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Abstract: In this report, we analyze a general model of large CDMA networks (Code Division Multiple Access) and we propose a new formalism to represent the workload in wireless networks. We focus on stability issues on the downlink when users in the network want to receive some data from the base stations. We derive the stability region of this data flows system when the network is either in full macrodiversity, in soft handover or in a cellular architecture. In particular, we prove, for the model we consider, that well-designed cellular networks achieve the same stability region as networks in macrodiversity. We extend also our results to networks when slow-fading and mobility are taken into account.

Key-words: CDMA networks, macrodiversity, spatial point processes, power control, stability region.

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Stabilité des flots de données dans les réseaux CDMA en macrodiversité

Résumé : Dans ce rapport, nous analysons un modèle général de réseau CDMA(Code Division Multiple Access) de grande taille et nous proposons un nouveau formalisme pour représenter la charge dans les réseaux sans fils. Nous nous concentrons sur les problèmes de stabilité sur la voie descendante lorsque les utilisateurs dans le réseau veulent recevoir des stations de base des données. Nous obtenons la région de stabilité de ce système de flot de données lorsque le réseau est en pleine macrodiversité, en transition progressive ou encore dans une architecture cellulaire plus traditionnelle. En particulier, nous prouvons, pour le modèle considéré, qu'un réseau cellulaire optimalement conçu a la même région de stabilité qu'un réseau en macrodiversité. Nous étendons également nos résultats pour des réseaux où le fading lent et la mobilité sont pris en compte.

Mots-clés : réseaux CDMA, macrodiversité,processus ponctuels spaciaux, contrôle de puissance, région de stabilité

1 Introduction

For next generation wireless networks, the communication channel will carry voice and also data. The specifications for this two types of services differ strongly. Indeed, for voice flows, the length of the connection is the duration of the call and the stake for the system is to guarantee to each user in the network a fixed bit rate to sustain the voice signal all along the call. Whereas for data flows, the bit rate is flexible and the connection ends when the user has obtained (for downlink) or sent (for uplink) the required data. The first concern of the network is thus to work out a policy which guarantees stability. Once stability is ensured, it is then an important issue to ensure a global fairness for users. However, it is out of the scope of this paper to discuss this issue. For a high-level overview of fairness in wireless networks, see in particular Le Boudec and Radunovic [25]. In this paper, our effort is concentrated on solving the stability problem on the downlink for data flows. The interference between users is of course the main effect which has to be taken into account to make the stability analysis.

In the network we are analyzing in this paper, base stations are deployed on the whole plane. Some users arrive in the network and they want to receive some data through a CDMA communication channel. The users stay is the network until they have received all the data they wanted. The base stations have attached cells and a base station can only send a signal toward the users in its attached cell. If the cell are non-overlapping, this is a traditional *cell network* . If the cells are overlapping, some users could possibly receive their data from two or more base stations. This system is called *soft handover*. If the cell attached to a base station is the whole plane for all base stations, then there is in fact no cell structure anymore and a user can receive its data from all base stations. This corresponds to a CDMA network in full *macrodiversity*. Macrodiversity supersedes the two other network structures.

Now, we can introduce the stability issue in our network. Suppose that at time 0, there are some users with data requirements. As time grows, newcomers arrive and leave when the base stations have finished to serve them. When time tends to infinity, what happens to the mean number of active users per surface unit ? If this density tends toward infinity the system is unstable : the network does not manage to serve everybody. If this density of active users tends toward a finite value, the system is stable. It implies for example that the mean number of incoming users (per time and surface unit) is equal to the mean number of departing users (per time and surface unit) : in other words, the system has a positive rate. So without stability, the system cannot practically operate.

The remainder of this paper is organized as follows. Section 2 summarizes the results of this paper through a commonly used example of large CDMA network. In Section 3, we describe a realistic and mathematically tractable model of macrodiversity for the downlink in a static framework. By static we mean that the system is not evolving with time. Here the set of users is fixed and each user require a connection at a minimal given bit rate. The key components of our model are the spatial location of base stations and users. In Section 4, we analyze the static model and establish a necessary and sufficient condition for the feasibility of the power control problem with macrodiversity and find some sufficient conditions. These results derive from the work of Baccelli et al. in [12]. Section 5 introduces the dynamical system. By dynamic, we mean that users arrive and leave over time : the network is seen as a dynamical system. The users arrival process is described with space-time point processes. The base stations enforce adaptative policy to serve the active users in the network. The stability region of this system is established in Section 6 with minimal hypothesis. In Section 7, we interpret the stability region in terms of an optimal network design. In particular, we state that the macrodiversity model we have used does not help much to increase the stability region. In Section 8, we extend our model to random environments : namely, we include a simple model of slow fading and mobility and derive the stability region. Lastly, Section 9 contains a discussion on all the assumptions done on our model. Since, the aim of this paper is to derive the stability region of a real network, it is essential to address the validity of the hypothesis done.

In the static framework, the problem of power control and load constraints in CDMA networks has drawn much attention. Most authors are only considering CDMA networks without macrodiversity. Gilhousen et al. in [20] have derived a simple sufficient condition when the emitted powers of the base stations is given. Zander [36], [35] has proved that the necessary and sufficient condition is linked to the spectral radius of a matrix which characterizes the communication channel. This result has been extended by Baccelli et al. in [12] and [13] to large networks and they have proposed a decentralized admission protocol. In particular, the authors have introduced the geometry as the key feature of the power control feasibility.

For macrodiversity CDMA networks in the static framework, Hanly [17], [18] has solved the power control problem on the uplink. Along this line, in [6], general results for macrodiversity both on uplink and downlink are given for large networks.

For TDMA networks, stability considerations are addressed by Bonald et al. in [30], [31]. In [30], a single cell is considered, they derive the stability region and they address the problem of fairness among active users. In [31], they include to their model the mobility of users inside the cell and they prove some bounds on the total workload.

Lastly, some authors have established the stability region of neighboring systems. Ephremides and Tassiulas in a pioneering paper [33] have computed the stability region of a model a finite multi-hop radio network. As we do in this paper, they define the stability region of a queuing system as the set of parameters for which there exists a stable policy. Along this line, Bambos et al. in [2] and [4] have considered a general kind of queuing system where a server can adapt the policy over time to serve a finite number of queues. Contrary to the network we consider in our paper, there is no spatial component in both works.

2 Summary of results through an Example

In order to fix ideas and to give an outline of the work done in this paper, in this section we illustrate our results on a classical example of CDMA network. Voluntarily we do not give any precise definition here.

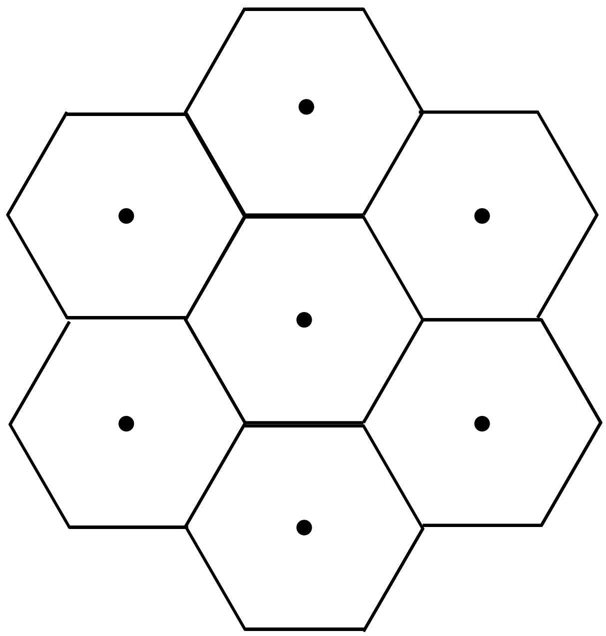

Figure 1: The regular networks with the hexagonal cells : large dots are base stations

Suppose that the base stations of our networks are located on an regular hexagonal grid. The distance between two adjacent sites is L km. A sequence of users arrives in the network. The user n arrives at time T_n, located at point X_n of the plane. We suppose that this user arrival process is a space-time Poisson point process of intensity $\lambda > 0$. λ is the spatial density of newcomers in a time unit. We suppose that each user wants to receive in mean σ bits of data from the base stations.

