Connectivity in Ad-Hoc Networks: an Infinite-Server Queue Approach

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Abstract: In this paper we present some extensions on previously published results regarding connectivity issues in one–dimensional ad–hoc networks. We show how an equivalent $GI|D|\infty$ queueing model may be used to address the issue, and present connectivity results on both infinite and finite networks for various node placement statistics. We then show how a $GI|G|\infty$ model may be used to study broadcast percolation problems in ad–hoc networks with general node placement and random communication range. In particular, we obtain explicit results for the case of nodes distributed according to a Poisson distribution operating in a fading environment. In case of nodes distributed according to a Poisson point process, heavy traffic theory is applied to derive the critical communication range for connectivity and the critical transmission power for broadcast percolation in dense networks. The analysis is then extended to the case of unreliable ad–hoc networks, with an in–depth discussion of asymptotic results.

Key-words: $GI|G|\infty$ queue, connectivity, ad–hoc networks, heavy traffic

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Connectivité des Réseaux Ad–Hoc: une Approche Basée sur les Files à une Infinité de Serveurs

Résumé : Dans cet article nous présentons des extensions des résultats dejà publiés sur la connectivité des réseaux ad–hoc unidimensionels. Nous montrons qu'une file d'attente $GI|D|\infty$ équivalente peut être utilisée pour étudier ce problème, et nous présentons des résultats pour des réseaux finis et infinis avec diverses distributions de placement des nœuds. De plus, nous montrons comment un modèle $GI|G|\infty$ peut être utilisé pour étudier les problèmes de *broadcast percolation* dans les réseaux ad–hoc avec un placement général des nœuds et une portée de communication aléatoire. En particulier, nous obtenions des résultats explicites pour une distribution de nœuds poissonnienne et un environnement avec *fading*. Pour le cas des nœuds distribués selon un processus ponctuel de Poisson, la théorie de forte charge est utilisée pour obtenir la portée critique de communication et la puissance critique de transmission pour des réseaux denses. L'analyse est ensuite étendue aux réseaux non fiables, avec une discussion approfondie des résultats asymptotiques.

Mots-clés : file d'attente $GI|G|\infty$, connectivité, réseaux ad–hoc, forte charge

1 Introduction

The growing interest in the field of self–organizing largely deployed networks has lead to a series of research regarding the limiting performance achievable, both in terms of connectivity and capacity. In this paper we address connectivity issues arising in one–dimensional ad–hoc networks. While some literature exists on the subject (see for instance [1, 2, 3, 4] and the references therein), we provide in this work a framework, based on queueing–theoretical tools, which allows for an extension of the existing results. While the relation between coverage problems and queueing tools is an old and well–estabilished one (see ad example [5]), we show that, using some results on $GI|G|\infty$ queues, extensions to previously published results may be found, both in terms of connectivity for networks with general node placement and for broadcast percolation problems in fading channels. For nodes placed according to a Poisson point process, the limiting performance for very dense networks may be studied by considering heavy traffic limits for the equivalent queuing model, leading to a critical transmission range (resp. transmission power) for the the connectivity (resp. broadcast percolation) problem. In particular, the critical transmission range in the $M|D|\infty$ model turns out to be the same found by Gupta and Kumar [6]. Eventually, we study the impact of node failures on connectivity issues. A general discussion of how to incorporate this issue into the general framework is provided, and an in–depth analysis for asymptotics is presented. In particular, for the $M|D|\infty$ case, we retrieve results equivalent to those find by Shakkottai et al. [7] for grid networks. A discussion of the obtained results and of some open issues concludes the paper.

2 A $GI|D|\infty$ Model for Connectivity in One–Dimensional Ad–Hoc Networks

Let us consider a one–dimensional network, where nodes are randomly placed along a semi–infinite line. Let us denote by X_n the position of the n–th node, with $X_0 = 0$. Further, we denote by $Y_n = X_{n+1} - X_n$ the distance between two successive nodes. In the following, we will assume that $\{Y_n\}$ forms a sequence of i.i.d. symbols, so that we may drop the index n. Consider the simplest model for propagation, where nodes are connected if and only if their distance is less than a given value $R > 0$. We call R the communication range of the nodes. In such a situation it is a well known result that, for any finite R, the resulting network will be disconnected P–almost surely [3]. Furthermore, the network will be P–almost surely divided into an infinite number of finite clusters, which will be referred to in the following as "spatial cluster". In order to characterize the cluster statistics, let us consider an equivalent $GI|D|\infty$ model, with i.i.d. interarrival times distributed as Y and fixed service time R. Then, a spatial cluster in the ad–hoc network corresponds to a busy period in the queueing model.

Let $F_Y(a)$ and $f_Y(a)$ be the cdf and the pdf for the interarrival time Y. We denote by B_n and N_n, respectively, the duration of the n–th busy period and the number of clients served therein. Since the busy periods are i.i.d., we drop the index n in the following. Then the LST of the busy period is given by [8]:

$$\mathcal{B}(s) = \frac{e^{-sR}(1 - F_Y(R))}{1 - \int_0^R e^{-st} f_Y(t)dt}. \tag{1}$$

While in general this expression may not be inverted directly, and one has to resort to numerical methods [9], the moments of any order may easily be computed by differentiation. For example, the average busy period duration is given by:

$$E[B] = -\frac{\partial \mathcal{B}(s)}{\partial s}\bigg|_{s=0} = R + \frac{\int_0^R t f_Y(t)dt}{1 - F_Y(R)} \tag{2}$$

Accordingly, the pgf of the number of clients served in a busy cycle is given by [8]:

$$\mathcal{N}(z) = z\frac{1 - F_Y(R)}{1 - zF_Y(R)}. \tag{3}$$

Inverting, we find the pmf of the random variable N:

$$P[N = k] = (1 - F_Y(R)) \cdot F_Y(R)^{k-1}, \quad k = 1, 2, \ldots \tag{4}$$

The average number of clients served is thus given by:

$$E[N] = \frac{1}{1 - F_Y(R)}.$$

Clearly, the number of nodes in a cluster is given by $\Psi = N$, whereas the area covered by a cluster is given by $T = R + B$. A parameter of interest is the probability p_I that a given point X is isolated, which corresponds to a customer which finds an empty queue upon its arrival. Thus:

$$p_I = P[\Psi = 1] = 1 - F_Y(R). \tag{5}$$

Note that, trivially, $p_I = E[N]^{-1}$. This means, roughly speaking, that the smaller the probability of a node being isolated, the bigger the average number of nodes in a cluster. Hence, the probability p_I may be taken as a meaningful metric for understanding the question of how one should place nodes to form large cluster. As an example, in Fig.1 we plotted p_I vs. the mean distance between adiacent nodes for various node placement statistics. Note that, for the normal distribution, also a negative inter–node distance is, in principle, feasible: it is thus clear that the obtained results are of interest only for an average value $E[Y]$ sufficiently large. The behavior of heavy–tailed distributions (in our case a Pareto r.v. with shape parameter β set to 1.5) is worth some comments. In fact, from (5) we get that p_I depends only on the value $F_Y(R)$. Let us analyze two extreme cases, "sparse" and "dense" networks, respectively. Basically, a sparse network is one in which the density of nodes is low with respect to the transmission range, i.e. $\lambda R \ll 1$, whereas, on the contrary, in a dense network we have that $\lambda R \gg 1$. In order to study the effect of the tail distribution on the network connectivity, let us focus on two distribution, namely exponential and Pareto. For a given transmission range R and average value $E[Y] = \lambda^{-1}$, we have that the two distribution cross at $\lambda^*(R)$, where $\lambda^*(R)$ is the unique solution of the equation:

$$e^{-\lambda R} = \left(\frac{\beta - 1}{\beta \cdot \lambda R}\right)^{\beta}, \tag{6}$$

for which also $\lambda R \geq \frac{\beta}{\beta-1}$ holds. For $\beta = 1.5$, ad example we get $\lambda^* = \frac{3.5452}{R}$. Then, a network is said to be dense if, for a given R, $\lambda > \lambda^*(R)$. Otherwise, it is said to be sparse. Let us start by analyzing the latter. In this case, Pareto distribution overcomes the exponential one. In dense networks, on the other hand, the tail distribution plays a central role in connectivity performance. In this case, the heavy–tailed behavior of Pareto leads to a penalty in performance. Still, it remains a fact that, as shown in Fig. 1, for a very low density of terminal, and Pareto distribution, a node is isolated P–almost surely, which is due to the bias in the Pareto distribution. In this sense, it seems that there are two threshold: for very low density, exponential overcomes Pareto, while the roles swap for "medium" density, inverting again over $\lambda^*(R)$.

