t \geq 0)$ is a $\mathcal{F}_T$-measurable process and B_t is a $\mathcal{F}_t$-Brownian motion. Suppose $\mathcal{H}_t \subset \mathcal{F}_t$. Then $a(t) = 0$, and the optimal portfolio is

$$\pi^*(t) = \frac{\mathbb{E}[\mu + X_t - \rho(t) | \mathcal{H}_t]}{\sigma^2}. \tag{20}$$

provided it is an element of $\mathcal{A}_{\mathcal{H}}$.

A further generalization to any enlargement of filtration is the following.

Proposition 5.6 *Consider the following model*

$$dS_t = \mu(t) S_t dt + \sigma S_t dB(t)$$

where σ is constant, $\mu(t)$ is $\mathcal{G}_t$-adapted, B is a $\mathcal{F}$-Brownian motion, $\mathcal{F} \subseteq \mathcal{G}$ and $\mathcal{H}$ is a general filtration. If $B_t = \tilde{B}_t + \int_0^t \beta(s) ds$ where $\tilde{B}_t$ is a $\mathcal{H}_t$-Brownian motion and β is an $\mathcal{H}$-adapted cadlag process with $\int_0^T |\beta(s)| ds < \infty$, then $a(t)$ defined in (16) exists and we have $\dfrac{a(t)}{\sigma} = \beta(t)$.

This proposition follows using the definition of $a(t)$ and provides a first "example" when $a(t) \neq 0$. Nevertheless, we are not aware of any explicit case of initial enlargement of filtration where Proposition 5.6 can be applied and that can not be framed into Proposition 5.2 as in Example 5.3. (For more on this, see next section). In this line, we end up this section with a remark pointing towards a converse statement.

Remark 5.7 *One may wonder if in the general case of three filtrations $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$ the existence of optimal portfolios imply that the set-up under consideration is an initial enlargement of filtration. This would also imply that the term $a(t)$ is artificial. These two issues will be answered by the example in the next section. In fact, one has from Theorem 3.1 that if there exists an optimal portfolio and if σ is constant, then $\mathbb{E}[B_t|\mathcal{H}_t]$ is a $\mathcal{H}$-semimartingale and*

$$\mathbb{E}[B_t|\mathcal{H}_t] = \tilde{B}_t + \frac{1}{\sigma}\int_0^t a(s)ds$$

where $\tilde{B}$ is a $\mathcal{H}$-Brownian motion. We shall illustrate this point in Remark 6.5.3.

6 An interpretation of $a(t)$

In this section we consider a first example that does not fit the framework of Proposition 5.2. That is, here we deal with an example where $\mathcal{H}_t \subsetneqq \mathcal{F}_t$. Therefore the results of the previous section can not be applied. We consider a small investor acting in a market influenced by an insider. The insider influences the prices using a Brownian motion in the original filtration $\mathcal{F}$. This will lead to an interesting interpretation of the additional term $a(t)$ which appeared in Corollary 3.2.

Suppose that the small investor can observe neither the Brownian motion B nor the drift μ, but only the stock price process S, that is

$$\mathcal{H}_t = \sigma(S_s, 0 \leq s \leq t) \tag{21}$$

is the filtration generated by the price process S. Then the quadratic variation process of S is given by

$$< S, S >_t = \int_0^t \sigma_s^2 S_s^2 ds, \quad 0 \leq t \leq T.$$

It follows that the process $(\sigma_t, 0 \leq t \leq T)$ is $\mathcal{H}_t$-adapted and thus

$$\mathbb{E}[\sigma^2(t)|\mathcal{H}_t] = \sigma^2(t).$$

The optimal portfolio if it exists, must then satisfy (see Corollary 3.2)

$$\pi^*(t)\sigma^2(t) = \mathbb{E}[(\mu(t) - \rho(t))|\mathcal{H}_t] + \lim_{h\to 0} \frac{1}{h}\mathbb{E}[\int_t^{t+h} \sigma(s)d^-B(s)|\mathcal{H}_t].$$

6.1 The particular case: $\mu(t) = \mu + bB_T$

We consider the case when the dynamics of the prices are given by

$$dS_t = S_t(\mu + bB_T)dt + \sigma S_t d^- B_t \tag{22}$$

where μ and b are real numbers, $\sigma > 0$. We suppose moreover that $\rho(t) = \rho =$ constant.

