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Abstract: We analyze the performance of connected dominating set (CDS)election protocols in wireless ad hoc networks. We compare the dominating set made from MPR and a new connected dominating set (NCDS) protocols issued from a straightforward generalization of the work of Wu and Li. We analyze the impact on OLSR protocol.

Key-words: multipoint relays, connected dominating set, ad hoc network

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Performance des ensemble dominants dans le protocole OLSR

Résumé : Nous analysons les performances de protocoles d'élection d'ensembles dominants (CDS) dans un réseau sans fil ad hoc. Nous comparons les ensembles dominants faits par les MPR et un nouvel algorithme de sélection d'ensemble dominant (NCDS) obtenu en généralisant les travaux de Wu et Li. Nous examinons l'impact sur le protocole OLSR.

Mots-clés : relais multipoint, diffusion, réseau ad-hoc, ensemble connexe dominant

1 Introduction

We analyze the performance of connected dominating set (CDS)election protocols in wireless ad hoc networks. We compare the dominating set made from MPR [4, 5] and a new connected dominating set (NCDS) protocols issued from a straightforward generalization of the work in [1]. Both protocols only need that every node in the network knows the neighborhood of their neighbors. This can be easily attained by making nodes advertizing a list of their neighbor identifiers in periodic hellos. It turns out that in a unit disk graph model, the MPR CDS outperform NCDS up to an average node degree m of 20. Above this average degree, NCS outperforms MPR CDS significantly. In particular when the density d of the network increases the average density of NCDS relay nodes tends to a constant limit, while the MPR CDS density slightly augments in $m^{1/3}$.

The paper is organized as follow. In the second section we presents the new CDS protocol with its proof of functioning, in the third section we provide simulation results on the performance of MPR CDS and NCDS protocols as well as analytical results about the asymptotic properties of these performances. In the fourth section we outline some applications in the OLSR protocol.

2 The New Connected Dominating Set protocol

In [1] WuLi present a sequence of CDS protocols that we call k-CDS. In a k CDS every host decides of its membership in CDS by scanning the database of its neighbor neighborhoods. It must check all subsets of k neighbors or less such that the following set of properties holds

1. all selected neighbors has ID larger than host ID;
2. the selected neighbors form a single connected component
3. the selected neighbors cover the whole host neighorhood, *i.e.* every host neighbor is neighbor of at least one selected neighbor

A node belongs to the k-CDS when no such subset exists satisfying the above properties. This k-CDS specification was coming as a full step generalization of the 1-CDS and 2-CDS protocols described in previous WuLi publications [2].

However searching for all subsets of k neighbors is an expensive procedure, needing $O(m^k)$ operations. We therefore propose a straightforward generalization equivalent to k-CDS with $k = \infty$, but with a much less expensive processing cost, and that we call NCDS.

The specification of NCDS protocol is the following: a node belongs to the NCDS set if and only if any the following properties is not satisfied by the set of all neighbor of ID larger than host ID, that we will call the ID-dominator neighbor set:

1. it forms a single connected component;
2. it covers all the host neighbor

It should be noticed that it takes less than $O(m^2)$ to check that the ID-dominator neighbor set is connected and that it covers the whole host neighborhood. In fact the two conditions for not being in the NCDS relay set is equivalent to the fact that the ID-dominator neighbor set is a CDS of the node's neighborhood.

Generalization to any kind of ID, priority, willingness, neighbor size. If one wants that NCDS relay set covers every node a minimum number of times, say ℓ times, for $\ell \geq 1$ one has to change the last conditions for NOT being a member of the NCDS into:

- the selected neighbors cover the whole host neighorhood, *i.e.* every host neighbor is neighbor of at least ℓ selected neighbors.

Notice that the parameter ℓ, NCDS-coverage, can be tuned locally and determined via local consideration of link qualities.

2.1 Proof of correctness of NCDS protocol

We have to prove that that the nodes that belongs to the NCDS set forms a CDS. For this purpose we take an arbitrary connected network. We will start from the whole set of neighbor node and manipulate it in a finite number of steps that will transform the set in the NCDS set, and show that each of this steps keeps the CDS property.

Each of this step consists in the following operation:

1. Select in the current set a node which does not belong to the NCDS set;
2. Remove this node from the current set;
3. insert in the current set the ID-dominator neighbor set of the removed node.

Notice that the ID-dominator neighbor set is established with respect to the original whole set of nodes of the networks, therefore new nodes maybe inserted in the current set. The decrease of the current set to the NCDS set is not completely a trivial matter.

