Nonlinear Cloth Simulation

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N° 5099

Janvier 2004

THÈME 3

Rapport de recherche

ISSN 0249-6399 ISRN INRIA/RR--5099--FR+ENG

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Thème 3 — Interaction homme-machine,
images, données, connaissances
Projet Alcove

Rapport de recherche n° 5099 — Janvier 2004 — 27 pages

Abstract: This research report describes a cloth simulation system including nonlinear behaviors. Our cloth model simulates the fabric as a complex yarn interlacing structure. This approach allows us to deal with the yarn scale problem by including the nonlinear interactions happening inside this structure. This model has three different components: the traction, the bending and the shearing. We also add new friction terms in order to reproduce the yarn interlacing structure and the nonlinear properties of a real cloth. Moreover, this representation avoids a detailed 3D geometric model of yarns which is almost always unusable in computer graphics.

Furthermore, our physical model is based on the Kawabata Evaluation System (KES). It reproduces the behavior of a cloth using the parameters coming from direct measurements on real clothes. Therefore we propose an identification procedure to compute all the physical parameters of our model from the KES curves. Since many textile manufacturers use this KES to design their fabrics, we are able to accurately simulate their real fabrics in computer graphics using the manufacturer's parameters. The simulation of a complete garment made of these fabrics could also be achieved in a virtual fashion show to visualize its characteristics and/or flaws for example.

Key-words: Nonlinear Deformation, Cloth Simulation, Physically Based Modeling, Parameter Identification, Hysteresis, Solid Friction, Numerical Computation

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Simulation de Tissus Non-Linéaires

Résumé : Ce rapport de recherche décrit un système de simulation de tissus intégrant des comportements non-linéaires. Notre modèle simule le tissu comme une structure complexe de fils entrelacés. Cette approche permet de prendre en compte le problème de l'échelle du fil en incluant des interactions non-linéaires à l'intérieur même de la structure. Ce modèle possède en fait trois composantes différentes: la traction, la flexion et le cisaillement. Nous avons également ajouté de nouveaux termes de frottement afin de reproduire la structure de fils entrelacés et les propriétés non-linéaires d'un vrai tissu. Par ailleurs, cette représentation évite une modélisation géométrique 3D détaillée des fils, pratiquement toujours inutilisable en synthèse d'images.

De plus, notre modèle physique est basé sur le Système d'Evaluation de Kawabata (KES). Il reproduit le comportement d'un tissu en utilisant les paramètres venant directement de mesures sur des tissus réels. C'est pourquoi nous proposons une technique d'identification de tous les paramètres physiques de notre modèle depuis les courbes de Kawabata. Dans la mesure où beaucoup de fabricants de textiles utilisent le Système d'Evaluation de Kawabata pour concevoir leurs tissus, nous sommes capables de simuler avec précision leurs tissus réels en synthèse d'images, et en utilisant les paramètres du fabricant. La simulation d'un vêtement complet conçu de ces tissus pourrait alors être effectuée dans un défilé de mode virtuel pour visualiser ses caractéristiques et/ou ses défauts par exemple.

Mots-clés : Déformation non-linéaire, Simulation de tissus, Modélisation physique, Identification de paramètres, Hysteresis, Frottement de solides, Calcul Numérique

1 Introduction

1.1 Overview of the problem

Many works have been proposed to simulate clothes in computer graphics [56, 20, 52, 53, 11, 46, 5, 24, 55, 17, 4]. However none of these works are really usable in a textile industrial context. Indeed, most of manufacturers would like to visualize their model using computer graphics techniques. But none of the existing physical models for image synthesis are able to take into account the real parameters that they use to design the fabrics. Many fabric designers use the Kawabata's Evaluation System (KES) to get the mechanical properties of their clothes. Actually, a KES is a powerful device that produces curves depending on the applied force (traction, bending or shearing). These curves accurately describe the real behavior of a fabric. In this research report, we introduce a new physical model for cloth simulation including the Kawabata's parameters coming from KES experiments. A real fabric always has a nonlinear behavior, and this is characterized by hysteresis (see section 3.1). Our cloth model includes this nonlinear phenomenon through a new friction model which simulates the physical contact between yarns without any explicit geometric modeling of fibers. We also propose a new stable and reliable method for implicit integration handling nonlinear deformations.

1.2 Organization of the research report

The research report is organized as follows. In the next section, we discuss previous work related to cloth simulation. Section 3 describes the bases and the tools necessary to understand our model. In particular we present the notion of hysteresis which is mainly responsible for the nonlinear behaviors of a cloth. We also explain the different existing friction models. Section 4 depicts our cloth model in details. Each of the three components characterizing our model is explained in details: the traction, the bending and the shearing. We also show how to integrate our own friction model into these components. Section 5 gives the description of our implicit integration method. In section 6, we describe an identification technique to match the real KES curves to the ones produced by our model. This validates our cloth model. Section 7 shows some computation times and images generated using our cloth model. The last section brings forward some conclusions and future research directions.

2 Background and Previous Work

Since this research report is concerned with cloth dynamics and particularly internal cloth dynamics, we do not address collisions or self-collisions. In this research report, we use our own simple collision model but any advanced model could be used [14].

We distinguish two categories of techniques of cloth models. The first one concerns the pure textile research area. The second one is about cloth in computer graphics.

2.1 Cloth Modeling in Textile Research

Many works have been done in textile research to simulate clothes. Pierce first developed a geometric and a force-geometry model [44] to describe a plain weave woven fabric. He assumed that the yarns within the fabric were perfectly flexible and circular in cross section. This model can be applied to a limited range of problems because it is either based on geometric relations or on a simple mechanical description. Kemp [38] modified Pierce's model introducing the yarn flattening. Many other authors proposed Pierce-like models adding extensions [43, 29, 36, 1, 28].

Strain energy methods simulate the cloth by creating and minimizing equations that define the energy in a textile structure. These models can be divided into two categories. The first one concerns the low-level structural models and they are based on the deformation of yarns [31]. The second one is a high-level structural model using the shell and plate theory [3].

These models from the textile research community allow a better understanding of the problem, but they use empiric parameters or assumptions that have to be validated. This is a recent area called parameter identification. We discuss this subject in section 2.3.

2.2 Cloth Modeling in Computer Graphics

There is a rich history about cloth simulation in computer graphics. The non-physical approaches [56, 20] focus on roughly approximations of cloth without any dynamics. Wei [56] first defined a geometric approach to approximate the folds in a cloth. Coquillart [20] has proposed the free form deformations to simulate fabrics. These two models have no real physical modeling regarding the forces and the energy of the deformable object.

On the other hand, physically-based modeling techniques use mechanic models to realistically simulate a cloth. We propose to split these techniques into two parts: the continuous models [52, 53] and the discrete models [11, 46, 5, 24, 55, 17, 4] which our contribution belongs to.

2.2.1 Continuous models

Terzopoulos et al. [52, 53] have proposed elasticity-based models to simulate a deformable structure such as cloth. This model is an approximation of the continuous dynamics that is solved by finite elements or finite differences. Eischen et al. [27] use a nonlinear shell theory to accurately simulate cloth. Finite elements methods do not take into account the yarn/fabric structure. They generally simulate the textile surface without describing the inner mechanic.

2.2.2 Discrete models

Discrete models use mass-spring or particle systems driven by energy functions and forces to depict a cloth. Our model belongs to this category.

Baraff et al. [5] describes an implicit integration scheme avoiding the stiffness problem. They have developed a method to constrain the cloth model with a low computation time. This ground-breaking approach is then used to improve cloth simulation. Volino et al. [55] and Ascher et al. [4] focus their work on the numerical analysis to enhance the performance and the accuracy of the simulation. In order to achieve real-time performance, Desbrun et al. [24] make further approximations and violate the local preservation of the angular moments. Provot [46] uses explicit integration and he avoids the stiff problem using weaker springs. Furthermore a post-processing step limiting the strain rate of stretching is necessary. He thus makes a coarse approximation of the cloth rigidity. Breen et al. [12, 11] use a set of K.E.S fabric tests to develop specific bending, shearing and traction models. However these models do not take into account the hysteresis and they are limited to the static case: only the final state of the cloth is used. Recently, Bridson et al. [13] proposed a new formulation for a bending model with interesting visual results. Choi et al. [17] reduce the over-damping due to implicit integration. Unfortunately these works suffer from a linearization of the cloth dynamics. They also introduced a specific bending energy to reproduce more accurately folds in cloth. These results are visually spectacular but they have a low physical interpretation. This signifies that they are unusable in our case.