Another metric of interest which may be easily calculated for general node placement statistics is the probability that the k–th node is connected to the first one. This, in turns, may be obtained as:

$$p_R(k) = P[N \geq k] = \sum_{t=k}^{\infty} P[N = t] = (1 - F_Y(R)) \sum_{t=k}^{\infty} F_Y(R)^{t-1} = F_Y(R)^{k-1}. \tag{7}$$

Note that, again, it may be rewritten in terms of p_I, leading to:

$$p_R(k) = (1 - p_I)^{k-1}.$$

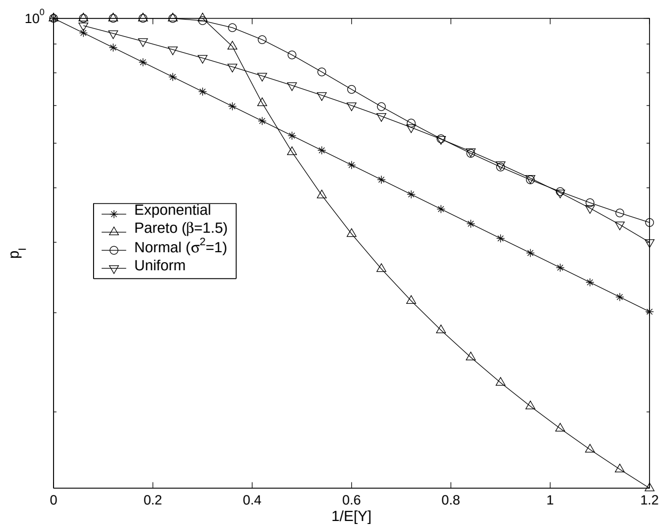

Figure 1: Probability of a generic node to be isolated for various node placement statistics vs. average node density, $R = 1$

The same considerations presented for p_I still holds for the cluster size distribution. As an example, for the exponential distribution we get:

$$p_R^{Exp}(k) = (1 - e^{-\lambda R})^{k-1},$$

whereas for Pareto–distributed inter–node distance we get:

$$p_R^{Par}(k) = \left(1 - \frac{E[Y](\beta-1)^{\beta}}{\beta R}\right)^{k-1}, \quad \text{for} R \geq \frac{E[Y](\beta-1)}{\beta},$$

where $E[Y]$ is the average distance between adjacent nodes.

In this case, as previously stated, the effect of the tail distribution becomes predominant as soon as the node density becomes sufficiently large: as an example we plotted the probability of a generic k–th node to be connected for both exponential and Pareto–distributed r.v. Y for two different node density. It is apparent that, whereas with an average of 1 nodes per transmission range Pareto–distributed nodes presents better connectivity performance, for the case of dense networks (in our case $\lambda R = 4$ nodes per transmission range) exponentially–distributed nodes tend to form bigger cluster, as expected.

To illustrate more in details the effect of the node density, in Fig.3 we plotted, for the case

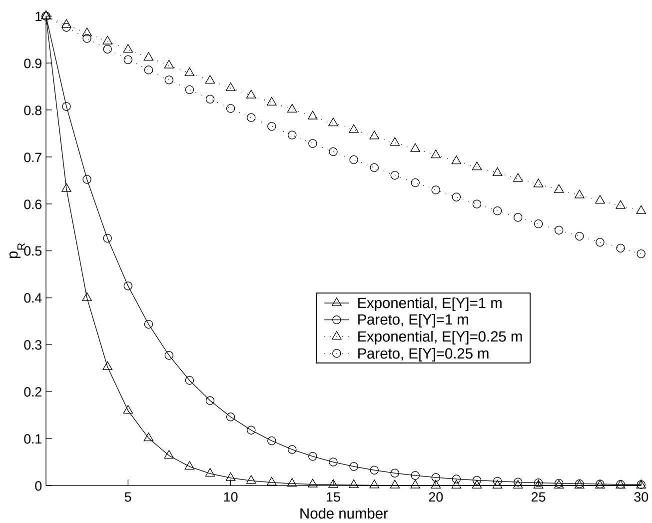

Figure 2: Probability of k–th node to be connected, $R = 1$, for Pareto and exponential inter–nodes distance

of Poissonian distribution, the probability of connection as a function of λR for various k.

The problem of connectivity in finite ad–hoc networks has been treated for the first time in [2], where the authors focus on a network with n nodes placed according to a Poisson process. We put ourselves in a different framework: given that there is a node at some point d in space, which is the probability that the node is connected to the node at $X_0 = 0$? This question may be reformulated in the equivalent queueing system by considering the pdf of the busy period duration:

$$p_C(d) = P[B > d] = 1 - F_B(d).$$

In general, to obtain the ccdf of the busy period one should perform the inversion of the pdf LST given in (1) or, directly, of the ccdf LST:

$$\hat{\mathcal{B}}(s) = \frac{1 - \mathcal{B}(s)}{s}. \tag{8}$$

In general, direct inversion of these LSTs is not possible, and, hence, one has to resort to numerical procedures to obtain the probability of being connected at a distance d [9].
One case of practical interest where direct inversion is possible is that of nodes distributed accorded to a Poisson distribution with average density λ. In this case the pdf LST is given

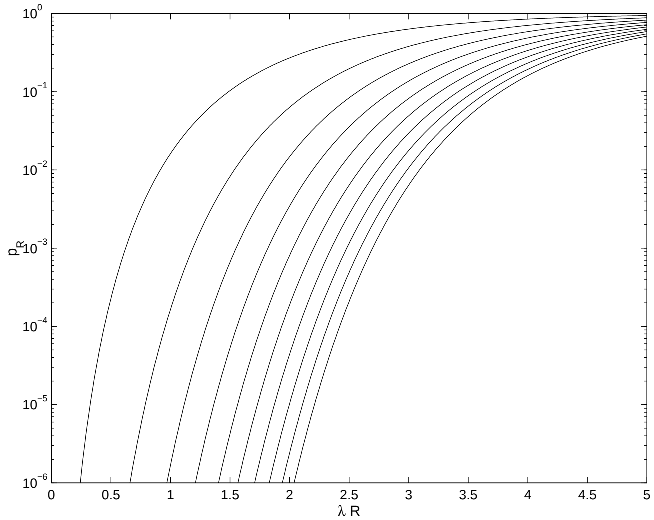

Figure 3: Probability of node k being connected ($k = 10, 20, \ldots, 100$) as a function of λR

by:

$$\mathcal{B}(s) = \frac{(s+\lambda)e^{-(s+\lambda)R}}{s + \lambda e^{-(s+\lambda)R}}, \tag{9}$$

whose inversion leads to [10, 11]:

$$f_B(t) = e^{-\lambda R}\delta(t-R) + 1(t-R)\cdot\lambda e^{-\lambda R} \sum_{k=0}^{\lfloor \frac{t}{b} \rfloor - 1} (-\lambda e^{-\lambda R})^k \cdot \frac{[t - R(k+1)]^k - e^{-\lambda R}\cdot \max\{0, [t-R(k+2)]^k\}}{k!}, \tag{10}$$

where $\delta(\cdot)$ is the Dirac delta function and $1(\cdot)$ represents the unit step function. Then, evaluation of $p_C(d)$ may be performed integrating (10), leading to the same expression obtained in [3] by solving a differential equation with appropriate boundary conditions. Some results are reported in Fig.4, Fig.5 and Fig.6. In particular, the arising of critical connectivity phenomena ([12, 6, 2, 13]) is evident from Fig.6. Note, however, that a critical connectivity range does arise in dense networks only, whereas for sparse networks the function tends to become much smoother. Furthermore, in Fig.7 we plotted the mean cluster extension, as a function of λ, for three different transmission ranges.