Lemma 6.1 *Suppose that S_t satisfies (22) and $\mathcal{H}_t$ is given by (21). Then the quantity $a(t)$ defined in (16) is explicitly given by*

$$a(t) \equiv \lim_{h\to 0^+} \frac{1}{h}\mathbb{E}[\sigma(B_{t+h} - B_t)|\mathcal{H}_t] = \frac{\sigma b(bB_T t + \sigma B_t)}{(b^2T + 2b\sigma)t + \sigma^2}. \tag{23}$$

PROOF. Integrating equation (22), we obtain

$$S_t = S_0 \exp(\mu t + btB_T - \frac{1}{2}\sigma^2 t + \sigma B_t).$$

Consequently,

$$\begin{aligned} \mathcal{H}_t &= \sigma(\mu s - \tfrac{1}{2}\sigma^2 s + bsB_T + \sigma B_s, 0 \leq s \leq t) \\ &= \sigma(bsB_T + \sigma B_s, 0 \leq s \leq t). \end{aligned}$$

and

$$\sigma\mathbb{E}[B_{t+h} - B_t|\mathcal{H}_t] = \sigma\mathbb{E}[B_{t+h} - B_t|bsB_T + \sigma B_s, 0 \leq s \leq t).$$

Consider the following partition

$$0 = s_0 < s_1 < \ldots < s_n = t \quad \text{with time interval } \Delta = s_{i+1} - s_i.$$

and denote $\mathcal{H}_t^n$ the σ-algebra generated by $\{bs_iB_T+\sigma B_{s_i}, i=0\dots n\}$.

Since $bs_iB_T+\sigma B_{s_i}$ is a Gaussian vector, the conditional expectation can be expressed as

$$\sigma\mathbb{E}[B_{t+h}-B_t|bs_iB_T+\sigma B_{s_i}, i=0,\dots n]=\sum_{i=0}^{n-1}\alpha_i(bB_T(s_{i+1}-s_i)+\sigma(B_{s_{i+1}}-B_{s_i}))$$

where the constant coefficients α_i have to be determined by using the correlations of each term with $bB_T(s_{j+1}-s_j)+\sigma(B_{s_{j+1}}-B_{s_j})$. Doing this calculations, one gets

$$\sigma bh\Delta=\sum_{i=0,i\neq j}^{n-1}\alpha_i(b^2T\Delta^2+2b\sigma\Delta^2)+\alpha_j(b^2T\Delta^2+2b\sigma\Delta^2+\sigma^2\Delta)$$

that is

$$\sigma bh=\sum_{i=0,i\neq j}^{n-1}\alpha_i(b^2T+2b\sigma)\Delta+\alpha_j(b^2T\Delta+2b\sigma\Delta+\sigma^2).$$

In matrix form this gives

$$\sigma bh\mathbf{1}_{n\times 1}=((bT+2\sigma)b\sigma\mathbf{1}_{n\times n}+\sigma^2I_{n\times n})\alpha,$$

where $\mathbf{1}_{a\times b}$ denotes a matrix of order $a\times b$ with all entries equal to 1, $I_{a\times a}$ denotes the identity matrix of order $a\times a$ and $\alpha=(\alpha_0,...,\alpha_{n-1})^T$. By linear combinations of these equations we get

$$\alpha_0=\alpha_1=\cdots\alpha_{n-1}\equiv\alpha$$

$$\sigma bh=\alpha(b^2T+2b\sigma)\Delta(n-1)+\alpha(b^2T\Delta+2b\sigma\Delta+\sigma^2)$$

which gives

$$\alpha=\frac{\sigma bh}{(b^2T+2b\sigma)\Delta n+\sigma^2}.$$

We thus get

$$\sigma\mathbb{E}[B_{t+h}-B_t|\mathcal{H}_t^n]=\frac{\sigma bh}{(b^2T+2b\sigma)\Delta n+\sigma^2}(bB_Tn\Delta+\sigma B_t).$$

Since $n\Delta = t$ the above expression is independent of n and

$$\sigma\mathbb{E}[B_{t+h} - B_t|\mathcal{H}_t] = \frac{\sigma bh}{(b^2T + 2b\sigma)t + \sigma^2}(bB_Tt + \sigma B_t). \tag{24}$$

Consequently

$$a(t) = \lim_{h\to 0^+}\frac{1}{h}\sigma\mathbb{E}[B_{t+h} - B_t|\mathcal{H}_t] = \frac{\sigma b}{(b^2T + 2b\sigma)t + \sigma^2}(bB_Tt + \sigma B_t).\square \tag{25}$$

Note that this is an example where $a(t) \neq 0$. Furthermore, as $\mathcal{H}_t = \sigma(bsB_T + \sigma B_s, 0 \leq s \leq t)$, the small investor can not decouple a fix part of the noise (B_T) out of the observed $B_T + \sigma B_s$, $s \leq t$. But as $s \to T$ the "decoupling" becomes easier. This is in the spirit of a continuous enlargement of filtration setting introduced in Corcuera et al. [CIKN].