There is no question that the initial current set before the first step forms a CDS. First we prove that each step keeps the CDS property. This is obvious because the removed node is replaced by a connected subset which covers the whole neighborhood of the removed node. If a pair of nodes is connected by the removed node then there exists a connected chain in the ID-dominator neighbor set that connects the pair.

Second we will prove that we can apply the steps only a finite number of times. In fact it is a well known result that any increasing order replacement automaton operating on a subset of a finite set of elements must terminate in a finite number of steps. An increasing order replacement automaton is an automaton which select an arbitrary element in the current subset and replace it by an arbitrary subset of elements of the original set which are of strictly higher order than the removed element.

Third since the steps sequence finishes when no longer no NCDS node remains, and since all NCDS node remain, the terminal status of the current set is exactly the NCDS set.

3 Quantitative results

In this section we provide results on the performance of the NCDS algorithm on networks that follow the unit disk graph model.

3.1 Simulation

In this section we simulate via Maple program the behavior of CDS made by MPR and CDS made by NCDS. We consider a unit disk graph model. We will consider two cases:

- Finite networks
- Infinite networks

In the finite network we consider a square of side length L, as map of the network and we randomly dispatch N nodes in the square. We will consider $N = 100$, 200, 1,000 and 2,000. We consider as parameter the average neighborhood size number m of a central nodes. We adapt L in such a way that $m = \pi N/L^2$. This model has been introduced in [3].

In the case of the infinite network we consider a random node and we compute its MPR set and the CDS nodes belonging to its neighborhood, based on a random one hop and two-hop neighborhood. In other words we simulate the network in a disk of radius 2 centered on this node.

3.1.1 Finite networks

In the simulation we select a random point in the square and we compute its MPR set according to a simulated neighborhood and two hop neighborhood that fit with the square limits. In [3] it has been checked that the probability that a node is involved in an MPR flooding is very close to the ratio beween its number of MPR selectors and the total number of its neighbor. Therefore it suffices to compute a per node average of the sum of its weighted MPR, the weight being the inverse of the number of neighbors of this MPR (including itself) in order to get the average ratio of nodes involved in an MPR CDS.

About the NCDS node the same operations apply. Instead of computing the MPR node of our randomly selected point, we compute the members of the NCDS relay set that belongs to the neighborhood of the point. For this we need to know the network graph up to two hops. The probability that a CDS node be neighbor of a random point being equal to the ratio of the number of neighbor of this node over the total number of nodes in the network. Therefore the average fraction of NCDS relay nodes in the network is equal to the per node average of the sum of the weighted NCDS relay nodes that belong to the node neighborhood, the weight being equal to the inverse of the neighborhood size of the NCDS node (including itself).

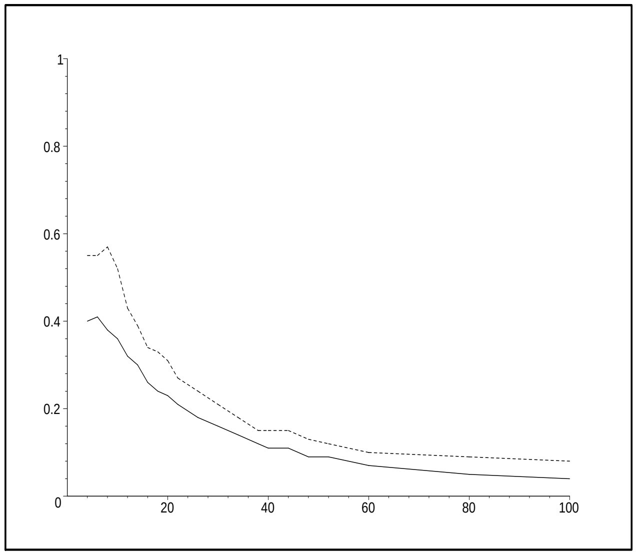

Figure 1: Percentage of nodes involved in an MPR CDS (plain) and in NCDS (dashed) versus m, in a square network of $N = 100$.

3.1.2 Infinite networks

In this subsection we show simulation results of the case where the network is unbounded. In this case the map is the whole plan and the node density is uniform and constant. The total number of nodes is infinite of course, but this is not a problem since the simulation are always performed by simulated the two-hop neighborhood of a random nodes. Instead of providing the percentage of nodes involved in the flooding we display the average number of CDS nodes per unit area.