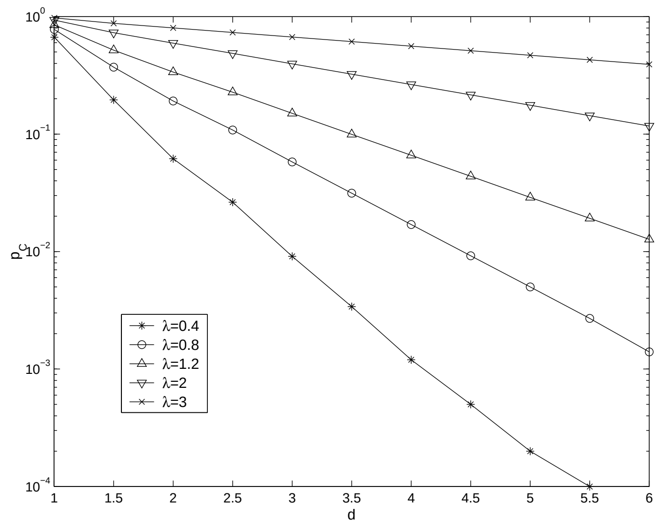

Figure 4: Probability of connection for a node placed at d for different values of λ, $R = 1$

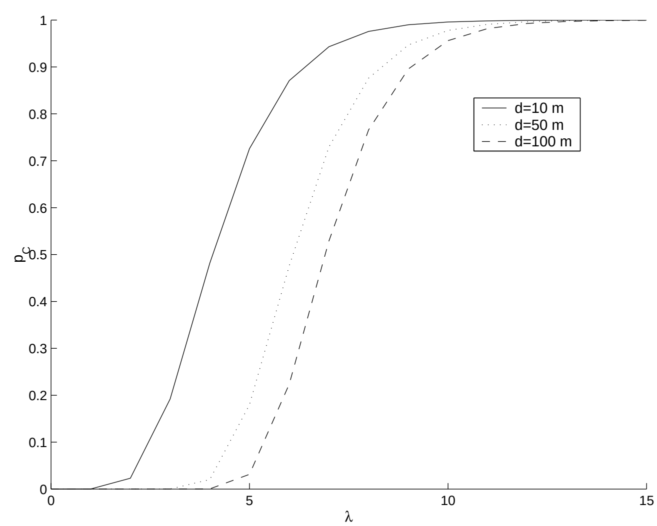

Figure 5: Probability of connection at d vs. λ, $R = 1$ m

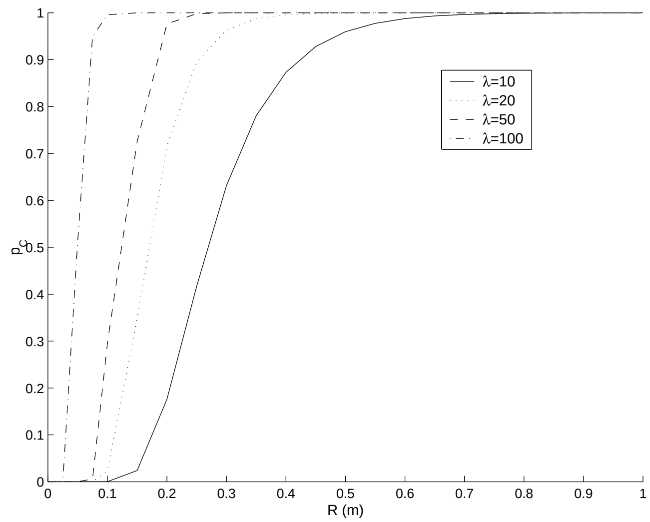

Figure 6: Probability of connection at $d = 1$ m vs. R for various node densities

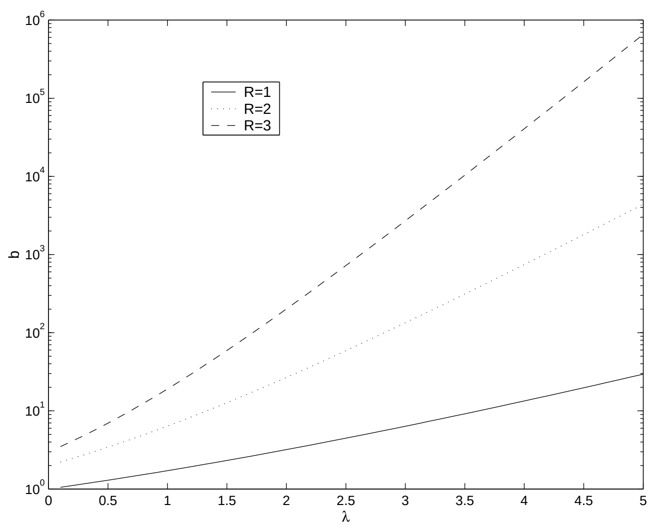

Figure 7: Mean cluster extension as a function of λ, $R = 1, 2, 3$.

2.1 Heavy Traffic Analysis for Dense Networks

Let us consider the limiting behavior of a $M|D|\infty$ network under heavy traffic; this corresponds, in the original system, to a network with a very high density of nodes. In this case, we know that the busy period, appropriately scaled, converges to an exponential distribution [11]:

Theorem 2.1. *As* $\lambda \to +\infty$,

$$F_B(aE[B]) \to 1 - e^{-a},\ a > 0. \tag{11}$$

Since the average busy period length is given by:

$$E[B] = \frac{e^{\lambda R} - 1}{\lambda}, \tag{12}$$

we have that a node at a distance D will be connected with probability

$$p_C(d) = P[B > d] = e^{-\frac{d}{E[B]}} = e^{-\frac{d\lambda}{e^{\lambda R}-1}}. \tag{13}$$

Thus, we may focus on the behavior of the function $f(\lambda) = \frac{d\lambda}{e^{\lambda R}-1}$. If $f(\lambda) \to 0$, then the network is asimptotically connected with high probability at a distance d. Now, let us assume that the transmission range is a function of λ of the type [13, 6]:

$$R(\lambda) = \frac{\ln \lambda + c(\lambda)}{\lambda}. \tag{14}$$

Then,

$$f(\lambda) = \frac{d\lambda}{e^{c(\lambda)}\lambda - 1}.$$

Taking the limit for $\lambda \to +\infty$, we get:

$$\lim_{\lambda \to +\infty} f(\lambda) = \frac{d}{e^c}, \tag{15}$$

where $c = \lim_{\lambda \to +\infty} c(\lambda)$. Thus, for any given d, a necessary and sufficient condition for asymptotical connectivity is that $c = +\infty$. Hence, we get an analogous of the result of Gupta and Kumar [6]:

Theorem 2.2. *Given a one–dimensional ad–hoc networks, with nodes placed according to a Poisson process of intensity* λ *and having a deterministic transmission range of* $R(\lambda) = \frac{\ln \lambda + c(\lambda)}{\lambda}$ *meters, the connectivity at a distance* d *is ensured whp as* $\lambda \to +\infty$ *iff* $\lim_{\lambda \to +\infty} c(\lambda) = +\infty$.

Similarly, we may compute the asymptotics for the vanishing of the probability of isolated nodes. As it may be easily expected, it turns out that the conditions are different, the one for connectivity being tighter, which shows that the widespread use of a sort of equivalence in the asymptotics for the two concepts may lead to erroneour results [13, 1].