Lemma 6.2

$$\mathbb{E}[B_T|\mathcal{H}_s] = \frac{(bT + \sigma)}{(b^2T + 2b\sigma)s + \sigma^2}(bB_Ts + \sigma B_s). \tag{26}$$

Proof. We proceed as before. Let $0 = s_0 < s_1 < \ldots s_n = t$ and $\Delta = s_{i+1} - s_i$.

$$\begin{aligned}\mathbb{E}(B_T|\mathcal{H}_t^n) &= \mathbb{E}(B_T|bs_iB_T + \sigma B_{s_i}, 0 \leq i \leq n)\\ &= \sum_{i=0}^{n-1}\alpha_i(bB_T\Delta + \sigma(B_{s_{i+1}} - B_{s_i})).\end{aligned}$$

By computing the correlation with $bB_T\Delta + \sigma(B_{s_{j+1}} - B_{s_j})$ we get

$$bT\Delta + \sigma\Delta = \sum_{i=0,i\neq j}^{n-1}\alpha_i(b^2\Delta^2T + 2\sigma b\Delta^2) + \alpha_j(b^2\Delta^2T + 2\sigma b\Delta^2 + \sigma^2\Delta).$$

In matrix form this leads to

$$(bT + \sigma)\mathbf{1}_{n\times 1} = ((bT + 2\sigma)b\Delta\mathbf{1}_{n\times n} + \sigma^2 I_{n\times n})\alpha.$$

As before, this gives

$$\alpha_0 = \alpha_1 = \cdots \alpha_{n-1} \equiv \alpha$$
$$\alpha = \frac{bT + \sigma}{(b^2T + 2b\sigma)t + \sigma^2}.$$

which implies (26).□

Theorem 6.3 *Suppose that S_t is given by (22) with $b \geq 0$ and $\mathcal{H}_t$ is given by (21). Then*

(i) The optimal portfolio for problem (6) exists and is given by

$$\pi^*(t) = \frac{\mathbb{E}\left[\mu(t) - \rho|\mathcal{H}_t\right]}{\sigma^2} + \frac{b(bB_Tt + \sigma B_t)}{\sigma((b^2T + 2b\sigma)t + \sigma^2)} \tag{27}$$

which can be rewritten as

$$\pi^*(t) = \frac{\mu - \mathbb{E}\left[\rho|\mathcal{H}_t\right]}{\sigma^2} + \frac{b(bB_Tt + \sigma B_t)(bT + \sigma + \sigma^{-1})}{\sigma^2((b^2T + 2b\sigma)t + \sigma^2)}.$$

(ii) The optimal utility is finite and is given by

$$J(\pi^*) = \frac{(\mu - \rho)^2 T}{2\sigma^2} + \frac{1}{2\gamma}(1 - \gamma \ln(1 + \frac{1}{\gamma})) \tag{28}$$

where we have set

$$\gamma \equiv \frac{\sigma^2}{bT(bT + 2\sigma)}. \tag{29}$$

Remark 6.4 *If $\rho(t)$ is not constant, then the optimal portfolio and utility are respectively given by*

$$\pi^*(t) = \frac{\mu - \mathbb{E}\left[\rho(t)|\mathcal{H}_t\right]}{\sigma^2} + \frac{b(bB_Tt + \sigma B_t)(bT + \sigma + \sigma^{-1})}{\sigma^2((b^2T + 2b\sigma)t + \sigma^2)},$$

$$\begin{aligned} J(\pi^*) = &\frac{1}{2\sigma^2}\mathbb{E}\left[\int_0^T \mathbb{E}[\mu(s) - \rho(s)|\mathcal{H}_s]^2 ds\right] - \frac{1}{\sigma}\int_0^T E\left[D_{s^+}\mathbb{E}[\rho(s)|\mathcal{H}_s]\right] ds \\ &+ \left(\frac{bT}{\sigma} + \frac{3}{2}\right)\left(\frac{(bT)^2\gamma}{\sigma^2}\right)\left(1 - \gamma \ln(1 + \frac{1}{\gamma})\right), \end{aligned}$$

provided sufficient hypotheses are assumed on $\rho(t)$ in order that $\pi^* \in \mathcal{A}_{\mathcal{H}}$ and the conditions of Theorem 4.2 are satisfied.

Proof. The expression (27) is obtained using (25) and Corollary 3.2. To check that the candidate π^* given by (27) is indeed an optimal portfolio, we have to prove that $M_{\pi^*}(t)$ is a $\mathcal{H}$-martingale (checking that $\pi^* \in \mathcal{A}_{\mathcal{H}}$ is straightforward). Plugging (27) into (7), we get

$$\begin{aligned} M_{\pi^*}(t) &= \mathbb{E}\left[\int_0^t (\mu(s)-\rho(s)-\mathbb{E}(\mu(s)-\rho(s)|\mathcal{H}_s))ds|\mathcal{H}_t\right] \\ &\quad -\mathbb{E}\left[\int_0^t \sigma b(bB_T s+\sigma B_s)((b^2T+2b\sigma)s+\sigma^2)^{-1}ds|\mathcal{H}_t\right]+\sigma\mathbb{E}[B_t|\mathcal{H}_t] \\ &\equiv M^1_{\pi^*}(t)+M^2_{\pi^*}(t)+M^3_{\pi^*}(t). \end{aligned}$$