3.2 Infinite density

A nice property of NCDS is that it produces CDS of a finite density. In other if $\mu(m)$ is the density of the CDS when the average node degree is m then

$$\lim_{m \to \infty} \mu(m) = \mu < \infty \tag{1}$$

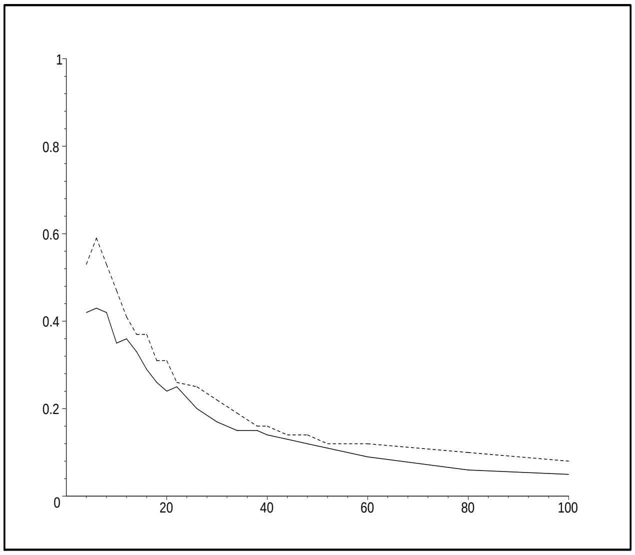

Figure 2: Percentage of nodes involved in an MPR CDS (plain) and in NCDS (dashed) versus m, in a square network of $N = 200$.

We will prove this property in the context of infinite network (unbounded map). This property contrast with CDS made of MPR where the density increase in $m^{1/3}$ [6].

In the sequel we assume, without loss of generality that the ID of a node is a random number uniformly selected in the interval $(0, 1)$. The number of neighbor of a random point follows a Poisson distribution of mean m. For a given point of ID $\theta \in (0, 1)$, the number k of neighbors of higher ID is independent of the number n of neighbors of lower ID. The first number follows a Poisson distribution of mean $(1 - \theta)m$ and the second number, a Poisson distribution of mean θm.

Let $p_{k,n}$ be the probability that k points randomly selected in the unit disk do not cover n other points randomly selected on the unit disk or do not form a single connected component. It should also be noted that $p_{k,n}$ is a decreasing functions in k but an increasing function in n. Let $P(x, y) = \sum_{k,n} p_{n,k} \frac{x^k y^n e^{-x-y}}{k!n!}$. The probability that a node of ID equal to θ belongs to the CDS is exactly $P((1 - \theta)m, \theta m)$. Therefore the density of CDS node

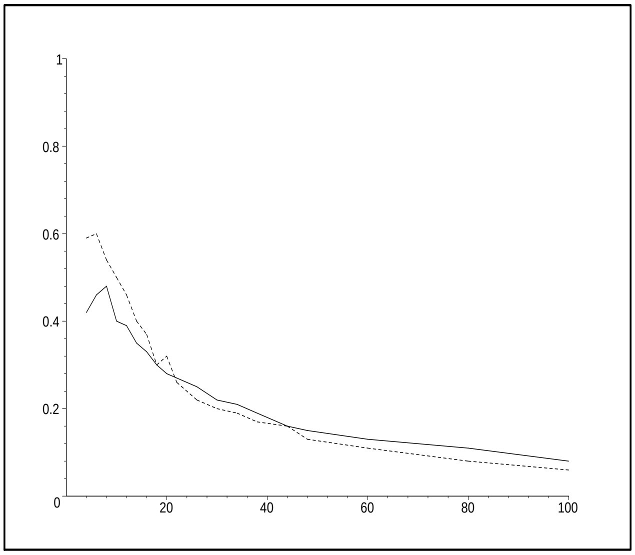

Figure 3: Percentage of nodes involved in an MPR CDS (plain) and in NCDS (dashed) versus m, in a square network of $N = 1000$.

regardless of ID is

$$\mu(m) = m \int_0^1 P((1-\theta)m, \theta m) d\theta \tag{2}$$

In order to simplify this formulation, we introduce the following parameter. Let p_k denote the probability that k points randomly selected in the unit disk does not form a single connected component or does not cover the whole unit disk, *i.e.* there are points in the unit disk which does not belong to the union of the unit disks centered on the selected points. We have $p_{k,n} < p_k$ for all n. Indeed $\lim_{n\to\infty} p_{k,n} = p_k$. We have $p_0 = p_1 = p_2 = 0$, $p_3 > 0$. Let ℓ such that $p_\ell \neq 0$.