Theorem 2.3. *Given a one–dimensional ad–hoc networks, with nodes placed according to a Poisson process of intensity* λ *and having a deterministic transmission range of* $R(\lambda) = \frac{\ln \lambda + c(\lambda)}{\lambda}$ *meters, the absence of isolated nodes at a distance* d *is ensured whp as* $\lambda \to +\infty$ *iff* $e^{-c(\lambda)} = o(\lambda)$[1].

Proof. The probability of a node being isolated is given by $P_I = e^{-\lambda R(\lambda)}$. Taking the limit, we get:

$$\lim_{\lambda \to +\infty} p_I = \lim_{\lambda \to +\infty} \frac{e^{-c(\lambda)}}{\lambda}, \tag{16}$$

and we are done. □

Trivially, we may conclude that the absence of isolated nodes is asymptotically ensured for any finite $c = \lim_{\lambda \to +\infty} c(\lambda)$.

3 A $GI|G|\infty$ Model for Broadcast Percolation in Ad–Hoc Networks

By applying much the same methods used in the previous section we may gain insight into the broadcast percolation problem in one–dimensional networks [12]. In this case, we may extend the framework to take into account a random characterization of the propagation channel, which, in turn, translates into a statistical characterization of the trasmission range R. In this case, no results can be drawn from the analysis in terms of connectivity. Since the transmission range is not deterministic any longer, nothing may be inferred in terms of the backward direction, or, equivalently, no reversibility property may be applied. In other words, if the message is able to propagate till to node k, this does not necessarily imply that a message may propagate from node k to node 0. In any case, we may get results for general trasmission range distribution and general node placement statistics, by applying results on the busy periods of $GI|G|\infty$ queue [8]. Note that, since we assume that a single message will be present in the network at any time, no interference issues arise in this framework [2].

[1] With the usual notation, $f(n) = o(g(n))$ if $\lim_{n \to +\infty} \frac{f(n)}{g(n)} = 0$.

[2] From an implementation perspective, our assumptions may be used to model either a system where transmitter are using orthogonal channels (either in the time, frequency or code domain) or, alternatively,

A characterization, in terms of distribution, of the transmission range for fading channels, which considers path loss, thermal noise, log–normal shadowing, Rayleigh fading and the impact of jamming nodes is provided in the appendix. Unless otherwise specified, the results of the following subsections will be derived, for the sake of clarity, upon the assumption of path loss of the form $d^{-\alpha}$ and Rayleigh fading.

3.1 A special case: $M|G|\infty$

Let us assume that the nodes are distributed along the semi–infinite line according to a Poisson point process with intensity λ. Furthermore, the transmission range is characterized by means of (52), or, accordingly, to any given cdf $F_R(a)$. Hence, from [8] the LST of the busy period of the equivalent $M|G|\infty$ queue is given by:

$$\mathcal{B}(s) = 1 + \frac{s}{\lambda} - \frac{1}{\lambda \mathcal{P}_0(s)}, \tag{17}$$

where $\mathcal{P}_0(s)$ is the Laplace transform of the function:

$$P_0(t) = e^{-\lambda \int_0^t P[R>a]da} = e^{-\lambda \int_0^t e^{-a^\alpha \beta} da}, \tag{18}$$

representing the probability that at time t the system is empty. Differentiating (17), we may easliy obtain the expression for the average broadcast extension:

$$b = \frac{1}{\lambda P_0} - \frac{1}{\lambda}, \tag{19}$$

where $P_0 = \lim_{t \to +\infty} P_0(t) = e^{-\lambda E[R]}$. The expression above may then be rewritten as:

$$b = \frac{e^{\lambda E[R]} - 1}{\lambda}. \tag{20}$$

Hence the average broadcast extension depends only on the average of the transmission range and not on its distribution. The average transmission range may be computed as [14]:

$$E[R] = \int_0^{+\infty} (1 - F_R(a))\, da = \int_0^{+\infty} e^{-a^\alpha \beta} da = \frac{\beta^{-\frac{1}{\alpha}}}{\alpha} \Gamma\left(\frac{1}{\alpha}\right). \tag{21}$$

Note, that, for $\alpha = 2$ (free space propagation) the expression above leads to:

$$E[R] = \frac{1}{2} \cdot \sqrt{\frac{\pi}{\beta}}.$$

the receiver present an infinite capture threshold. It should thus be clear that our assumptions lead to optimistic results with respect to a real–world situation. Note also that the system we model exploits the diversity provided by the fading channel.

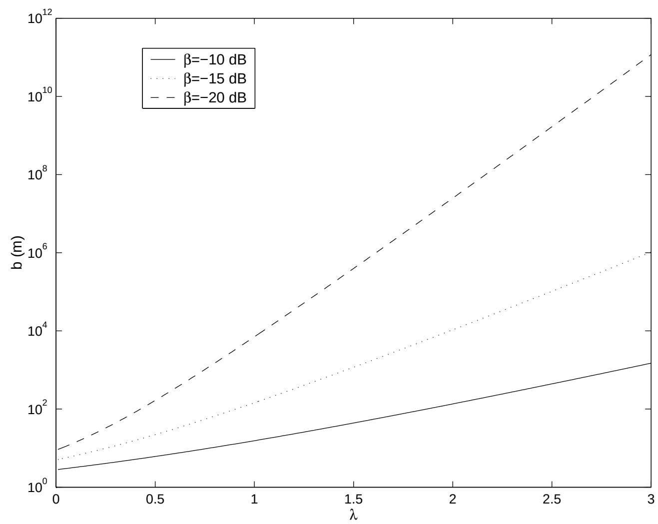

Figure 8: Average broadcast extension vs. average node density, $\alpha = 2$

The average broadcast extension in Rayleigh fading channel is plotted in Fig.8 for various values of β and $\alpha = 2$. The impact of the path loss factor may be seen in Fig.9, where the average broadcast extension is plotted for various values of α for $\beta = -15$ dB. Otherwise, we may proceed as in [10], getting directly the expression for the busy period cdf:

$$F_B(a) = 1 - \frac{\sum_{n=1}^{\infty} f^{*n}(a)}{\lambda}, \tag{22}$$

where $f^{*n}(\cdot)$ denotes the n–fold convolution of:

$$f(a) = \lambda e^{-a^{\alpha}\beta} P_0(a). \tag{23}$$

Hence, we may substitute in (22) and solve numerically, in order to get the statistics of the distance a broadcast message will travel. The analysis for the number of hosts reached by the broadcast turns out to be extremely difficult (see the note in the appendix of [10] for more details).

3.2 Heavy traffic analysis for dense networks

As already done for the connectivity problem, we apply the heavy traffic limit for the queueing process, in order to study the behavior of broadcast percolation in dense networks.

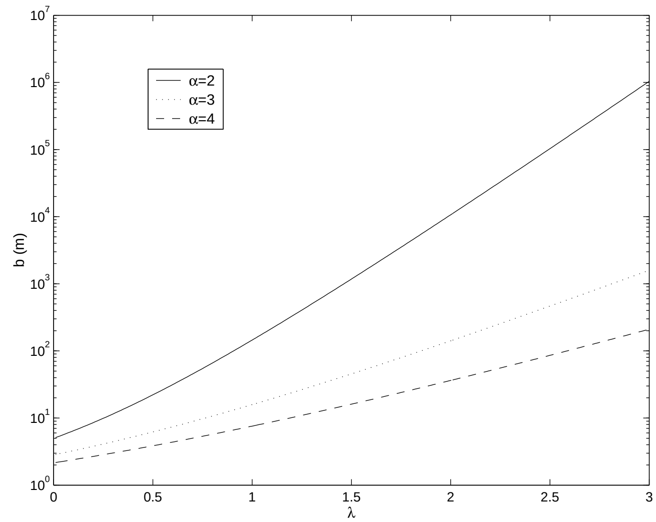

Figure 9: Average broadcast extension vs. average node density, $\beta = -15$ dB

Let us consider that nodes are placed upon a semi–infinite line according to a Poisson process of intensity λ, and that the channel model gives rise to a random transmission range with cdf $F_R(\cdot)$. We recall the following result from [11]:

Theorem 3.1. *If $E[R^2]$ is finite, then as $\lambda \to +\infty$,*

$$F_B(aE[B]) \to 1 - e^{-a}, \ a > 0. \tag{24}$$

Note that the condition stated above is only sufficient; a necessary and sufficient condition is reported in [11]. Let us consider Rayleigh fading channels; then, by using results from [14]:

$$E[R^2] = \int_0^{+\infty} \alpha a^{\alpha+1} e^{-a^\alpha \beta} da \leq \alpha \int_0^{+\infty} \alpha a^{\alpha+1} e^{-a\beta} da = \frac{\alpha}{\beta^{\alpha+2}} \Gamma(\alpha+2) < +\infty. \tag{25}$$

We may thus apply the proposition above, obtaining the following theorem:

Theorem 3.2. *Given a one–dimensional ad–hoc networks, with nodes placed according to a Poisson process of intensity λ, with a channel characterized by a path loss of the form $\left(\frac{1}{d}\right)^\alpha$ and by Rayleigh fading, in which all nodes transmit at a fixed power $P_{tx}(\lambda) = \left[\frac{\ln \lambda + c(\lambda)}{\lambda}\right]^\alpha$, a broadcast message generated by a node placed at the origin reaches a node placed at distance d whp as $\lambda \to +\infty$ iff $\lim_{\lambda \to +\infty} c(\lambda) = +\infty$.*

Proof. The proof follows along the lines of Theorem 2.2. Indeed, it is sufficient to notice that the expression is basically the same, apart from the fact that here we have to consider the average communication range. Then a necessary and sufficient condition is that, if $E[R(\lambda)] \sim \frac{\ln \lambda + c(\lambda)}{\lambda}$, $\lim_{\lambda \to +\infty} c(\lambda) = +\infty$. Recalling that we may write $\beta = \frac{K}{P_{tx}}$, where K is a constant, from (21) we get

$$E[R(\lambda)] = [P_{tx}(\lambda)]^{\frac{1}{\alpha}} \cdot \left[\frac{K^{-\frac{1}{\alpha}}}{\alpha} \Gamma\left(\frac{1}{\alpha}\right) \right]. \tag{26}$$

Then, $E[R] = \frac{\ln \lambda + c(\lambda)}{\lambda} \cdot \left[\frac{K^{-\frac{1}{\alpha}}}{\alpha} \Gamma\left(\frac{1}{\alpha}\right) \right]$ and we are done. □

4 Unreliable Ad–Hoc Networks

In largely deployed sensor network, it is mandatory to take into account for the possible unreliability of the devices. We consider, as above, a one–dimensional network, where the distance between successive devices is given by indipendent random variables with a common distribution $F_Y(\cdot)$, and each node is active with probability $p(\lambda)$, where $\lambda^{-1} = E[Y]$. We aim thus at generalizing the results obtained in the previous sections for the case of reliable nodes. In particular, we will focus on asymptotics, in order to find scaling laws for $p(\lambda)$ and $R(\lambda)$ jointly.
As a starting point, consider the interdistance between two successive active nodes, which will be denoted by $\tilde{Y}$. In particular, under our assumptions, $\tilde{Y} = \sum_{i=1}^{K} Y_i$, where K is a geometrically distributed random variable, having pmf $p_K(a) = P[K = a] = p(\lambda) \cdot (1 - p(\lambda))^{a-1}$, $a = 1, 2, \ldots$.
A special case arises when Y is exponentially distributed with mean λ^{-1}. In that situation, we end up with a statistical sampling of a Poisson process, which turns out to be still a Poisson process. Hence, $F_{\tilde{Y}}(a) = 1 - e^{-\lambda p(\lambda) a}$. For this case, the generalization of the connectivity and broadcast percolation results is thus trivial. To see the effect of p on network connectivity we reported some results in Fig.10, where the $M|D|\infty$ case is depicted for various values of p and $\lambda R = 2$.

In the more general case, we have for the Laplace–Stjelties transform of the $\tilde{Y}$ pdf:

$$\tilde{\mathcal{Y}}(s) = \frac{\mathcal{Y}(s) p(\lambda)}{1 - (1 - p(\lambda))\, \mathcal{Y}(s)}. \tag{27}$$

Then numerical inversion may be performed to get the pdf of $\tilde{Y}$ and the rest of the analysis follows accordingly.

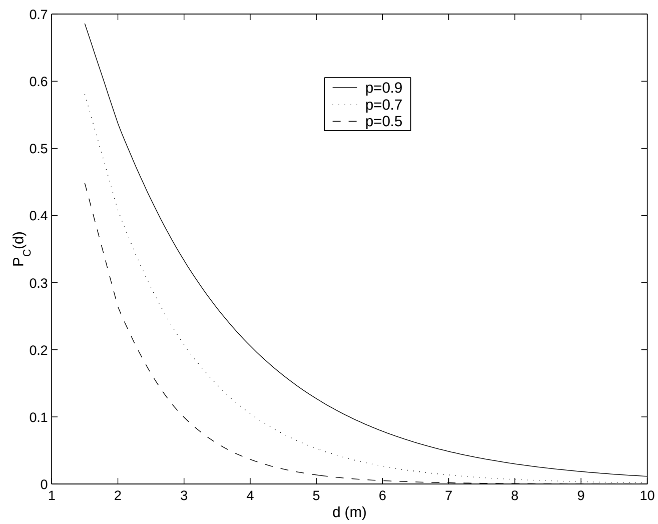

Figure 10: Probability of connectivity at distance d for various p, $\lambda R = 2$

It is thus eviden that the analysis of the connectivity of unreliable networks represents a straightforward generalization of what done in the previous section for reliable nodes. However, some interesting phenomena do arise in the asymptotics. Let us consider nodes distributed according to a Poisson point process of intensity λ, which are active with a given probability $p(\lambda)$. The reason for keeping p as a function of λ comes directly from some possible application features. For example, we may think of a network deployed by throwing sensors from an airplane. In such a situation, it is clear that the terrain configuration may well influence the probability of a node to succesfully "land", so that p may, in general, be thought as a decreasing function of λ. In any case, the resulting process is still Poisson with intensity $\lambda p(\lambda)$. The problem arises from the fact that, from a network designer point of view, $p(\lambda)$ cannot assumed to be known *a priori*. As a consequence the transmission power (and consequently the transmission range) has to be dimensioned by relying only on the knowledge of λ, a parameter that can be controlled in the deployment phase. In this case, even for a fixed $p(\lambda)$, a different scaling law for $R(\lambda)$ is necessary to provide asymptotic connectivity. The asymptotic analysis provides hence results which may be succesfully employed as guidelines for the dimensioning of robust dense ad–hoc networks.
As previously stated, we limit ourselves to the case of nodes distributed according to a Poisson point process of intensity λ, and study scaling laws as the density $\lambda \to +\infty$. For $\lim_{\lambda \to +\infty} p(\lambda) = p > 0$, the condition found for the connectivity in reliable networks is

clearly only necessary. A stronger condition for sufficiency is provided. The case $p(\lambda) \to 0$ is also investigated, with particular attention for the case $p(\lambda) = \lambda^{-\gamma}$. The results we will find closely ressemble those obtained by Shakkottai et al. [7] for regular sensor grids. This analogy leads to conjecture that the results in [7] may be generalized to a random node placement.