We assume that the available bandwidth on the channel is ΔHz and that there is no noise. The attenuation function (or path loss function) from an emitting point y to a receiving point x supposed to be equal to : $K|x-y|^{-\alpha}$, where K is a positive constant and $|\bullet|$ denotes the Euclidean distance.

The results presented in this paper prove that if :

$\lambda < \lambda_c$ *then the network is stable, if* $\lambda > \lambda_c$ *the network is unstable.*

The analytical expression of the critical density λ_c is known, using an approximation given in [21], [13] :

$$\lambda_c \approx \frac{2}{\log(2)\sqrt{3}} \frac{\Delta}{\sigma L^2} (1 + \frac{0.94}{\alpha - 2})^{-1}. \tag{1}$$

For $\Delta = 1$ MHz, $L = 1$ km, $\alpha = 4$, $\sigma = 1$ Mo, the above formula gives : $\lambda_c \approx 510$ users per square kilometer and per hour.

An other interpretation of this numerical result gives a good insight. $\lambda\sigma$ is the mean number of bits pumped per surface unit per time unit : this is the bit rate per surface unit of our network. For our numerical example, the maximal rate per surface unit is : $1,13$ bits per square kilometer and per second.

In this paper, we prove also that this critical intensity λ_c is the same for the network in macrodiversity and for the traditional hexagonal cell networks. Therefore, macrodiversity has not increased the maximal density of users the system is able to handle.

If instead of the hexagonal grid, we had supposed that the stations sites was a Poisson point process then :

$$\lambda_c = 0. \ a.s.$$

The system is never stable, whatever the density of base stations is.

3 Static Model Description

We consider a set of base stations indexed by $\mathbb{N}$ scattered on the plane. The j^{th} base station is located at Y_j in $\mathbb{R}^2$. Some users located at points $\{X_i\}_{i\in\mathbb{N}}$ are active in the network. Each user requires to receive a signal at a constant bit rate, say R_i.

In a macrodiversity network on the downlink, the base stations can cooperate and they can emit toward any user. They are jointly coding a signal for each user and users are decoding independently. This kind of channel is known as multiple input multiple output

(MIMO) broadcast channel (see [15],[1]). The capacity region of such network is unknown. However, there exists some simple lower bonds.

Let S_{ij} be the power of the signal sent by base station j dedicated to user i. The channel from Y_j to X_i is characterized by an attenuation function $L(X_i, Y_j) > 0$, a Gaussian white noise of spectral density η_i and a spectral bandwidth of Δ Hz.

Along this paper, we will suppose that for all x, j :

$$I(x) = \sum_k L(x, Y_k) < +\infty \quad \text{and} \quad \lim_{|x| \to +\infty} L(x, Y_j) = 0,$$

where $|\bullet|$ is the Euclidean norm.

Classical information theoretics results imply that there will exist a code which will guarantee a bit rate of R_i bits per second for all users if :

$$R_i \leq \Delta \log_2 \big(1 + \frac{\sum_j L(X_i, Y_j) S_{ij}}{\Delta \eta_i + \sum_j L(X_i, Y_j) \sum_{m \neq i} S_{mj}}\big). \tag{2}$$

In Equation (2), we have a classical signal to noise ratio. This bound is actually computed by assuming that the base stations are sending uncorrelated signals and for a given user, they treat what they send to all other users as noise. Indeed, it is possible to achieve better rates. In Section 9, we discuss the validity and the limits of all the assumptions and approximations which are done in our model.

In a CDMA network, the power allocation $(S_{ij})_{i,j \in \mathbb{N}}$ is dynamically computed. The power control problem can be simply stated as :

Does there exists a finite power allocation $(S_{ij})_{i,j \in \mathbb{N}}$ such that Equation (2) holds for all i ?

The answer to this question is not straightforward, in particular, it heavily depends on the geometry of the network, that is the coefficients $L(X_i, Y_j)$. In the next section we analyze this problem.

Remark 1. *In our model, we have not included any power limitation : a station can emit at any finite power. A more realistic model should include constraints such that $\sum_i S_{ij} < S_{\max}$. However, we claim that the main limitation in wireless networks comes from interference not from the background noise, thus our model fully tackles the special features of wireless communications. This issue is discussed in Section 9.*

Remark 2. *As explained in Introduction, we have described a network in full macrodiversity : a user can receive a useful signal from any base station. In Subsection 6.2 we solve the stability of cellular networks and networks in soft handover as a special case of our model.*

4 Power Allocation Algebra

Equation (2) can be conveniently rewritten as a signal to noise ratio inequality :

$$\frac{\sum_j L(X_i,Y_j)S_{ij}}{\Delta\eta_i + \sum_j L(X_i,Y_j)\sum_m S_{mj}} \geq h_i,$$

where $h_i = 1 - 2^{-\frac{R_i}{\Delta}}$. With an abuse of language, h_i will also be called bit rate requirement of user i.

The user i will get the sufficient signal to noise ratio if and only if the sum of the individual signal to noise ratios $\frac{L(X_i,Y_j)S_{ij}}{\Delta\eta_i+\sum_j L(X_i,Y_j)\sum_m S_{mj}}$ is above h_i. The macrodiversity network can thus be divided into an usual cellular network where user i requires a bit rate h_{ij} to the base station j. Formally let :

$$\mathcal{H} = \{H = (h_{ij}) \in \mathbb{R}^{\mathbb{N}\times\mathbb{N}},\, H \geq 0,\, \forall i \sum_j h_{ij} = h_i\}.$$

$\frac{h_{ij}}{h_i}$ can be understood as the proportion of user i which is attached to station j. An element H of $\mathcal{H}$ is called a *bit rate matrix*. For a given bit rate matrix H, we consider the matrix :

$$T(H) = \sum_i h_{ij} \frac{L(X_i,Y_k)}{L(X_i,Y_j)}.$$

For an infinite non-negative matrix, let $\rho(T)$ be its spectral radius (refer to [28] for details). The following results states that the answer of the power control problem is contained in the matrices $T(H)$.

Proposition 1. Let,$J = \inf_{H\in\mathcal{H}} \rho(T(H))$.
Equation (2) has a solution $J < 1$. If $J > 1$ Equation (2) has no solution.

Proof. The proof follows from the results of Baccelli and al. in [12]. For a complete proof see [6]. In the following, we give the simple main idea. First, we consider a fixed bit rate matrix H : the base station j wants to guarantee an individual signal to noise ratio of at least h_{ij} to user i. We define S_j has the total power emitted by station j : $S_j = \sum_i S_{ij}$. Let $S = (S_j)_{j\in\mathbb{N}}$ be the vector of total emitted powers, we can show, by elementary calculations that Equation (2) implies component-wise : $S \geq T(H)S + b$, where b contains the noise of the channel. This inequality is classical and the existence of a non-negative vector S relies on whether or not the spectral radius of $T(H)$ is less than one. Since all the bit rate matrices in $\mathcal{H}$ are possible we get the infimum. It remains to prove that if the inequality for the total emitted powers S has a solution, then it is possible to compute the individual powers S_{ij}. □

The spectral radius of a matrix is difficult to compute, even for finite dimensional matrices. However, it is well-known that if a matrix is sub-stochastic then its spectral radius is less than 1.