In their book Breen et al. [30] conclude that developing a new detailed structural model of the cloth is an important challenge: "from the yarn behavior, together with a weaving or knitting plan, it should be possible to predict the behavior of finished fabric". This approach raises several tough problems especially the internal friction interactions between yarns. Our model actually tries to solve this problem (see section 4). But a more detailed cloth model is not only required in CAD systems [42]. It is also a possible answer to the problem adressed by Bhat et al. [6] about choosing the right parameters of a desired fabric (see next section and section 6).

2.3 Parameter Identification

Having a mechanical model is essential to produce realistic cloth simulations. However, these simulations are difficult to tune due to the many parameters that must be adjusted to achieve the look of a particular fabric [6]. Therefore many works have been proposed to identify the parameters of a given fabric using specific data or input images [12, 11, 16, 39, 33, 19, 35, 26, 6]. However very few of these papers have validated their techniques on real data [48, 49, 19]. This research report attempts to solve this problem, identifying the Kawabata's parameters from the KES data on real fabrics (see section 6).

In general, people validate their results with a simple visual comparison. Colier et al. [19] use a finite-element cloth model with measured mechanical properties. They estimate the accuracy of their simulated draping position using a drapemeter [18]. However the drapemeter is not accurate enough for a true mechanical validation and for textile industry applications. Louchet et al. [39] and Joukhadar et al. [33] use an evolutionary algorithm to identify the parameters of a deformable object.

Chen et al. [16] study the influence of the parameters and highlighted their fundamental role in cloth simulation. Realff et al. [48, 49] propose a fabric model to simulate the uniaxial tension (traction). Their approach uses the original fabric geometry and the yarn measured properties from mechanical experiments to identify and validate the fabric model. The identified parameters are then compared to the real tensile tests [49]. However, even if this approach shows accurate results, it only deals with the traction. No bending or shearing is performed.

Breen et al. [12, 11] compute energy functions from Kawabata's experiments on fabrics. They identify the parameters only from a part of the Kawabata's curve (no hysteresis) using a polynomial approximation and a linear fitting. This technique results in a set of non-physical parameters.

Eberhardt et al. [26] follow the previous approach but they used a different method. They also extend the technique to the dynamic case and identify the whole Kawabata's curve (hysteresis). They used the KES experimental data and they fit the curve applying successive piecewise linear functions. Since they have developed an empirical and linear approach, the authors roughly approximate the hysteresis.

Recently, Bhat et al. [6] presented a promising algorithm to identify the parameters from a set of real images of a cloth simulation. The major advantage of this technique is to allow the estimation of the parameters of Baraff's model [5] from videos. However, it is not suitable for textile industry applications that require a much more accurate cloth model to simulate real fabrics. An extension of Bhat et al.'s method to a nonlinear cloth model simulating hysteresis would be extremely useful.

3 Cloth Physics

In this section we describe the two important concepts that are required to understand our model: the yarn interlacing structure and the friction models.

3.1 The hysteresis

When deformation of a fabric occurs (for small strains like drape), traction, bending as well as shearing phenomena have to be simulated. They are characterized by a nonlinear behavior and especially hysteresis. The hysteresis is difficult to handle because it is a subtle physical phenomenon. However, it is essential to a realistic cloth simulator using computer graphics for textile industrial applications. The hysteresis corresponds to a delay in the evolution of a physical system that tends to preserve its initial state. In many cases, this is commonly considered as a memory form effect. The fact that the fabric is a yarn interlacing structure creates the illusion that the structure and especially the geometry of the fabric are generating this behavior. However, the study of yarn clearly shows that the yarn and the assemblage of fibers are responsible of these non-linearities. The geometry of the fabric does not create any major nonlinear phenomenon. These fibers are maintained together by a twisting process better known as torsion. Their organization makes them slightly move inside the assemblage, thus creating an internal friction effect and hysteresis.

3.2 Internal Friction

Friction forces play a major role in everyday life. Without it, we would not be able to walk or to hold our pants. Friction occurs in every mechanical system especially when there is a physical interaction between two surfaces brought into contact. A wide range of physical phenomena cause friction. This includes elastic traction, plastic deformations, wave phenomena, etc.. The friction is the tangential reaction force between two surfaces brought into contact. Actually these reaction forces are the combination of many different mechanisms. They depend on the contact geometry and topology, the materials of the surface, the displacement speed, the relative velocity and the presence of lubricant.

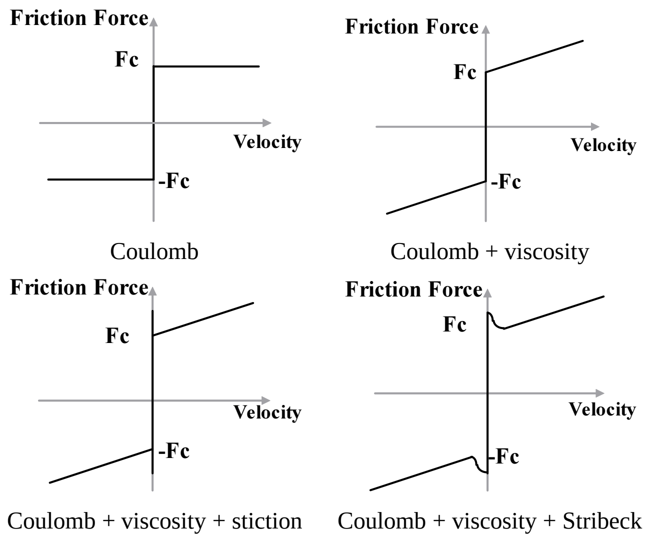


Figure 1: Classical Friction Models

Most friction models are based on Leonardo da Vinci's laws of friction. The physicist Amonton made simlilar studies and he defined two laws: friction is directly proportional to the applied load, and friction is independent of the obvious area of contact [2]. Coulomb published one of the most famous and comprehensive studies about friction [21]. Lately Karnopp [34] and Stribeck [50] improved the Coulomb's model in order to take into account the stiction and the Stribeck effect (see figure 1). These laws provide useful estimates and qualitative predictions for a wide range of behaviors associated with friction. They are looked upon as static models because friction is not considered as a dynamic phenomenon. This feature presents fundamental and practical drawbacks especially with low velocity. More advanced models may be required in specialized area like cloth modeling.

As described in section 3.1, a yarn is a fibrous assemblage. When a deformation occurs, the contact between two fibers or between the weft and the warp produce friction. Therefore we consider that hysteresis happens during the fabric deformation. But the observation of this effect shows three specific characteristics especially when the cloth dynamic is in its transition phase (acceleration for example):

- Pre-sliding effect which corresponds to a spring-like response in order to characterize limited friction start,

- Static friction effect which is the friction intensity when velocity is null,

- Stribeck effect which is a diminution of the friction force for small velocity. This is caused by a transition between lubricated friction and partial lubricated friction [50].

Classical friction models are not able to reproduce these characteristics because the friction force does not depend on the velocity. The friction is not considered as a dynamic phenomenon and consequently it does not model a dynamic system. Creating a general friction model using basic physical laws is not powerful enough. Approximated models exist for certain configurations. But we rather look for a general model for cloth simulation, including the friction phenomena. This also means that we are not interested in a microscopic friction model which would increase the computational cost. Experiments have been performed under ideal conditions to develop a new class of friction models focusing on the dynamic properties. These models really fit experimental observations regarding friction[47, 22]. Classical models do not. Another advantage of this model is its ability to depict the three previous physical characteristics (pre-sliding, static friction and stribeck effects) in textile modeling. It also depends on identifiable physical parameters. Therefore we use these works to develop our internal friction model for cloth simulation (see section 4.2).