Let us consider the connectivity problem with nodes distributed according to a Poisson process of intensity λ. Thus, the following holds:

Theorem 4.1. *For a general $R(\lambda)$ with nonzero support, a necessary and sufficient condition for asymptotic connectivity at a distance d is that:*

$$\lim_{\lambda \to +\infty} \lambda p(\lambda) e^{-\lambda p(\lambda) R(\lambda)} = 0. \tag{28}$$

Proof. The probability of connectivity at d is given by:

$$p_C(d) = e^{-\frac{d\lambda p(\lambda)}{e^{\lambda p(\lambda) R(\lambda)} - 1}}. \tag{29}$$

Then, we may study the behavior of the function $f(\lambda)$:

$$f(\lambda) = \frac{\lambda p(\lambda)}{e^{\lambda p(\lambda) R(\lambda)} - 1}. \tag{30}$$

A necessary and sufficient condition for asymptotical connectivity is that $\lim_{\lambda \to +\infty} f(\lambda) = 0$. The statement follows straightforwardly. □

Now we consider a particular expression for $R(\lambda)$ and a finite non–zero p.

Theorem 4.2. *If $p = \lim_{\lambda \to +\infty} p(\lambda)$ satisfies $1 < p < 0$ and $R(\lambda) = \frac{\ln \lambda + c(\lambda)}{\lambda}$, the network is asymptotically connected at distance d if $c(\lambda)$ satisfies*

$$c(\lambda) \geq \xi \ln \lambda, \tag{31}$$

where

$$\xi > \frac{1-p}{p}. \tag{32}$$

Furthemore, the network is asymptotically connected only if $c = \lim_{\lambda \to +\infty} c(\lambda) = +\infty$.

Proof. We start with the proof of the sufficient condition. From (30), considering the particular expression of $R(\lambda)$, we obtain:

$$f(\lambda) = \frac{p(\lambda)\lambda}{e^{-p(\lambda)[\ln \lambda + c(\lambda)]} - 1} \tag{33}$$

Taking the limit and applying (31), the former equation becomes:

$$\lim_{\lambda\to+\infty} f(\lambda) = \lim_{\lambda\to+\infty} p(\lambda)\lambda^{1-p(\lambda)}e^{-p(\lambda)c(\lambda)} = \lim_{\lambda\to+\infty} p\lambda^{1-p}e^{-pc(\lambda)} \leq$$
$$\leq \lim_{\lambda\to+\infty} p\lambda^{1-p}e^{-p\xi\ln\lambda} = \lim_{\lambda\to+\infty} p\lambda^{1-p-p\xi}, \quad (34)$$

which clearly converges to 0 for $\xi > \frac{1-p}{p}$.
The necessary condition is proven by *reductio ad absurdum.* Let us assume $c < +\infty$. Then,

$$\lim_{\lambda\to+\infty} f(\lambda) = \lim_{\lambda\to+\infty} p\lambda^{1-p}e^{-pc}, \quad (35)$$

which clearly diverges, and the network is not asymptotically connected. □

Note that there is a big gap between the necessary and sufficient condition.
As done in [7] let us consider a particular case of $p(\lambda)$ and a general $R(\lambda)$.

Theorem 4.3. *Let $p(\lambda)$ be $p(\lambda) = \lambda^{-\gamma}$, where $0 < \gamma < 1$. Then a sufficient condition for asymptotic network connectivity at distance d is given by:*

$$p(\lambda)R(\lambda) \geq \xi\frac{\ln\lambda}{\lambda}, \quad (36)$$

where the constant ξ satisfies:

$$\xi > 1-\gamma. \quad (37)$$

Proof. From (30), applying (36) we have:

$$f(\lambda) \leq \frac{\lambda p(\lambda)}{e^{\xi\ln\lambda}-1} = \frac{\lambda p(\lambda)}{\lambda^{\xi}-1} \sim \lambda^{1-\xi}p(\lambda) = \lambda^{1-\xi-\gamma}, \quad (38)$$

which tends to 0 iff $\gamma > 1-\xi$. □

Considering a particular expression for $R(\lambda)$, the previous result may be particularized as follows:

Corollary 1. *Let $p(\lambda)$ be $p(\lambda) = \lambda^{-\gamma}$, where $0 < \gamma < 1$ and $R(\lambda) = \frac{\ln\lambda + c(\lambda)}{\lambda}$. Then, a sufficient condition for asymptotic connectivity is that*

$$c(\lambda) \geq (\xi\lambda^{\gamma}-1)\ln\lambda, \quad (39)$$

where ξ satisfies:

$$\xi > 1-\gamma. \quad (40)$$

Proof. Under the condition (39), we clearly have:

$$p(\lambda)R(\lambda) = \frac{\ln \lambda + c(\lambda)}{\lambda^{1+\gamma}} \geq \frac{\xi \lambda^{\gamma} \ln \gamma}{\lambda^{1+\gamma}} = \frac{\xi \ln \gamma}{\lambda}, \tag{41}$$

and Theorem 4.3 may be applied to conclude the proof. □

A necessary condition may also be given:

Proposition 1. *Let $p(\lambda)$ be $p(\lambda) = \lambda^{-\gamma}$, where $0 < \gamma < 1$ and $R(\lambda) = \frac{\ln \lambda + c(\lambda)}{\lambda}$. Then, a necessary condition for asymptotic connectivity is that $c = \lim_{\lambda \to +\infty} c(\lambda) = +\infty$.*

Proof. The proof follows by a *reductio ad absurdum* argument. Let us assume $c < +\infty$ and consider the limiting behavior of $f(\lambda)$:

$$\lim_{\lambda \to +\infty} f(\lambda) = \lim_{\lambda \to +\infty} \lambda^{1-\gamma} e^{-\lambda^{-\gamma}[\ln \lambda + c(\lambda)]} = \lim_{\lambda \to +\infty} \lambda^{1-\gamma} e^{-\frac{\ln \lambda}{\lambda^{\gamma}}} e^{-\frac{c(\lambda)}{\lambda^{\gamma}}}. \tag{42}$$

Now, if $c < +\infty$, we have

$$f(\lambda) \sim \lambda^{1-\gamma}, \tag{43}$$

which diverges, and hence the network is not asymptotically connected. □

Similar results hold for the broadcast percolation problems for nodes distributed according to a Poisson point process and Rayleigh fading. The easy proofs are not reported.

Theorem 4.4. *For a general $P_{tx}(\lambda)$, a necessary and sufficient condition for a broadcast message to asymptotically percolate to distance d is that:*

$$\lim_{\lambda \to +\infty} \lambda p(\lambda) e^{-\lambda p(\lambda)[P_{tx}(\lambda)]^{\frac{1}{\alpha}}} = 0. \tag{44}$$

Theorem 4.5. *If $p = \lim_{\lambda \to +\infty} p(\lambda)$ satisfies $1 < p < 0$ and $P_{tx}(\lambda) = \left(\frac{\ln \lambda + c(\lambda)}{\lambda}\right)^{\alpha}$, a broadcast message asymptotically percolates to distance d if $c(\lambda)$ satisfies*

$$c(\lambda) \geq \xi \ln \lambda, \tag{45}$$

where

$$\xi > \frac{1-p}{p}. \tag{46}$$

Furthemore, the network asymptotically supports broadcast percolation only if $c = \lim_{\lambda \to +\infty} c(\lambda) = +\infty$.