Corollary 1. Equation (2) has a finite solution if

$$\inf_{H\in\mathcal{H}} \sup_j \sum_i h_{ij} \frac{I(X_i)}{L(X_i, Y_j)} < 1, \tag{3}$$

This corollary gives an intuitive insight to the feasibility of the power control problem. Indeed, for a fixed bit rate matrix, $\sum_i h_{ij} \frac{I(X_i)}{L(X_i,Y_j)}$ has to be understood as the load of base station j. $\frac{I(X_i)}{L(X_i,Y_j)}$ is the processing cost of user i for the base station j. Equation (3) says that the load is less than 1 for all stations. For a given bit rate matrix H, each base station can check independently of the others whether or not they can solve their power control problem.

5 Dynamic Model Description

5.1 Users' Arrival Process

We now describe a model of data flows on a CDMA network in macrodiversity. As above we suppose we have a set of base stations deployed on the plane.

The users arrive in the network with a service requirement and they leave when it as been fulfilled. A given user, say n, arrives in the network, at time T_n, located in point X_n of the plane and it requires to receive a amount a bits σ_n. Our user n can receive its data from any base station in the network. The set of our user is a simple marked point process A, we denote by $A([0,t]\times B)$ the number of users arriving in the set $B \subset \mathbb{R}^2$ between times 0 and t. We suppose that :

$$E(A([0,t]\times B)) = t\int_B \lambda(dx), \tag{4}$$

where $\lambda(dx)$ is a measure finite on all bounded set (i.e. a *Radon measure*) and E is the expectation. $\lambda(dx)$ is the *spatial intensity* of our space-time point process A.

In particular, Equation (4) implies that A is time-stationary. We make however a stronger probabilistic assumption on A, namely we suppose that A is *time-ergodic*, in particular (from Birkhoff's Theorem) it implies that almost surely (a.s.) :

$$\lim_{t\to+\infty} \frac{1}{t} \sum_n \mathbb{1}(T_n \in [0,t], X_n \in B)\sigma_n = \int_B \bar{\sigma}_x \lambda(dx),$$

where $\bar{\sigma}_x$ is the mean number of bits required by a *typical user* arriving at x. $\bar{\sigma}_x$ is the mean number of bits required by a user ariving at x under the Palm distribution. In the scope of this paper, it is not necessary to be more precise on the definition of $\bar{\sigma}_x$. For example, if the bit requirement σ_n is independent of (T_n, X_n), then $\bar{\sigma}_x$ is the usual mean and does not depend on x. We assume that the bits requirements of users satisfy : $\bar{\sigma}_x < +\infty$.

For example, if the spatial intensity is λdx, then the arrival of users is also *space-stationary.* For example A could be a space-time homegeneous Poisson point process. In applications, there can also be some hot-spots where the density of users is high, a spatial intensity measures of type $\lambda(x)dx$ would describe this kind of users' patterns. We could also consider a scenario where there are some fixed entry points to other networks : in this case, $\lambda(dx)$ would have a Dirac mass at entry points.

Let T'_n be the time of departure of the user n of our arrival point process A and for t in the interval $[T_n, T'_n)$, $\sigma_n(t)$ denotes the remaining number of bits at time t the user n wants to receive : $\sigma(T_n) = \sigma_n$ and $\sigma(T'_n) = 0$.

At time t the workload in the network is a represented as a measure on $\mathbb{R}^2$ with atoms at users locations. More formally, the workload at time t, W_t, is defined as :

$$W_t = \sum_n \sigma_n(t) \mathbb{1}(t \in [T_n, T'_n)) \delta_{X_n}, \tag{5}$$

where δ_x is a unit mass at x. For a set B, $W_t(B)$ is the total number of bits required by users in B at time t.

Remark 3. *The assumptions made so far are very weak, in particular note that in a finite amount of time on all the plane, an infinite number of users can possibly arrive. Our model is thus well adapted to large wireless networks. Moreover note also that no independence is required.*

5.2 Base Station Adaptative Policy

We are now going to describe how the base stations are serving the users. In a macrodiversity network, the base stations can serve any user, wherever it is.

Let $r_n(t)$ be the bit rate achieved for user n in the system at time t. With the notation of section 4, let $h_{nj}(t)$ be the instantaneous bit rate given by station j to user n :

$$\sum_j h_{nj}(t) = 1 - 2^{-\frac{r_n(t)}{\Delta}}.$$

For consistency, we require that $h_{nj}(t)$ can be positive only when t lies in $[T_n, T'_n]$.

In CDMA networks, the signal bandwidth is large (Δ is around 1.5 MHz). In view of Equation (2), for large bandwidth the capacity of the channel becomes linear with the power of the signal. The following approximation is thus justified :

$$\begin{aligned} \tfrac{d}{dt}\sigma_n(t) \quad = -r_n(t) \quad &= \Delta \log_2(1 - \textstyle\sum_j h_{nj}(t)) \\ &\approx -\tfrac{\Delta}{\log(2)} \textstyle\sum_j h_{nj}(t) \end{aligned}$$

In our system, at any given time, the base stations can adapt the bit rates $H(t) = \{h_{nj}(t)\}$, provided that they can solve the power allocation problem. In view of Corollary 1, it is sufficient to guarantee that $T(H(t))$ is sub-stochastic :

$$\forall j, \quad \sum_n h_{nj}(t) \frac{I(X_n)}{L(X_n, Y_j)} < 1, \tag{6}$$

Since we have defined the workload as an atomic measure, along this line, we also define the policy enforced at time t by the base station j as an atomic measure with atoms at users' locations :

$$\pi_j(t) = \sum_n h_{nj}(t) \frac{I(X_n)}{L(X_n, Y_j)} \delta_{X_n}.$$

From Equation (6), the constraint on the policy is :

$$\int_{\mathbb{R}^2} \pi_j(t)(dx) < 1.$$

All the computation is unchanged if we replace the strict inequality in this last equation by a less or equal. For a subset of the plane, $\pi_j(t)(B)$ is the ratio of processing power that the base station dedicates to users in B. Note in particular that for a set B :

$$\textit{if } W_t(B) = 0 \textit{ then for all } j,\ \pi_j(t)(B) = 0, \tag{7}$$

which states that the base stations are sharing their processing power among active users. From a mathematical point of view, it asserts that the policy is absolutely continuous with respect to W.

From the discussion above, we can define a policy as :

Definition 1. A *policy* is a mapping $t \mapsto (\pi_j(t))_{j \in \mathbb{N}}$ such that for all j and t, $\pi_j(t)$ is a measure of total mass less or equal than one which satisfies Condition (7).

Here is a natural example of a policy :

$$\pi_j^+(t) = \begin{cases} 0 & \text{if} \quad W_t \text{ is the measure zero} \\ \delta_{x_j^+} & \text{otherwise .} \end{cases}$$

where, $x_j^+ = \arg\max\left\{x : \frac{I(x)}{L(x,Y_j)} 1(W_t(\{x\}) > 0)\right\}$. If multiple choices of x are possible, choose the first in the lexicographic order.

With this policy, the base station serves the user with the best channel first. Note in particular that this policy is *work conserving* : if W_t is not empty, the station is active.

For a given policy, we can then rewrite the Workload Equation (5) in a bounded set B, for $t, h \in \mathbb{R}_+$:

$$
\begin{aligned}
W_{t+h}(B) \quad &= W_t(B) + \sum_n \sigma_n 1(T_n \in [t, t+h), X_n \in B) \\
&- \frac{\Delta}{\log(2)} \sum_j \int_t^{t+h} \int_B \frac{L(x, Y_j)}{I(x)} \pi_j(s)(dx)ds,
\end{aligned} \tag{8}
$$

In words : the total number of bits at time $t+h$ brought by users in the set B is equal to the total number of bits which was brought at time t plus the bits brought by the newcomers in set B between t and $t+h$ and minus all the work done by the base stations in this set during this time.