4 Cloth Model

4.1 General structure

The main idea of our model is to simulate the nonlinear properties of the cloth using internal friction terms. Actually we use three fundamental components to build our model: the shearing, the bending and the traction (see figure 2). Every component integrates a friction term in order to preserve hysteresis. The moments and the forces of each component are described in the section 4.3.

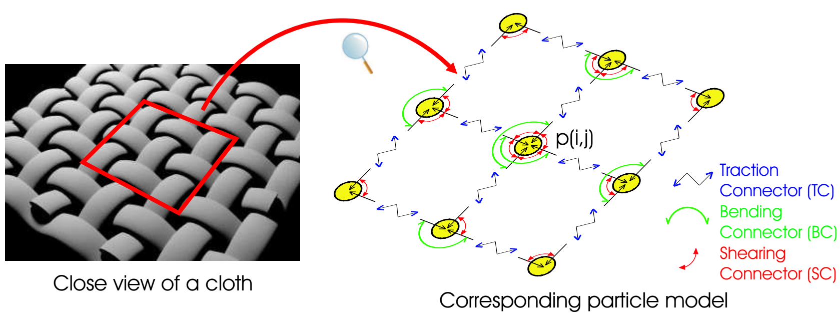


Figure 2: Our cloth model.

Our cloth model is based on a mesoscopic analysis of cloth. It is approximated by a quadrilateral mesh of interconnected particles $p(i, j)$ [10], because it can handle the exact geometry of cloth (e.g. crossing warp and weft yarns). This representation simulates fabrics by modeling the low level structure of the material and the anisotropic behavior due to different warp and weft properties. The Joukhadar connector system [32] is used to depict the connections between particles, as shown in figure 2. However the three types of connectors (bending(BC), traction(TC), shearing (SC)) have different mathematical and physical formulations in our case (see section 4.3).

4.2 Internal friction model

Our model of internal friction is based on a generalization of the classical Coulomb's model. We show here how to include this friction into the three components of our model.

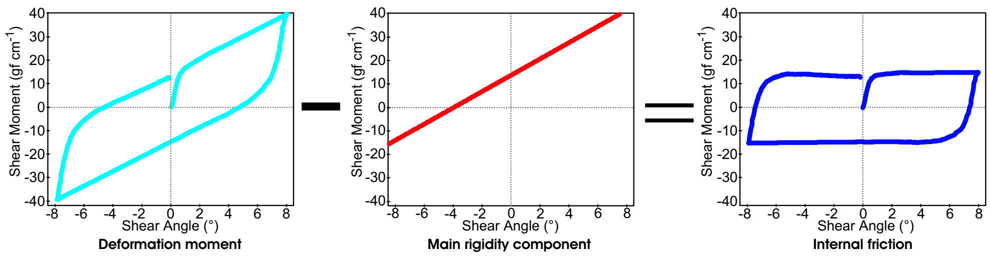


Figure 3: Friction Extraction (during shearing)

First of all, we apply the KES on different fabric samples using three different deformations (bending, traction and shearing). We now obtain a Kawabata's curve depicting the general shape of the internal friction. For traction, it is a function of force and relative elongation. For bending and shearing, this curve is a function of moment and either deformation curvature (bending) or deformation angle (shearing). Then we compute the main rigidity component corresponding to the main slope of these curves. Finally, we subtract this main rigidity component to the deformation moment (bending and shearing) or the force (traction) to estimate the internal friction (see figure 3) of the cloth.

We simulate the internal friction applying an extended version of the Dahl's model [23]. It was originally designed for control systems with friction. It is a simple and efficient model that has been developed from experimental observations about friction in servomotor systems with ball bearings. Using the stress-strain curve in classical dynamics (see left figure 4), Dahl proposed a differential equation to simulate the friction force from the displacement:

$$\frac{dF}{dx} = \sigma\left(1 - \frac{F}{F_c}\mathrm{sgn}(v)\right)^{\alpha}$$

where:

σ	is the stiffness coefficient,
F_c	is the the Coulomb's friction force,
v	is the displacement speed,
α	is a parameter that controls the shape of the stress-strain curve. $\alpha = 1$ is used except for plastic materials where $\alpha > 1$.

To get a time friction function the model is simply rewritten as follow :

$$\frac{dF}{dt} = \frac{dF}{dx}\frac{dx}{dt} = \sigma(1 - \frac{F}{F_c}\text{sgn}(v))^{\alpha} v \tag{1}$$

This model is a generalization of ordinary Coulomb's friction with the possibility to reproduce hysteresis, pre-sliding and zero-slip displacement. It also only depends on the displacement and the sign of the velocity.

The shape of the friction curve presented in figure 4 shows the phenomena described in the previous sections (pre-sliding, Coulomb's friction and Stribeck effect).The Dahl's model might be used as a first approximation to capture the friction behavior. However it does not take into account the stiction and the Stribeck effect which is a rate dependent phenomenon. Therefore we propose to use an extension of the Dahl's model, as the one developed by Bliman and Sorine[8, 7]. This model is also built on the Rabinowicz's experimental observations and the theory of the hysteresis operator.

Bliman and Sorine defined the space variable s : $s = \int_o^t |v(\tau)| d\tau$

Using this space variable, they stress that friction is a function of the path only. It does not depend on how fast the system moves along the path. The models are then defined by a linear state model of the variable s:

$$\begin{array}{rcl} \frac{dx_s}{ds} & = & Ax_s + Bv_s \\ F & = & Cx_s \end{array}$$

where v_s is $sgn(v)$, the sign of v.

This model has different complexities according to its order. The first order model is given by:

$$A = -1/\varepsilon_f, \quad B = f_1/\varepsilon_f, \quad C = 1$$

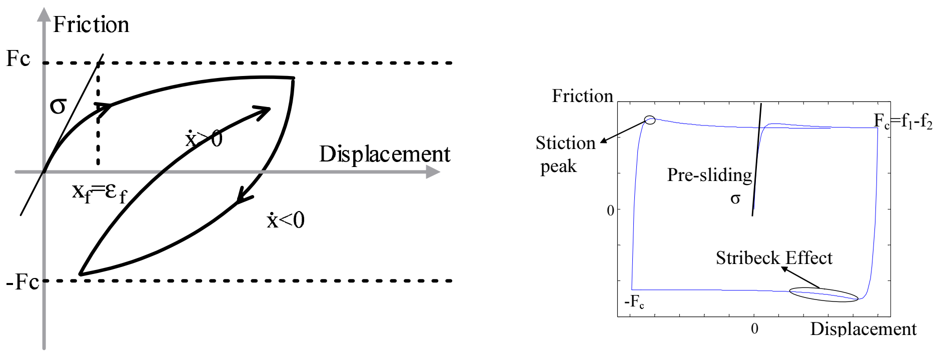
where ε_f is the displacement giving the pre-sliding slope σ.
It can also be written as:

$$\frac{dF}{dt} = \frac{dF}{ds}\frac{ds}{dt} = f_1/\varepsilon_f \; (1 - \frac{F}{F_c}\text{sgn}(v)) \; v$$

Actually, this first order model corresponds to the Dahl's Model (1) with $f_1 = F_c$, $\sigma = f_1/\varepsilon_f$ and $\alpha = 1$. Unfortunately, it still does not handle the stiction and the Stribeck effect. This can be achieved with the second order model:

$$\begin{array}{l} A = \begin{pmatrix} -1/(\eta\varepsilon_f) & 0 \\ 0 & -1/\varepsilon_f \end{pmatrix} \\ B = \begin{pmatrix} f_1/(\eta\varepsilon_f) \\ -f_2/\varepsilon_f \end{pmatrix} \\ C = \begin{pmatrix} 1 & 1 \end{pmatrix} \end{array} \tag{2}$$

where $f1 - f2$ corresponds to the kinetic friction reached exponentially as $s \to \infty$ [9]. This model can be viewed as a parallel connection of a fast and a slow Dahl's model. The fast model has higher Coulomb's friction than the slow model. The force from the slow model is subtracted from the fast model, which results in a stiction peak. Both the first and second order models can be shown to be dissipative. Bliman and Sorine also show that, as ε_f goes to zero, the first order model behaves as



Dahl's Dynamic Model

Enhanced Dynamic Model
(Bliman-Sorinne)

Figure 4: Cloth Internal Friction

a classical Coulomb's friction model. The second order model behaves as a classical model with Coulomb's friction and stiction.