Theorem 4.6. *Let $p(\lambda)$ be $p(\lambda) = \lambda^{-\gamma}$, where $0 < \gamma < 1$. Then a sufficient condition for a broadcast message to asymptotically percolate to distance d is given by:*

$$p(\lambda)\,(P_{tx}(\lambda))^{\frac{1}{\alpha}} \geq \xi \frac{\ln \lambda}{\lambda}, \tag{47}$$

where the constant ξ satisfies:

$$\xi > 1 - \gamma. \tag{48}$$

Corollary 2. *Let $p(\lambda)$ be $p(\lambda) = \lambda^{-\gamma}$, where $0 < \gamma < 1$ and $P_{tx}(\lambda) = \left(\frac{\ln \lambda + c(\lambda)}{\lambda}\right)^{\alpha}$. Then, a sufficient condition for a broadcast message to asymptotically percolate is that*

$$c(\lambda) \geq (\xi \lambda^{\gamma} - 1) \ln \lambda, \tag{49}$$

where ξ satisfies:

$$\xi > 1 - \gamma. \tag{50}$$

Proposition 2. *Let $p(\lambda)$ be $p(\lambda) = \lambda^{-\gamma}$, where $0 < \gamma < 1$ and $P_{tx}(\lambda) = \left(\frac{\ln \lambda + c(\lambda)}{\lambda}\right)^{\alpha}$. Then, a necessary condition for a broadcast message to asymptotically percolate is that $c = \lim_{\lambda \to +\infty} c(\lambda) = +\infty$.*

5 Conclusions

In this paper we presented some novel results, obtained by means of queueing theoretical tools, on connectivity issues in wireless ad–hoc networks. We showed that the spatial constraints involved in a one–dimensional ad–hoc network can be translated into an equivalent system in the time domain, which may be analyzed by means of classical queueing tools. For a fixed transmission range, we showed the influence of the nodes displacement statistics on some metrics of interest, such as the probability of a node to be isolated, the average cluster size and the probability of the generic k–th node to be connected, showing how heavy–tailed distribution may degrade performance in dense networks. The similar problem of broadcast percolation may as well be reduced to an equivalent queueing system, in which case we can extend the framework to account also for random communication range, such as the one induced by Rayleigh fading channels. For both problems, we showed how heavy traffic theorems may be used to derive the critical tranmsission range and the critical transmission power in this kind of networks. Finally, the framework has been extended to account for unreliable nodes, with a particular emphasis on the asymptotic behavior of such systems, which shows a notable difference to the fully reliable case.
Two research issues appears of major interest. First, the asymptotic analysis provides scaling

laws for the Poisson case only, and it is not clear at the moment whether different distributions (keeping out of contest the deterministic one) may yield better scaling laws. Second, it would be interesting to incorporate in our framework some cooperative diversity schemes [15, 16], and study their impact on connectivity performance.

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APPENDIX

A Modelling The Communication Range in Presence of Fading

We consider nodes trasmitting at a fixed power P_{tx}, with a channel characterized by a path loss of the type $\left(\frac{1}{d}\right)^{\alpha}$, where $2 \leq \alpha \leq 4$. We assume that an additive gaussian noise with power P_{noise} is present at the receiver and that the channel is characterized by flat slow fading, with no line–of–sight (this clearly represents a pessimistic assumption). In Rayleigh fading channels, the impact of the randomness due the gaussian noise is usually negligible compared to the variation in signal–to–noise ratio (SNR) due to the fading process. We assume that the fading is constant over the transmission of a frame and subsequent fadings are i.i.d. (hence we obtain what is called a block–fading channel). Hence, taking the fading power equal to 1, the average signal–to–noise ratio may be written as $\overline{\gamma} = \frac{P_{tx}}{d^{\alpha} P_{noise}} = \frac{\overline{\gamma_0}}{d^{\alpha}}$. The pdf of the SNR is thus given by [17]:

$$f_{\gamma}(a) = \frac{1}{\overline{\gamma}} e^{-\frac{a}{\overline{\gamma}}} = \frac{d^{\alpha}}{\overline{\gamma_0}} e^{-\frac{a d^{\alpha}}{\overline{\gamma_0}}}. \tag{51}$$

In our model, the transmitted message can be correctly decoded if and only if the SNR γ is greater than a given threshold Ψ [3]. Hence the transmission range is not any more fixed, but is a random variable which depends on both the transmitter–receiver distance d and the

[3] In the asymptote for "good" long codes, the probability of corrected reception as a function of the SNR tends to a step function.

SNR γ. Hence the probability that the message is correctly received at a distance d is given by:

$$P[\gamma(d) \geq \Psi] = \int_{\Psi}^{\infty} f_{\gamma}(a)da = e^{-\frac{\Psi}{\overline{\gamma}}} = e^{-d^{\alpha}\frac{\Psi}{\overline{\gamma_0}}}.$$

Assuming $\Psi = \beta\overline{\gamma_0}$, we thus get:

$$P[\gamma(d) \geq \Psi] = e^{-d^{\alpha}\beta}.$$

The transmission range statistics may then be obtained by means of:

$$F_R(a) = P[R \leq a] = 1 - P[R > a] = 1 - P[\gamma(a) > \Psi] = 1 - e^{-a^{\alpha}\beta}. \tag{52}$$

The function $F_R(a)$ is plotted in Fig.11 for some values of α.
While Rayleigh fading represents an useful model for dimensioning purposes, in many situation it is not able to reflect the behavior of wireless channels. In such cases, more complex models have to be used. In particular, a widely used model is the m–Nakagami, which represents a generalization of Rayleigh fading, and is acknowledged to accurately characterize a wide range of multipath mobile channels. The presence of a parameter, m, makes it possible to adjust the model to fit experimental data. As a further application, Nakagami channels may be used to analyze an m–antennas based diversity assisted system employing maximal ratio combining [18]. In m–Nakagami fading, the istantaneous signal–to–noise ratio pdf is given by:

$$f_{\gamma}(a) = \left(\frac{m}{\overline{\gamma}}\right)^{m} \frac{a^{m-1}}{\Gamma(m)} e^{-m\frac{a}{\overline{\gamma}}},$$

where $\Gamma(\cdot)$ is the usual Gamma function. Note that for $m = 1$ we obtain the usual expression for Rayleigh fading. The ccdf is given by:

$$P[\gamma > a] = 1 - F_{\gamma}(a) = \frac{\Gamma\left(m, \frac{m\gamma}{\overline{\gamma}}\right)}{\Gamma(m)}, \tag{53}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function [14]. In the following we will assume m to be an integer, in which case, after some algebra, the expression above simplifies as:

$$P[\gamma > a] = e^{-m\frac{a}{\overline{\gamma}}} \sum_{i=0}^{m-1} \left(\frac{ma}{\overline{\gamma}}\right)^{i} \cdot \frac{1}{(i+1)!}. \tag{54}$$

Thus, proceeding as above, the cdf of the transmission range turns out to be given by:

$$F_R(a) = 1 - P[\gamma(a) > \Psi] = 1 - e^{-m\beta a^{\alpha}} \sum_{i=0}^{m-1} (m\beta a^{\alpha})^{i} \cdot \frac{1}{(i+1)!}. \tag{55}$$

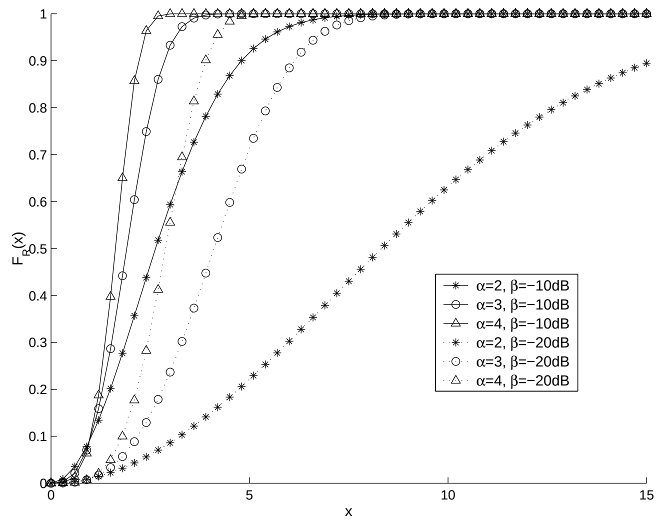

Figure 11: Cumulative distribution function of R for various values of α and β

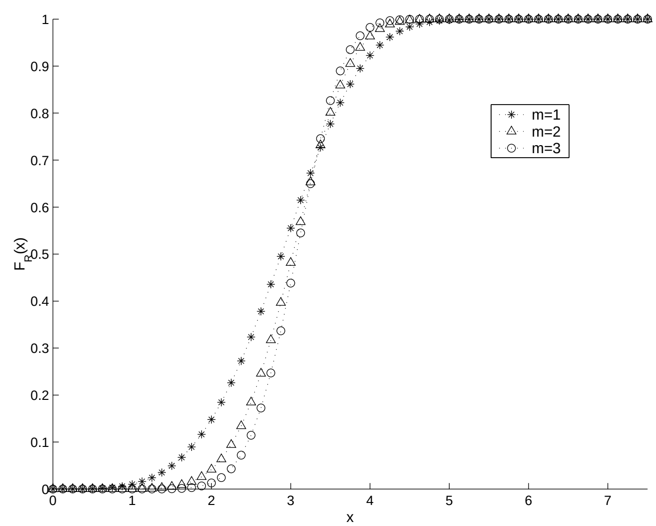

Figure 12: Cumulative distribution function of R for various values of m, $\beta = -20$ db, $\alpha = 4$

As an example, in Fig.12 we plotted the transmission range cdf for various m.