6 Stability Region

6.1 Main Result

In this section, we answer the following question :

Does there exist a policy scheme which leads to a stable queuing system ?

First of all, we must make clear what kind of stability we are looking for :

Definition 2. A policy π is *stable* if there exists a finite stationary workload $\{Z_t\}$, $t \in \mathbb{R}$, satisfying Equation (8). The system is *stable* if there exists a stable policy scheme.

As discussed in Introduction, stability is based on the stabilization of the workload W_t as time tends toward $+\infty$ when the initial workload at time 0 was a measure m. In this section, we prove the existence of a policy scheme which is stable in the sense of definition 2 under some assumptions. If this policy is enforced, we can prove that for any initial workload and any bounded set B, $W_t(B)$ couples with $Z_t(B)$ the stationary solution. In particular, it implies for any real a : $\lim_{t \to +\infty} P(W_t(B) < a) = P(Z_0(B) < a)$. The link with stability is then clear.

We want to find the minimal assumption under which there exists a stable policy. To this end, we define :

$$
\mathcal{F} = \Big\{ \quad f = (x \mapsto f_j(x))_{j \in \mathbb{N}} \text{ such that, for all } j, \\
f_j \text{ is non-negative and } \lambda(dx)\text{-a.e. } \sum_j f_j(x) = 1 \Big\}.
$$

An f in $\mathcal{F}$ can be interpreted as a spatial allocation : for a given policy scheme, $f_j(x)$ is the ratio of bits which are sent by the base station j for a typical user arriving in x.

The parameter of our queuing system is the arrival marked point process A. Let $\mathcal{N}$ be the set of possible point processes A and :

$$
\mathcal{N}^s = \Big\{ A \in \mathcal{N} : \quad \exists f \in \mathcal{F} \text{ such that : } \forall j, \\
\int_{\mathbb{R}^2} \frac{I(x)\bar{\sigma}_x f_j(x)}{L(x, Y_j)} \lambda(dx) < \frac{\Delta}{\log(2)} \Big\},
$$

$$\bar{\mathcal{N}}^s = \Big\{ A \in \mathcal{N} : \quad \exists f \in \mathcal{F} \text{ such that} : \forall j, \\ \int_{\mathbb{R}^2} \frac{I(x)\bar{\sigma}_x f_j(x)}{L(x,Y_j)} \lambda(dx) \leq \frac{\Delta}{\log(2)} \Big\},$$

Note that $\int_{\mathbb{R}^2} \frac{\bar{\sigma}_x f_j(x) I(x)}{L(x,Y_j)} \lambda(dx)$ is a traffic load : $\bar{\sigma}_x \lambda(dx)$ is the mean number of bits per unit of surface and $\frac{L(x,Y_j)}{I(x)}$ is the processing rate at x for station j. We can now state the stability region of our data flow system.

Theorem 1. For the CDMA system in macrodiversity,

- if $A \in \mathcal{N}^s$, then there exists a stable policy,
- if there is a stable policy then $A \in \bar{\mathcal{N}}^s$.

The proof of this result can be found in [7]. A sketch of proof is done in Appendix. The second statement comes from a conservation equation. To prove the first statement, we build a policy which is stable. The policy π^+ defined in the previous section can be unstable when A is in $\mathcal{N}^s$.

Note that the stability region depends on the distribution of the point process A only through its means. The conclusions of Theorem 1 are not surprising. To fix ideas, suppose that there is only one base station and that all users arrive at the same place, say 0. Then the theorem says that the stability region is given by $\bar{\sigma}_0 \lambda(0) < \frac{\Delta}{\log(2)}$. This result is the usual condition $\rho < 1$ for G/G/1 queues.

In the proof of the theorem, we establish that to a given stable policy π corresponds a set of functions $(f_j)_{j \in \mathbb{N}}$ in $\mathcal{F}$ such that :

$$\int_{\mathbb{R}^2} \frac{I(x)\bar{\sigma}_x f_j(x)}{L(x,Y_j)} \lambda(dx) \leq \frac{\Delta}{\log(2)}. \tag{9}$$

As explained above, $f_j(x)$ is for the policy π the proportion of service carried by station j for users in x in the stationary regime.

And conversely for a set of functions $(f_j)_{j \in \mathbb{N}}$ in $\mathcal{F}$ such that Equation (9) is satisfied (with a strict inequality), there exists a stable policy. In fact, this last assertion is only true for a dense subset of functions of $\mathcal{F}$ (refer to [7]).

For example, suppose once more that all users arrive at 0, and suppose that the policy enforced by the stations is work-conserving. Then, $f_j(0) = \frac{L(0,Y_j)}{I(0)}$, the proportion of data received at 0 sent by base station j is proportional to the attenuation function at 0.

6.2 Soft Handover

From the discussion above on the mapping from an f in $\mathcal{F}$ to a stable policy scheme, we can extend easily our stability result to a more restrictive class of macrodiversity.

We suppose now that a cell $\{V_j\}_{j\in\mathbb{N}}$ is attached to each base station. The base station j can only sent some data to users located in the cell V_j. We suppose that the cells V_j cover the plane. If the sets V_j are not overlapping then we have a usual *cellular network*, if the cells are overlapping, we have a network in *soft handover*.

In such a network, the class of available policies is more restrictive, in our measure formalism, this limitation can be conveniently written as :

$$B \cap V_j = \emptyset \text{ implies for all } t \,:\, \pi_j(t)(B) = 0.$$

If *supp* stands for the support of a function, the corresponding $(f_j)_{j\in\mathbb{N}}$ will satisfy : $supp(f_j) \subset V_j$.

To extend Theorem 1 to the soft handover case, we set :

$$\mathcal{F}_{soft} = \Big\{ \begin{array}{l} f = (x \mapsto f_j(x))_{j\in\mathbb{N}} \text{ such that,} \\ \forall j,\ f_j \text{ is non-negative, } supp(f_j) \subset V_j \text{ and} \\ \lambda(dx)\text{-a.e. } \sum_j f_j(x) = 1 \Big\}. \end{array}$$

We define similarly $\mathcal{N}^s_{soft}$ and $\bar{\mathcal{N}}^s_{soft}$:

$$\mathcal{N}^s_{soft} = \{A \in \mathcal{N} : \begin{array}{l} \exists f \in \mathcal{F}_{soft} \text{ such that : } \forall j, \\ \int_{\mathbb{R}^2} \frac{I(x)\bar{\sigma}_x f_j(x)}{L(x,Y_j)} \lambda(dx) < \frac{\Delta}{\log(2)} \}, \end{array}$$

We have the following stability result :

Theorem 2. For the CDMA system with soft handover,

- if $A \in \mathcal{N}^s_{soft}$, then there exists a stable policy,
- if there is a stable policy then $A \in \bar{\mathcal{N}}^s_{soft}$.

As explained, the proof is similar to the proof of Theorem 1. For cellular networks, the forthcoming Theorem 3 will give a stronger statement on stability.

7 Spatial Network Optimization

7.1 Network Workload

In this section, we make some additional assumptions on our model :

- The arrival intensity can be written as $\lambda(dx) = \lambda(x)dx$.
- The attenuation function is radial and positive : $L(x, Y_j) = l(|x - Y_j|)$, where $l(r) > 0$ represents the attenuation at distance r.

In most applications, this attenuation function satisfies : $l(r) \sim r^{-\alpha}$, for an $\alpha > 2$.