We now have an internal friction model capable of simulating nonlinear forces during the fabric deformation. In the next section, we explain how to integrate this friction into each of the three components of our cloth model.

4.3 Moments and Forces

As described in the section 4.2 we characterize the cloth hysteresis by subtracting an elastic force. The traction, the bending and the shearing have an elastic force coupled with other forces. We can write the following formulation for the internal force F:

$$
\begin{aligned}
F &= F_{TC} + F_{BC} + F_{SC} \\
&= \sum_{i,j} F_{tract\ p(i,j)} + F_{bend\ p(i,j)} + F_{shear\ p(i,j)}
\end{aligned}
$$

where F_{TC}, F_{BC} and F_{SC} are the traction, the bending and the shearing forces respectively.

4.3.1 Traction

Our model has been made possible by many scientific contributions about the yarn and the fabric traction [54, 15, 49]. We have also performed several traction tests on different fabric samples to capture the general behavior of this force. From these experiments we extract a general shape split into four different parts (see left figure 6):

- The first part is the priming zone of yarns. The fibers are partially organized as the initial state (see right figure 6). When traction occurs, theses fibers receive longitudinal efforts aligning

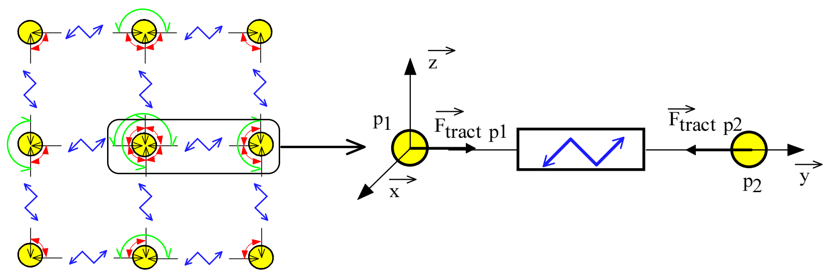
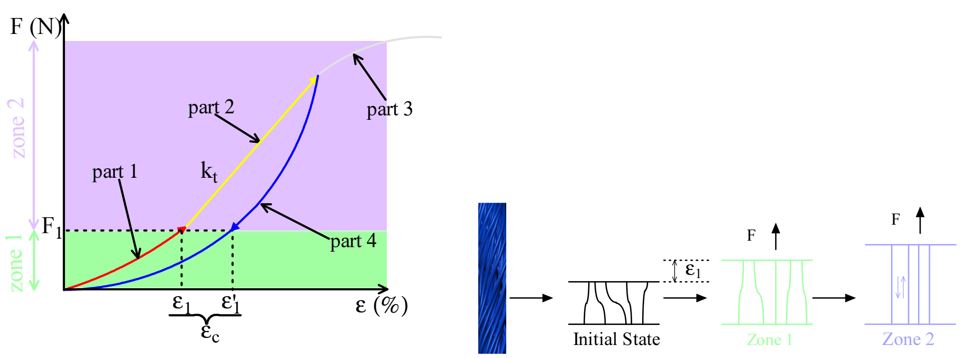


Figure 5: Traction Connector

them in the same direction. This effect leads to a lower traction resistance. We model it by a modified rigidity using a distribution function. This function characterizes the progressive alignment of the fibers. The internal friction caused by the fiber interactions is also affected the progressive alignment. Moreover, the yarn is blocked in a fabric structure. It cannot be longitudinally compressed. We use a geometric constraint to avoid any compression, but Choi et al.'s method [17] could also be used.

- The second part is the linear elastic response of the fabric. It includes a stiffness coefficient k_t known as the linear rigidity, and a friction term.
- The third part corresponds to the fabric damaging. In the case of cloth draping, this can be neglected.
- The fourth part is the hysteresis. It takes into account the friction between fibers during an elastic elongation.



Fabric traction shape Fiber deformation during traction.

Figure 6: Description of fabric and fiber behaviors during a traction process.

We propose the following model for the Traction Connector (TC) which links two particles (p_1 and p_2) (see figure 2c):

$$\vec{F}_{tract\ p_1} = \begin{cases} \text{if } |\vec{F}_{tract\ p_1}| \leq F_1 \\ \quad \left[\left(1 - \frac{F_1}{k_t \varepsilon_c}\right)\varepsilon + \frac{2F_1}{k_t} - \varepsilon_c\right]\left(k_t \frac{\varepsilon}{\varepsilon_c} + F_{tract_{fr}}(v_s)\right)\vec{k}_{p_1} \\ \text{else} \\ \quad k_t(\varepsilon - \varepsilon_c) + F_1 + F_{tract_{fr}}(v_s)\,\vec{k}_{p_1} \end{cases} \tag{3}$$

where:

ε	is the relative elongation between the two particles,
$\varepsilon_c = (\varepsilon_1 \text{ or } \varepsilon_1')$	is the relative elongation defined by the deformation of fibers during traction (with initial value ε_1),
$F_{tract_{fr}}$	is a friction force (N) defined by a first order Bliman-Sorine's model (see eq. 1) with: $F_{tract_{fr}} = Cx_s$, $v_s = sign(\dot{\varepsilon}(t)) \quad s(t) = \int_0^t |\dot{\varepsilon}(\tau)|d\tau$, $A = -1/\varepsilon_{fr_{tract}}, \quad B = F_{c_{tract}}/\varepsilon_{fr_{tract}}, \quad C = 1.$
$F_{c_{tract}}$	is the Coulomb's friction force (N),
$\varepsilon_{fr_{tract}}$	is the relative elongation defining the pre-sliding,
$\dot{\varepsilon}$	is the relative elongation speed,
k_t	is the linear rigidity for the relative elongation,
F_1	is the force (N) defining a change between zone 1 and zone 2 (see figure 6),
$\vec{k}_{p_1}$	is a unit vector defined as $\vec{k}_{p_1} = \frac{\vec{p2} - \vec{p1}}{\lvert\vec{p2} - \vec{p1}\rvert}$.

4.3.2 Bending

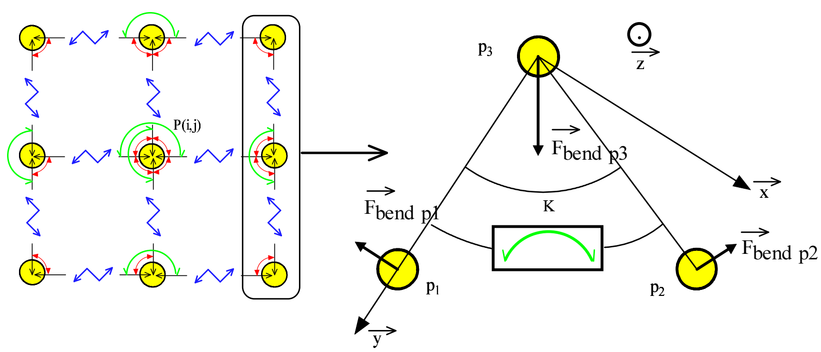


Figure 7: Bending Connector

The Bending Connector (BC) represents the constraint applied to the three particles p_1, p_2 and p_3 (see figure 2b). The force applied by this connector to the particles p_1 and p_2 is given by the following equation:

$$\vec{F}_{bend\ p_1} = \frac{k_b(K - K_0) + M_{fr_{bend}}(v_s)}{|\vec{p}_3 - \vec{p}_1|}\ \vec{F}_{p_1} \tag{4}$$

where:

K	is the curvature (m^{-1}) of the three particles computed from Breen's equation [10],
K_0	is the residual curvature (m^{-1}) for plastic deformation (for example a permanent fold) ,
k_b	is the flexural rigidity, representing linear elastic relation between curvature and bending moment ($N.m^2$),
$\vec{F}_{p_1}$	is the unit vector perpendicular to the direction $\overrightarrow{p_1 p_3}$,
$M_{fr_{bend}}$	is the friction moment for bending ($N \cdot m^{-1}$) due to fiber collisions.