Let $u<t$. We want to prove

$$\mathbb{E}[M_{\pi^*}(t)-M_{\pi^*}(u)|\mathcal{H}_u]=0.$$

First, we show that $M^1_{\pi^*}$ satisfies the martingale property. For $u<t$,

$$\begin{aligned} \mathbb{E}[M^1_{\pi^*}(t)-M^1_{\pi^*}(u)|\mathcal{H}_u] &= \mathbb{E}\Big[\mathbb{E}\left[\int_0^t (\mu(s)-\rho(s)-\mathbb{E}(\mu(s)-\rho(s)|\mathcal{H}_s))ds|\mathcal{H}_t\right]|\mathcal{H}_u\Big] \\ &\quad -\mathbb{E}\left[\int_0^u (\mu(s)-\rho(s)-\mathbb{E}(\mu(s)-\rho(s)|\mathcal{H}_s))ds|\mathcal{H}_u\right] \\ &= \mathbb{E}\Big[\mathbb{E}\left[\int_0^u (\mu(s)-\rho(s)-\mathbb{E}(\mu(s)-\rho(s)|\mathcal{H}_s))ds|\mathcal{H}_t\right]|\mathcal{H}_u\Big] \\ &\quad +\int_u^t \left(\mathbb{E}[\mu(s)-\rho(s)|\mathcal{H}_u]-\mathbb{E}[\mu(s)-\rho(s)|\mathcal{H}_u]\right)ds \\ &\quad -\int_0^u \left(\mathbb{E}[\mu(s)-\rho(s)|\mathcal{H}_u]-\mathbb{E}[\mu(s)-\rho(s)|\mathcal{H}_u]\right)ds \\ &= 0 \end{aligned}$$

Next we prove that $M^2_{\pi^*}+M^3_{\pi^*}$ is a $\mathcal{H}$ martingale. We have using Lemmas 6.1 and 6.2,

$$\begin{aligned} -\mathbb{E}[M^2_{\pi^*}(t)-M^2_{\pi^*}(u)|\mathcal{H}_u] &= \mathbb{E}\Big[\mathbb{E}\left[\int_u^t \sigma b(bB_T s+\sigma B_s)((b^2T+2b\sigma)s+\sigma^2)^{-1}ds|\mathcal{H}_t\right]|\mathcal{H}_u\Big] \\ &= \mathbb{E}\left[\int_u^t \sigma b(bB_T s+\sigma B_s)((b^2T+2b\sigma)s+\sigma^2)^{-1}ds|\mathcal{H}_u\right] \end{aligned}$$

and

$$\mathbb{E}[M^3_{\pi^*}(t)-M^3_{\pi^*}(u)|\mathcal{H}_u]=\sigma\mathbb{E}\left[\mathbb{E}(B_t|\mathcal{H}_t)-B_u|\mathcal{H}_u\right]=\sigma\mathbb{E}[B_t-B_u|\mathcal{H}_u].$$

Let $u = t_0 < t_1 < \ldots < t_n = t$ be a partition of $[u,t]$ with time interval $\Delta = s_{i+1} - s_i$. We have

$$\begin{aligned}\sigma\mathbb{E}[B_t - B_u|\mathcal{H}_u] &= \sigma\mathbb{E}[\textstyle\sum_{i=0}^{n-1}(B_{t_{i+1}} - B_{t_i})|\mathcal{H}_u] \\ &= \sigma\textstyle\sum_{i=0}^{n-1}\mathbb{E}[B_{t_{i+1}} - B_{t_i}|\mathcal{H}_u] \\ &= \sigma\textstyle\sum_{i=0}^{n-1}\mathbb{E}[\mathbb{E}(B_{t_{i+1}} - B_{t_i}|\mathcal{H}_{t_i})|\mathcal{H}_u] \\ &= \sigma\textstyle\sum_{i=0}^{n-1}\mathbb{E}[b\Delta(bB_Tt_i + \sigma B_{t_i})((b^2T + 2b\sigma)t_i + \sigma^2)^{-1}|\mathcal{H}_u]\end{aligned} \tag{30}$$

by using (26), and this last expression converges to

$$\mathbb{E}\left[\int_u^t \sigma b(bB_Ts + \sigma B_s)((b^2T + 2b\sigma)s + \sigma^2)^{-1}ds|\mathcal{H}_u\right]$$

when $n \to \infty$. Consequently

$$\mathbb{E}[M^2_{\pi^*}(t) + M^3_{\pi^*}(t) - M^2_{\pi^*}(u) - M^3_{\pi^*}(u)|\mathcal{H}_u] = 0$$

and M_{π^*} is a $\mathcal{H}$- martingale.