We have seen in Theorem 1 that the stability of the CDMA network relies on the value of :

$$\rho = \inf_{f \in \mathcal{F}} \sup_{j \in \mathbb{N}} \int_{\mathbb{R}^2} \frac{\bar{\sigma}_x f_j(x) I(x)}{l(|x - Y_j|)} \lambda(x) dx. \tag{10}$$

If $\rho < \frac{\Delta}{log(2)}$, the system is stable, if $\rho > \frac{\Delta}{log(2)}$, the system is unstable. In this section, we analyse this optimization problem. To this end, we define :

$$\rho_j(f) = \int_{\mathbb{R}^2} \frac{\bar{\sigma}_x f_j(x) I(x)}{l(|x - Y_j|)} \lambda(x) dx \text{ and } \rho(f) = \sup_j \rho_j(f).$$

$\mathcal{F}$ is a convex closed set and $f \to \rho(f)$ is a convex function, thus the minimum of Equation (10) is reached. In this section, we take interest to the optimal subset of $\mathcal{F}$ defined as :

$$\mathcal{F}^* = \{f \in \mathcal{F} : \rho(f) = \rho\}.$$

The extremal points of the convex set $\mathcal{F}$ are the measurable functions such that $f_j(x) = \mathbb{1}(x \in V_j)$, for a Borel set V_j. This class of function is called *tessellation.* A tessellation is a partition of the plane : each point $x \in \mathbb{R}^2$ is affiliated to a unique base station j.

We know that there is a mapping from an $(f_j)_{j \in \mathbb{N}}$ to a stable policy π. Therefore, in Equation (10) we are looking for a policy scheme which maximizes the intensity of bits requirements. The policy scheme which corresponds to a tessellation is a cellular type policy : a user receives a usable signal only from one base station.

Proposition 2. If ρ is finite there is a f such that :

$$\forall j, \quad \rho_j(f) = \int_{\mathbb{R}^2} \frac{\bar{\sigma}_x f_j(x) I(x)}{l(|x - Y_j|)} \lambda(x) dx = \rho.$$

If there is a finite number of base station, all $f \in \mathcal{F}^*$ satisfy the above equation.

Proof. Let $f \in \mathcal{F}$ and suppose for example, $\rho_1(f) < \rho_2(f)$, since, $\rho_2(f) > 0$, f_2 is not a.e. equal to 0. Thus, there exists a measurable non negative function $x \mapsto \epsilon(x)$ such that $f_2^\epsilon(x) = f_2(x) - \epsilon(x) \geq 0$, $f_1^\epsilon(x) = f_1(x) + \epsilon(x) \leq 1$ and $\rho_2(\epsilon) > 0$. Let $f_j^\epsilon(x) = f_j(x)$, for $j \notin \{1, 2\}$. $f^\epsilon \in \mathcal{F}$ and we have $\rho_1(f^\epsilon) = \rho_1(f) + \rho_1(\epsilon)$ and $\rho_2(f^\epsilon) = \rho_2(f) - \rho_2(\epsilon)$. Thus for ϵ small enough, $\sup_{j \in \{1,2\}} \rho_j(f^\epsilon) < \sup_{j \in \{1,2\}} \rho_j(f)$ and $\rho(f^\epsilon) \leq \rho(f)$.

Suppose now that $f \in \mathcal{F}^*$, then f^ϵ is also in $\mathcal{F}^*$. By iterating the construction above for all j, j', such that $\rho_{j'}(f) < \rho_j(f)$, the proposition follows. □

Proposition 2 as an intuitive meaning : for an optimal spatial allocation, the traffic load which is the same on each base station. Along the same line, we can prove a more surprising result :

Proposition 3. Suppose ρ is finite. If for all $j \neq k$, $Y_j \neq Y_k$ and $r \mapsto l(r)$ is a convex and decreasing function, then there is an $f \in \mathcal{F}^*$ which is a tessellation.

If there is finite number of base stations, all $f \in \mathcal{F}^*$ are tessellations.

This proposition gives a counter-intuitive result : we prove that the stability region of fixed cell networks is identical to the stability region of networks in macrodiversity. In terms of stability, macrodiversity does not help. This result is similar to the negative result presented in [6] on the feasibility of the power control problem on the downlink. However, note that all the difficulty is to find the optimal tessellation which reaches the optimum. This result is not very surprising from the point of view of convex optimization : it only asserts that the extremum is reached at an extremal point.

Proof. The proof is technically simple but uses some classical results of integration theory. We consider the $f \in \mathcal{F}^*$ given by proposition 2. Let $E = f_1(]0,1[)^{-1} \cap f_2(]0,1[)^{-1}$. In this proof, μ will denote the Lebesgue measure. We want to show that $\mu(E) = 0$.

Suppose instead that $\mu(E) > 0$, we can suppose without loss of generality that $\mu(E) < +\infty$. Let A, B be disjoint compact sets of positive Lebesgue measure included in E, these sets exist in view of Theorem 2.14 in Rudin [26] (Riesz Representation Theorem). We consider the mapping $\phi(x) = \mathbb{1}(x \in A) - \nu \mathbb{1}(x \in B)$, $\nu > 0$.

Let $f_1^\epsilon(x) = f_1(x) + \epsilon\phi(x)$, $f_2^\epsilon(x) = f_2(x) - \epsilon\phi(x)$ and $f_i^\epsilon(x) = f_i(x)$ for $i \notin \{1,2\}$. If $\epsilon > 0$ is small enough, f^ϵ and $f^{-\epsilon}$ are in $\mathcal{F}$ and for $i \in \{1,2\}$:

$$\rho_i(f^{\pm\epsilon}) = \rho_i(f) \pm \epsilon\rho_i(\phi) = \rho \pm \epsilon\rho_i(\mathbb{1}_A) \mp \nu\epsilon\rho_i(\mathbb{1}_B).$$

Since $f \in \mathcal{F}^*$, $\max(\rho_1(f^{\pm\epsilon}), \rho_2(f^{\pm\epsilon})) \geq \rho$ and we deduce that $sign(\rho_1(\phi)) = sign(\rho_2(\phi))$, where $sign$ is the sign function ($sign(0) = 0$, and for $x \neq 0$, $sign(x) = \frac{x}{|x|}$). It follows that for all real ν, $\rho_1(\mathbb{1}_A) - \nu\rho_1(\mathbb{1}_B)$ and $\rho_2(\mathbb{1}_A) - \nu\rho_2(\mathbb{1}_B)$ have the same sign. Therefore the vector $(\rho_1(\mathbb{1}_A), \rho_1(\mathbb{1}_B))$ and $(\rho_2(\mathbb{1}_A), \rho_2(\mathbb{1}_B))$ are colinear : exists $C_{A,B}$ such that : $\rho_1(\mathbb{1}_A) = C_{A,B}\rho_2(\mathbb{1}_A)$: $C_{A,B}$ cannot depend on B and by symmetry does not depend neither on A. Thus, exists $C > 0$ such that :

$$\rho_1(\mathbb{1}_A) = C\rho_2(\mathbb{1}_A).$$

This last equality has been proved for any compact set included in E. From Theorem 2.14 in [26], it can be extended to any Borel set included in E. Thus, for all A included E, such that $\mu(A) > 0$, $\frac{1}{\mu(A)} \int_A (\frac{\bar{\sigma}_x I(x)}{l(|x-Y_1|)}\lambda(x) - C\frac{\bar{\sigma}_x I(x)}{l(|x-Y_2|)}\lambda(x))dx = 0$. We can apply Theorem 1.40 of [26] and conclude that a.e. :

$$Cl(|x - Y_1|) = l(|x - Y_2|).$$

This contradicts our hypothesis on $l(r)$. Therefore $\mu(E) = 0$. and we have proved that there exists an f in $\mathcal{F}^*$ such that a.e. $f_j = \mathbb{1}(x \in V_j)$ and $V_j \cap V_k$ is a set of measure 0. Since for two functions equal a.e., $\rho_j(f)$ has the same value, $(\mathbb{1}(V_j))_{j\in\mathbb{N}}$ is a tessellation in $\mathcal{F}^*$. □

7.2 Cellular Networks

We have seen in Proposition 3, that the cellular policies reaches the stability region under mild assumptions. In this subsection, we focus on this subclass of policies.