Now let us extend the above assumption, to take into account a random characterization of the path loss (lognormal shadowing [19]) and the possible presence of interfering nodes, transmitting on the same frequency band, threading the footprints of [20]. Focusing on a CDMA network, let us denote by G the processing gain; transmission is succesfull if and only if the signal to interference ratio (SIR) γ is above a given threshold Ψ. If P_{rx} is the received power of the useful signal and P_{int} is the total power of the interference signals, then we have:

$$\gamma = \frac{P_{rx}}{\frac{P_{int}}{G} + P_{noise}}.$$

Let us denote by η_i the path loss for the i–th path, where η_0 represents the path loss the information signal undergoes, while for $i \geq 1$ we refer to the path loss experienced by the i–th interference signal. Jamming nodes are scattered over the infinite plane according to a Poisson process of intensity λ, and transmit at fixed power $P_I = \xi P_{tx}$. Besides, we assume that interferers form a dense field with respect to the devices of the ad–hoc networks, in the sense that the interference levels experienced by different hosts are given by independent random variables. Assuming that the i–th signal is undergoing Rayleigh fading, its received power turns out from (51) to be exponentially distributed, with mean given by $P_0 = P_{tx}\eta_0$ and $P_i = P_I \eta_i = \xi P_{tx} \eta_i$ for $i \geq 1$, respectively,

$$f_{\tilde{P}_i}(a) = \frac{1}{P_i} e^{-\frac{a}{P_i}}.$$

Denoting by $N|R$ the (conditioned) r.v. indicating the number of interfering nodes placed in a circle of radius R, we have that the transmission is succesful with probability:

$$P_S = P[\gamma \geq \Psi] = \lim_{c \to +\infty} \sum_{k=0}^{\infty} P[N = k | R = c] \int_0^{+\infty} da_0 \dots$$

$$\dots \int_0^{+\infty} da_k \int_{\Psi}^{\infty} db f_{\gamma|\eta_0,\dots,\eta_k,N}(b|a_0,\dots,a_k,k) \prod_{i=0}^{k} f_{\eta_i|R}(a_i|c). \quad (56)$$

Thus, using the definition of γ and conditioning on the power levels of the interfering signals we get:

$$\int_{\Psi}^{\infty} db f_{\gamma|\eta_0,\ldots,\eta_k,N}(b|a_0,\ldots,a_k,k) =$$

$$= \int_0^{+\infty} db_1 \ldots \int_0^{+\infty} db_k \int_{\frac{\Psi}{G}\sum_{i=1}^k b_i + \Psi P_{noise}}^{\infty} db f_{\tilde{P}_0|\eta_0,\ldots,\eta_k,N}(b|a_0,\ldots,a_k,k) \prod_{i=1}^k f_{\tilde{P}_i}(b_i) =$$

$$= e^{-\frac{\Psi P_{noise}}{P_{tx} a_0}} \int_0^{+\infty} db_1 \ldots \int_0^{+\infty} db_k e^{-\frac{\Psi \sum_{i=1}^k b_i}{G P_{tx} a_0}} \prod_{i=1}^k f_{\tilde{P}_i}(b_i) =$$

$$= e^{-\frac{\Psi P_{noise}}{P_{tx} a_0}} \int_0^{+\infty} db_1 \ldots \int_0^{+\infty} db_k \prod_{i=1}^k \frac{1}{P_i} e^{-\frac{b_i}{P_i} - \frac{\Psi b_i}{G P_{tx} a_0}} =$$

$$= e^{-\frac{\Psi P_{noise}}{P_{tx} a_0}} \prod_{i=1}^k \int_0^{+\infty} db_i e^{-\frac{b_i}{P_{tx} a_i \xi}\left(1 - \frac{\Psi a_i \xi}{G a_0}\right)} =$$

$$= e^{-\frac{\Psi P_{noise}}{P_{tx} a_0}} \prod_{i=1}^k \frac{1}{1 - \frac{\Psi \xi a_i}{G a_0}}. \quad (57)$$

Next step is to average over the link attenuations (conditioned on the circle radius R), whose pdf is given by:

$$f_{\eta_i|D_i}(a_i|d_i) = \frac{e^{-\frac{(\ln a_i + \alpha \ln d_i)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2} a_i}, \quad (58)$$

where D_i is the distance between the i–th transmitter and the destination node, and $D_0 = d$. Since the nodes are uniformly distributed within the circle of radius R, we have:

$$f_{\eta_i|R}(a_i|c) = \int_0^c \frac{2 d_i d d_i}{c^2} f_{\eta_i|D_i}(a_i|d_i). \quad (59)$$

By letting $x_i = \ln(a_i) + \alpha \ln(d_i)$ and $\tau = \ln(a_0) - \alpha \ln(d)$, we get, after some standard algebra:

$$\int_0^{+\infty} da_0 \ldots \int_0^{+\infty} da_k \int_{\Psi}^{\infty} db f_{\gamma|\eta_0,\ldots,\eta_k,N}(b|a_0,\ldots,a_k,k) \prod_{i=0}^k f_{\eta_i}(a_i) =$$

$$= \int_{-\infty}^{+\infty} da \frac{1}{\sigma} \phi\left(\frac{a}{\sigma}\right) \left[1 - e^{-mue^{-a}} (I_R(a))^k\right], \quad (60)$$

where $\mu = \frac{\Psi \xi P_{noise} d^{\alpha}}{P_{tx}}$, $\phi(\cdot)$ is the standard normalized gaussian pdf and

$$I_R(a) = \int_{-\infty}^{+\infty} dx \frac{1}{\sigma} \phi\left(\frac{x}{\sigma}\right) \int_0^R \frac{2r dr}{R^2} \frac{1}{1 + \frac{\Psi \xi}{G} e^{x-R} \left(\frac{d}{r}\right)^{\alpha}}.$$

Averaging over the Poisson distribution of the number of jamming nodes, we get:

$$\sum_{k=0}^{\infty} P[N=k|R=c] \int_0^{+\infty} da_0 \ldots \int_0^{+\infty} da_k \int_{\Psi}^{\infty} db f_{\gamma|\eta_0,\ldots,\eta_k,N}(b|a_0,\ldots,a_k,k) \prod_{i=0}^{k} f_{\eta_i}(a_i) =$$
$$= \int_{-\infty}^{+\infty} da \frac{1}{\sigma}\phi\left(\frac{a}{\sigma}\right)\left[1 - e^{-\mu e^{-a}} e^{-\lambda\pi R^2[1-I_R(a)]}\right]. \quad (61)$$

The limit for $R \to +\infty$ does exist [20, 21] and thus we get:

$$P[\gamma(d) > \Psi] = \int_{-\infty}^{+\infty} da \frac{1}{\sigma}\phi\left(\frac{a}{\sigma}\right)\left[1 - e^{-\mu e^{-a} - \lambda\pi d^2 \left(\frac{\Psi\xi}{G}\right)^{\frac{2}{\alpha}} H(a)}\right], \quad (62)$$

where

$$H(a) = \frac{2\pi}{\alpha}\text{cosec}\left(\frac{2\pi}{\alpha}\right) e^{2\left(\frac{\sigma}{\alpha}\right)^2 - \frac{2a}{\alpha}}.$$

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