We compute now the value function.We use (17) together with equalities (23) and (26). We have

$$\begin{aligned}-\frac{1}{2\sigma^2}\mathbb{E}\int_0^T a(s)^2ds &= -\frac{b^2}{2}\int_0^T \frac{1}{((b^2T + 2\sigma b)s + \sigma^2)^2}\mathbb{E}\left(b^2s^2B_T^2 + \sigma^2B_s^2 + 2b\sigma sB_sB_T\right)ds \\ &= -\frac{b^2}{2}\int_0^T \frac{b^2s^2T + \sigma^2s + 2b\sigma s^2}{((b^2T + 2\sigma b)s + \sigma^2)^2}ds \\ &= -\frac{b^2}{2}\int_0^T \frac{s}{(b^2T + 2b\sigma)s + \sigma^2}ds.\end{aligned}$$

Moreover we have

$$\begin{aligned}D_{s^+}\mathbb{E}[B_T|\mathcal{H}_s] &= \frac{(bT + \sigma)bs}{(b^2T + 2b\sigma)s + \sigma^2} \\ D_{s^+}a(s) &= \frac{\sigma b^2 s}{(b^2T + 2b\sigma)s + \sigma^2}\end{aligned}$$

so that

$$D_{s^+}\left[\frac{\mathbb{E}[\mu(s)-\rho|\mathcal{H}_s]+a(s)}{\sigma}\right] = \frac{1}{\sigma}\left(\frac{b^2 s(bT+\sigma)}{(b^2T+2\sigma b)s+\sigma^2}+\frac{\sigma b^2 s}{(b^2T+2\sigma b)s+\sigma^2}\right)$$
$$=\frac{b^2 s(bT+2\sigma)}{\sigma((b^2T+2\sigma b)s+\sigma^2)}.$$

We thus get

$$J(\pi^*) = \frac{1}{2\sigma^2}\mathbb{E}\int_0^T \mathbb{E}[\mu(s)-\rho|\mathcal{H}_s]^2 ds - \frac{b^2}{2}\int_0^T \frac{s}{(b^2T+2b\sigma)s+\sigma^2}ds$$
$$+\frac{b^2}{\sigma}(bT+2\sigma)\int_0^T \frac{s}{(b^2T+2b\sigma)s+\sigma^2}ds$$
$$=\frac{1}{2\sigma^2}\mathbb{E}\int_0^T \mathbb{E}[\mu(s)-\rho|\mathcal{H}_s]^2 ds + b^2(\frac{bT}{\sigma}+\frac{3}{2})\int_0^T \frac{s}{(b^2T+2b\sigma)s+\sigma^2}ds.$$

We now use that $b \geq 0$ and by integration we have

$$\int_0^T \frac{s}{(b^2T+2b\sigma)s+\sigma^2}ds = \frac{T}{b^2T+2b\sigma}\left(1-\frac{\sigma^2}{(b^2T+2b\sigma)T}\ln(1+\frac{b^2T+2b\sigma}{\sigma^2}T)\right)$$

which can also be written as

$$\frac{T^2\gamma}{\sigma^2}(1-\gamma\ln(1+\frac{1}{\gamma}))$$

which is positive. Similarly, one computes $\frac{b^2}{2\sigma^2}\int_0^T \mathbb{E}[\mathbb{E}[B_T|\mathcal{H}_s]^2]ds$. We thus get (28). □

Remark 6.5 1. *The coefficient* $\frac{1}{\gamma}$ *(see (29)) can be interpreted as the insider effect on the utility of the* $\mathcal{H}$ *investor. When* $\gamma \to +\infty$ *(which is implied by* $b \to 0$*, that is the insider effect vanishes) the utility of the* $\mathcal{H}$ *investor is closer to the optimal utility in the classical Merton problem. A similar interpretation can be applied for* $\gamma \to 0$*. From* (28)*, we obtain*

$$\lim_{b\to 0} J(\pi^*) = \frac{(\mu-\rho)^2 T}{2\sigma^2} \quad \textit{(Merton problem)}$$
$$\lim_{b\to\infty} J(\pi^*) = +\infty \quad \textit{(Strong drift problem)}.$$

2. *The restriction $b \geq 0$ has a practical interpretation in our model. In this situation when $\mu(t) = \mu + bB_T$, the insider introduces a higher appreciation rate in the stock price if $B_T > 0$. Therefore given the linearity of the equation of S this indicates that the higher the final stock price the bigger the value of the drift of the equation driving S. Some cases of negative values for b can be studied but the practical character of such an study is dubious.*