Let V_j be the cell attached to the base station j. For $j \neq k$, $V_j \cap V_k = \emptyset$ and $\cup_j V_j = \mathbb{R}^2 \setminus \mathcal{O}$, where $\mathcal{O}$ is a set of measure 0. In a cellular network, the base stations are not sharing the users, they are divided among them. We say that a cellular policy is *work-conserving* if $W_t(V_j) > 0$ implies $\pi_j(t)(V_j) = 1$.

Theorem 3. For a fixed cell network with cells $\{V_j\}, j \in \mathbb{N}$, governed by Equation (8) with bounded cells, *any work conserving policy* is stable if :

$$\forall j, \quad \int_{V_j} \frac{\bar{\sigma}_x I(x)}{L(x, Y_j)} \lambda(dx) < \frac{\Delta}{\log(2)}.$$

If there is a j such that : $\int_{V_j} \frac{\bar{\sigma}_x I(x)}{L(x,Y_j)} \lambda(dx) > \frac{\Delta}{\log(2)}$ then *any cellular policy* is unstable.

Proof. We only give an idea of the proof, for a complete proof see [7]. The second part of the theorem follows from Theorem 2. To prove the first assertion is enough to prove that the following policy is stable :

$$\pi_j^-(t) = \begin{cases} \delta_{x_j^-} & \text{if} \quad W_t(V_j) > 0 \\ 0 & \text{if} \quad W_t(V_j) = 0. \end{cases}$$

with $x_j^- = \arg\min\{x : \frac{I(x)}{L(x,Y_j)} 1(x \in V_j) 1(W_t(\{x\}) > 0)\}$. If multiple choices of x are possible, choose the first in the lexicographic order.

π^- is the "slowest" policy : if π is a work-conserving cellular policy, we have $W_t^\pi(V_j) \geq W_t^{\pi^-}(V_j)$, where $W_t^\pi(V_j)$ is the total workload at time t in the cell V_j for a given initial workload condition at time 0 when the policy π is enforced. Therefore, one can show that if π^- is stable then any other work-conserving policy is stable. □

This theorem is along the line of the result on single server queue which asserts that the stability does not depend on the discipline, provided it is work-conserving.

7.3 Homogeneous Networks

7.3.1 Spatially Ergodic Network

From now on, we have only assumed a time-stationarity of our system. In this subsection, we take interest to space-time stationarity networks.

Now, the arrival point process A is supposed to be stationary in time and space. The intensity of A is denoted by λ. The bits of data requirements of users are supposed to be identically distributed, independent and independent of A, in this setting : $\bar{\sigma}_x = E(\sigma) = \bar{\sigma}$. We assume that the attenuation function is radial and positive as in Subsection 7.1. $rl(r)$ is supposed to be $L^1(\mathbb{R}_+)$.

We have never specified what is the geometry of the network, that is the base stations positions. In this subsection we suppose that the point pattern $\{Y_j\}_{j\in\mathbb{N}}$ is a realization of an ergodic point process on the plane $\mathbb{R}^2$ of intensity $\nu > 0$. From Campbell Formula, we have : $E(I(x)) = \nu \int_{\mathbb{R}_+} rl(r)dr < +\infty$, (to simplify notations, E denotes also the expectation with respect to the point process of base stations).

The stability of the system still relies on the optimization problem given by Equation (10).

Lemma 1. ρ is a.s. constant. Thus, provided $\rho \neq \frac{\Delta}{\log(2)}$ a.s., the system is stable or instable a.s..

Proof. $(f_j)_{j\in\mathbb{N}} \mapsto (f_j(\bullet - y))_{j\in\mathbb{N}}$ is a bijection on $\mathcal{F}$. It follows that for all y, ρ is invariant under translations by y. Thus, for all $a \geq 0$, from ergodicity, $P(\rho > a) \in \{0, 1\}$. □

It is important to notice, that infinity is a possible constant value for ρ.

7.3.2 Periodic Network

We suppose that the set of base stations is located on an regular hexagonal grid of radius R. We index our base station by $\mathbb{Z}^2$ and with a complex representation of $\mathbb{R}^2$, the base station (p, q) is located at $Y_{p,q} = R(p + qe^{i\frac{\pi}{3}})$. Let $\{V_j\}, j \in \mathbb{Z}^2$, be the Voronoi Tessellation of the Hexagonal Network (that is, $x \in V_j$ if for all $j' \neq j$, $|x - Y_j| < |x - Y_{j'}|$).

Proposition 4.

$$\rho = \lambda\bar{\sigma} \int_{V_{0,0}} \frac{I(x)}{l(|x|)} dx.$$

A simple argument on the symmetry of the hexagonal grid leads to the proof.

Proposition 4 implies that the Voronoi cellular network is optimal for the hexagonal grid.

To retrieve Formula (1) given in Section 2, we have used an analytical approximation used in [13] of $\int_{V_{0,0}} \frac{I(x)}{l(|x|)} dx$.

7.3.3 Poisson Network

The hexagonal grid is the most regular point pattern we can imagine. On the other side, it is interesting to analyze what do we get when the base stations' point pattern is much more irregular. To this end, we suppose now that the base stations are located according to a realization of a Poisson Process of finite intensity $\nu > 0$.

Proposition 5. Suppose $\limsup_{r\to+\infty} \frac{l(nr)}{l(r)} > 0$ for $n \in \mathbb{N}$, l is non-increasing and $r^2 l(r) \in L^1(\mathbb{R}_+)$. If $\{Y_j\}_{j\in\mathbb{N}}$ is an homogeneous Poisson point process of finite intensity $\nu > 0$ then, a.s., $\rho = +\infty$.

Thus, in the homogeneous Poisson case, the network cannot be stable. Note that if $l(r) \sim r^{-\alpha}$, $\alpha > 3$, the assumption of the proposition holds. Whatever the intensity of the base stations is, a local behavior of the Poisson point pattern will lead to a global instability. This negative result is similar to the results in the static case given in [12], [6].

The proof is done in the Appendix.

8 Slow Fading and Mobility

8.1 General Setting

We now extend our stability result to a more complex model. We suppose that the attenuation depends on the time. Namely we suppose that at time t, the attenuation function is equal to :

$$L(x, Y_j; t).$$

The shot-noise I depends on the time and is then equal to $I(x;t) = \sum_k L(x, Y_k; t)$. Let $\mu_j(x;t) = \frac{L(x,Y_j;t)}{I(x;t)}$ be the processing rate at time t for the base station j. For technical reasons, we require that for all j, the mappings $(\mu_j(\bullet;t))_{t\in\mathbb{R}}$ are taking value into a *countable set* of processing rates mappings : $(\mu_j^n)_{n\in\mathbb{N}}$.

We define $Z_j(t) \in \mathbb{N}$ as the random variable which drives the state of the processing rate : $\mu_j(x;t) = \mu_j^{Z_j(t)}(x)$. We assume that the random variables $Z_j(t)$ are *time-ergodic*. Let

$$p_j^n = P(\mu_j(\bullet;t) = \mu_j^n) = P(Z_j(t) = n).$$

Following the results of Subsection 6.1, we define the following sets in our time varying setting :

$$\mathcal{F}_{time} = \Big\{ \quad f = (x \mapsto f_{j,n}(x))_{j\in\mathbb{N}} \text{ such that, } \forall j,n,\ f_{j,n} \text{ is non-negative and } \lambda(dx)\text{-a.e.} \sum_{j,n} f_{j,n}(x) = 1 \Big\}.$$

$$\mathcal{N}^s_{time} = \Big\{ A \in \mathcal{N} : \quad \exists f \in \mathcal{F} \text{ such that} : \forall j, n, \\ \int_{\mathbb{R}^2} \frac{\bar{\sigma}_x f_j(x)}{\mu_j^n(x)} \lambda(dx) < p_j^n \frac{\Delta}{\log(2)} \Big\},$$

and $\bar{\mathcal{N}}^s_{time}$ is defined similarly with $\leq$ instead of $<$.

