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Abstract: This paper investigates how to design feedback controls supporting TCP (Transmission Control Protocol) based on modern control theories. The present paper studies a simplified version of this problem: the linearized version of the well-known AIMD (Additive Increase Multiplicative Decrease) dynamic model under the assumption that the network information is known. Since we formulate the feedback control design problem as state-space models without assuming its structure in advance, we get three interesting and important results that have not been observed in the previous studies on the congestion control problem:

1. In order to fully support TCP, we need a PD-type (Proportional-Derivative) state-feedback control structure in terms of queue length (or RTT: Round Trip Time) which backs up the conjecture in the networking literature that the AQM RED is not enough to control TCP dynamic behavior, where RED can be classified as a P-type AQM (or as an output feedback control for the AIMD model);

2. In order to fully support TCP in the presence of delays, we derive delay-dependent feedback control structures to compensate for delays explicitly from the knowledge of RTT, capacity and number of sources, where all existing AQMs including RED, REM/PI and AVQ are delay-independent controls;

3. In order to analyze different AQM structures in a unified manner, rather than comparing them via simulations, we propose a PID-type mathematical framework using integral control action.

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As a performance index to measure the deviation of the closed-loop system from the equilibrium, we use a LQ (linear quadratic) cost of the transients on state and control variables such as queue length, aggregate rate, jitter in the aggregate rate and congestion measure. Stabilizing gains of the proposed feedback control structures are obtained minimizing the LQ cost. Then, the impact of the control structure on performance is discussed from the study of the stabilizing gain design for the proposed mathematical framework. All results are extended to the case of multiple links and heterogeneous delays.

Key-words: TCP, AQM, feedback control, stability, delay compensation, optimal performance, flow control, AIMD.

Conception de mécanismes de contrôle en boucle supportant TCP et basés sur la théorie du contrôle moderne

Résumé : Nous proposons des mécanismes de contrôle en boucle supportant TCP (Transmission Control Protocol) et basés sur la théorie du contrôle moderne. Cet article étudie la version linéarisée du modèle dynamique bien connu qu'est l'AIMD (Additive Increase Multiplicative Decrease) en supposant que l'information sur le réseau est disponible. Nous formulons ce problème comme un modèle à espace d'état sans présumer des structures et nous obtenons trois propriétés intéressantes et importantes, qui sont ces suivantes:

1. Pour supporter pleinement TCP, la structure de contrôle doit être de type P.D. (Proportionnel-Dérivé) par rapport à la taille de la file d'attente (ou du RTT). Ceci confirme l'hypothèse souvent formulée dans la littérature des réseaux que RED ne suffit pas pour contrôler le comportement dynamique de TCP, en effet RED est une structure du contrôle de type P;

2. Pour supporter pleinement TCP dans le cas où il y a des délais, nous proposons des structures de contrôle qui compensent explicitement les retards en utilisant l'information sur le RTT, sur la capacité, ainsi que sur le nombre de sources. Toutes les structures de contrôle existant, y compris RED, REM/PI, et AVQ sont quant-à elles indépendantes du retard d'action;

3. Pour tenter d'interpréter les différentes structures AQM de manière unifiée plutôt que par des simulations, nous proposons un cadre mathématique de type P.I.D. (Proportionnel-Intégral-Dérivé) en ajoutant un contrôle intégral.

Comme critère de performance pour mesurer la déviation de l'équilibre du système en boucle fermée, nous utilisons un coût LQ (linéaire quadratique) des éléments transitoires sur les variables d'état et de contrôle comme la taille de la file d'attente, le taux agrégé, la gigue dans le taux et la mesure de congestion. Les gains stabilisateurs de nos structures de contrôle en boucle sont obtenus en minimisant le coût LQ. Nous pouvons alors étudier l'impact de la structure de contrôle sur les performances en nous appuyant sur l'étude de la conception du gain stabilisateur dans le cadre mathématique proposé. Tous ces résultats sont étendus au cas de réseaux multi-liens avec des retards hétérogènes.

Mots-clés : TCP, AQM, contrôle en boucle, stabilité, compensation de retard, performance optimale, contrôle de flux, AIMD.

1 Introduction

In the Internet, congestion control enables end-users to fully utilize the allocated capacity with the help of queuing at routers. Since end-users do not know the allocated capacity, a dynamic window-based mechanism TCP (Transmission Control Protocol) acting at a packet level has been developed, so that each flow from an end-user obeys a ‘conservation of packets’ principle, which means that for a connection ‘in equilibrium’, i.e., running stably with a full window of data in transit, a new packet isn’t put into the network until an old packet leaves. Since congestion information occurs at routers, interaction between TCP mechanism and the congestion information cannot avoid delays which make the closed-loop dynamics difficult to deal with, where signaling the congestion information by dropping or marking packets at routers is called AQM (Active Queue Management). Since TCP Reno/AQM Droptail has been proposed in [6], the current internet is still using this protocol and its variants as a congestion control strategy.

Droptail can cause a large variation of queuing delay since it drops packets when the queue is full. More importantly, Droptail can often cause the global synchronization [7, 8] and thus have low throughput. In order to overcome these problems, RED (Random Early Detection) has been suggested in [9, 10]. RED drops packets with a probability proportional to the average queue length. Since then, there have been a lot of investigations about how to tune the design parameters in RED [11, 12, 13, 14]. The experimental results show that RED is not enough to control TCP and thus not easy to fully utilize the given network resources. As a result, new AQM algorithms such as BLUE [15], AVQ (Adaptive Virtual Queue) [16] and REM (Random Exponential Marking) [17] have been suggested. These AQM structures are designed to achieve additional performances such as queue clearing. However, none of these papers appropriately address how to tune the gains of their AQM structures for closed-loop stability since they lack of a TCP dynamic model, even though stabilizing the closed-loop system is necessary to get the maximum utilization. So, their AQM algorithms are only compared through simulations in the literature.

In order to address this problem, paper [18] has developed a dynamic model to reflect AIMD (Additive Increase Multiplicative Decrease) mode of TCP for a single link and homogeneous sources and paper [19] has applied the transfer function approach to the problem of stabilizing RED design based on the AIMD model. As a follow up, papers [20, 21] have investigated how to scale gains of PI-type (Proportional-Integral) REM in terms of queue length and P-type AVQ in terms of aggregate, respectively, for closed-loop stability. Papers [22, 23] have suggested PI-type AQM in terms of aggregate in an inner loop. Papers [24, 25] have proposed P-type AQM with a low-pass filter in terms of aggregate. Although all these papers suggest stabilizing conditions derived from the linearized systems, they have not focused on what kind of control structures are necessary to control the closed-loop system.

In addition, none of those papers consider how to compensate for delays explicitly even if they know the previous dynamic information and delays. This kind of control strategy is called delay-independent (or memoryless) control in the literature. It is well-known that the delay-independent control has a limit on performance in the presence of a large delay [26, 27] compared with the delay-dependent (or memory) control since the delay-independent controls cannot regulate the delayed closed-loop dynamics arbitrarily. In the literature, previous studies on feedback control design in the presence of delays have focused on deriving stabilizing conditions, while our work studies an explicit feedback control design.

The present paper tries to address these issues as follows: we investigate what kind of feedback control structures are necessary to regulate the given TCP arbitrarily without and with delays. Then, we attempt to interpret the impact of different AQM structures on performance rather than to compare their performances via simulations. This work builds upon the AIMD model introduced in [18, 19]. As a first step to get basic results for more realistic cases, we study a simplified version of this problem as in [19, 20, 22, 23]. First, we consider the linearized version of the TCP and queue dynamics around the equilibrium, so stability means local stability near equilibrium, and the variables denote transients from their equilibrium points. The study on the linearized version can be justified by the fact that congestion occurs near the equilibrium point since the main role of feedback control is to keep the closed-loop system around the equilibrium. Second, when we try to compensate for delays explicitly and to get stabilizing optimal gains for the derived feedback control structure, we assume that information of the networks is known accurately. In addition, we do not study the case of input/state constrained uncertain systems which are intrinsically included in real networks and network simulators, although all cases need to be studied for implementation of the feedback control in real networks. Instead, we study the case of multiple links and heterogeneous delays which to our knowledge, is a first in the TCP Reno literature [2].

The main new feature of the present paper is to formulate the feedback control design problem as state-space models[1]. It allows us to investigate what is a natural state-feedback control, how to compensate for delays explicitly and what is the impact of different feedback control structures on performance.

The well-known TCP AIMD model in [18, 19] and its extension to multiple links and heterogeneous sources are introduced in Section 2.

In Section 3, we derive the state-space model of the AIMD and queue dynamics for the feedback control design. Thereby, we obtain the PD-type state-feedback control structure in terms of queue length (or RTT: Round Trip Time) which implies the structural deficiency of P-type RED for closed-loop stability. We show that this procedure can easily be extended to the case of additional dynamics like the low-pass filter of RED. By applying integral control action, we propose a mathematical framework to include PI-type REM and PI as well as P-type REM, from which the impact of each structure on performance will be studied in Section 5.

In Section 4, we suggest a delay-dependent feedback control to compensate for the delay in the congestion measure (feedback control) explicitly under the assumption that the forward delay from source to router is zero. This assumption is relaxed by adding a modified virtual queue so that we can still compensate for delays explicitly in the presence of both forward and backward delays. As a subsidiary result of this study, we verify that a simplified AVQ, which is P-type delay-independent AQM in terms of aggregate, is a state-feedback control for the AIMD model based on the virtual queue dynamics.

[1] The control strategy based on a state-space model is called modern control in the literature, while that based on a transfer function model as in [19, 20, 21, 22, 23] is called classical control. Since the state-space approach was developed in 1950s, it has been widely investigated in the literature due to many advantages over the transfer function approach (refer to any control literature for more details [28]).

In Section 5, we obtain stabilizing gains of the control structures by minimizing a linear quadratic (LQ) cost of the transients of state and control variables of the state-space model. Thus, the optimal control framework enables us to measure deviation of transients from the equilibrium point in the form of quadratic cost with the given weighting matrices of the cost function. For example, a slower transient will incur a higher cost. Inverse optimal control and the design of stabilizing gains by eigenvalues are also studied. Then, we discuss the impact of each structure on performance using the results of this study. As a by-product of this study, we show that it is possible to obtain stabilizing gains with *one* design parameter by setting all eigenvalues of the closed-loop system to be the same.

In Section 6, the results obtained for the case of single link and homogeneous sources are extended to the case of multiple links and heterogeneous sources. First, an equivalent nonlinear system of the general network and its linearized system are derived. Then, a state-space model, a delay compensation and a stabilizing optimal gain design are also studied. This study shows that the design procedures and resulting control structures for single link and homogeneous sources hold for general networks.

The validation of this work is addressed via *ns* simulations in the companion conference papers of the present paper [2, 3, 4, 5]; the simulation results via the non-deterministic nonlinear network simulator (*ns*) cannot be considered as a direct illustration of our theoretical results, but one can observe that they support our theoretical results qualitatively. For the linearized system, a direct illustration is shown in [1].

2 Preliminary: The Well-Known AIMD model of TCP

In this section, we introduce the well-known AIMD model of TCP in [18, 19] and its extension to multiple links and heterogeneous sources. From the next section, the present paper will consider the single link and homogeneous sources. Its extension to multiple links and heterogeneous delays will be studied in Section 6.

2.1 Notation

For describing the well-known AIMD model, we will use the following notations.

- N is the number of TCP sources, which we assume to be constant with time;
- c_l is the capacity of link l in packets/sec;
- d_s is the round trip propagation delay of source s;
- $b_l(t)$ is the real queue length of link l at time t (the average queue and virtual queue lengths are denoted as $\bar{b}_l(t)$ and $\tilde{b}_l(t)$, respectively);
- $\tau^f_{ls}(t)$ is the forward delay from source s to link l at time t;
- $\tau^b_{ls}(t)$ is the backward delay from link l to source s at time t;

- R_{ls} is a $\{0,\ 1\}$-valued variable with value 1 if source s uses link l, 0 otherwise;
- $\tau_s(t)$ is the RTT (Round Trip Time) of source s at time t $\left(\tau_s(t) = d_s + \sum_l R_{ls} \frac{b_l(t)}{c_l}\right)$;
- $w_s(t)$ is the window size of source s at time t;
- $p_l(t)$ is the feedback control at time t for TCP (it is mainly assumed to be the loss probability in this paper, but it can have other quantity depending on what kind of congestion information is used for the feedback control);
- $y_l(t)$ is the aggregate of link l at time t.

Equilibrium of each variable will be denoted as b_l^*, τ_{ls}^{f*}, τ_{ls}^{b*}, τ_s^*, w_s^* and p_l^*.

Whenever round-trip time, or forward and backward delay, appear in the *argument* of a variable, we will replace it by its equilibrium value τ_s^*, τ_{ls}^{f*}, τ_{ls}^{b*}. However, when round-trip time appears in the dependent variable, we will consider it time-varying. This avoids recursive time-arguments, but is admittedly an approximation, done exclusively for model tractability. Using this rule, $y_l(t)$ is approximated by $y_l(t) \approx \sum_s R_{ls} \frac{w_s(t-\tau_{ls}^{f*})}{\tau_s(t-\tau_{ls}^{f*})}$ (the rationale for the approximation is that the source rate, which is defined by $x_s(\cdot)$, and the window size are linked by a Little like law: $x_s(t) \approx \frac{w_s(t)}{\tau_s(t)}$).

In order to represent the linearized variables at the equilibrium, we will add the notation δ to each variable, so $\delta b_l(t) = b_l(t) - b_l^*$, $\delta \dot{b}_l(t) = \dot{b}_l(t)$, $\delta \ddot{b}_l(t) = \ddot{b}_l(t)$, $\delta \bar{b}(t) = \bar{b}(t) - b^*$, $\delta \dot{\bar{b}}(t) = \dot{\bar{b}}(t)$, $\delta \ddot{\bar{b}}(t) = \ddot{\bar{b}}(t)$, $\delta \dddot{\bar{b}}(t) = \dddot{\bar{b}}(t)$, $\delta \ddot{\tilde{b}}(t) = \ddot{\tilde{b}}(t)$, $\delta \dot{\tilde{b}}(t) = \dot{\tilde{b}}(t)$, $\delta \tau_s(t) = \tau_s(t) - \tau_s^*$, $\delta \dot{\tau}_s(t) = \dot{\tau}_s(t)$, $\delta w_s(t) = w_s(t) - w_s^*$, $\delta \dot{w}_s(t) = \dot{w}_s(t)$, $\delta p_l(t) = p_l(t) - p_l^*$ and $\delta \dot{p}_l(t) = \dot{p}_l(t)$.