Denoting $P(x) = \sum_k p_k \frac{x^k e^{-x}}{k!}$. Therefore

$$\mu(m) \leq \int_0^m P(x) dx \tag{3}$$

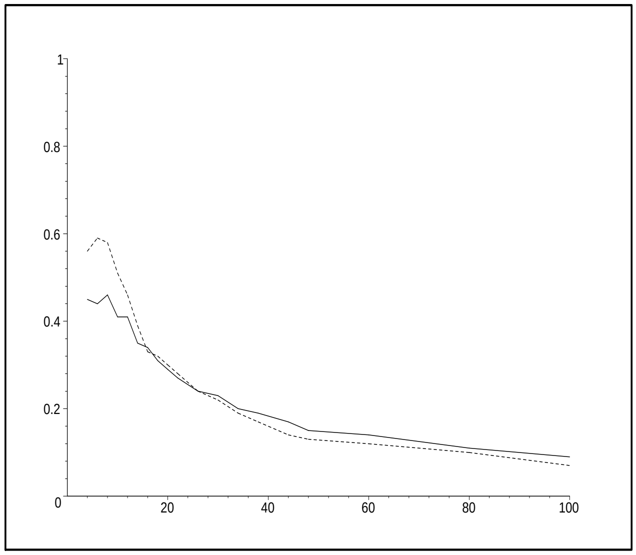

Figure 4: Percentage of nodes involved in an MPR CDS (plain) and in NCDS (dashed) versus m, in a square network of $N = 100$.

Remains to prove that $P(x)$ can be integrated on the whole positive axis and to provide some quantitative estimate.

It is clear that if a set of k points doesn't form a single connected component AND does not cover the unit disk then it contains no subset of ℓ elements which form a connected component AND cover the whole unit disk. Under this consideration it comes that $p_k \leq p_\ell^{\langle k/\ell \rangle}$. Consequently $p_k \leq p_\ell^{k/\ell - 1}$ and $P(x) \leq \exp(-(1 - \frac{p_\ell}{\ell})x)\frac{1}{p_\ell}$. Thus

$$\mu(m) \leq \frac{1}{(1 - \frac{p_\ell}{\ell})p_\ell} \tag{4}$$

4 Impact on OLSR

The OLSR protocol uses MPR for two purposes:

- To form an optimized CDS

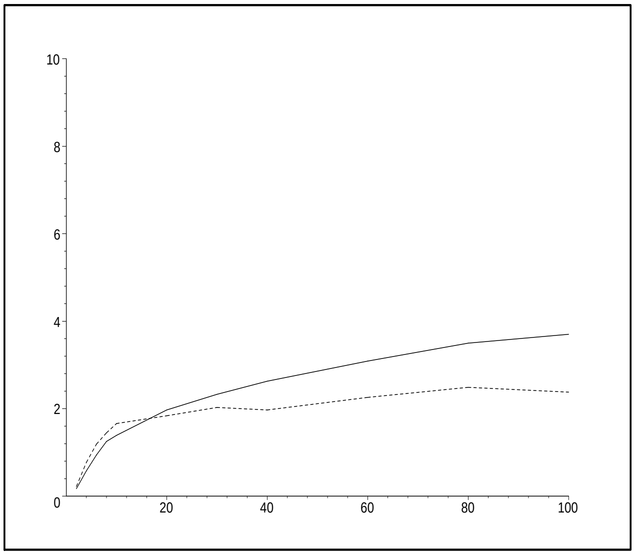

Figure 5: Density of nodes per square unit area involved in an MPR CDS (plain) and in NCDS (dashed) versus m in an infinite network.

- to optimize link state advertizement: nodes only advertize their MPR selectors set.

One can use the NCDS set in order to perform the optimized flooding of data and control. The advantage over genuine MPR flooding is that a node does not need to identify the last hop transmitter in order to decide about retransmission. Anyhow the node will still need to perform duplicate detection in order to avoid broadcast loops. Notice that the NCDS relay node are not selected by their neighbors since the selection is made comparing neighborhoods via hello packet. In other words NCDS relay nodes select themselves as relay.

However one must keep the MPR selection process in order to keep the advertized link state information to a minimal level. The MPR are necessary in order to route length to optimal. An increase of route lengths by a factor of 10advertize only links to NCDS nodes then the route optimality will be greatly affected. The stretch factor of the NCDS relay set is unknown but probably is in the typical range of 2 and 3. The only way to keep route optimality is to advertize kinda MPR selector links.

An interesting feature is that a node which is close to a node can inform the latter whether or not it belongs to the NCDS relay set. In particular a neighbor can detect if a node does not belong to the NCDS relay set. For example, node A has a lot of processing resource but very little transmit power. On the contrary node B has full transmit power but very little processing power. By default dumb node B will consider itself as member of the NCDS relay set. Node A will be enable to act as a relay node but by receiving the hellos from B's neighbors it will be able to determine whether or not node B belongs or not to the NCDS relay set. Doing the processing in place of node B and realizing that node B does not belong to the NCDS relay set it will consistently inform node B in a special hello.

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