The friction moment is given by a Bliman-Sorine's second order model with:

$$M_{fr_{bend}} = Cx_s$$

The space variable is now: $$v_s = sign(\dot{K}(t)) \quad s(t) = \int_0^t |\dot{K}(\tau)| d\tau$$

where $\dot{K}$ is the curvature speed.

If we do not need to simulate the Stribeck effect, we can use a first order model with:

$$A = -1/K_{fr_{bend}}, \quad B = M_{c_{bend}}/K_{fr_{bend}}, \quad C = 1$$

where:

$M_{c_{bend}}$	is the friction moment ($N \cdot m^{-1}$), e.g. the Coulomb's friction
$K_{fr_{bend}}$	is the curvature representing the pre-sliding effect (m^{-1}).

The force $\vec{F}_{bending\ p_3}$ applied to the particle p_3 is the result of the action/reaction principle. It is simply given by:

$$\vec{F}_{bending\ p_3} = -(\vec{F}_{bending\ p_1} + \vec{F}_{bending\ p_2})$$

4.3.3 Shearing

The cloth shearing is very different from traction and bending. The shearing is caused by the deformation of the cloth structure. The yarn deformation does not generate any shearing. Therefore it only depends on the cloth geometry and yarn friction.
This phenomenon leads into the following conclusions:

- The small deformations produced by shearing are mainly due to frictions between warp and weft yarns.

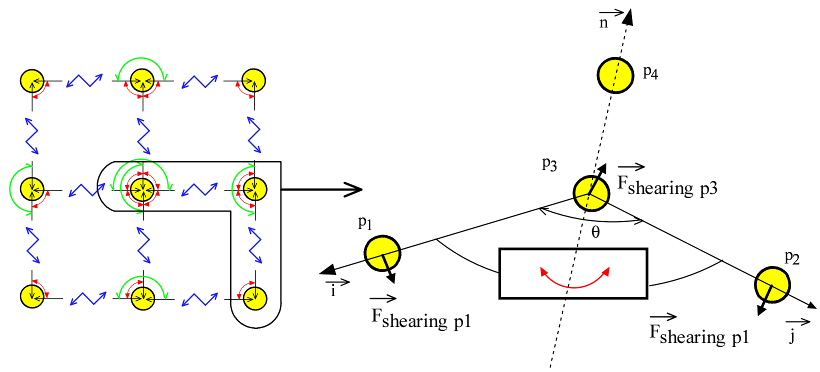


Figure 8: Shearing Connector

- For high stress, the shearing is caused by the yarn jamming. Therefore we have to constraint the cloth geometry with a maximum shearing angle. In practice, the bending effect occurs before this angle is reached. this is due to the important rigidity of a cloth.

Analyzing the data of the Kawabata's curves (see figure 10), we propose a shearing model for small deformations based on a second order Bliman-Sorine's model:

$$\begin{aligned} M_{shear} &= k_s(\theta - \theta_0) + M_{fr_{shear}} \\ M_{fr_{shear}} &= Cx_s \end{aligned}$$

with $v_s = sign(\dot{\theta}(t)) \quad s(t) = \int_0^t |\dot{\theta}(\tau)| d\tau$.

If the Stribeck effect is not required, a first order model with the following parameters is enough (see equation 2):

$$A = -1/\theta_f, \quad B = M_{c_{shear}}/\theta_{fr_{shear}}, \quad C = 1$$

where:

k_s	is the shearing rigidity $N \cdot m^{-1} \cdot rad^{-1}$ corresponding to the linear elastic response,
θ_0	is the initial angle between particles (usually $\frac{\pi}{2}$),
$M_{c_{shear}}$	is the moment $(N \cdot m^{-1})$ for the Coulomb's friction,
$\theta_{fr_{shear}}$	is the angle characterizing the pre-sliding shearing friction (see 4.2).

The force applied to a non-central particle (for example p_1) for a shearing connector (see figure 2a) is :

$$\vec{F}_{shear\ p_1} = \frac{k_s(\theta - \theta_0) + M_{fr_{shear}}(v_s)}{|\vec{p}_3 - \vec{p}_1|} \vec{k}_{p_1} \tag{5}$$

where $\vec{k}_{p_1} = \vec{n} \wedge \vec{j}$.

To respect the action/reaction principle, the force applied on p_3 is the opposite sum of $\vec{F}_{shear\ p_1} + \vec{F}_{shear\ p_2}$, as in the bending case.

5 Numerical Integration

The acceleration $\ddot{x}_{i,j}$ of the particle $p(i,j)$ combines the internal forces with the external forces; It can be written as $\ddot{x}_{i,j} = F_{i,j}/m_{i,j}$, where $m_{i,j}$ is the particle mass. The equations of motion lead to the state equation:

$$\frac{d}{dt}\begin{pmatrix} x \\ v \end{pmatrix} = \begin{pmatrix} v \\ M^{-1}F(x,v) \end{pmatrix}$$

where:

x	is the vector containing the particle position
v	is the particle velocity.

To solve these equations, a reliable integration method is needed. Implicit methods are a reference choice by many authors [5, 51, 53]). [5] use the backward Euler method :

$$\frac{1}{\Delta t}\begin{pmatrix} x^{t+1} - x^t \\ v^{t+1} - v^t \end{pmatrix} = \begin{pmatrix} v^{t+1} \\ M^{-1}F^{t+1} \end{pmatrix} \tag{6}$$

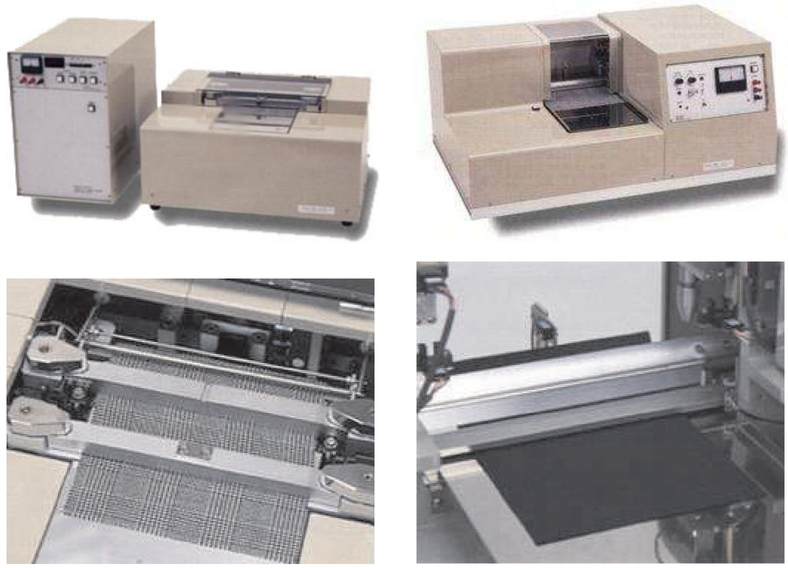
In order to reduce the computational cost, they simplify the solution by a first order linear approximation. The system is solved at each time step with a modified conjugate gradient (CG). Many authors also have extended this approach[24, 41, 17]. [25] have proposed to mix the implicit scheme with an explicit scheme.

The methods simulating cloth models usually require the computation of the Jacobian of the function F, or specific computations (CG) which need positive symmetric matrices. Consequently these algorithms are slow or they only approximate the dynamics of the cloth. Instead, we prefer to use Broyden's method [37] which solves nonlinear systems with a more efficient computation time.

Using the Euler's backward method (see eq. 6), we have to solve a nonlinear system $\phi(x) = 0$. Broyden's method avoids the evaluation of the Jacobian and the linearization of the equations. This scheme is a quasi Newton's method, but the Jacobian is incrementally approximated. During one iteration of the Broyden's method, we only perform one evaluation of the function ϕ. This is faster than any Newton-like methods. This advantage increases with the size of the equation system.