2.2 AIMD model for Multiple Links and Heterogeneous sources

TCP Reno experiencing multiple links and heterogeneous delays, which extends the case of single link and homogeneous sources in [18, 19], is given by

$$\begin{aligned} \dot{w}_s(t) &= \frac{w_s(t-\tau_s^*)}{\tau_s(t-\tau_s^*)} \left(1 - \sum_l R_{ls} p_l(t-\tau_{ls}^{b*})\right) \frac{1}{w_s(t)} \\ &\quad - \frac{w_s(t-\tau_s^*)}{\tau_s(t-\tau_s^*)} \sum_l R_{ls} p_l(t-\tau_{ls}^{b*}) \frac{w_s(t)}{2}. \end{aligned} \tag{1}$$

In (1), the above and below equations represent AI (Additive Increase) and MD (Multiplicative Decrease) behaviors of the congestion avoidance mode of TCP, respectively.

For the purposes of linearization, we note that non-bottleneck links (with empty equilibrium queues) can be ignored. For bottleneck links whose dimension is L in this paper, we make the assumption that rate increase of a source affects all bottlenecks in its path, and write

$$\begin{aligned}\dot{b}_l(t) &= -c_l + \sum_s R_{ls} \frac{w_s(t-\tau_{ls}^{f*})}{\tau_s(t-\tau_{ls}^{f*})} \\ &= -c_l + \sum_s R_{ls} \frac{w_s(t-\tau_{ls}^{f*})}{\left(d_s + \sum_k R_{ks} \frac{b_k(t-\tau_{ls}^{f*})}{c_k}\right)} \\ &\approx -c_l + y_l(t).\end{aligned} \tag{2}$$

Note that $w_s(\cdot)$, $b_l(\cdot)$, and $p_l(\cdot)$ cannot have negative values. But, the present paper does not consider the effect of input/state constraints,[2].

2.3 AIMD model for Single Link and Homogeneous Sources

>From (1) and (2), homogeneous TCP Reno sources with the same window size ($d_s = d$, $\tau_{ls}^{f*} = \tau^{f*}$, $\tau_{ls}^{b*} = \tau^{b*}$, $w_s(t) = w(t)$) sharing a common bottleneck router ($c_l = c$, $b_l(t) = b(t)$, $p_l(t) = p(t)$) can be modeled by

$$\dot{w}(t) = \frac{w(t-\tau^*)}{\left(d + \frac{b(t-\tau^*)}{c}\right)} \left(1 - p(t-\tau^{b*})\right) \frac{1}{w(t)} - \frac{w(t-\tau^*)}{\left(d + \frac{b(t-\tau^*)}{c}\right)} p(t-\tau^{b*}) \frac{w(t)}{2} \tag{3}$$

with the queue dynamics

$$\dot{b}(t) = -c + N \frac{w(t-\tau^{f*})}{\left(d + \frac{b(t-\tau^{f*})}{c}\right)} \approx -c + y(t). \tag{4}$$

3 Feedback Control Structure Based on A State-Space Model

We are now in a position to derive a state-space model of the given TCP and queue dynamics allowing us to obtain interesting and important results as presented in the following sections. The mathematical derivations of this section focus on the case of single link and homogeneous sources with the assumption $\tau^f(\cdot) = 0$, so that $\tau(t) = \tau^b(t)$ (We will study how to relax this assumption in Subsection 4.2).

>From the derived model, in this section, we obtain PD-type state-feedback control in terms of queue length or RTT. We show how to extend this procedure in the presence of an additional dynamics like a low-pass filter of RED. A mathematical framework is proposed as a trial to interpret existing AQM algorithms in a unified manner which will be discussed in detail in Section 5.

[2]For design of stabilizing controls in the presence of input/state constraints, see [29, 30, 31].

3.1 State-Feedback Control for The Well Known AIMD Model of TCP

The first key step to deriving a state-feedback control structure is to convert two coupled dynamics (3) and (4) to the equivalent single dynamical system by differentiating (4) and rearranging the differentiated equation with (3) and (4)

$$\begin{aligned}
\ddot{b}(t) &= N\frac{\dot{w}(t-\tau^{f*})}{\left(d+\frac{b(t-\tau^{f*})}{c}\right)} - N\frac{\dot{b}(t-\tau^{f*})}{c}\frac{w(t-\tau^{f*})}{\left(d+\frac{b(t-\tau^{f*})}{c}\right)^2} \\
&= N\frac{\dot{w}(t-\tau^{f*})}{\left(d+\frac{b(t-\tau^{f*})}{c}\right)} - \frac{\dot{b}(t-\tau^{f*})}{c}\frac{(\dot{b}(t)+c)}{\left(d+\frac{b(t-\tau^{f*})}{c}\right)} \\
&= \frac{N\left(\dot{b}(t-\tau^*)+c\right)}{\left(d+\frac{b(t)}{c}\right)^2\left(\dot{b}(t)+c\right)} - \frac{\left(\dot{b}(t)+c\right)\dot{b}(t)}{\left(d+\frac{b(t)}{c}\right)c} \\
&\quad - \left\{\frac{N\left(\dot{b}(t-\tau^*)+c\right)}{\left(d+\frac{b(t)}{c}\right)^2\left(\dot{b}(t)+c\right)} + \frac{\left(\dot{b}(t-\tau^*)+c\right)\left(\dot{b}(t)+c\right)}{2N}\right\}p(t-\tau^*) \qquad (5) \\
&= f\left(b(t),\dot{b}(t),\dot{b}(t-\tau^*),p(t-\tau^*)\right).
\end{aligned}$$

In general, it is difficult to systematically find a nonlinear function $p(t)$ which guarantees the global asymptotic stability for nonlinear dynamical systems with delays (see the equivalent nonlinear system (69) for multiple links and heterogeneous sources). As a starting point to address these problems and as a method to avoid the nonnegative constraint, the present paper considers an equilibrium point with positive values and studies the linearized version of the derived dynamical system (5) on $b(t)$, $\dot{b}(t)$, $\dot{b}(t-\tau^*)$ and $p(t-\tau^*)$ near the equilibrium point. We can justify this study by the objective of a feedback control that keeps the closed-loop system around the equilibrium, where congestion occurs near the equilibrium.

>From (5), we can derive the following model of the linearized TCP and queue dynamics:

$$\delta\ddot{b}(t) = A_1\delta b(t) + A_2\delta\dot{b}(t) + B_1\delta p(t-\tau^*), \qquad (6)$$

where $\delta b(0)$, $\delta\dot{b}(0)$ and $\{\delta p(\sigma),\ \sigma\in[-\tau^*,0]\}$ are given, $A_1 = -\frac{2cN}{\tau^*(2N^2+c^2\tau^{*2})}$, $A_2 = -\frac{(2cN\tau^*+2N^2+c^2\tau^{*2})}{\tau(2N^2+c^2\tau^{*2})}$, and $B_1 = -\frac{(2N^2+c^2\tau^{*2})}{2\tau^{*2}N}$. Refer to Appendix 8.1 for derivation of (6). For presentation of the linearized variables, refer to the end of Subsection 2.1.

The differential equation (6) can be represented as the following state-space model:

$$\dot{z}(t) = Az(t) + B\delta p(t-\tau^*), \qquad (7)$$

where

$$z(t) = \begin{bmatrix}\delta b(t)\\ \delta\dot{b}(t)\end{bmatrix}, \quad A = \begin{bmatrix}0 & 1\\ A_1 & A_2\end{bmatrix}, \quad B = \begin{bmatrix}0\\ B_1\end{bmatrix}. \qquad (8)$$

The above state-space model is a minimal representation with state variables ($\delta b(t)$, $\delta \dot{b}(t)$) of (6). Note that the open-loop system of (7) (i.e., $\dot{z}(t) = Az(t)$) is asymptotically stable since its system matrix A has negative eigenvalues $\frac{(A_2+\sqrt{A_2^2+4A_1})}{2}$ and $\frac{(A_2-\sqrt{A_2^2+4A_1})}{2}$, which means that the feedback control $\delta p(\cdot) = 0$ (i.e, $p(\cdot) = p^*$ which means the constant drop probability) makes the system stable under the assumption of $\tau^f(\cdot) = 0^3$.

>From the above state-space model, we can naturally get a PD-type state-feedback control

$$\delta p(t) \;\; = \;\; H\, z(t) \; = \; H_P \delta b(t) \; + \; H_D \delta \dot{b}(t) \tag{9}$$

if we ignore the time delay τ^* (i.e., $\tau^* = 0$) in the control $\delta p(t - \tau^*)$. How to deal with the delay and how to obtain a pair of a stabilizing gain (H_P, H_D) will be discussed in Section 4 and Section 5, respectively.

The derived PD-type control structure, which we also did not expect, is very interesting in the following sense: it supports the conjecture of the networking literature in terms of the feedback control structure for the first time to our knowledge that AQM RED is not enough to regulate the given TCP [11, 14], where RED can be classified as a P-type AQM (i.e., $H_D = 0$) or as an output feedback control[4]. When we ignore the time delay τ^*, the difference between the state- and output-feedback controls is as follows. With P-type RED with $H_D = 0$, the closed-loop system (7) is $\dot{z}(t) = \begin{bmatrix} 0 & 1 \\ A_1 + B_1 H_P & A_2 \end{bmatrix} z(t)$, while the closed-loop system with (9) is $\dot{z}(t) = \begin{bmatrix} 0 & 1 \\ A_1 + B_1 H_P & A_2 + B_1 H_D \end{bmatrix} z(t)$. From these equations, it is easy to see that we cannot adjust eigenvalues of the closed-loop system arbitrarily with the output feedback control, while we can do that with the state-feedback control. Thus, the advantage of the state-feedback control comes from having the same degree of freedom as that of the system, while the output-feedback control has a less one. The case of REM/PI, which adds an integral structure to RED, will be discussed in Subsection 3.3.

Remark 1 *In order to relate the derived structure with the previous ones based on the transfer function approach, we consider the transfer function from $\delta b(t)$ to $\delta w(t)$ for the linearized system of (3) ($\delta \dot{w}(t) = -k_1 \delta w(t) + k_2 \delta p(t)$) with control (9). Then, the resulting transfer function is equal to the lead-lag compensator [28] with the form*

$$\frac{\delta w(s)}{\delta b(s)} \;\; = \;\; \frac{k_2(H_P + H_D s)}{s + k_1}.$$

For more details about the relationship with other papers based on the transfer function approach, refer to Remark 1 in [3].

Remark 2 *The PD-type structure obtained in this paper can also be implemented at sources as follows. Similarly to (5), using $\tau(t) = d + \frac{b(t)}{c}$, (3) and (4) can be converted to the equivalent single*

[3] In this paper, “stable" means “asymptotically stable", not “marginally stable" which corresponds to “oscillating".

[4] See [28] for the concept of the state-feedback and output-feedback controls.

dynamical system:

$$\begin{aligned}\ddot{\tau}(t) &= \frac{N(\dot{\tau}(t-\tau^*)+1)}{\left(d+\frac{b(t)}{c}\right)^2(c\dot{\tau}(t)+c)} - \frac{(\dot{\tau}(t)+1)\dot{\tau}(t)}{\left(d+\frac{b(t)}{c}\right)} \\ &\quad - \left\{\frac{N(\dot{\tau}(t-\tau^*)+1)}{\left(d+\frac{b(t)}{c}\right)^2(c\dot{\tau}(t)+c)} + \frac{(\dot{\tau}(t-\tau^*)+1)(c\dot{\tau}(t)+c)}{2N}\right\} p(t-\tau^*).\end{aligned}$$

From this equivalent form, in the same way as derivation of (6)-(8), the state-space model of the linearized TCP and queue dynamics is given by (7) and (8) with $z(t)$ and B_1 replaced by $z(t) = \begin{bmatrix}\delta\tau(t)\\ \delta\dot{\tau}(t)\end{bmatrix}$ and $B_1 = -\frac{(2N^2+c^2\tau^{*2})}{2c\tau^{*2}N}$, respectively. From this state-space model, we can naturally get another PD-type state-feedback control in terms of RTT

$$\delta p(t) \;=\; H\, z(t) \;=\; H_P\delta\tau(t) \;+\; H_D\delta\dot{\tau}(t) \tag{10}$$

if we ignore the time delay τ^* (i.e., $\tau^* = 0$) in the control $\delta p(t-\tau^*)$. Thus, our design procedures and control structures based on the state $\delta b(t)$ and $\delta\dot{b}(t)$ hold.

For implementation, (9) can be rewritten as the equivalent form

$$p(t) \;=\; p^* \;+\; H_P\big(b(t)-b^*\big) \;+\; H_D\big(y(t)-c\big).$$

Thus, our PD-type structure can be considered as combination of queue and rate controls. The implementation requires values of p^* and b^*. Since the AIMD model (3) is developed under the assumption that the congestion measure $p(t)$ is small, the equilibrium probability p^* should be small if dropping is used as the congestion measure. A high p^* can cause frequent re-transmissions and time-out which lead to the mismatch between the AIMD model and the congestion mode of TCP. However, it can be large if marking like ECN (Explicit Congestion Notification) is used as the congestion measure [32, 33] which will not be studied in the current paper. p^* should also not be so close to zero in order not to make $p(t)$ saturated, where saturation can be a cause of instability or chaos as shown in [34]. For the same reason, b^* should not be so close to zero. Selection of b^* should consider two more things. First, a very large queuing delay can make the nominal-stable system oscillate if the delay is not compensated appropriately. Second, congestion avoidance phase at TCP does not control a lot of small packets which can be considered as noise or disturbance for the dynamical model (3)[5]. Thus, b^* should be selected so that small packets go through networks without causing congestion. Thus, if b^* is very small or closed to the maximum queue size, it is not easy to stabilize system (3) with (9).

In this subsection, we derived the state-space model and its state-feedback control structure, where the derived PD-type feedback control implies the structural deficiency of P-type RED for controlling the given TCP. However, this subsection did not consider average queuing dynamics of RED which is used to make the dynamic behavior of TCP smooth for the burst traffic [9]. This case is studied in the following subsection.

[5]For one way to model and deal with the disturbance based on the state-space approach, see [35].