The interpretation follows : $f_{j,n}(x)$ is the proportion of service which is performed by the base station j for a typical user arriving in x when the station is in state n. We can now state the natural extension of Theorem 1.

Theorem 4. For the time varying system,

- if $A \in \mathcal{N}^s_{time}$, then there exists a stable policy,
- if there is a stable policy then $A \in \bar{\mathcal{N}}^s_{time}$.

This theorem is appealing since it gives a simple characterization of the stability region. For example, with the proper hypothesis, the results of Section 7.

8.2 Slow Fading

There are two types of fading in wireless communications, slow and fast fading. Fast fading is caused by multi-path propagation of the signal from a base station to a user. In a CDMA setting, fast fading is not a relevant feature. Fast fading is relevant for TDMA schemes and can be used in opportunistic scheduling. On the contrary, slow fading has an impact in CDMA networks. Slow fading or shadowing is due to the random environment between base stations and users, for examples moving obstacles on the propagation path. Fading can be characterized by a collection of random field processes of mean one :

$$G_j(t,x) \in \mathbb{R}_+.$$

The attenuation function is then taken to be equal to :

$$L(x, Y_j; t) = G_j(t,x) l(|x - Y_j|),$$

where $l(r)$ is the usual radial attenuation function used in the simple model.

For slow fading, $(G_j(\bullet; t))_{j \in \mathbb{N}}$ is well approximated by independent log-normal fields. A model for its correlation function is given by Gudmundson in [16].

8.3 Mobility

Our general setting can also be used to include mobility. Consider a random field process with value on $\mathbb{R}^2$:

$$X(x,t) \in \mathbb{R}^2.$$

$X(x,t)$ receives the following interpretation : $X(x,t)$ is the position at time t of all users which arrived in x at any time. In particular the user (T_n, X_n) is located in $X(t, X_n)$ at time t. Note then that X_n is not anymore the position of the user n. This model is not fully realistic but leads to a computable stability region.

The attenuation function is then taken to be equal to :

$$L(x, Y_j; t) = l(|X(x,t) - Y_j|).$$

As a simple example, $X(x,t)$ could take a few distinct value around x.

9 Discussion

We discuss here the various hypothesis that have been done on all along the paper the model. It is important to discuss each of them to understand the scope of our results.

9.1 Channel Capacity : Equation (2)

As already stressed, Equation (2) is only a sufficient condition for the capacity of the network. The computation of the information capacity of this type of network is an open problem. Some advanced coding schemes are doing better that the bound we have used. On an application layer, however we claim that our bound is fairly realistic. To get our bound, we have supposed that a base station is coding independently the signals they dedicate to each active user. In particular, a base station treats all its users in the same way and does not have to adapt the code when new users arrive or when the environment changes. If it would be possible to overcome this practical limitation, higher rates could be achieved. For example, with the dirty paper coding (see Costa [9]), for a given indexation of the users, the following bit rate is achievable :

$$R_i \leq \Delta \log_2 \big(1 + \frac{\sum_j L(X_i, Y_j) S_{ij}}{\Delta \eta_i + \sum_j L(X_i, Y_j) \sum_{m>i} S_{mj}}\big). \tag{11}$$

The second assumption we have done to get the Equation (2) is that the base stations are sending uncorrelated signals. This is a natural assumption in a CDMA framework with the pseudo-orthogonal signatures and we think few can be said on this assumption.

At last, we have also derived in this paper some results on cellular networks. It is important to notice that if only one base station is sending a valuable signal toward each user, the bound given by Equation (11) is the exact information capacity.

9.2 Power Control Feasibility : Corollary 1

The second approximation we have done is linked to the feasibility of the power control. We have assumed that the power control was feasible when the condition given in Corollary 1 is fulfilled. This last condition is only sufficient, the necessary and sufficient condition is given by Proposition 1 and involves the spectral radius of the matrix $T(H)$. The sufficient condition is the sub-stochasticity of $T(H)$. Investigate the gap between the two conditions is a difficult problem. However, some qualitative and numerical comments can be done.

As it is pointed in paragraph VI-A of [12], if we assume that the model is perfectly periodic, namely if the network is hexagonal and there is the same distribution of users inside each hexagonal cell, then the sufficient condition given by Corollary 1 is also necessary. Therefore, if the network is hexagonal and if the active users point pattern is well balanced, the sufficient condition should not be to far from being necessary. Ideally, this qualitative statement should of course be made more precise.

For voice signals in cellular networks, some numerical validation are currently being done by France Telecom, refer to the forthcoming paper, [14]. Preliminary results tend to show that the probability of rejection of a user as it is analytically computed in [13] using the sub-stochasticity of T and the real probability of rejection differ from less that 10%. Therefore, from an engineering point of view, we claim that this sufficient condition sticks well to reality.

9.3 Signal to Noise Ratio

Two numerical approximations related to the signal to noise ratio have be done.

- *Maximal power limitation :* we have not supposed that the total emitted power of base stations was bounded by a maximal value S_{max}. Proposition 1 gives only the existence of a finite collection of emitted powers. First a simple comment : suppose that the noise in the network is zero. Then there is a scale invariance of the power control feasibility : if a collection of emitted powers $(S_{ij})_{i,j\in\mathbb{N}}$ is a solution of the power control problem then for any $\lambda > 0$, $(\lambda S_{ij})_{i,j\in\mathbb{N}}$ is trivially also a solution. Therefore setting a maximal value is a constraint which relies on the noise in the network, not from the interference between users.

  In the hexagonal network, numerical results given in [13] (fig. 3) show clearly that if the radius of the cells is less than 2 km, then the maximal power limitation is not limitative. Thus we claim that this maximal power limitation is almost irrelevant in a urban environment. Fortunately, this is precisely when the density of active user is high that the stability issue is critical.

- *Order of magnitude of the signal to noise ratio :* in Subsection 5.2 we have made the linearization :

$$\Delta \log_2 \big(1 + \frac{\sum_j L(X_i,Y_j)S_{ij}}{\Delta\eta_i + \sum_j L(X_i,Y_j)\sum_{m\neq i} S_{mj}}\big) \approx \frac{\Delta}{\log(2)}\frac{\sum_j L(X_i,Y_j)S_{ij}}{\Delta\eta_i + \sum_j L(X_i,Y_j)\sum_{m\neq i} S_{mj}}.$$

It is simple at least for voice signals to compute the error made. A signal to noise ratio of -16dB is required by each user in a cell. With this value, the error made by the above approximation is 1.25%, this is reasonable.

10 Concluding Remarks

In this paper, we have fully characterized the stability region of a data flow wireless network on the downlink in three scenario : cellular architecture, soft handover and macrodiversity. Our model is suited to large networks since, we have considered a network on all the plane with an infinite number of users. Our model is also robust, independence nor space homogeneity are required. Some prospective lines of research have appeared.

It would be interesting to derive the stability region when sophisticated coding scheme are used. In particular, the stability given by Equation (11) is a promising extension of our work. If the stability region is much larger, it would be essential to implement these codes for data flows.