6 Parameter Identification

Our cloth model is able to simulate nonlinear clothes from real data. It may require up to 32 parameters (16 for the warp and 16 for the weft) in the most complicated case. However, many clothes can be simulated using 12 parameters if there is no noticeable difference between warp and weft yarns.



Traction-shearing Module Bending Module

Figure 9: Kawabata Evaluation System (KES)

This is still complex enough to make almost impossible any manual estimation of these parameters. Therefore we propose here an automatic procedure to compute all of them from experiments characterizing a cloth behavior.

The Kawabata Evaluation System (see figure 9) evaluates traction, shearing and pure bending behaviors of a fabric. It also provides dynamic measurements. All the parameters of our cloth model are computable from the KES curves (see figure 10).

Using the KES data, we define a χ^2 merit function of M unknown parameters. We then determine the best-fit parameters by minimization this function (see tables 1,2 and 3). Usual methods are inappropriate because of the nonlinear characteristics. We choose the nonlinear least square algorithm based on the Levenberg-Marquardt method [40]. This method works very well in practice and has become the standard of nonlinear least-square routines [45]. In our case χ^2 is the quadratic normal distance between the KES data and the outputs of our three sub-models TC, BC and SC defined in equation 3, 4 and 5 respectively.

The initial parameters values are directly estimated from the curves. For the traction connector, the rigidity constant is evaluated by the general shape of the curve. The priming zone parameters are computed using the slope change of the traction curve. Finally, the friction parameters are given by the extraction procedure described in section 4.2. We apply the same evaluation process for the bending and shearing connectors, except that we do not have any priming zone term to approximate.

These initial values are close to the final identified parameters. Therefore the minimization of χ^2 is instantaneous.

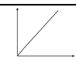
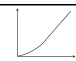
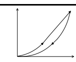
Traction Connector (TC)		
Physical Properties	**Parameters (M)**	**General Shape**
Linear Rigidity	k_t	
Priming Zone	F_1, ε_1, ε_1'	
Coulomb Friction	$F_{c_{tract}}$, $\varepsilon_{fr_{tract}}$	

Table 1: Traction parameters to identify. There is no Stribeck effect for a traction connector. The identification procedure may require up to 12 parameters if the cloth has anisotropic properties (warp and weft yarns have different behaviors).

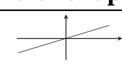
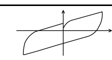
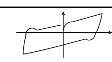
Shearing Connector (SC)		
Physical Properties	**Parameters (M)**	**General Shape**
Linear Rigidity	k_s	
Coulomb Friction	$M_{c1_{shear}}$, $\theta_{fr1_{shear}}$	
Stribeck Effect	$M_{c2_{shear}}$, $\theta_{fr2_{shear}}$	

Table 2: Shearing parameters to identify. If there is no hysteresis the number of parameters is reduced from 5 to 3. An anisotropic cloth doubles all the parameters up to a maximum of 10.

As an example, we identified the fabric parameters for an acrylic twill weave fabric. The Kawabata's curves do not present any Stribeck effect so we could use for every friction model a first order model (e.g. Dahl Model). Furthermore, we identify the parameters for the warp and weft direction as we have an anisotropic behavior.

Figure 10 shows the identified models and their accuracy. While identifying the unknown parameters, we obtained a satisfactory 2%. This error is caused by the high sensitivity of the Kawabata Evaluation System and the nonhomogeneous nature of the fabric material. Furthermore, it is not significant as for different cloth material (e.g. silk, coton, wool) traction, shearing and bending behaviors are varying much more than 2%.

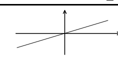
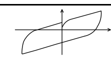
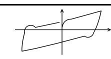
Bending Connector (BC)		
Physical Properties	**Parameters (M)**	**General Shape**
Linear Rigidity	k_b	
Coulomb Friction	$M_{c1_{bend}}$, $K_{fr1_{bend}}$	
Stribeck Effect	$M_{c2_{bend}}$, $K_{fr2_{bend}}$	

Table 3: Shearing parameters to identify. As for bending, if there is no hysteresis the number of parameters is reduced from 5 to 3. An anisotropic cloth doubles all the parameters up to a maximum of 10.

7 Results and Applications

We have performed several cloth simulations using the parameters computed from the KES curves of section 6. Table 4 summarizes the performance of our algorithm on an Intel Pentium 4-2Ghz.

number of particles	**CPU sec/frame**	**time step Δt**
400	0.115	0.001
2500	1.321	0.001
4900	6.302	0.001

Table 4: Computation times for nonlinear cloth simulation

The indicated CPU times correspond to the full time required to compute one frame: evaluation of forces, Broyden's solving method and collisions. The choice $\Delta t = 1e^{-3}s$ is driven by the need of accurate calculations. A higher time step would make the implicit method create too much artificial damping for realistic cloth simulation. Moreover, in our case the computational cost is highly influenced by the parameter values, especially the mass and the rigidity. Indeed, in practice, using parameters computed from real data generate very strong forces leading to important simulation computational costs.

One immediate application of our work concerns textile design. None of the existing works in Computer Graphics are usable in the textile industry because they require too many assumptions or approximations or they have not been validated on real fabrics using real industrial data (KES for example). Indeed, the textile industry often uses the KES to get the mechanical properties of the fabrics that they design. However, they are not able to put any of the Kawabata's parameters in a physical model to visualize the fabric or the cloth in computer graphics. Being able to display clothes in image synthesis from accurate KES data coming is an excellent way for textile industrials