3.2 With Additional Dynamics: Low-Pass Filter of RED

Consider the dynamics of average queuing $\bar{b}(t)$ in RED which is called the low-pass filter in the control literature:

$$\dot{\bar{b}}(t) \;=\; -P_1\bar{b}(t) \;+\; P_1 b(t), \qquad P_1 \;>\; 0. \tag{11}$$

Here, P_1 is a design parameter which decides the cut-off frequency [28]. From (6) and (11), we can get

$$\delta\dddot{\bar{b}}(t) \;=\; \overline{A}_2\delta\ddot{\bar{b}}(t) \;+\; \overline{A}_1\delta\dot{\bar{b}}(t) \;+\; \overline{A}_3\delta\bar{b}(t) \;+\; P_1B_1\delta p(t-\tau^*), \tag{12}$$

where $\overline{A}_1 \;=\; A_1 \;+\; A_2P_1$, $\overline{A}_2 \;=\; A_2 \;-\; P_1$, $\overline{A}_3 \;=\; A_1P_1$. The differential equation (12) can be represented as the following state-space model:

$$\dot{\bar{z}}(t) \;=\; \overline{A}\bar{z}(t) \;+\; \overline{B}\delta p(t-\tau^*), \tag{13}$$

where $\bar{z}(0)$ and $\{\delta p(\sigma),\ \sigma \in [-\tau^*, 0]\}$ are given,

$$\bar{z}(t) = \begin{bmatrix} \delta\bar{b}(t) \\ \delta\dot{\bar{b}}(t) \\ \delta\ddot{\bar{b}}(t) \end{bmatrix}, \quad \overline{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \overline{A}_3 & \overline{A}_1 & \overline{A}_2 \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} 0 \\ 0 \\ P_1B_1, \end{bmatrix}.$$

The above state-space model is a minimal representation of (12). From the above state-space model, we can naturally get the PD-type state-feedback control in the presence of a low-pass filter

$$\delta p(t) \;=\; \overline{H}z(t) \;=\; \overline{H}_1\delta\bar{b}(t) \;+\; \overline{H}_2\delta\dot{\bar{b}}(t) \;+\; \overline{H}_3\delta\ddot{\bar{b}}(t) \tag{14}$$

if we ignore the time delay τ^*. Note that RED with a low-pass filter can be represented as $\delta p(t) = \overline{H}_1\delta\bar{b}(t)$, where the derived structure (14) also shows that RED is not enough to control TCP dynamics of the AIMD model arbitrarily.

For implementation, (14) can be rewritten using (4) and (11) as the equivalent form, which does not require the estimation of $\dot{\bar{b}}(t)$ and $\ddot{\bar{b}}(t)$,

$$\begin{aligned} p(t) \;=\;& p^* \;+\; \overline{H}_1\big(\bar{b}(t) - b^*\big) \;+\; \overline{H}_2\big(-P_1\hat{b}(t) + P_1b(t)\big) \;-\; \overline{H}_3P_1\big(-P_1\hat{b}(t) + P_1b(t)\big) \\ &+ \overline{H}_3P_1\big(y(t) - c\big). \end{aligned}$$

In this subsection, we showed how to design a state-feedback control in the presence of a low-pass filter. In fact, the results in Subsection 3.1 can also be applied to other cases of additional dynamics. In the following subsection, we derive a state-feedback control in the presence of an integral structure.

3.3 A Unified Mathematical Framework Using Integral Control Action

This subsection applies integral control action technique in [36] to the system (7) as a trial to interpret other AQM algorithms such as REM and PI in a unified mathematical framework rather than to compare them via simulation, where impact of each structure on performance will be discussed in Section 5. Similarly to REM and PI, this subsection does not consider the low-pass filter of RED for ease of discussion.

The key step to applying the technique is to have another derivative of system (3) as follows:

$$\begin{aligned} \dddot{b}(t) &= \frac{\partial f}{\partial b(t)}\dot{b}(t) + \frac{\partial f}{\partial \dot{b}(t)}\ddot{b}(t) + \frac{\partial f}{\partial \dot{b}(t-\tau^*)}\ddot{b}(t-\tau^*) + \frac{\partial f}{\partial p(t-\tau^*)}\dot{p}(t-\tau^*) \qquad (15) \\ &= g\big(b(t), \dot{b}(t), \ddot{b}(t), \dot{b}(t-\tau^*), \ddot{b}(t-\tau^*), p(t-\tau^*), \dot{p}(t-\tau^*)\big). \end{aligned}$$

>From (15), we can derive the following third-order linearized TCP model:

$$\dot{z}_e(t) = A_e z_e(t) + B_e \delta\dot{p}(t-\tau^*), \qquad (16)$$

where $z_e(0)$ and $\{\delta\dot{p}(\sigma), \sigma \in [-\tau^*, 0]\}$ are given,

$$z_e(t) = \begin{bmatrix} z_0(t) \\ \dot{z}(t) \end{bmatrix}, \quad A_e = \begin{bmatrix} 0 & I_e \\ 0 & A \end{bmatrix}, \quad B_e = \begin{bmatrix} 0 \\ B \end{bmatrix},$$

with $z(t)$, A and B in (8), $z_0(t) = I_e z(t)$, and $I_e = [1,\ 0]$. Refer to Appendix 8.1 for derivation of (16). It is easy to check that the pair (A_e, B_e) is stabilizable if $B_1 \neq 0$.

>From the above state-space model, we can naturally get a PID-type[6] state-feedback control

$$\delta\dot{p}(t) = H_e\, z_e(t) = H_I \delta b(t) + H_P \delta\dot{b}(t) + H_D \delta\ddot{b}(t), \qquad (17)$$

if we ignore the time delay τ^*. As discussed in Subsection 3.1, setting $H_P = 0$ or $H_D = 0$ has a limitation on adjusting closed-loop eigenvalues, i.e., controlling the closed-loop dynamic behavior. The implication of $H_I = 0$ is that the augmented system (16) reduces to the original second-order system (7).

For implementation, (17) can be rewritten as the equivalent form

$$p(t) = p(t_0) + H_I \int_{t_0}^{t} \big(b(\sigma) - b^*\big) d\sigma + H_P\big(b(t) - b(t_0)\big) + H_D\big(y(t) - y(t_0)\big).$$

Note that the PID-type feedback control does not require the equilibrium point p^* even for the case that $H_I = 0$, while the PD-type feedback control needs p^*.

Now, we discuss the implication of integral action on the closed-loop dynamics.

[6] Just before submitting this paper, we have noticed that inner and outer loop PI-type AQM to the aggregate rate in [22] also has the PID-type delay-independent AQM to the queue length. Note that the PID-type AQM in this paper is derived from the state-space model differently from [20, 22, 23] based on the transfer function approach and is reported in [1] before paper [22] is published. Another delay-independent PID-type AQM is proposed in [37].

First, we consider its effect at the steady state. Without integral action, the stabilizing feedback control $\delta p(t)$ makes $z^T(t)z(t)$ approach zero which in turn makes $|\delta p(t)|$ approach zero since $\delta p(t) = Hz(t)$. As a result, the small $\delta p(t)$ makes $z^T(t)z(t)$ approach zero slowly, although this effect may be marginal for the global performance. With integral action, the resulting feedback control $\delta \dot{p}(t) = H_e z_e(t)$ at the time t reflects the accumulation of $\big(b(\sigma) - b^*\big)$ additionally, i.e., $p(t) = p(t_0) + H_e \int_{t_0}^{t} z_e(\sigma) d\sigma = p(t_0) + H_I \int_{t_0}^{t} \big(b(\sigma) - b^*\big) d\sigma + H\big(z(t) - z(t_0)\big)$ which makes the steady-state tracking error of $z(t)$ approach zero faster.

In the same way, at the transient state, this action makes the system approach the equilibrium faster when the queue $b(t)$ and rate $y(t)$ are below the equilibrium point since H_I is non-negative and H_D is positive for stability as shown in Section 5. However, since the integral structure accumulates the previous state information, it can make the system go over the equilibrium point easily, i.e., cause an overshoot. Then, the system goes back slowly to the equilibrium due to the damping property of the integral action. If the overshoot exceeds the maximum queue size, it causes a windup phenomenon due to the saturation which can severely degrade the performance (For anti-windup techniques, refer to [38, 39]). Derivative structure of the state-feedback controls reduces this damping phenomenon and makes the system go and back to the equilibrium fast. This is another main reason why we need the PID-type state-feedback control instead of PI-type REM/PI, where REM/PI can be classified as output feedback controls like P-type RED.

Until now, we ignored the time delay in the control (congestion measure), i.e., we did not compensated for delays explicitly. Let's assume that we use $\delta p(t) = Hz(t)$ in (9) for the delayed system (7) (or (17) for (16)). This kind of control including RED, REM, PI, AVQ and AQMs in [22, 23, 24, 25] is called delay-independent (or memoryless) control in the literature. Then, the closed-loop system is given by

$$\dot{z}(t) \;=\; Az(t) + BHz(t-\tau^*) \quad (\text{or } \dot{z}_e(t) = A_e z_e(t) + B_e H_e z_e(t-\tau^*))$$

and thus has infinite number of eigenvalues. As the delay value τ^* increases over some finite value (depending on A, B and H), the number of positive eigenvalues increases, i.e., the closed-loop system oscillates. The way to solve this problem is to have a small control gain H as done in [22, 23, 24, 25] or setting $H = 0$ so that the system (7) with $\delta p(\cdot) = 0$ dominates, where the system (7) with $\delta p(\cdot) = 0$ is asymptotically stable under the assumption $\tau^f(\cdot) = 0$. With the small control gain, however, the closed-loop system cannot approach the equilibrium much faster than the system (7) with $\delta p(\cdot) = 0$, i.e., delay-independent controls cannot regulate the delayed dynamics arbitrarily in the presence of large delays like the output-feedback controls for the nominal system. This is the main reason why we need to develop a delay-dependent control which compensates for delays explicitly. The following section investigates how to design a delay-dependent control that uses not only the current dynamic information at time t but also the accumulated control information from $t - \tau^*$ to t. Note that the integral control in REM/PI, which use the accumulated state information, is not constructed for the delay compensation.

4 Derivation of the Delay-Dependent State-Feedback Control

In the first subsection, we propose how to compensate for the delay in the congestion measure explicitly under the same assumption of Section 3 that the forward delay from source to router is zero. This assumption is relaxed by applying a modified virtual queue dynamics in the second subsection. For simplicity of notation, throughout the rest of this paper, we define

$$\begin{aligned}
e_1 &= e^{-a_1\tau^*} - e^{-a_2\tau^*}, \quad e_2 = a_1 e^{-a_1\tau^*} - a_2 e^{-a_2\tau^*}, \quad e_3 = a_2 e^{-a_1\tau^*} - a_1 e^{-a_2\tau^*} \\
e_4 &= -\frac{1}{e_1}\left[\frac{a_1}{a_2}e^{-a_1\tau^*} - \frac{a_2}{a_1}e^{-a_2\tau^*} + \frac{(a_2^2 - a_1^2)}{a_1 a_2}e^{-A_2\tau^*}\right] \\
e_5 &= \frac{1}{e_1}\left[\frac{1}{a_2}e^{-a_1\tau^*} - \frac{1}{a_1}e^{-a_2\tau^*} + \frac{(a_2 - a_1)}{a_1 a_2}e^{-A_2\tau^*}\right] \\
a_1 &= \frac{(A_2 + \sqrt{A_2^2 + 4A_1})}{2}, \quad a_2 = \frac{(A_2 - \sqrt{A_2^2 + 4A_1})}{2}, \quad \hat{B}_1 = \frac{B_1(a_2 - a_1)e^{-A_2\tau^*}}{e_1}
\end{aligned} \tag{18}$$

4.1 Compensation for the Delay in Feedback Control

The key to deriving an explicit memory control for the delayed system (7) is to transform the delayed system (7) to the equivalent nominal system

$$\dot{s}(t) = As(t) + \hat{B}\delta p(t), \tag{19}$$

where $s(t) = [s_1(t),\ s_2(t)]^T$, $\hat{B} = [0,\ \hat{B}_1]^T$,

$$s_1(t) = -\frac{e_2}{e_1}\big(\delta b(t) + u_{1\tau}(t)\big) + \big(\delta\dot{b}(t) + u_{2\tau}(t)\big) \tag{20}$$

$$s_2(t) = A_1\big(\delta b(t) + u_{1\tau}(t)\big) + \frac{e_3}{e_1}\big(\delta\dot{b}(t) + u_{2\tau}(t)\big) \tag{21}$$

$$\begin{bmatrix} u_{1\tau}(t) \\ u_{2\tau}(t) \end{bmatrix} = \frac{B_1}{(a_1 - a_2)}\int_{-\tau^*}^{0}\begin{bmatrix} e^{-(\sigma+\tau^*)a_1} - e^{-(\sigma+\tau^*)a_2} \\ a_1 e^{-(\sigma+\tau^*)a_1} - a_2 e^{-(\sigma+\tau^*)a_2}\end{bmatrix}\delta p(\sigma + t)d\sigma. \tag{22}$$

Refer to Appendix 8.2 for derivation of (19). It is easy to see that if $\tau^* = 0$, then $s(t) = z(t)$ and $\hat{B} = B$. Note that the pair $(A, \hat{B})$ is stabilizable, and the closed-loop system of (19) is asymptotically stable if and only if the transformed system (7) is asymptotically stable. Since $A_2^2 + 4A_1 > 0$ for system matrices of (7), we have $a_1 \neq a_2 \neq 0$, $e_1 < 0$ and $\hat{B}_1 < 0$ for $\tau^* > 0$, while the linearized model of [19] has $a_1 = a_2$ when $2N = \tau^* c$ even for $\tau^* > 0$.

Thus, by adding the memory control structure $u_{i\tau}(t)$, we can handle the delay in congestion measure explicitly. From the above state-space model, we can get a PD-type delay-dependent state-feedback control

$$\delta p(t) = H_P^\tau\big(\delta b(t) + u_{1\tau}(t)\big) + H_D^\tau\big(\delta\dot{b}(t) + u_{2\tau}(t)\big). \tag{23}$$

Equivalently, it can be rewritten as $p(t) = p^* + H_P^\tau\big(b(t) - b^* + u_{1\tau}(t)\big) + H_D^\tau\big(y(t) - c + u_{2\tau}(t)\big)$.