3. *From* (24), *we get*

$$\mathbb{E}[B_{t+h} - B_t - \frac{b(bB_T t + \sigma B_t)}{(b^2T + 2b\sigma)t + \sigma^2} h | \mathcal{H}_t] = 0.$$

Consequently, by a similar argument to (30), we have

$$\mathbb{E}[B_t - \int_0^t \frac{b(bB_T u + \sigma B_u)}{(b^2T + 2b\sigma)u + \sigma^2} du | \mathcal{H}_s] = \mathbb{E}[B_s - \int_0^s \frac{b(bB_T u + \sigma B_u)}{(b^2T + 2b\sigma)u + \sigma^2} du | \mathcal{H}_s]$$

and the process

$$\tilde{B}_t \equiv \mathbb{E}[B_t | \mathcal{H}_t] - \int_0^t \frac{b(bB_T u + \sigma B_s)}{(b^2T + 2b\sigma)u + \sigma^2} du = \mathbb{E}[B_t | \mathcal{H}_t] - \frac{1}{\sigma} \int_0^t a(u) du$$

is thus a $\mathcal{H}_t$-Brownian motion by Lévy's characterization theorem of Brownian motions. Essentially, a similar calculation leads to Remark 5.7.

4. *Consider an investor who estimates the appreciation rate of the prices by using the best linear estimate given by $\mathbb{E}[\mu(t)|\mathcal{H}_t]$ and builds his price model as*

$$d\tilde{S}_t = \mathbb{E}[\mu(t)|\mathcal{H}_t] \tilde{S}_t dt + \sigma \tilde{S}_t d\tilde{B}_t, \tag{31}$$

where $\tilde{B}$ is a $\mathcal{H}$-Brownian motion. If this investor decides to select portfolios by using this model, the admissible portfolios are $\mathcal{H}$-adapted processes and the investor faces the following logarithmic utility portfolio optimization problem:

$$J_0(\pi_0^*) = \max_{\pi \in \mathcal{H}} J_0(\pi)$$

where

$$J_0(\pi) = \mathbb{E}(\ln(\tilde{X}(T)))$$

and

$$d\tilde{X}(t) = \tilde{X}(t)\Big[(\mathbb{E}[\mu(t)|\mathcal{H}_t] - \rho)\pi(t)dt + \pi(t)\sigma d\tilde{B}(t)\Big] \qquad \tilde{X}(0) = x > 0.$$

The solution of this optimization problem will be similar to the"classical" Merton case. The optimal portfolio is here

$$\pi_0^*(t) = \frac{\mathbb{E}[\mu(t) - \rho|\mathcal{H}_t]}{\sigma^2}$$

which is different from (27) *and the optimal utility for this investor is*

$$\begin{aligned} J_0(\pi_0^*) = \;& \frac{1}{2\sigma^2}\int_0^T \mathbb{E}[\mathbb{E}[\mu(s) - \rho|\mathcal{H}_s]^2]ds \\ &= \frac{1}{2\sigma^2}\int_0^T \mathbb{E}[\mu - \rho + \frac{b(bT+\sigma)}{(b^2T + 2b\sigma)s + \sigma^2}(bB_Ts + \sigma B_s)]^2 ds \\ &= \frac{(\mu-\rho)^2 T}{2\sigma^2} + \frac{(bT+\sigma)^2}{2(bT+2\sigma)^2\gamma}(1 - \gamma\ln(1+\frac{1}{\gamma})) \\ &< J(\pi^*) \text{ (given by (28) under the model (22))}. \end{aligned} \tag{32}$$

The utility generated by the portfolio π_0^* *in the "real" model* (22), $J(\pi_0^*)$, *will be different from* $J_0(\pi_0^*)$ *obtained in* (32).

In fact, $J(\pi_0^*) - J_0(\pi_0^*) = \frac{\sigma(bT+\sigma)}{(bT+2\sigma)^2\gamma}(1 - \gamma\ln(1+\frac{1}{\gamma}))$ *represents the difference between the actual earnings of the policy* π_0^* *under the model (22) and the projected earning of the small investor using the model (31). Notably this quantity is positive.*

Moreover, $J(\pi^*) - J(\pi_0^*) = \frac{\sigma^2}{2(bT+2\sigma)^2\gamma}(1 - \gamma\ln(1+\frac{1}{\gamma}))$ *represents the difference between the optimal earnings if the small investor uses* π^* *recognizing an anticipating model* (28) *and the actual earnings of the small investor that uses portfolio* π_0^* *taking* (31) *as the model for the underlying. This difference comes from considering* $a \equiv 0$ *or not in Theorem 4.2. The difference in utility is obviously positive due to the optimal property of the portfolio with* $a(t) \neq 0$.