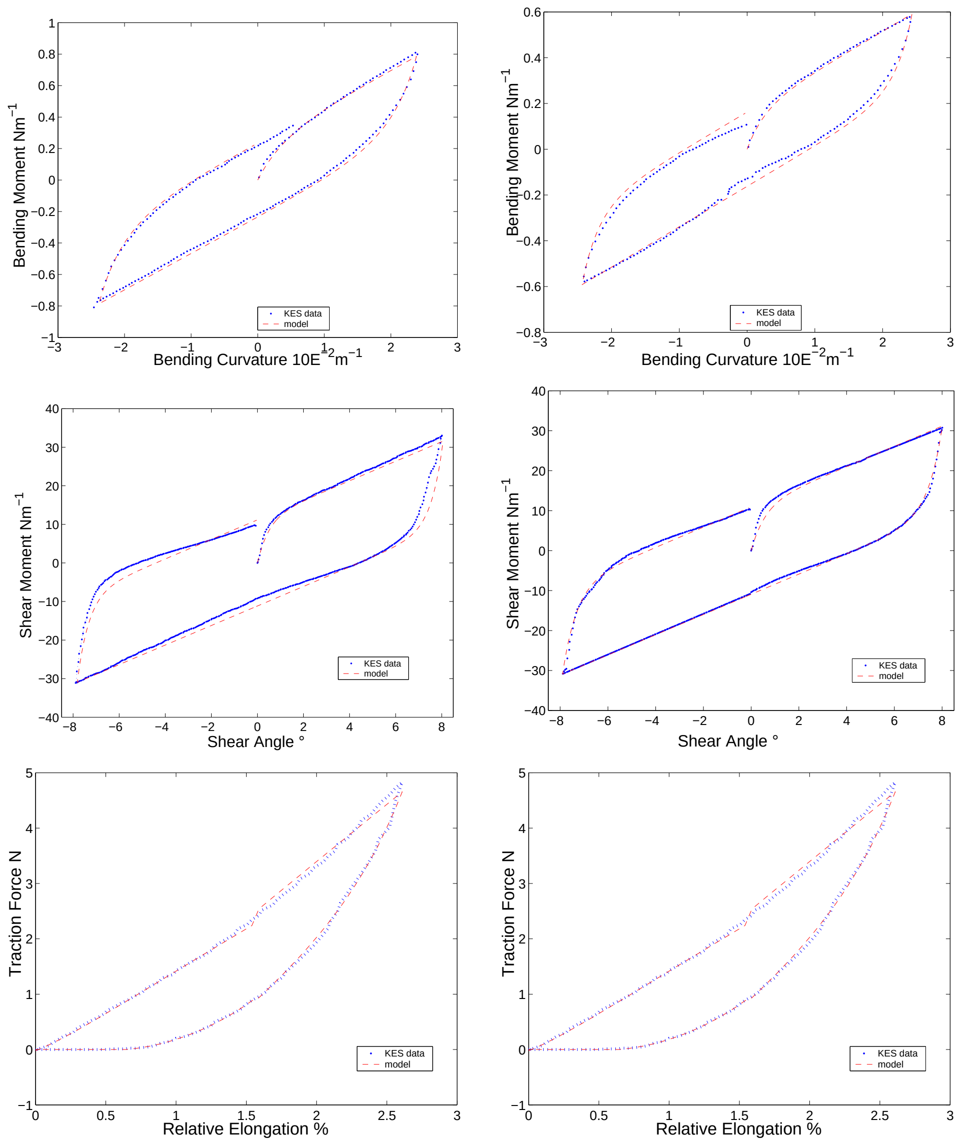


Figure 10: Parameter identification results on the KES curves. The left and right columns respectively correspond to the warp and weft. The blue curves represent the real data coming from KES experiments. Using the initialization values we compute a set of parameters for our model. We are then able to produce a new KES curve. The final parameters of our cloth model are obtained by minimizing the error between this curve and the original blue curve. The red curve is generated using the final solution (320 iterations were needed). The remaining error ($\sim$ 2%)is due to the high sensitivity of the Kawabata Evaluation System and the nonhomogeneous nature of the fabric.

to examine the properties and flaws of their fabrics. This could lead to more realistic virtual fashion shows because none of our physical parameters are empirically determined.

8 Conclusion and Future Work

We have proposed a simple and accurate physical model to simulate non-linearities and hysteresis in cloth modeling. It is a particle system driven by force functions and identifiable parameters based on the Kawabata Evaluation System (KES). It includes an internal friction model to simulate the organization of yarns inside a fabric without using any complex geometric model. This friction term is integrated into the three components of our cloth model: the traction, the bending and the shearing. To solve the inherent equations, we use a rapid integration technique that has the advantages of standard implicit methods. Efficient nonlinear solving is thus possible. This leads to accurately simulate the cloth behavior without any approximations.

This physical model is only one step to achieve realistic cloth simulation. Indeed, we would like now to focus on parameter identification especially from video data. We believe that capturing the cloth behavior and the Kawabata's parameters from image sequences is a tough but very promising challenge.

Many other enhancements are possible to make our cloth model much more powerful. For example, when a cloth is too much stretched it does not always return to its initial state. The elasticity does not act anymore and the fabric is damaged. We have already added a damaging term to our model. However many further tests are required for its validation.

Contents

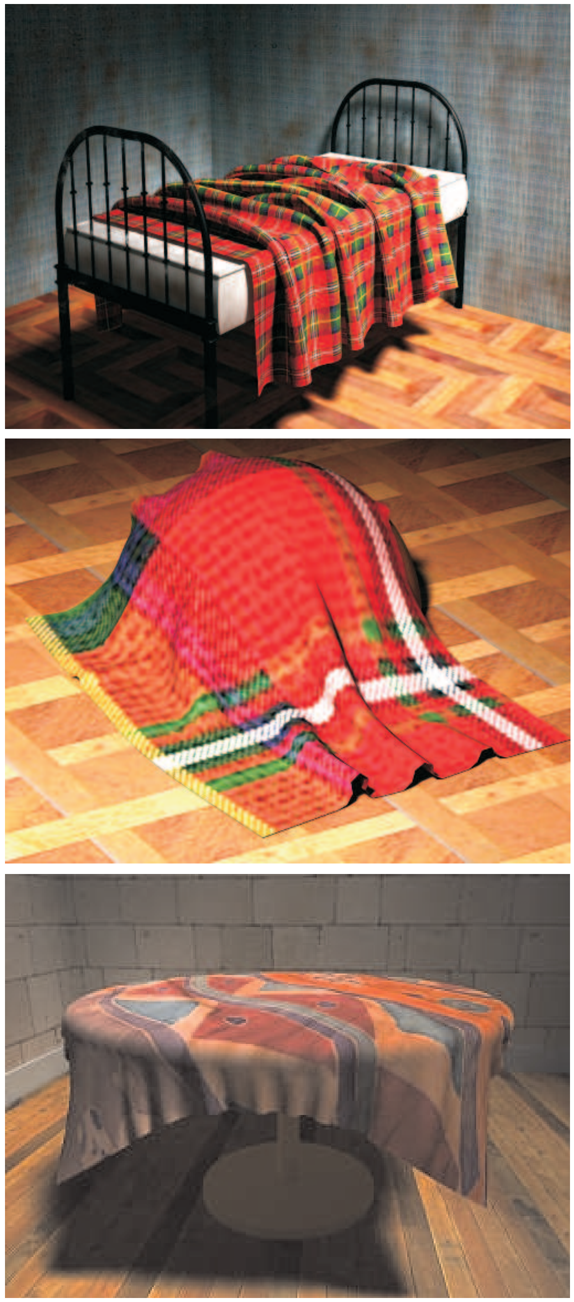


Figure 11: Simulation of clothes using our technique. Since hysteresis is a very subtle effect it is not possible to display any images having this characteristic. However, these results show that our physical model is able to produce synthetic images directly using the KES curves. Indeed, these images have been generated using the KES curves of figures 10.

References

[1] G. M. Abbot, P. Grosberg, and G.A.V. Leaf. The elastic resistance to bending of plain-woven fabrics. 64(6):346, June 1973.

[2] G. Amontons. On the resistance originating in machines. In *French Royal Academy of Sciences*, pages 206–222, 1699.

[3] J. Armibayat and J.W.S. Hearle. The anatomy of buckling of textile fabrics: drape and conformability. 80:51–69, 1989.

[4] U. Ascher and E. Boxerman. On the modified conjugate gradient method in cloth simulation. *The Visual Computer*, (to appear).

[5] D. Baraff and A. P. Witkin. Large steps in cloth simulation. In *Computer Graphics (Proceedings of SIGGRAPH 98)*, Computer Graphics Proceedings, Annual Conference Series, pages 43–54, Orlando, Florida, July 1998. ACM Press.

[6] Kiran S. Bhat, Christopher D. Twigg, Jessica K. Hodgins, Pradeep K. Khosla, Zoran Popovic', and Steven M. Seitz. Estimating cloth simulation parameters from video. In *Proceedings of the 2003 ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, pages 37–51. Eurographics Association, 2003.

[7] P.-A. Bliman. Mathematical study of the dahl's friction model. *european Journal of Mechanics. A/Solids*, 11(6):835–848, 1992.

[8] P.-A. Bliman and M. Sorine. Fricition modelling by hysteresis operators. application to dahl, stiction and stribeck effects. In *Proceedings of the conference "Models of Hysteresis"*, Trento, Italy, 1991.

[9] P.-A. Bliman and M. Sorine. Easy-to-use realistic dry friction models for automatic control. In *Proceedings of 3rd European Control Conference*, pages 3788–3794, Rome, Italy,, 1995.

[10] D. E. Breen and D. H. House. A physically based model of woven cloth. *the Visual Computer*, 8(5-6):264–277, June 1992.

[11] D.E. Breen, D.H. House, and M.J. Wozny. A particle-based model for simulating the draping behavior of woven cloth. *Textile Research Journal*, 64(11):663–685, November 1994.

[12] D.E. Breen, D.H. House, and M.J. Wozny. Predicting the drape of woven cloth using interacting particles. In *Computer Graphics (Proceedings of SIGGRAPH 94)*, Computer Graphics Proceedings, Annual Conference Series, pages 365–372, Orlando (Floride), July 1994. ACM Press.

[13] R. Bridson, S. Marino, and R. Fedkiw. Simulation of clothing with folds and wrinkles. In *Proceedings of the 2003 ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, pages 28–36. Eurographics Association, 2003.