Similarly, (16) can be transformed to the equivalent nominal system

$$\dot{s}_e(t) \;=\; A_e s_e(t) \;+\; \hat{B}_e \delta\dot{p}(t), \tag{24}$$

where $s_e(t) = [s_1(t),\ s_2(t),\ s_3(t)]^T$, $\hat{B}_e = [0,\ \hat{B}^T]^T$,

$$s_1(t) \;=\; \frac{(a_2-a_1)e^{-A_2\tau^*}}{e_1}\big(\delta b(t)+\dot{u}_{1\tau}(t)\big) \;+\; e_4\big(\delta\dot{b}(t)+\dot{u}_{2\tau}(t)\big) \;+\; e_5\big(\delta\ddot{b}(t)+\dot{u}_{3\tau}(t)\big) \tag{25}$$

$$s_2(t) \;=\; -\frac{e_2}{e_1}\big(\delta\dot{b}(t)+\dot{u}_{2\tau}(t)\big) \;+\; \big(\delta\ddot{b}(t)+\dot{u}_{3\tau}(t)\big) \tag{26}$$

$$s_3(t) \;=\; A_1\big(\delta\dot{b}(t)+\dot{u}_{2\tau}(t)\big) \;+\; \frac{e_3}{e_1}\big(\delta\ddot{b}(t)+\dot{u}_{3\tau}(t)\big) \tag{27}$$

$$\begin{bmatrix}\dot{u}_{1\tau}(t)\\ \dot{u}_{2\tau}(t)\\ \dot{u}_{3\tau}(t)\end{bmatrix} \;=\; \frac{B_1}{(a_1-a_2)}\int_{-\tau^*}^{0}\begin{bmatrix}\frac{(a_1-a_2)}{a_1a_2}+\frac{e^{-(\sigma+\tau^*)a_1}}{a_1}-\frac{e^{-(\sigma+\tau^*)a_2}}{a_2}\\ e^{-(\sigma+\tau^*)a_1}-e^{-(\sigma+\tau^*)a_2}\\ a_1e^{-(\sigma+\tau^*)a_1}-a_2e^{-(\sigma+\tau^*)a_2}\end{bmatrix}\delta\dot{p}(\sigma+t)d\sigma. \tag{28}$$

Refer to Appendix 8.2 for derivation of (24). The pair (A_e, B_e) is stabilizable (or controllable) if the pair (A, B) is stabilizable (or controllable).

>From the above model, we can get a PID-type delay-dependent state-feedback control

$$\delta\dot{p}(t) \;=\; H_I^\tau\big(\delta b(t)+\dot{u}_{1\tau}(t)\big) \;+\; H_P^\tau\big(\delta\dot{b}(t)+\dot{u}_{2\tau}(t)\big) \;+\; H_D^\tau\big(\delta\ddot{b}(t)+\dot{u}_{3\tau}(t)\big). \tag{29}$$

The equivalent form is $p(t) = p(t_0)+H_I^\tau\left(\int_{t_0}^t(b(\sigma)-b^*)d\sigma+u_{1\tau}(t)\right)+H_P^\tau\big(b(t)-b(t_0)+u_{2\tau}(t)\big)+H_D^\tau\big(y(t)-y(t_0)+u_{3\tau}(t)\big)$, where $\begin{bmatrix}u_{1\tau}(t)\\ u_{2\tau}(t)\\ u_{3\tau}(t)\end{bmatrix} = \frac{B_1}{(a_1-a_2)}\int_{-\tau^*}^{0}\begin{bmatrix}\frac{(a_1-a_2)}{a_1a_2}+\frac{e^{-(\sigma+\tau^*)a_1}}{a_1}-\frac{e^{-(\sigma+\tau^*)a_2}}{a_2}\\ e^{-(\sigma+\tau^*)a_1}-e^{-(\sigma+\tau^*)a_2}\\ a_1e^{-(\sigma+\tau^*)a_1}-a_2e^{-(\sigma+\tau^*)a_2}\end{bmatrix}\big(p(\sigma+t)-p(\sigma+t_0)\big)d\sigma$.

The procedures in this subsection can directly be applied to the case of a low-pass filter which has one more dimension as shown in Subsection 3.2. For more details, refer to our companion paper [4].

Next, we propose how to relax the assumption made in the derivation of (5) that the forward delay from source to router is zero, still compensating for delays explicitly.

4.2 Arbitrary Delay Compensation Based on A Modified Virtual Queue

The forward delay $\tau_s^f(t)$ from source to router produces a state-delay for the linearized system of the coupled dynamics (3) and (4) that makes it impossible to compensate for delays in a closed-form since the state-delayed system is infinite-dimensional (infinite number of eigenvalues).

In order to overcome this problem, we add the following modified virtual queue dynamics ($\dot{\tilde{b}}(t)$) at the router[7] to the AIMD model (3):

$$\dot{\tilde{b}}(t) \;=\; \gamma_1 \left(-\gamma_2 c \;+\; N\frac{w(t-\tau^{f*})}{\tau(t-\tau^{f*})}\right) \;\approx\; \gamma_1 \;(-\gamma_2 c + y(t))\,,$$

where $\tilde{b}(t)$ is the virtual queue length, and $\gamma_1(>0)$ and γ_2 are free design parameters.

If $0<\gamma_2<1$, $N\frac{w^*}{\tau^*}=\gamma_2 c$ and $b^*=0$. Hence, we have $\tau^*=\tau^{f*}+\tau^{b*}=d$, $w^*=\frac{d}{N}\gamma_2 c$, and $p^*=\frac{2}{(2+w^{*2})}=\frac{2N^2}{\left(2N^2+d^2\gamma_2^2c^2\right)}$.

Since the virtual queue dynamics is dominant near the equilibrium point, we approximate that the real queue $b(\cdot)$ is zero near the operating point in this subsection thus, the round trip time $\tau(t-\tau^{f*})$ is approximated as the round trip propagation delay d.

Then, (3) based on the above virtual queue dynamics is converted to the equivalent form

$$\begin{aligned}\ddot{\tilde{b}}(t) \;&=\; \frac{\gamma_1 N\left(\dot{\tilde{b}}(t-d)+\gamma_1\gamma_2 c\right)}{d^2\left(\dot{\tilde{b}}(t)+\gamma_1\gamma_2 c\right)} \;-\\ &\quad \left\{\frac{\gamma_1 N\left(\dot{\tilde{b}}(t-d)+\gamma_1\gamma_2 c\right)}{d^2\left(\dot{\tilde{b}}(t)+\gamma_1\gamma_2 c\right)}+\frac{\left(\dot{\tilde{b}}(t-d)+\gamma_1\gamma_2 c\right)\left(\dot{\tilde{b}}(t)+\gamma_1\gamma_2 c\right)}{2\gamma_1 N}\right\}p(t-d)\\ &=\; f\left(\dot{\tilde{b}}(t),\dot{\tilde{b}}(t-d),p(t-d)\right).\end{aligned}$$

Similarly to the previous sections, its linearized state-space model is given by

$$\delta\ddot{\tilde{b}}(t) \;=\; \tilde{A}_2\delta\dot{\tilde{b}}(t) \;+\; \tilde{B}_1\delta p(t-d), \tag{30}$$

where $\delta\dot{\tilde{b}}(0)$ and $\{\delta p(\sigma),\sigma\in[-d,0]\}$ are given,

$$\tilde{A}_2 \;=\; -\frac{2\gamma_2 cN}{(2N^2+\gamma_2^2c^2d^2)},\quad \tilde{B}_1 \;=\; -\frac{\gamma_1(2N^2+\gamma_2^2c^2d^2)}{2d^2N}. \tag{31}$$

Note that the above linearized model does not include the term $\dot{\tilde{b}}(t-d)$, where it has the term $\dot{\tilde{b}}(t-d)$ in general. This interesting and non-intuitive property allows us to use the same delay compensation technique as that in the previous subsection.

The state-space model (30) leads to the following P-type state-feedback control in terms of aggregate:

$$\delta p(t) \;=\; \tilde{H}_P\left(\delta\dot{\tilde{b}}(t)+u_{1d}(t)\right) \quad \Big(\text{or } p(t)=p^*+\tilde{H}_P\left(\gamma_1(-\gamma_2 c+y(t))+u_{1d}(t)\right)\Big)\,, \tag{32}$$

where $u_{1d}(t)=\int_{-d}^{0}e^{-\tilde{A}_2(\sigma+d)}\tilde{B}_1\delta p(\sigma+t)d\sigma$ (or $u_{1d}(t)=\int_{-d}^{0}e^{-\tilde{A}_2(\sigma+d)}\tilde{B}_1(p(\sigma+t)-p^*)d\sigma$).

[7] If $\gamma_1=\gamma_2=1$, the virtual queue dynamics is equal to the real queue dynamics, and the AVQ in [21] assumes $\gamma_1<0$ while we assumes $\gamma_1>0$ in this paper.

It is interesting to see that the delay-independent AVQ in [16, 21] seems to be a P-type control in terms of aggregate since we have $\delta p(t) = \tilde{H}_P^a \delta\dot{\tilde{b}}(t)$ linearizing the AVQ $p(t) = p(\dot{\tilde{b}}(t))$, which is the same as (32) in the absence of the delay d. Thus, our result verifies that the simplified AVQ is the delay-independent state-feedback control for the AIMD model when the virtual queue dynamics is dominant.

Similarly to Subsection 3.3, if one wants to make the steady-state tracking error approach zero fast, by applying integral control action technique we can derive extended state-space model

$$\dot{\tilde{z}}(t) \;=\; \tilde{A}\tilde{z}(t) \;+\; \tilde{B}\delta\dot{p}(t-d), \tag{33}$$

where $\tilde{z}(0)$ and $\{\delta\dot{p}(\sigma), \sigma \in [-d, 0]\}$ are given, $\tilde{z}(t) = \begin{bmatrix} \delta\dot{\tilde{b}}(t) \\ \delta\ddot{\tilde{b}}(t) \end{bmatrix}$, $\tilde{A} = \begin{bmatrix} 0 & 1 \\ 0 & \tilde{A}_2 \end{bmatrix}$, $\tilde{B} = \begin{bmatrix} 0 \\ \tilde{B}_1 \end{bmatrix}$.

The equivalent nominal system of the delayed system (33) is given by

$$\dot{\tilde{s}}_e(t) \;=\; \tilde{A}\tilde{s}_e(t) \;+\; \hat{\tilde{B}}\delta\dot{p}(t), \tag{34}$$

where $\tilde{s}_e(t) = [\tilde{s}_1(t),\ \tilde{s}_2(t)]^T$, $\hat{\tilde{B}} = [0,\ \hat{\tilde{B}}_1]^T$,

$$\tilde{s}_1(t) \;=\; \frac{\tilde{A}_2 e^{-\tilde{A}_2 d}}{\left(1 - e^{-\tilde{A}_2 d}\right)}\left(\delta\dot{\tilde{b}}(t) + \dot{u}_{1d}(t)\right) \;+\; \left(\delta\ddot{\tilde{b}}(t) + \dot{u}_{2d}(t)\right) \tag{35}$$

$$\tilde{s}_2(t) \;=\; \frac{\tilde{A}_2}{\left(1 - e^{-\tilde{A}_2 d}\right)}\left(\delta\ddot{\tilde{b}}(t) + \dot{u}_{2d}(t)\right), \qquad \hat{\tilde{B}}_1 \;=\; \frac{\tilde{B}_1 \tilde{A}_2 e^{-\tilde{A}_2 d}}{\left(1 - e^{-\tilde{A}_2 d}\right)} \tag{36}$$

$$\begin{bmatrix} \dot{u}_{1d}(t) \\ \dot{u}_{2d}(t) \end{bmatrix} \;=\; -\frac{\tilde{B}_1}{\tilde{A}_2}\int_{-d}^{0} \begin{bmatrix} 1 - e^{-(\sigma+d)\tilde{A}_2} \\ -\tilde{A}_2 e^{-(\sigma+d)\tilde{A}_2} \end{bmatrix} \delta\dot{p}(\sigma+t)d\sigma, \tag{37}$$

which leads to

$$\delta\dot{p}(t) \;=\; \tilde{H}_I^d\left(\delta\dot{\tilde{b}}(t) + \dot{u}_{1d}(t)\right) \;+\; \tilde{H}_P^d\left(\delta\ddot{\tilde{b}}(t) + \dot{u}_{2d}(t)\right). \tag{38}$$

Equivalently, $p(t) = p(t_0) + \tilde{H}_I^d\left(\int_{t_0}^t \gamma_1[-\gamma_2 c + y(\sigma)]d\sigma + u_{1d}(t)\right) + \tilde{H}_P^d\left(\gamma_1(y(t) - y(t_0)) + u_{2d}(t)\right)$, where $\begin{bmatrix} u_{1d}(t) \\ u_{2d}(t) \end{bmatrix} = -\frac{\tilde{B}_1}{\tilde{A}_2}\int_{-d}^{0} \begin{bmatrix} 1 - e^{-(\sigma+d)\tilde{A}_2} \\ -\tilde{A}_2 e^{-(\sigma+d)\tilde{A}_2} \end{bmatrix} (p(\sigma+t) - p(\sigma+t_0))d\sigma$.

Up to now, we studied what is a state-feedback control structure for the TCP AIMD model and its variants with additional dynamics, and how to compensate for delays explicitly. The natural next question is how to obtain a stabilizing gain of the feedback control structure. Although there are many ways to do that, as a trial to compare impact of different AQM structures on performance, the present paper applies optimal control framework that allows us to measure deviation of the transients of state and control variables from the equilibrium.

5 A Stabilizing Gain Design and Its Impact on Performance

In this section, we show how to get stabilizing optimal gains of the feedback control structures for the linearized systems (19) and (24) which can easily be applied to the cases of additional dynamics such as a low-pass filter and a modified virtual queue dynamics (see our companion papers [4, 5] for the cases of additional dynamics). And then, the impact of each structure on performance is discussed from the results of the optimal control framework.

5.1 A Stabilizing Optimal Gain for PD-Type Feedback Control

As a performance measure for (19), we consider the following optimization problem:

$$\min_{\delta p(\cdot)} \; J(s(t), \delta p(\cdot)) \;=\; \int_t^{t+\infty} \left(s^T(\sigma) Q s(\sigma) \;+\; \delta p^T(\sigma) R_c \delta p(\sigma) \right) d\sigma, \tag{39}$$

where the state weighting matrix Q is nonnegative, the control weighting matrix R_c is positive and the pair $(A, Q^{\frac{1}{2}})$ is observable (R_c should not be confused with the routing matrix R). By solving the above optimization problem, we can get stabilizing optimal and inverse optimal gains of the PD-type control.

Even if Q is negative, we can get a stabilizing control if the system is stabilizable. However, we do not consider the detailed case. Without loss of generosity, the current paper sets the weighting matrices Q as $Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}$ and R_c as $R_c = 1$, where $Q_1 > 0$ and $Q_2 \geq 0$.