We have proved that under mild assumptions, there is an optimal policy which is a traditional cellular policy. The corresponding optimal tessellation is unfortunately unknown. In the simplest case, namely for the hexagonal grid, we have proved that this optimal tessellation was the Voronoi tessellation. It would be very appealing to find an algorithm which builds this optimal tessellation for more complex base stations patterns.

At last, in this paper we have defined the workload in a spatial network as a measure with atoms. This representation has appeared to be convenient and it could be used for any type of wireless network, such as ad-hoc networks.

Appendix : Sketch of proof of Theorem 1

10.1 if there is a stable policy then $A \in \bar{\mathcal{N}}^s$

Suppose the policy scheme π is stable and let W_t be the stationary workload associated to policy π. For a bounded set B, we can then define $\bar{\pi}_j(B) = E(\pi_j(t)(B))$. Using the ergodicity of our system and the Campbell Formula, we prove by taking expectation in Equation (8) that the following conservation equation holds :

$$\int_B \bar{\sigma}_x \lambda(dx) = \sum_j \int_B \mu_j(x) \bar{\pi}_j(dx).$$

Let $\tilde{\pi}_j(x)$ as the Radon-Nicodym derivative of $\bar{\pi}_j(dx)$ with respect to $\lambda(dx)$. We define $f_j(x) = \frac{L(x,Y_j)\tilde{\pi}_j(x)}{I(x)\bar{\sigma}_x}$.

It appears easily that $f_j(x)$ has all the required properties : $\lambda(dx)$-a.e. $\sum_j f_j(x) = 1$ and $\int_{\mathbb{R}^2} \frac{I(x)\bar{\sigma}_x f_j(x)}{L(x,Y_j)} \lambda(dx) \leq 1$.

10.2 if $A \in \mathcal{N}^s$, then there exists a stable policy

We only give the definition of the policy which is stable when $A \in \mathcal{N}^s$. For a given policy, $t \in \mathbb{R}_+$ and a bounded set B, we define $W_t^j(B)$ as the total number of bits sent by the server station j toward users located in x between time 0 and t. Formally, W_t^j is a measure defined by : $W_t^j(B) = \int_0^t \int_B \frac{I(x)}{L(x,Y_j)} \pi_j(s)(dx)ds$.

The admissible set for the base station j is defined as :

$$\mathcal{A}_j(t) = \{x : W_t^j(\{x\}) \leq f_j(x) \sum_n \sigma_n 1(T_n \in [0,t), X_n = x)\}.$$

$x \in \mathcal{A}_j(t)$ means that the ratio of work performed by base station j at x is less than $f_j(x)$. We define the following policy :

$$\pi_j(t) = \begin{cases} \delta_{x_j^*} & \text{if} \quad \mathcal{A}_j(t) \neq \emptyset \\ 0 & \text{if} \quad \mathcal{A}_j(t) = \emptyset. \end{cases}$$

where,$x_j^* = \arg\max\{x : \frac{I(x)}{L(x,Y_j)} 1(x \in \mathcal{A}_j(t), W_t(\{x\}) > 0)\}$, if multiple choices of x are possible, choose the first in the lexicographic order.

This policy scheme divides the total workload among the base station in proportion $(f_j)_{j\in\mathbb{N}}$. Under some hypothesis on $(f_j)_{j\in\mathbb{N}}$ which can be done without loss of generality, this policy is stable as soon as $A \in \mathcal{N}^s$.

Remark 4. *From an engineering point of view, this policy is not satisfactory : the policy depends on the parameters f_j which is a priori unknown. Moreover, through $\mathcal{A}_j(t)$ the policy depends on the past of the system and this is not desirable to implement a protocol. Thus, Theorem 1 has to be seen as an existence theorem, it is not constructive. On the other hand, Theorem 3 gives concrete policy schemes which lead to practical protocols.*

Appendix : Proof of Proposition 5

Lemma 2. Let N be a Poisson random variable of parameter $\nu > 0$. Let $\theta > e$, then for ν large enough : $P(N > \theta\nu) > 1 - \frac{e^{-\nu(\theta \ln \theta - \theta + 1)}}{1 - e\theta^{-1}}$.

Proof. The proof relies on Stirling Formula. □

We now turn to the proof of Proposition 5 and suppose ρ is finite.

Let $B(0,R)$ denotes the open ball of radius R centered on the origin, we define : $\mathcal{C}_n = B(0,(n+1)R)\backslash B(0,nR)$. The surface of $\mathcal{C}_n$ is denoted by $|\mathcal{C}_n| = (2n+1)\pi R^2$. Let $\theta > e$ and $S_R = \theta\rho \sum_{n\geq 4} |\mathcal{C}_n| \frac{l((n-1)R)}{l(4R)} < \infty$.

For a Poisson point process Φ, the event :

$$\begin{array}{rcl} A_R & = & \{\Phi(B(0,2R))=0\}\cap\{\Phi(\mathcal{C}_2)>2S_R\} \\ & & \cap_{n>2}\{\Phi(\mathcal{C}_n)\leq\theta|\mathcal{C}_n|\nu\} \end{array}$$

has positive probability in view of Lemma 2. Indeed, since the sets $\mathcal{C}_n$ are disjoint we have : $P(\cap_n \Phi(\mathcal{C}_n)\leq\theta|\mathcal{C}_n|\nu)=\prod_n P(\Phi(\mathcal{C}_n)\leq\theta|\mathcal{C}_n|\nu)>\prod_n(1-\frac{e^{-|\mathcal{C}_n|\nu(\theta\ln\theta-\theta+1)}}{1-e^{\theta-1}})>0$.

Let $N=\Phi(\mathcal{C}_2)$. On A_R, for $x\in B(0,R)$ we have $I(x)=\sum_j l(|x-Y_j|)\geq Nl(4R)$. Moreover if $Y_j\in\mathcal{C}_n$, since : $\int_{B(0,R)} f_j(x)\frac{I(x)}{l(|x-Y_j|)}dx\leq\rho$, we deduce :

$$\int_{B(0,R)} f_j(x)dx\leq\frac{\rho}{N}\frac{l((n-1)R)}{l(4R)}.$$

It follows :

$$\begin{array}{rcl} \sum_j \mathbb{1}(|Y_j|\geq 3R)\int_{B(0,R)} f_j(x)dx & = & \sum_{n>2}\sum_{Y_j\in\mathcal{C}_n}\int_{B(0,R)} f_j(x)dx \\ & \leq & \frac{\rho}{N}\theta\sum_n|\mathcal{C}_n|\frac{l((n-1)R)}{l(4R)} \\ & \leq & \frac{1}{2}. \end{array}$$

Therefore, since $\sum_j\int_{B(0,R)} f_j(x)dx=\pi R^2$, we have :

$$\sum_j \mathbb{1}(Y_j\in\mathcal{C}_2)\int_{B(0,R)} f_j(x)dx\geq\frac{\pi R^2}{2}.$$

Finally we notice that :

$$\begin{array}{rcl} \rho & \geq & \frac{1}{N}\sum_j \mathbb{1}(|Y_j|\in\mathcal{C}_2)\int_{B(0,R)} f_j(x)\frac{I(x)}{l(|x-Y_j|}dx \\ & \geq & \frac{1}{N}\sum_j N\frac{l(4R)}{l(R)}\mathbb{1}(|Y_j|\in\mathcal{C}_2)\int_{B(0,R)} f_j(x)dx \\ & \geq & \frac{l(4R)}{l(R)}\frac{\pi R^2}{2}. \end{array}$$

From the hypothesis on l, $\frac{l(4R)}{l(R)}$ is lower bounded as R tends toward infinity for a suitable choice of the sequence R. Thus, if R large enough, we find a contradiction.

We have done our reasoning on the event A_R, since $P(A_R)>0$, by ergodicity of the Poisson process, the result is extended on the whole σ-algebra.

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