[14] Robert Bridson, Ronald P. Fedkiw, and John Anderson. Robust treatment of collisions, contact, and friction for cloth animation. *ACM Transactions on Graphics*, 21(3):594–603, July 2002.

[15] Z. CAI. A nonlinear viscoelastic model for describing the deformation behavior of braided fiber seals. 65(8):461–470, August 1995.

[16] B. Chen and M. Govindaraj. A parametric study of fabric drape. *Textile Research Journal*, 66(1):17–24, January 1996.

[17] Kwang-Jin Choi and Hyeong-Seok Ko. Stable but responsive cloth. *ACM Transactions on Graphics*, 21(3):604–611, July 2002.

[18] C. C. Chu, C. L. Cumings, and N. A. Teixeira. Mechanic of elastic performance of textile material, part V: A study of the factors affecting the drape fabric - the development of a drape meter. 20(1):66–67, January 1950.

[19] J. R. Colier, B. J. Collier, G. O'Toole, and S. M. Sargand. Drape prediction by means of finite-element analysis. 82(1):96–107, 1991.

[20] Sabine Coquillart. Extended free-form deformation: A sculpturing tool for 3d geometric modeling. In *Computer Graphics (Proceedings of SIGGRAPH 90)*, volume 24, pages 187–196, August 1990.

[21] C. A. Coulomb. Théorie des machines simples. *Mémoires de Mathématiques et de Physique de l'Académie des Sciences*, pages 161–331, 1785.

[22] J. Courtney-Pratt and E. Eisner. The effect of a tangential force on the contact of metallic bodies. In *Proceedings of the Royal Society*, volume A238, pages 529–550, 1957.

[23] P. Dahl. A solid friction model. Technical report, The aerospace Corporation, El Segundo, CA, 1968.

[24] Mathieu Desbrun, Peter Schröder, and Alan Barr. Interactive animation of structured deformable objects. In *Graphics Interface*, pages 1–8, Kingston, Ontario, June 1999.

[25] B. Eberhardt, O. Etzmuss, and M. Hauth. Implicit-explicit schemes for fast animation with particles systems. pages 137–151, 200.

[26] B. Eberhardt, A. Weber, and W. Strasser. A fast, flexible, particle system model for cloth draping. *IEEE Computer Graphics and Applications*, 16(5):52–59, September 1996.

[27] J. W. Eischen, S. Deng, and T. G. Clapp. Finite-element modeling and control of flexible fabric parts. *IEEE Computer Graphics and Applications*, 16(5):52–59, September 1996.

[28] T.K. Gosh, S.K. Batra, and R.L. Baker. The bending behavior of plain-woven fabric, part III: Thread-bending behavior and the effect of fabric set. 81(3):272–287, 1990.

[29] P. Grosberg, S. Kedia, N. M. Swani, and B. J. Park. The mechanical properties of woven fabrics part I to VI. *Textile Research Journal*, 36:part I 71–79, part II 205–211, part III 332–337, part IV 338–345, part V 421–431,part VI 1085–1100, 1966.

[30] D. H. House and D. E. Breen, editors. *Cloth Modeling and Animation*. A K Peters, 2000.

[31] S. De Jong and R. Postle. A general analysis of fabric mechanics using optimal control theory. 48(2):125–137, February 1978.

[32] A. Joukhadar, C. Bard, and C. Laugier. Combining geometric and physical models, the case of a dextrous hand. In *Proc. of the IEEE-RSJ Int. Conf. on Intelligent Robots and Systems*, Munchen (DE), September 1994.

[33] A. Joukhadar, F. Garat, and Ch. Laugier. Parameter identification for dynamic simulation. In *IEEE Int. Conf. on Robotics and Automation*, pages 1928–1933, Albuquerque, NM, USA, 1997.

[34] D. Karnopp. Computer simulation of slipstick friction in mechanical dynamic systems. *Journal of Dynamics Systems, Measurement, and Control*, 107(1):100–103, 1985.

[35] S. Kawabata. The standardisation and analysis of hand evaluation. Technical report, The textile machinery society of Japan, Osaka, July 1980.

[36] S. Kawabata, M. Niwa, and H. Kwai. The finite deformation theory of plain weave part I to III. (2):21–85, February 1973.

[37] C.T. Kelley, editor. *Iterative Methods for Linear and Nonlinear Equations (Frontiers in Applied Mathematics, Vol. 16)*. Society for Industrial & Applied Mathematics, 1995.

[38] A. Kemp. An extension of peirce's cloth geometry to the treatment of non-circular threads. 49(1):44–48, January 1958.

[39] Jean Louchet, Xavier Provot, and David Crochemore. Evolutionary identification of cloth animation models. In Dimitri Terzopoulos and Daniel Thalmann, editors, *Computer Animation and Simulation '95*, pages 44–54. Springer-Verlag, 1995.

[40] D. W. Marquardt. An algorithm for least-squares estimation of nonlinear parameters. *Journal of the Society for Industrial and Applied Mathematics*, 11:431–441, 1963.

[41] M. Meyer, G. Debunne, M. Desbrun, and A. H. Barr. Interactive animation of cloth-like objects in virtual reality. *The Journal of Visualization and Computer Animation*, 12(1):1–12, may 2001. animation, implicit integration, cloth.

[42] Hidehiko Okabe, Haruki Imaoka, Takako Tomiha, and Haruo Niwaya. Three dimensional apparel cad system. In *Computer Graphics (Proceedings of SIGGRAPH 92)*, volume 26, pages 105–110, July 1992.

[43] B. Olofsson and N. Oguchi. A theory of elasto-plastic buckling. *Textile Research Journal*, 36(4):295–309, April 1966.

[44] F. T. Peirce. The handle of cloth as measurable quantity. 21:377–416, May 1930.

[45] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery. *Numerical recipe in C*. Cambridge University Press, second edition, 1992.

[46] Xavier Provot. Deformation constraints in a mass-spring model to describe rigid cloth behavior. In Wayne A. Davis and Przemyslaw Prusinkiewicz, editors, *Graphics Interface '95*, pages 147–154. Canadian Human-Computer Communications Society, 1995.

[47] E. Rabinowicz. The nature of the static and kinetic coefficients of friction. *Journal of applied physics*, 22(11):1373–1379, 1951.

[48] M.L. Realff. Identifying local deformation phenomena during woven fabric uniaxial tensile loading. *Textile Research Journal*, 64(3):135–141, March 1994.

[49] M.L. Realff, M.C. Boyce, and S. Backer. A micromechanical model of the tensile behavior of woven fabric. *Textile Research Journal*, 67(6):445–459, June 1997.

[50] R. Stribeck. The key qualities of sliding and roller bearings. *Zeitschrift des Vereines Seutscher Ingenieure*, 46(28,39):1342–1348,1432–1437, 1902.

[51] D. Terzopoulos and K. Fleischer. Deformable models. *The Visual Computer*, 1988(4):306–331, 1988.

[52] D. Terzopoulos, J. Platt, A. Barr, and K. Fleischer. Elastically deformable models. In *Computer Graphics (Proceedings of SIGGRAPH 87)*, volume 21 of *Computer Graphics Proceedings, Annual Conference Series*, pages 205–214, Anaheim, California, July 1987. ACM Press.

[53] D. Terzopoulos and A. Witkin. Physically based model with rigid and deformable components. *IEEE Computer Graphics and Applications*, pages 41–51, December 1988.

[54] L. Vangheluwe. Relaxation and inverse relaxation of yarn after dynamic loading. 63(9):552–556, September 1993.

[55] Pascal Volino and Nadia Magnenat-Thalmann. Comparing efficiency of integration methods for cloth simulation. In *Computer Graphics International 2001*, pages 265–272, July 2001.

[56] Jerry Weil. The synthesis of cloth objects. In *Computer Graphics (Proceedings of SIGGRAPH 86)*, volume 20 of *Computer Graphics Proceedings, Annual Conference Series*, pages 49–54, Dallas, Texas, August 1986. ACM Press.

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