For ease of explanation of the above performance index, assume that $\tau^* = 0$ (i.e., $s(t) = z(t)$ and $\hat{B} = B$). Then, we define the stabilizing optimal gain design problem as the problem of choosing a feedback control $\delta p(t)$ that minimizes the cost of transient around an equilibrium:

$$\min_{\delta p(\cdot)} \; J(z(t), \delta p(\cdot)) \;=\; \int_t^{t+\infty} \left(Q_1 \delta b^2(\sigma) + Q_2 \delta \dot{b}^2(\sigma) + \delta p^2(\sigma) \right) d\sigma.$$

Each term in the integrand penalizes transients on the queue length, queue length rate and the fluctuation of the loss probability, respectively. Hence, the cost is a weighted sum of transients on queue, queue rate and fluctuation in probability, weighted by Q_1, Q_2 and 1, respectively. For the given weighting, a slower transient incurs a higher cost.

Throughout the rest of this subsection, for simplicity of notation, we define

$$F_1 \;=\; A_1^2 + \hat{B}_1^2 Q_1, \quad F_2 \;=\; A_2^2 + \hat{B}_1^2 Q_2, \quad a_3 \;=\; \frac{1}{(a_1 - a_2)} \log_e \frac{a_2}{a_1}. \tag{40}$$

Proposition 1 *The stabilizing optimal gain of the PD-type delay-dependent control (23), which minimizes the transient cost (39) for system (19), is given by*

$$\begin{aligned} H_P^\tau &= \frac{\left(A_1 e_3 - e_2\sqrt{F_1} + A_1 e_1 \sqrt{F_2 + 2A_1 + 2\sqrt{F_1}}\right)}{(a_1 - a_2)B_1} \\ H_D^\tau &= \frac{\left(e_1\left(A_1 + \sqrt{F_1}\right) + e_3\left(A_2 + \sqrt{F_2 + 2A_1 + 2\sqrt{F_1}}\right)\right)}{(a_1 - a_2)B_1} \end{aligned} \tag{41}$$

and the resulting optimal cost is given by

$$J^* \;=\; s^T(0)Ks(0), \tag{42}$$

where K satisfies $0 = A^T K + KA + Q - K\hat{B}\hat{B}^T K$.

When $\tau^* = 0$ in (7), the stabilizing optimal gain of (23) (or (9)) is given by

$$H_P = -\frac{\left(A_1 + \sqrt{A_1^2 + B_1^2 Q_1}\right)}{B_1}, \quad H_D = -\frac{\left(A_2 + \sqrt{A_2^2 + B_1^2 Q_2 + 2A_1 + 2\sqrt{A_1^2 + B_1^2 Q_1}}\right)}{B_1}.$$

If the state and input constraints are not violated, then $s_1(t)$ ($\delta b(t)$ when $\tau^* = 0$) is given by

$$\begin{aligned} s_1(t) &= \frac{1}{(\lambda_2 - \lambda_1)}\left[\big(\lambda_2 s_1(0) - s_2(0)\big)e^{\lambda_1 t} - \big(\lambda_1 s_1(0) - s_2(0)\big)e^{\lambda_2 t}\right] \quad \textit{when } \lambda_1 \neq \lambda_2 \\ &= \left[s_1(0) + t\big(s_2(0) - s_1(0)\lambda_1\big)\right]e^{\lambda_1 t} \quad \textit{when } \lambda_1 = \lambda_2 \end{aligned} \tag{43}$$

$$\lambda_1,\ \lambda_2 \;=\; \frac{-\sqrt{F_2 + 2(A_1 + \sqrt{F_1})} \pm \sqrt{F_2 + 2(A_1 - \sqrt{F_1})}}{2}. \tag{44}$$

Proof: The optimal control that minimizes (39) and the resulting optimal cost are given by

$$\delta p^*(t) \;=\; -\hat{B}^T Ks(t), \quad J^*(s(t)) \;=\; s^T(t)Ks(t). \tag{45}$$

Note that K is a symmetric positive definite matrix [40] and the resulting closed-loop system is asymptotically stable since the pairs $(A, \hat{B})$ and $(A, Q^{\frac{1}{2}})$ are controllable and observable, respectively [41, 42].

By solving (45), we get (41). >From the closed-loop system, we get (43) and (44). ■

Proposition 1 implies that the solution of problem (39) is a stabilizing feedback control, specified by (H_P, H_D). Conversely, given any AQM of this structure, it solves problem (39) with appropriate weights Q_i as the next result says. It can be easily proved from Proposition 1.

Proposition 2 *Given a stabilizing control $\delta p(t) = [H_1,\ H_2]s(t)$ that satisfies $A_1 + \hat{B}_1 H_1 < 0$ and $A_2 + \hat{B}_1 H_2 < 0$, it solves problem (39) with weights:*

$$Q_1 \;=\; \frac{(H_1^2\hat{B}_1 + 2H_1 A_1)}{\hat{B}_1}, \quad Q_2 \;=\; \frac{(H_2^2\hat{B}_1 + 2H_2 A_2 + 2H_1)}{\hat{B}_1}. \tag{46}$$

The corresponding closed-loop eigenvalues λ_1 and λ_2 are given by

$$\lambda_1,\ \lambda_2 = \frac{(A_2 + \hat{B}_1 H_2) \pm \sqrt{(A_2 + \hat{B}_1 H_2)^2 + 4(A_1 + \hat{B}_1 H_1)}}{2}. \tag{47}$$

Proposition 3 *Given eigenvalues λ_1 and λ_2 of the closed-loop system (19) with (41), where real parts of λ_1 and λ_2 are negative, it solves problem (39) with weights:*

$$Q_1 = \frac{(\lambda_1^2\lambda_2^2 - A_1^2)}{\hat{B}_1^2}, \quad Q_2 = \frac{(\lambda_1^2 + \lambda_2^2 - A_2^2 - 2A_1)}{\hat{B}_1^2}. \tag{48}$$

>From Proposition 3, an easy way to design Q_1 and Q_2 is to make λ_1 equal to λ_2 and increase the value of $|\lambda_1|$. It can be done by setting $Q_1 = \frac{(\lambda^4 - A_1^2)}{\hat{B}_1^2}$, $Q_2 = \frac{\left(2\sqrt{A_1^2 + \hat{B}_1^2 Q_1} - A_2^2 - 2A_1\right)}{\hat{B}_1^2}$. Then, we have only to design one parameter for the second-order system.

Remark 3 *Actually, $\delta p(t)$ is constrained as $-p^* \leq \delta p(t) \leq 1 - p^*$. Thus, it is necessary to check the extremum of $\delta p(t)$. To this end, we define*

$$\begin{aligned}
t_a^* &= \frac{1}{(\lambda_1 - \lambda_2)} log_e \frac{-\lambda_2\beta_2}{\lambda_1\beta_1}, \quad t_b^* = \frac{\beta_3}{\left(s_2(0) - \lambda_1 s_1(0)\right)} \\
\beta_1 &= \left(A_1 + A_2\lambda_1 - \lambda_1^2\right)\left(\lambda_2 s_1(0) - s_2(0)\right), \quad \beta_2 = \left(A_1 + A_2\lambda_2 - \lambda_2^2\right)\left(s_2(0) - \lambda_1 s_1(0)\right) \\
\beta_3 &= \frac{\left(\lambda_1^2(A_2 - 2\lambda_1)s_1(0) - (A_1 + 2A_2\lambda_1 - 3\lambda_1^2)s_2(0)\right)}{\lambda_1\left((A_2 - 2\lambda_1)\lambda_1 + A_1 + \lambda_1^2\right)}.
\end{aligned}$$

From (41) and (43), if $\lambda_1 \neq \lambda_2$, the extremum of $\delta p(t)$ is given by

$$\begin{aligned}
\delta p(t_a^*) &= \frac{1}{\hat{B}_1(\lambda_1 - \lambda_2)} \left\{ \beta_1 \left(\frac{-\lambda_2\beta_2}{\lambda_1\beta_1}\right)^{\frac{\lambda_1}{(\lambda_1 - \lambda_2)}} + \beta_2 \left(\frac{-\lambda_2\beta_2}{\lambda_1\beta_1}\right)^{\frac{\lambda_2}{(\lambda_1 - \lambda_2)}} \right\} \ \textit{when}\ \ t_a^* > 0 \\
\delta p(0) &= -\frac{1}{\hat{B}_1}\left((A_1 + \lambda_1\lambda_2)s_1(0) + (A_2 - \lambda_1 - \lambda_2)s_2(0)\right) \ \textit{when}\ \ t_a^* \leq 0.
\end{aligned}$$

If $\lambda_1 = \lambda_2$, the extremum of $\delta p(t)$ is given by

$$\begin{aligned}
\delta p(t_b^*) &= -\left\{ \frac{(A_1 + \lambda_1^2)}{\hat{B}_1}\left(s_1(0) + t\beta_3\right) + \frac{(A_2 - 2\lambda_1)}{\hat{B}_1}\left(s_2(0) + \lambda_1 t\beta_3\right) \right\} e^{\lambda_1 t^*} \ \textit{when}\ \ t_b^* > 0 \\
\delta p(0) &= -\frac{1}{\hat{B}_1}\left((A_1 + \lambda_1^2)s_1(0) + (A_2 - 2\lambda_1)s_2(0)\right) \ \textit{when}\ \ t_b^* \leq 0.
\end{aligned}$$

Similarly, we show how to get a stabilizing optimal gain of the PID-type control for the linearized system (24) in the next section.

5.2 A Stabilizing Optimal Gain for PID-Type Feedback Control

As a performance measure for (24), similarly to the previous section, consider

$$\min_{\delta\dot{p}(\cdot)} \ J(s_e(t), \delta\dot{p}(\cdot)) \ = \ \int_t^{t+\infty} \left(s_e^T(\sigma)Qs_e(\sigma) + \delta\dot{p}^T(\sigma)R_c\delta\dot{p}(\sigma)\right) d\sigma, \tag{49}$$

where $Q = Q^T \geq 0$ and the pair $(A_e, Q^{\frac{1}{2}})$ is observable.

For simplicity, throughout the rest of this subsection, we also define

$$K_e \ = \ \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{12} & K_{22} & K_{23} \\ K_{13} & K_{23} & K_{33} \end{bmatrix}, \quad Q \ = \ \begin{bmatrix} Q_1 & 0 & 0 \\ 0 & Q_2 & 0 \\ 0 & 0 & Q_3 \end{bmatrix} \tag{50}$$

$$\hat{\lambda}_1 \ = \ \lambda_1 + \lambda_2 + \lambda_3, \quad \hat{\lambda}_2 \ = \ \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_1\lambda_3, \quad \hat{\lambda}_3 \ = \ \lambda_1\lambda_2\lambda_3. \tag{51}$$

Proposition 4 *The stabilizing optimal gain of the PID-type delay-dependent control (29), which minimizes transient cost (49) for system (24), is given by*

$$\begin{aligned} H_I^\tau &= -\hat{B}_1 K_{13}\frac{(a_2 - a_1)e^{-A_2\tau^*}}{e_1} \\ H_P^\tau &= -\hat{B}_1\left(K_{13}e_4 - K_{23}\frac{e_2}{e_1} + K_{33}A_1\right) \\ H_D^\tau &= -\hat{B}_1\left(K_{13}e_5 + K_{23} + K_{33}\frac{e_3}{e_1}\right) \end{aligned} \tag{52}$$

and the resulting optimal cost is given by

$$J^*(s_e(0)) \ = \ s_e^T(0)K_e s_e(0), \tag{53}$$

where K_e is a symmetric positive definite matrix satisfying $0 = A_e^T K_e + K_e A_e + Q - K_e\hat{B}_e\hat{B}_e^T K_e$,

$$\begin{aligned} K_{11} &= \left(A_1 - \hat{B}_1^2 K_{23}\right)\frac{\sqrt{Q}_1}{\hat{B}_1}, \quad K_{12} \ = \ \left(A_1 - \hat{B}_1^2 K_{33}\right)\frac{\sqrt{Q}_1}{\hat{B}_1}, \quad K_{13} \ = \ -\frac{\sqrt{Q}_1}{\hat{B}_1} \\ K_{22} &= \frac{\sqrt{Q}_1}{\hat{B}_1} - A_1 K_{33} - (A_2 - \hat{B}_1^2 K_{33})K_{23}, \quad K_{23} \ = \ \frac{\left(-2A_2K_{33} + \hat{B}_1^2 K_{33}^2 - Q_3\right)}{2}, \end{aligned} \tag{54}$$

and K_{33} is the positive solution of the following fourth order polynomial, that makes K_{23} greater than $\frac{A_1}{\hat{B}_1^2}$ and makes K_{22} positive:

$$\begin{aligned} -\hat{B}_1^7 K_{33}^4 + 4A_2\hat{B}_1^5 K_{33}^3 + (4A_1\hat{B}_1^3 - 4A_2^2\hat{B}_1^3 + 2\hat{B}_1^5 Q_3)K_{33}^2 + (-8\hat{B}_1^2\sqrt{Q_1} - 8A_1A_2\hat{B}_1 \\ -4A_2\hat{B}_1^3 Q_3)K_{33} + 8A_2\sqrt{Q_1} - 4A_1\hat{B}_1 Q_3 + 4\hat{B}_1 Q_2 - \hat{B}_1^3 Q_3^2 \ = \ 0. \end{aligned} \tag{55}$$

When $\tau^* = 0$ in (16), the stabilizing optimal gain of (29) (or (17)) is given by

$$H_I = -B_1 K_{13}, \quad H_P = -B_1 K_{23}, \quad H_D = -B_1 K_{33}.$$

If the state and input constraints are not violated, then $s_1(t)$ ($\delta b(t)$ when $\tau^* = 0$) is given by

$$\begin{aligned} s_1(t) &= b_{11}e^{\lambda_1 t} + b_{12}e^{\lambda_2 t} + b_{13}e^{\lambda_3 t} \;\; \textit{when} \;\; \lambda_1 \neq \lambda_2 \neq \lambda_3 && (56)\\ &= b_{21}e^{\lambda_1 t} + b_{22}e^{\lambda_2 t} + b_{23}te^{\lambda_2 t} \;\; \textit{when} \;\; \lambda_1 \neq \lambda_2 = \lambda_3 && (57)\\ &= b_{31}e^{\lambda_1 t} + b_{32}te^{\lambda_1 t} + b_{33}t^2e^{\lambda_1 t} \;\; \textit{when} \;\; \lambda_1 = \lambda_2 = \lambda_3, && (58)\end{aligned}$$

where

$$\begin{aligned} b_{11} &= \frac{\lambda_2\lambda_3 s_1(0) - (\lambda_2+\lambda_3)s_2(0) + s_3(0)}{(\lambda_1-\lambda_3)(\lambda_1-\lambda_2)}\\ b_{12} &= \frac{\lambda_1\lambda_3 s_1(0) - (\lambda_1+\lambda_3)s_2(0) + s_3(0)}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}\\ b_{13} &= \frac{\lambda_1\lambda_2 s_1(0) - (\lambda_1+\lambda_2)s_2(0) + s_3(0)}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)} && (59)\\ b_{21} &= b_{11}|_{\lambda_2=\lambda_3}, \quad b_{22} = \frac{(\lambda_1^2 - 2\lambda_1\lambda_2)s_1(0) + 2\lambda_2 s_2(0) - s_3(0)}{(\lambda_2-\lambda_1)^2}\\ b_{23} &= \frac{\lambda_1\lambda_2 s_1(0) - (\lambda_1+\lambda_2)s_2(0) + s_3(0)}{(\lambda_2-\lambda_1)} && (60)\\ b_{31} &= s_1(0), \quad b_{32} = s_2(0) - \lambda_1 s_1(0), \quad b_{33} = \frac{\lambda_1^2 s_1(0) - 2\lambda_1 s_2(0) + s_3(0)}{2} && (61)\end{aligned}$$

and

$$\hat{\lambda}_1 = A_2 - \hat{B}_1^2 K_{33}, \;\; \hat{\lambda}_2 = -A_1 + \hat{B}_1^2 K_{23}, \;\; \hat{\lambda}_3 = \hat{B}_1\sqrt{Q_1}. \quad (62)$$

The proof of the above proposition follows that of Proposition 1.