6.2 General case: $\mu(t) = \mu + bX, \quad X \in \mathcal{F}_T$

We consider the generalization of the previous section to the case when $\mu(t) = \mu + bX$, where X is a general smooth $\mathcal{F}_T$-measurable random variable. The dynamics of the prices is

$$dS_t = S_t(\mu + bX)dt + \sigma S_t d^- B_t$$

where μ and b are real numbers, $\sigma > 0$. In this section rather than using direct calculation we use the integration by parts formula of Malliavin Calculus in order to obtain the "explicit" expression of $a(t)$. The goal in this section is just to show that $a(t)$ can also be computed explicitly in other situations. We shall not write down here the long and tedious expressions for the optimal portfolio and optimal utility.

Lemma 6.6 *The quantity $a(t)$ defined in (16) is then given by*

$$a(t) \equiv \lim_{h \to 0^+} \frac{1}{h} \mathbb{E}[\sigma(B_{t+h} - B_t)|\mathcal{H}_t] = \sigma \mathbb{E}[\int_t^T \frac{D_v X D_t X}{\int_t^T (D_r X)^2 dr} \delta B_v | \mathcal{H}_t],$$

if the right hand side above is well defined and right continuous in t.

PROOF. Consider the following partition

$$0 = s_0 < s_1 < \ldots < s_n = t \quad \text{with time interval } \Delta = s_{i+1} - s_i.$$

and denote $\mathcal{H}_t^n$ the σ-algebra generated by $\{bs_i X + \sigma B_{s_i}, i = 0 \ldots n\}$. We have for a smooth bounded function f

$$\begin{aligned} &\mathbb{E}[B_{t+h} - B_t | bs_i X + \sigma B_{s_i}, i = 0, \ldots n] \\ &\quad = \mathbb{E}[(B_{t+h} - B_t) f(bX(s_n - s_{n-1}) + \sigma(B_{s_n} - B_{s_{n-1}}), \ldots, bXs_1 + \sigma B_{s_1})]. \end{aligned}$$

Denote

$$Z = (bX(s_n - s_{n-1}) + \sigma(B_{s_n} - B_{s_{n-1}}), \ldots, bXs_1 + \sigma B_{s_1}).$$

By duality formula, we can write

$$\begin{aligned} \mathbb{E}[(B_{t+h} - B_t) f(Z)] = \; & \mathbb{E}[\int_t^{t+h} \sum_{i=1}^n \frac{\partial f}{\partial x_i}(Z) b(s_i - s_{i-1}) D_u X du] \\ = & \int_t^{t+h} \mathbb{E}[\sum_{i=1}^n \frac{\partial f}{\partial x_i}(Z) b(s_i - s_{i-1}) D_u X] du. \end{aligned} \tag{33}$$

Now, we have for $\alpha_2 > \alpha_1 \geq t$

$$\int_{\alpha_1}^{\alpha_2} D_v X D_v f(Z) dv = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(Z) b(s_i - s_{i-1}) \int_{\alpha_1}^{\alpha_2} (D_v X)^2 dv.$$

Multiplying both sides by $\dfrac{D_u X}{\int_{\alpha_1}^{\alpha_2} (D_u X)^2 du}$, we get

$$\mathbb{E} \int_{\alpha_1}^{\alpha_2} \frac{D_v X D_u X}{\int_{\alpha_1}^{\alpha_2} (D_r X)^2 dr} D_v f(Z) dv = \mathbb{E} \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(Z) b(s_i - s_{i-1}) D_u X. \tag{34}$$

The duality principle in Malliavin calculus implies that

$$\mathbb{E} \int_{\alpha_1}^{\alpha_2} \frac{D_v X D_u X}{\int_{\alpha_1}^{\alpha_2} (D_v X)^2 dv} D_v f(Z) dv = \mathbb{E} \left[f(Z) \int_{\alpha_1}^{\alpha_2} \frac{D_v X D_u X}{\int_{\alpha_1}^{\alpha_2} (D_v X)^2 dv} \delta B_v \right]. \tag{35}$$

Here δ denotes the Skorohod integral. Combining (33), (34), (35), we get

$$\begin{aligned} \mathbb{E}((B_{t+h} - B_t) f(Z)) &= \int_t^{t+h} \mathbb{E} \left[f(Z) \int_{\alpha_1}^{\alpha_2} \tfrac{D_v X D_u X}{\int_{\alpha_1}^{\alpha_2} (D_v X)^2 dv} \delta B_v \right] du \\ &= \int_t^{t+h} \mathbb{E} \left[f(Z) \mathbb{E}[\int_{\alpha_1}^{\alpha_2} \tfrac{D_v X D_u X}{\int_{\alpha_1}^{\alpha_2} (D_v X)^2 dv} \delta B_v | \mathcal{H}_u] \right] du \end{aligned}$$

since $f(Z)$ is $\mathcal{H}_t$-measurable.