Proposition 4 implies that the solution of problem (49) is a feedback control algorithm, specified by (K_{13}, K_{23}, K_{33}). Conversely, given any AQM of this structure, it solves problem (49) with appropriate weights Q_i, as the next result states.

Proposition 5 *Given a stabilizing control $\delta\dot{p}(t) = [H_1 \; H_2 \; H_3]s_e(t)$, it solves problem (49) with weights*

$$Q_1 = H_1^2, \;\; Q_2 = H_2^2 - 2\frac{(A_2H_1 + \hat{B}_1H_1H_3 - A_1H_2)}{\hat{B}_1}, \;\; Q_3 = H_3^2 + 2\frac{(A_2H_3 + H_2)}{\hat{B}_1} \quad (63)$$

Then, K_{11}, K_{12}, K_{22}, K_{13}, K_{23}, and K_{33} are given by

$$\begin{aligned} K_{11} &= \frac{(A_1 + \hat{B}_1H_2)H_1}{\hat{B}_1}, \;\; K_{12} = \frac{(A_2 + \hat{B}_1H_3)H_1}{\hat{B}_1}, \;\; K_{22} = H_2H_3 + \frac{(A_1H_3 + A_2H_2 + H_1)}{\hat{B}_1}\\ K_{13} &= -\frac{H_1}{\hat{B}_1}, \;\; K_{23} = -\frac{H_2}{\hat{B}_1}, \;\; K_{33} = -\frac{H_3}{\hat{B}_1} && (64)\end{aligned}$$

and $\hat{\lambda}_1$, $\hat{\lambda}_2$, $\hat{\lambda}_3$ *are given by*

$$\hat{\lambda}_1 \;=\; A_2 + \hat{B}_1 H_3, \quad \hat{\lambda}_2 \;=\; -(A_1 + \hat{B}_1 H_2), \quad \hat{\lambda}_3 \;=\; \hat{B}_1 H_1. \tag{65}$$

Proposition 6 *Given eigenvalues* λ_1, λ_2, *and* λ_3 *of the closed-loop system (24) with (52), where real parts of* λ_1, λ_2, *and* λ_3 *are negative,* $\delta\dot{p}(t)$ *solves problem (49) with weights*

$$Q_1 \;=\; \frac{\hat{\lambda}_3^2}{\hat{B}_1^2}, \quad Q_2 \;=\; \frac{(-A_1^2 + \hat{\lambda}_2^2 - 2\hat{\lambda}_1\hat{\lambda}_3)}{\hat{B}_1^2}, \quad Q_3 \;=\; \frac{(-A_2^2 - 2A_1 + \hat{\lambda}_1^2 - 2\hat{\lambda}_2)}{\hat{B}_1^2}. \tag{66}$$

Then, K_{13}, K_{23}, K_{33}, K_{11}, K_{12}, *and* K_{22} *are given by*

$$\begin{aligned} K_{11} \;&=\; -\frac{\hat{\lambda}_2\hat{\lambda}_3}{\hat{B}_1^2}, \quad K_{12} \;=\; \frac{\hat{\lambda}_1\hat{\lambda}_3}{\hat{B}_1^2}, \quad K_{22} \;=\; \frac{(-A_1A_2 - \hat{\lambda}_1\hat{\lambda}_2 + \hat{\lambda}_3)}{\hat{B}_1^2} \\ K_{13} \;&=\; -\frac{\hat{\lambda}_3}{\hat{B}_1^2}, \quad K_{23} \;=\; \frac{(A_1 + \hat{\lambda}_2)}{\hat{B}_1^2}, \quad K_{33} \;=\; \frac{(A_2 - \hat{\lambda}_1)}{\hat{B}_1^2}. \end{aligned} \tag{67}$$

Similarly to the previous section, an easy way to design Q_1, Q_2, and Q_3 is to make λ_1, λ_2, and λ_3 equal, and increase the value of λ_1. It can be done by setting $Q_1 = \left(\frac{\lambda^3}{\hat{B}_1}\right)^2$, $Q_2 = \frac{(-A_1^2+3\lambda^4)}{\hat{B}_1^2}$, $Q_3 = \frac{(-A_2^2-2A_1+3\lambda^2)}{\hat{B}_1^2}$, and increasing the value of λ. Since $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$, λ decides the decaying rate of the closed-loop system.

We now interpret the impact of each structure on performance from the results of the PID-type optimal control framework. The current paper mainly focus on AQM algorithms such as RED, REM and PI based on the real-queue dynamics. For a brief discussion about AVQ based on the virtual queue dynamics, see Subsection 4.2.

5.3 Impact of Different AQM Structures on Performance

For ease of comparison, we assume that $\tau^* = 0$ (i.e., $s_e(t) = z_e(t)$, $\hat{B}_e = B_e$) for the linearized model and we do not consider the low-pass filter of RED (For more details, see [4]).

Then, the linear models of RED and REM/PI are

$$\begin{aligned} \text{simplified RED:} \quad & \delta\dot{p}^r(t) \;=\; H_2^r \delta\dot{b}(t) \\ \text{REM/PI:} \quad & \delta\dot{p}^m(t) \;=\; H_1^m \delta b(t) \;+\; H_2^m \delta\dot{b}(t) \end{aligned}$$

for some nonnegative constants H_2^r, H_1^m, H_2^m. The linear models of RED and REM/PI roughly capture the models in the original papers [9, 17, 20].

By Proposition 4, the stabilizing optimal AQM has a strictly positive gain $K_{33} > 0$. Since this condition is satisfied by none of RED, REM and PI, none of them can be made optimal, in the sense of minimizing (49), by tuning its parameters. We can also interpret their structural deficiency of D-type control as follows.

>From (65) in Proposition 5, the sum of eigenvalues of the closed-loop system with REM and PI is given by

$$\hat{\lambda}_1 = A_2,$$

while our PID-type AQM has

$$\hat{\lambda}_1 = A_2 + B_1 H_3.$$

Thus, we cannot adjust the sum of eigenvalues without D-type control structure, i.e., cannot control the dynamic behavior of the closed-loop system arbitrary while we can do that with the state-feedback control structures. For example, $\hat{\lambda}_1$ is less negative when $H_3 = 0$ than when $H_3 > 0$. This suggests that the decay rate is smaller with $H_3 = 0$. We can similarly interpret the structural deficiency of P-type RED, compared with the PD-type state-feedback AQM from Propositions 1 and 2.

As shown in (53) and (64), transient costs of a simplified RED and REM/PI can be obtained from (53) by setting some elements of K to zero, with $H_1 = H_3 = 0$ (i.e., $K_{11} = K_{12} = K_{13} = K_{33} = 0$) and with $H_3 = 0$ (i.e., $K_{33} = 0$), respectively. Note that the costs of RED and REM/PI are always greater than that of the stabilizing optimal AQM since (53) is the optimal cost for the given system and weighting matrices.

Until now, we considered single link and homogeneous sources. In the next section, we come back to the general networks of TCP with multiple links and heterogeneous delays of Subsection 2.2.

6 Extension to Multiple Links and Heterogeneous Sources

In this section, we extend the results in the previous sections to the case of multiple links and heterogeneous sources. First, we derive an equivalent nonlinear dynamics and a linearized state-space model to this general case. Then, delay compensation and stabilizing optimal gain design are studied.

6.1 State-Space Model for General Networks

To derive a state-space model for the general case, we define

$$w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_N(t) \end{bmatrix}, \quad b(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_L(t) \end{bmatrix}, \quad p(t) = \begin{bmatrix} p_1(t) \\ p_2(t) \\ \vdots \\ p_L(t) \end{bmatrix}, \quad R_s = \begin{bmatrix} R_{1s} \\ R_{2s} \\ \vdots \\ R_{Ls} \end{bmatrix}$$

$$R_l = [R_{l1},\ R_{l2},\ \cdots,\ R_{lN}], \quad D_1 = \text{diag}\left(\frac{1}{c_l}\right), \quad D_2 = \text{diag}\left(\frac{1}{\tau_s^*}\right)$$

$$A_{1l} = -R_l\ \text{diag}\left(\frac{2w_s^{*2}}{\tau_s^{*3}(2+w_s^{*2})} R_s^T D_1\right), \quad A_{2l} = -R_l\ \text{diag}\left(\frac{2w_s^*}{\tau_s^*(2+w_s^{*2})}\right) R_l^{-1}$$

$$A_{3l} = -R_l\ \text{diag}\left(\frac{w_s^*}{\tau_s^{*2}} R_s^T D_1\right), \quad B_l = -R_l\ \text{diag}\left(\frac{(2+w_s^{*2})}{2\tau_s^{*2}} R_s^T\right)$$

$$\alpha_{ls} = \text{diag}(A_{1l}) E_l^1 E_s^2, \quad \beta_{ls} = \text{diag}(A_{3l}) E_l^1 E_s^2, \quad \gamma_{ls} = \text{diag}(B_l) E_l^1 E_s^2, \tag{68}$$

where R_l^{-1} is a vector satisfying $R_l R_l^{-1} = 1$, $E_l^1 \in R^{(L\times L\times N)\times(L\times N)}$ and $E_s^2 \in R^{(N\times L)\times L}$ have identity matrices $I_l \in R^{(L\times N)\times(L\times N)}$ and $I_s \in R^{L\times L}$ only at l-th and s-th block, respectively, and zero matrices at the other blocks. As an exceptional definition, when τ_s^{b*} is used inside the feedback control variable $p(\cdot)$ like $p(-\tau_s^{b*})$ and $p(t-\tau_{ls}^{f*}-\tau_s^{b*})$, we mean that $p(-\tau_s^{b*}) = [p_1(-\tau_{1s}^{b*}), p_2(-\tau_{2s}^{b*}), \cdots, p_L(-\tau_{Ls}^{b*})]^T$ and $p(t-\tau_{ls}^{f*}-\tau_s^{b*}) = [p_1(t-\tau_{ls}^{f*}-\tau_{1s}^{b*}), p_2(t-\tau_{ls}^{f*}-\tau_{2s}^{b*}), \cdots, p_L(t-\tau_{ls}^{f*}-\tau_{Ls}^{b*})]^T$.

Differentiating (2) and rearranging the differentiated equation with (1) and (2), we have

$$\ddot{b}_l(t) = \sum_s R_{ls}\left[\frac{\dot{w}_s(t-\tau_{ls}^{f*})}{\tau_s(t-\tau_{ls}^{f*})} - \frac{w_s(t-\tau_{ls}^{f*})}{\tau_s^2(t-\tau_{ls}^{f*})}\left(\sum_k R_{ks}\frac{\dot{b}_k(t-\tau_{ls}^{f*})}{c_k}\right)\right] \tag{69}$$

$$\begin{aligned} &= \sum_s R_{ls}\Bigg[\frac{w_s(t-\tau_{ls}^{f*}-\tau_s^*)}{w_s(t-\tau_{ls}^{f*})}\frac{\left(1-\sum_k R_{ks}p_k(t-\tau_{ls}^{f*}-\tau_{ks}^{b*})\right)}{\tau_s(t-\tau_{ls}^{f*})\tau_s(t-\tau_{ls}^{f*}-\tau_s^*)} - \frac{1}{2}\frac{w_s(t-\tau_{ls}^{f*}-\tau_s^*)}{\tau_s(t-\tau_{ls}^{f*}-\tau_s^*)} \\ &\quad \frac{w_s(t-\tau_{ls}^{f*})}{\tau_s(t-\tau_{ls}^{f*})}\left(\sum_k R_{ks}p_k(t-\tau_{ls}^{f*}-\tau_{ks}^{b*})\right) - \frac{w_s(t-\tau_{ls}^{f*})}{\tau_s^2(t-\tau_{ls}^{f*})}\left(\sum_k R_{ks}\frac{\dot{b}_k(t-\tau_{ls}^{f*})}{c_k}\right)\Bigg] \\ &= \sum_s f_s\big(w_s(t-\tau_{ls}^{f*}-\tau_s^*),\ w_s(t-\tau_{ls}^{f*}),\ b(t-\tau_{ls}^{f*}),\ \dot{b}(t-\tau_{ls}^{f*}),\ b(t-\tau_{ls}^{f*}-\tau_s^*), \\ &\quad p(t-\tau_{ls}^{f*}-\tau_s^{b*})\big). \end{aligned} \tag{70}$$

>From (70), we can get the following linearized TCP model of the general case:

$$\dot{z}(t) = \begin{bmatrix} 0 & I \\ 0 & \text{diag}(A_{2l}) \end{bmatrix} z(t) + \sum_s\sum_l \begin{bmatrix} 0 & 0 \\ \alpha_{ls} & \beta_{ls} \end{bmatrix} z(t-\tau_{ls}^{f*}) + \sum_s\sum_l \begin{bmatrix} 0 \\ \gamma_{ls} \end{bmatrix} \delta p(t-\tau_{ls}^{f*}-\tau_s^{b*}), \tag{71}$$

where $\{z(\sigma), \sigma \in [-\tau_{ls}^{f*}, 0]\}$ and $\{\delta p(\sigma), \sigma \in [-\tau_{ls}^{f*} - \tau_s^{b*}, 0]\}$ are given, $z(t) = \begin{bmatrix} \delta b(t) \\ \delta \dot{b}(t) \end{bmatrix}$.

If we want to apply integral action, we have another differentiation as follows:

$$\begin{aligned} \dddot{b}_l(t) &= \sum_s \Bigg[\frac{\partial f_s}{\partial w_s(t-\tau_{ls}^{f*}-\tau_s^*)} \dot{w}_s(t-\tau_{ls}^{f*}-\tau_s^*) + \frac{\partial f_s}{\partial w_s(t-\tau_{ls}^{f*})} \dot{w}_s(t-\tau_{ls}^{f*}) \\ &\quad + \frac{\partial f_s}{\partial b(t-\tau_{ls}^{f*})} \dot{b}(t-\tau_{ls}^{f*}) + \frac{\partial f_s}{\partial \dot{b}(t-\tau_{ls}^{f*})} \ddot{b}(t-\tau_{ls}^{f*}) \\ &\quad + \frac{\partial f_s}{\partial b(t-\tau_{ls}^{f*}-\tau_s^*)} \dot{b}(t-\tau_{ls}^{f*}-\tau_s^*) + \frac{\partial f_s}{\partial p(t-\tau_{ls}^{f*}-\tau_s^{b*})} \dot{p}(t-\tau_{ls}^{f*}-\tau_s^{b*}) \Bigg] \\ &= \sum_s g_s\big(w_s(t-\tau_{ls}^{f*}-\tau_s^*),\ \dot{w}_s(t-\tau_{ls}^{f*}-\tau_s^*),\ w_s(t-\tau_{ls}^{f*}),\ \dot{w}_s(t-\tau_{ls}^{f*}), \\ &\qquad b(t-\tau_{ls}^{f*}), \dot{b}(t-\tau_{ls}^{f*}),\ \ddot{b}(t-\tau_{ls}^{f*}),\ b(t-\tau_{ls}^{f*}-\tau_s^*),\ \dot{b}(t-\tau_{ls}^{f*}-\tau_s^*), \\ &\qquad p(t-\tau_{ls}^{f*}-\tau_s^{b*}),\ \dot{p}(t-\tau_{ls}^{f*}-\tau_s^{b*})\big). \end{aligned} \tag{72}$$

>From (72), we can get the extended linearized TCP model of the general case

$$\dot{z}_e(t) = A_e z_e(t) + \sum_s \sum_l A_{ls} z_e(t-\tau_{ls}^{f*}) + \sum_s \sum_l B_{ls} \delta \dot{p}(t-\tau_{ls}^{f*}-\tau_s^{b*}), \tag{73}$$

where $\{z_e(\sigma), \sigma \in [-\tau_{ls}^{f*}, 0]\}$ and $\{\delta \dot{p}(\sigma), \sigma \in [-\tau_{ls}^{f*} - \tau_s^{b*}, 0]\}$ are given,

$$z_e(t) = \begin{bmatrix} \delta b(t) \\ \delta \dot{b}(t) \\ \delta \ddot{b}(t) \end{bmatrix},\ A_e = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & 0 & \mathrm{diag}(A_{2l}) \end{bmatrix},\ A_{ls} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \alpha_{ls} & \beta_{ls} \end{bmatrix},\ B_{ls} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{ls} \end{bmatrix}. \tag{74}$$

For detailed derivation of the linearized TCP models (71) and (73), refer to Appendix 8.3. As shown in the above equations, the state-feedback controls of (71) and (73) also have PD-type and PID-type state-feedback control structures as in (9) and (17) if we ignore all delays.

6.2 Delay Compensation for General Networks

In the same way as Section 4, we first consider how to compensate for delays in the feedback control. If $\tau_{ls}^{f*} = 0$ for all l and s, (73) can be converted to the equivalent system

$$\dot{z}_e(t) = A_e z_e(t) + \sum_s B_s \delta \dot{p}(t-\tau_s^{b*}), \tag{75}$$

where $z_e(0)$ and $\{\delta\dot{p}(\sigma), \sigma \in [-\tau_s^{b*}, 0]\}$ are given, $A_e = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & A_1 & A_2 \end{bmatrix}$ and $B_s = \begin{bmatrix} 0 \\ 0 \\ \eta_s \end{bmatrix}$, $A_1 = -R\,\mathrm{diag}\left(\frac{2w_s^{*2}}{\tau_s^{*3}(2+w_s^{*2})}\right) R^T D_1$, $A_2 = \mathrm{diag}(A_{2l}) - R\,\mathrm{diag}\left(\frac{w_s^*}{\tau_s^{*2}}\right) R^T D_1$, $\eta_s = \begin{bmatrix} -R_{1s}\frac{(2+w_s^{*2})}{2\tau_s^{*2}} R_s^T \\ \vdots \\ -R_{Ls}\frac{(2+w_s^{*2})}{2\tau_s^{*2}} R_s^T \end{bmatrix}$.

Define $h_e(t) = z_e(t) + \sum_s \sum_k \int_t^{t+\tau_{ks}^{b*}} e^{A_e(t-\sigma)} \begin{bmatrix} 0 \\ 0 \\ \eta_s E_k \end{bmatrix} \delta\dot{p}_k(\sigma - \tau_{ks}^{b*}) d\sigma$, where E_k is a L-column vector that has 1 in l-th row and 0 in other rows. Then, the delayed system (75) can be converted to the equivalent nominal system

$$\begin{aligned} \dot{h}_e(t) &= A_e h_e(t) + \sum_s \left[\Phi_{A_e}(t, t+\tau_{1s}^{b*}) \begin{bmatrix} 0 \\ 0 \\ \eta_s E_1 \end{bmatrix}, \quad \cdots, \Phi_{A_e}(t, t+\tau_{Ls}^{b*}) \begin{bmatrix} 0 \\ 0 \\ \eta_s E_L \end{bmatrix} \right] \delta\dot{p}(t) \\ &= A_e h_e(t) + \tilde{B}_e \delta\dot{p}(t). \end{aligned} \tag{76}$$

To get a more simplified equivalent nominal system, we have one more transformation. Let $s_e(t) = T_e h_e(t)$ and $\tilde{B}_e = \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \\ \tilde{B}_3 \end{bmatrix}$, where $T_e = \begin{bmatrix} -\tilde{B}_3\tilde{B}_2^{-1} - e_1 A_1 & I - e_1 A_2 & e_1 \\ 0 & -\tilde{B}_3\tilde{B}_2^{-1} & I \\ 0 & A_1 & A_2 - \tilde{B}_3\tilde{B}_2^{-1} \end{bmatrix}$, $e_1 = (\tilde{B}_2 - \tilde{B}_3\tilde{B}_2^{-1}\tilde{B}_1)\ (A_1\tilde{B}_1 + A_2\tilde{B}_2 - \tilde{B}_3)^{-1}$, $\tilde{B}_3\tilde{B}_2^{-1} + e_1 A_1 \neq 0$, and $A_1 + (A_2 - \tilde{B}_3\tilde{B}_2^{-1})\tilde{B}_3\tilde{B}_2^{-1} \neq 0$.

Then, (76) can be rewritten as

$$\dot{s}_e(t) = \hat{A}_e s_e(t) + \hat{B}_e \delta\dot{p}(t), \tag{77}$$

where

$$\begin{aligned} \hat{A}_e &= \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & \hat{A}_1 & \hat{A}_2 \end{bmatrix}, \quad \hat{B}_e = \begin{bmatrix} 0 \\ 0 \\ \hat{B}_1 \end{bmatrix}, \quad \hat{A}_1 = \hat{B}_1 e_2 A_1 \tilde{B}_2 \tilde{B}_3^{-1}, \quad \hat{A}_2 = A_2 - \tilde{B}_3\tilde{B}_2^{-1} + \hat{B}_1 e_2 \\ \hat{B}_1 &= A_1\tilde{B}_2 + A_2\tilde{B}_3 - \tilde{B}_3\tilde{B}_2^{-1}\tilde{B}_3, \quad e_2 = (A_2\tilde{B}_2 + A_1\tilde{B}_2\tilde{B}_3^{-1}\tilde{B}_2 - \tilde{B}_3)^{-1}. \end{aligned} \tag{78}$$

If $\tau_{ls}^{b*} = 0$ for all l and s, then $s_e(t) = z_e(t)$ and $\hat{B}_1 = -R\,\mathrm{diag}\left(\frac{(2+w_s^{*2})}{2\tau_s^{*2}}\right) R^T$.

>From this state-space model, we can get a PID-type delay-dependent state-feedback control as in (29) (in the same way, we can get a PD-type delay-dependent state-feedback control). It is also easy to see that we can relax the assumption $\tau_{ls}^{f*} = 0$ using the virtual queue dynamics as in Subsection 4.2.

6.3 A Stabilizing Optimal Gain Design for General Networks

Similarly to the previous section, to get a stabilizing optimal gain, we consider (49) as a performance measure under the assumption that the system is controllable and observable.

Proposition 7 *The stabilizing optimal gain of (29), which minimizes (49), is given by*

$$H_I^\tau = -R_c^{-1}\hat{B}_1 K_{13}, \quad H_P^\tau = -R_c^{-1}\hat{B}_1 K_{23}, \quad H_D^\tau = -R_c^{-1}\hat{B}_1 K_{33} \tag{79}$$

and the resulting optimal cost is given by $J^*(s_e(0)) = s_e^T(0)K_e s_e(0)$, *where*

$$\begin{aligned}
K_{11} &= Q_1^{\frac{1}{2}}\theta, \quad K_{12} \;=\; K_{33}\hat{B}_1 R_c^{-\frac{1}{2}}Q_1^{\frac{1}{2}} - \hat{A}_2^T\hat{B}_1^{-T}R_c^{\frac{1}{2}}Q_1^{\frac{1}{2}}, \quad K_{13} \;=\; -\hat{B}_1^{-T}R_c^{\frac{1}{2}}Q_1^{\frac{1}{2}} \\
K_{22} &= \theta(R_c^{-\frac{1}{2}}\hat{B}_1^T K_{33} - R_c^{\frac{1}{2}}\hat{B}_1^{-1}\hat{A}_2) - \hat{A}_1^T\hat{B}_1^{-T}R_c\hat{B}_1^{-1}\hat{A}_2^T + Q_1^{\frac{1}{2}}R_c^{\frac{1}{2}}\hat{B}_1^{-1} \\
K_{23} &= \hat{B}_1^{-T}R_c\hat{B}_1^{-1}\hat{A}_1 + \hat{B}_1^{-T}R_c^{\frac{1}{2}}\theta \\
\theta &= (\hat{A}_1^T\hat{B}_1^{-T}R_c\hat{B}_1^{-1}\hat{A}_1 + Q_1^{\frac{1}{2}}R_c^{-\frac{1}{2}}\hat{B}_1^T K_{33} - Q_1^{\frac{1}{2}}R_c^{\frac{1}{2}}\hat{B}_1^{-1}\hat{A}_2 + K_{33}\hat{B}_1 R_c^{-\frac{1}{2}}Q_1^{\frac{1}{2}} \\
&\quad -\hat{A}_2^T\hat{B}_1^{-T}R_c^{\frac{1}{2}}Q_1^{\frac{1}{2}} + Q_2)^{\frac{1}{2}}
\end{aligned} \tag{80}$$

and K_{33} *is the symmetric positive definite solution of the following algebraic equation, that makes* K_{11} *and* K_{22} *symmetric and positive definite:*

$$\begin{aligned}
0 \;=\;& \hat{A}_2^T K_{33} + K_{33}\hat{A}_2 + \hat{A}_1^T\hat{B}_1^{-T}R_c\hat{B}_1^{-1} + \hat{B}_1^{-T}R_c\hat{B}_1^{-1}\hat{A}_1 + \theta R_c^{\frac{1}{2}}\hat{B}_1^{-1} + \hat{B}_1^{-T}R_c^{\frac{1}{2}}\theta \\
&+Q_3 - K_{33}\hat{B}_1 R_c^{-1}\hat{B}_1^T K_{33}.
\end{aligned}$$

The proof of the above proposition follows that of Proposition 1.

Proposition 8 *Given* $\delta\dot{p}(t) = [H_1\ H_2\ H_3]s_e(t)$, *it solves problem (49) with weights*

$$\begin{aligned}
Q_1 &= H_1^T R_c H_1 \\
Q_2 &= H_2^T R_c H_2 + A_1^T\hat{B}_1^{-T}R_c H_2 + H_2^T R_c\hat{B}_1^{-1}\hat{A}_1 - H_1^T R_c H_3 \\
&\quad -H_3^T R_c H_1 - H_1^T R_c\hat{B}_1^{-T}\hat{A}_2 - \hat{A}_2^T\hat{B}_1^{-T}R_c H_1 \\
Q_3 &= H_3^T R_c H_3 + H_2^T R_c\hat{B}_1^{-1} + \hat{B}_1^{-T}R_c H_2 + \hat{A}_2^T\hat{B}_1^{-T}R_c H_3 + \hat{B}_1^{-T}R_c H_3\hat{A}_2.
\end{aligned} \tag{81}$$

Then, K_{11}, K_{12}, K_{22}, K_{13}, K_{23}, *and* K_{33} *are given by*

$$\begin{aligned}
K_{11} &= H_1^T R_c H_2 + H_1^T R_c\hat{B}_1^{-1}\hat{A}_1, \quad K_{12} \;=\; H_3^T R_c H_1 + \hat{A}_2^T\hat{B}_1^{-T}R_c H_1 \\
K_{22} &= H_2^T R_c H_3 + H_1^T R_c\hat{B}_1^{-1} + \hat{A}_1^T\hat{B}_1^{-T}R_c H_3 + H_2^T R_c\hat{B}_1^{-1}\hat{A}_2 \\
K_{13} &= -\hat{B}_1^{-T}R_c H_1, \quad K_{23} \;=\; -\hat{B}_1^{-T}R_c H_2, \quad K_{33} \;=\; -\hat{B}_1^{-T}R_c H_3.
\end{aligned} \tag{82}$$

>From the study in this section, we can see that the design procedures and structural properties for the case of single link and homogeneous sources hold for the general network case.

7 Conclusion

The present paper studied how to design a stabilizing feedback control based on state-space models for the given TCP and its variants with additional dynamics, where the feedback control can be implemented either at sources or at routers with different type of congestion signal. We derived state-feedback control and explicit delay compensation structures which are necessary to regulate the given dynamical system arbitrarily. As a subsequent result, we proposed a mathematical framework allowing us to interpret RED and REM/PI as different approximations of the PID-type mathematical framework, and discussed the impact of each structure on performance from the results of the stabilizing optimal control framework.