The process

$$\tilde{B}_t \equiv \mathbb{E}[B_t | \mathcal{H}_t] - \int_0^t \mathbb{E} \left[\int_{\alpha_1}^{\alpha_2} \frac{D_v X D_u X}{\int_{\alpha_1}^{\alpha_2} (D_v X)^2 dv} \delta B_v | \mathcal{H}_u \right] du$$

is a $\mathcal{H}$-martingale. We deduce under the continuity hypothesis that for any $t \leq \alpha_1 < \alpha_2 \leq T$

$$\lim_{h \to 0^+} \frac{1}{h} \mathbb{E}[B_{t+h} - B_t | \mathcal{H}_t] = \mathbb{E}[\int_{\alpha_1}^{\alpha_2} \frac{D_v X D_t X}{\int_{\alpha_1}^{\alpha_2} (D_v X)^2 dv} \delta B_v | \mathcal{H}_t]. \tag{36}$$

We can take in the above formulas $\alpha_1 = t$ and $\alpha_2 = T$.□

Remark 6.7 *If $X = B_T$, then we find* (27). *Indeed in this case, the r.h.s. of* (36) *is equal to* $\mathbb{E}[\frac{B_T - B_t}{T-t}|\mathcal{H}_t]$ *which can be rewritten as*

$$\mathbb{E}[\frac{B_T - B_t}{T-t}|\mathcal{H}_t] = (T-t)^{-1}((1+\frac{bt}{\sigma})\mathbb{E}[B_T|\mathcal{H}_t] - \frac{1}{\sigma}\mathbb{E}[\sigma B_t + btB_T|\mathcal{H}_t]).$$

Now, using the fact that $\sigma B_t + btB_T$ is $\mathcal{H}$-adapted and the expression (26), *we obtain*

$$\begin{aligned}\mathbb{E}[\frac{B_T - B_t}{T-t}|\mathcal{H}_t] &= (T-t)^{-1}((1+\frac{bt}{\sigma})\frac{(bT+\sigma)}{(b^2T+2b\sigma)t+\sigma^2}(bB_Tt+\sigma B_t) - \frac{1}{\sigma}(\sigma B_t + btB_T)) \\ &= (T-t)^{-1}(\sigma B_t + btB_T)\frac{(\sigma+bt)(bT+\sigma) - (b^2T+2b\sigma)t - \sigma^2}{\sigma((b^2T+2b\sigma)t+\sigma^2)} \\ &= \frac{b(\sigma B_t + btB_T)}{(b^2T+2b\sigma)t+\sigma^2}.\end{aligned}$$

Concluding remarks: In this article we have studied markets where insiders are also large traders and therefore have an influence on the drift of the price dynamics. This leads naturally to the study of optimization problems in an anticipative framework. We believe that this formalism goes beyond the classical formulation of markets with insiders using initial enlargement of filtration approach. In future work, we shall consider the case when the drift is continuously influenced by the insider.

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References

[BØ] F. Biagini and B. Øksendal: A general stochastic calculus approach to insider trading. Preprint Series, Dept. of Mathematics, Univ. of Oslo, 17/2002.

[CIKN] J.M. Corcuera, P. Imkeller, A. Kohatsu-Higa and D. Nualart: Additional utility of insiders with imperfect dynamical information. *Finance and Stochastics*, 8, 437-450, 2004.

[GP] A. Grorud and M. Pontier: Asymmetrical information and incomplete markets. Information modeling in finance, Int. J. Theor. Appl. Finance 4 (2001), no. 2, 285–302.

[HØ] Y. Hu and B. Øksendal: Optimal smooth portfolio selection for an insider. Preprint Series, Dept. of Mathematics, Univ. of Oslo 12/2003.

[Im] P. Imkeller: Malliavin's calculus in insiders models: additional utility and free lunches, Math. Finance, January 2003, 1, 153–169

[KP] Karatzas, I. and Pikovsky,I. Anticipative Portfolio Optimization. Advances in Applied Probability 28 (1996), 1095-1122.

[N] D. Nualart: The Malliavin Calculus and Related Topics. Springer-Verlag 1995.

[Ø] B. Øksendal: An Introduction to Malliavin Calculus with Applications to Economics. Working Paper 3/1996, Norwegian School of Economics and Business Administration.

[ØS] B. Øksendal and A. Sulem: Partial observation in an anticipative environment. Preprint University Oslo 31/2003.

[RV1] F. Russo and P. Vallois: Forward, backward and symmetric stochastic integration. Probab. Th. Rel. Fields 97 (1993), 403–421.

[RV2] F. Russo and P. Vallois: Stochastic calculus with respect to continuous finite quadratic variation processes. Stochastics and Stochastics Reports 70 (2000), 1–40.

[RV3] F. Russo and P. Vallois: The generalized covariation process and Itô formula. Stochastic Process. Appl. 59 (1995), 81–104.

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