One can extend this work to more realistic congestion control problems by applying modern control theories or stochastic theories. We also expect that our feedback control design procedures can be applied to the future TCP protocol or other dynamical systems.

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8 Appendix

8.1 Derivation of (16)

Let (w^*, b^*, p^*) be the equilibrium point. Then, the linearized model of TCP Reno (5) is

$$\delta\ddot{b}(t) = \frac{\partial f}{\partial b(t)}|_*\delta b(t) + \frac{\partial f}{\partial \dot{b}(t)}|_*\delta\dot{b}(t) + \frac{\partial f}{\partial \dot{b}(t-\tau^*)}|_*\delta\dot{b}(t-\tau^*) + \frac{\partial f}{\partial p(t-\tau^*)}|_*\delta p(t-\tau^*),$$

where

$$\begin{aligned} \frac{\partial f}{\partial b(t)}|_* &= -\frac{2N}{\tau^{*3}c}(1-p^*), \qquad \frac{\partial f}{\partial \dot{b}(t)}|_* = -\frac{N}{\tau^{*2}c}(1-p^*) - \frac{1}{\tau} - \frac{p^*c}{2N} \\ \frac{\partial f}{\partial \dot{b}(t-\tau^*)}|_* &= 0, \qquad \frac{\partial f}{\partial p(t-\tau^*)}|_* = -\frac{N}{\tau^{*2}} - \frac{c^2}{2N}, \qquad p^* = \frac{2N^2}{2N^2 + c^2\tau^{*2}}. \end{aligned} \tag{83}$$

>From (83), the linearized model of TCP Reno (5) can be converted to (6).

Similarly, the linearized model of TCP Reno (15) is

$$\begin{aligned} \delta\dddot{b}(t) &= \frac{\partial g}{\partial b(t)}|_*\delta b(t) + \frac{\partial g}{\partial \dot{b}(t)}|_*\delta\dot{b}(t) + \frac{\partial g}{\partial \ddot{b}(t)}|_*\delta\ddot{b}(t) + \frac{\partial g}{\partial \dot{b}(t-\tau^*)}|_*\delta\dot{b}(t-\tau^*) \\ &\quad + \frac{\partial g}{\partial \ddot{b}(t-\tau^*)}|_*\delta\ddot{b}(t-\tau^*) + \frac{\partial g}{\partial p(t-\tau^*)}|_*\delta p(t-\tau^*) + \frac{\partial g}{\partial \dot{p}(t-\tau^*)}|_*\delta\dot{p}(t-\tau^*) \\ &= \frac{\partial f}{\partial b(t)}|_*\delta\dot{b}(t) + \frac{\partial f}{\partial \dot{b}(t)}|_*\delta\ddot{b}(t) + \frac{\partial f}{\partial p(t-\tau^*)}|_*\delta\dot{p}(t-\tau^*). \end{aligned}$$

From (83), the linearized model of TCP Reno (15) can be converted to (16).

8.2 Derivation of (19) and (24)

We have only to derive (24) since we can handle (19) as a special case of (24).

Note that system (16) can be written as

$$\begin{aligned} z_e(t+\tau^*) &= e^{A_e\tau^*}z_e(t) + \int_t^{t+\tau^*} e^{A_e(t+\tau^*-\sigma)}B_e\delta\dot{p}(\sigma-\tau^*)d\sigma \\ &= e^{A_e\tau^*}\left[z_e(t) + \int_{-\tau^*}^{0} e^{-A_e(\sigma+\tau^*)}B_e\delta\dot{p}(\sigma+t)d\sigma\right], \end{aligned} \tag{84}$$

where

$$e^{A_e t} = \frac{1}{(a_2-a_1)}\begin{bmatrix} (a_2-a_1) & \frac{(a_1^2-a_2^2)}{a_1a_2}+\frac{a_2}{a_1}e^{a_1t}-\frac{a_1}{a_2}e^{a_2t} & \frac{(a_2-a_1)}{a_1a_2}-\frac{1}{a_1}e^{a_1t}+\frac{1}{a_2}e^{a_2t} \\ 0 & a_2e^{a_1t}-a_1e^{a_2t} & -e^{a_1t}+e^{a_2t} \\ 0 & a_1a_2(e^{a_1t}-e^{a_2t}) & -a_1e^{a_1t}+a_2e^{a_2t} \end{bmatrix} \tag{85}$$

Define the insider part of the above second equation as

$$h_e(t) = z_e(t) + \int_{-\tau^*}^{0} e^{-A_e(\sigma+\tau^*)}B_e\delta\dot{p}(t+\sigma)d\sigma = z_e(t) + \begin{bmatrix}\dot{u}_{1\tau}(t)\\ \dot{u}_{2\tau}(t)\\ \dot{u}_{3\tau}(t)\end{bmatrix}. \tag{86}$$

Using (84) and (86), system (16) can be rewritten as the following nominal system:

$$\dot{h}_e(t) = A_eh_e(t) + \tilde{B}_e\delta\dot{p}(t), \tag{87}$$

where

$$\tilde{B}_e = \begin{bmatrix}\tilde{B}_1\\ \tilde{B}_2\\ \tilde{B}_3\end{bmatrix} = e^{-A_e\tau^*}B_e = \frac{B_1}{(a_1-a_2)}\begin{bmatrix}\left(\frac{(a_1-a_2)}{a_1a_2}+\frac{1}{a_1}e^{-a_1\tau^*}-\frac{1}{a_2}e^{-a_2\tau^*}\right)\\ (e^{-a_1\tau^*}-e^{-a_2\tau^*})\\ (a_1e^{-a_1\tau^*}-a_2e^{-a_2\tau^*})\end{bmatrix}. \tag{88}$$

To get the explicit stabilizing optimal gain as shown in Propositions 1 and 4, we have another transformation as follows.

Let $s_e(t) = T_e h_e(t)$, where $T_e = \begin{bmatrix} -\frac{(A_1\tilde{B}_2^2+A_2\tilde{B}_2\tilde{B}_3-\tilde{B}_3^2)}{\tilde{B}_2(A_1\tilde{B}_1+A_2\tilde{B}_2-\tilde{B}_3)} & \frac{(A_1\tilde{B}_1\tilde{B}_2+A_2\tilde{B}_1\tilde{B}_3-\tilde{B}_2\tilde{B}_3)}{\tilde{B}_2(A_1\tilde{B}_1+A_2\tilde{B}_2-\tilde{B}_3)} & \frac{(\tilde{B}_2^2-\tilde{B}_1\tilde{B}_3)}{\tilde{B}_2(A_1\tilde{B}_1+A_2\tilde{B}_2-\tilde{B}_3)} \\ 0 & -\frac{\tilde{B}_3}{\tilde{B}_2} & 1 \\ 0 & A_1 & \frac{(A_2\tilde{B}_2-\tilde{B}_3)}{\tilde{B}_2} \end{bmatrix}$.

T_e can be rewritten as $T_e = \begin{bmatrix} \frac{(a_2-a_1)e^{-(a_1+a_2)\tau^*}}{e_1} & e_4 & e_5 \\ 0 & -\frac{e_2}{e_1} & 1 \\ 0 & A_1 & \frac{e_3}{e_1} \end{bmatrix}$ using the following equations

$$\begin{aligned}
\frac{(A_1\tilde{B}_2^2 + A_2\tilde{B}_2\tilde{B}_3 - \tilde{B}_3^2)}{\tilde{B}_2} &= -\frac{B_1^2}{\tilde{B}_2}e^{-(a_1+a_2)\tau^*} = \frac{B_1(a_2-a_1)}{e_1}e^{-(a_1+a_2)\tau^*} \\
(A_1\tilde{B}_1 + A_2\tilde{B}_2 - \tilde{B}_3) &= -B_1\frac{(A_1\tilde{B}_1\tilde{B}_2 + A_2\tilde{B}_1\tilde{B}_3 - \tilde{B}_2\tilde{B}_3)}{\tilde{B}_2} \\
&= \frac{B_1}{e_1}\left[\frac{a_1}{a_2}e^{-a_1\tau^*} - \frac{a_2}{a_1}e^{-a_2\tau^*} + \frac{(a_2^2-a_1^2)}{a_1a_2}e^{-(a_1+a_2)\tau^*}\right] \\
\frac{(\tilde{B}_2^2 - \tilde{B}_1\tilde{B}_3)}{\tilde{B}_2} &= -\frac{B_1}{e_1}\left[\frac{1}{a_2}e^{-a_1\tau^*} - \frac{1}{a_1}e^{-a_2\tau^*} + \frac{(a_2-a_1)}{a_1a_2}e^{-(a_1+a_2)\tau^*}\right].
\end{aligned}$$

Since $a_2 < a_1 < 0$ for system matrices of (16), note that $\tilde{B}_1 < 0$, $\tilde{B}_2 > 0$, and $\tilde{B}_3 < 0$ for $\tau^* > 0$. Since $\det(S) = \frac{B_1^3}{\tilde{B}_2^3} \neq 0$, there exists S^{-1}. Thus, using the transformation $s_e(t) = T_e h_e(t)$, we can rewrite system (87) as

$$\dot{s}_e(t) = SA_eS^{-1}s(t) + S\tilde{B}_e\delta\dot{p}(t) = A_es_e(t) + \begin{bmatrix} 0 \\ 0 \\ \frac{B_1(a_2-a_1)}{e_1}e^{-(a_1+a_2)\tau^*} \end{bmatrix}\delta\dot{p}(t).$$

Similarly, we get (19).

8.3 Derivation of (71) and (73)

We have only to show how to derive (73) since we can handle (71) as a special case of (73). Linearizing (72) and using $R_s^T p^* = \frac{1}{\left(1+\frac{w_s^{*2}}{2}\right)}$ and $R_l D_2 w^* = c_l$, we have

$$\begin{aligned}
\delta \dddot{b}_l(t) &= \sum_s \Bigg[\frac{\partial g_s}{\partial w_s(t-\tau_{ls}^{f*}-\tau_s^*)}|_* \delta w_s(t-\tau_{ls}^{f*}-\tau_s^*) + \frac{\partial g_s}{\partial \dot{w}_s(t-\tau_{ls}^{f*}-\tau_s^*)}|_* \delta \dot{w}_s(t-\tau_{ls}^{f*}-\tau_s^*) \\
&\quad + \frac{\partial g_s}{\partial w_s(t-\tau_{ls}^{f*})}|_* \delta w_s(t-\tau_{ls}^{f*}) + \frac{\partial g_s}{\partial \dot{w}_s(t-\tau_{ls}^{f*})}|_* \delta \dot{w}_s(t-\tau_{ls}^{f*}) \\
&\quad + \frac{\partial g_s}{\partial b(t-\tau_{ls}^{f*})}|_* \delta b(t-\tau_{ls}^{f*}) + \frac{\partial g_s}{\partial \dot{b}(t-\tau_{ls}^{f*})}|_* \delta \dot{b}(t-\tau_{ls}^{f*}) + \frac{\partial g_s}{\partial \ddot{b}(t-\tau_{ls}^{f*})}|_* \delta \ddot{b}(t-\tau_{ls}^{f*}) \\
&\quad + \frac{\partial g_s}{\partial b(t-\tau_{ls}^{f*}-\tau_s^*)}|_* \delta b(t-\tau_{ls}^{f*}-\tau_s^*) + \frac{\partial g_s}{\partial \dot{b}(t-\tau_{ls}^{f*}-\tau_s^*)}|_* \delta \dot{b}(t-\tau_{ls}^{f*}-\tau_s^*) \\
&\quad + \frac{\partial g_s}{\partial \dot{p}(t-\tau_{ls}^{f*}-\tau_s^{b*})}|_* \delta \dot{p}(t-\tau_{ls}^{f*}-\tau_s^{b*}) \Bigg], \qquad (89)
\end{aligned}$$

where

$$\frac{\partial g_s}{\partial w_s(t-\tau_{ls}^{f*}-\tau_s^*)}|_* = 0, \quad \frac{\partial g_s}{\partial \dot{w}_s(t-\tau_{ls}^{f*}-\tau_s^*)}|_* = 0, \quad \frac{\partial g_s}{\partial w_s(t-\tau_{ls}^{f*})}|_* = 0 \qquad (90)$$

$$\frac{\partial g_s}{\partial \dot{w}_s(t-\tau_{ls}^{f*})}|_* = -R_{ls}\frac{2w_s^*}{\tau_s^{*2}(2+w_s^{*2})}, \quad \frac{\partial g_s}{\partial \dot{b}(t-\tau_{ls}^{f*})}|_* = 0, \quad \frac{\partial g_s}{\partial \dot{b}(t-\tau_{ls}^{f*})}|_* = 0 \qquad (91)$$

$$\frac{\partial g_s}{\partial \ddot{b}(t-\tau_{ls}^{f*})}|_* = -R_{ls}\frac{w_s^*}{\tau_s^{*2}}R_s^T D_1, \quad \frac{\partial g_s}{\partial b(t-\tau_{ls}^{f*}-\tau_s^*)}|_* = 0, \quad \frac{\partial g_s}{\partial \dot{b}(t-\tau_{ls}^{f*}-\tau_s^*)}|_* = 0 \qquad (92)$$

$$\frac{\partial g_s}{\partial \dot{p}(t-\tau_{ls}^{f*}-\tau_s^{b*})}|_* = -R_{ls}\frac{(2+w_s^{*2})}{2\tau_s^{*2}}R_s^T.$$

Linearizing (70), we have

$$\delta \ddot{b}_l(t) = \sum_s R_{ls}\frac{\delta \dot{w}_s(t-\tau_{ls}^{f*})}{\tau_s^*} - \sum_s R_{ls}\frac{w_s^*}{\tau_s^{*2}}R_s^T D_1 \delta \dot{b}(t-\tau_{ls}^{f*})$$

which leads to

$$\delta \dot{w}(t-\tau_{ls}^{f*}) = D_2^{-1} R_l^{-1}\left[\delta \ddot{b}_l(t) + \sum_s R_{ls}\frac{w_s^*}{\tau_s^{*2}}R_s^T D_1 \delta \dot{b}(t-\tau_{ls}^{f*})\right]. \qquad (93)$$

From (93), (89) can be rewritten as

$$\begin{aligned}
\delta \dddot{b}_l(t) &= -R_l \mathrm{diag}\left(\frac{2w_s^*}{\tau_s^*(2+w_s^{*2})}\right) R_l^{-1}\left[\delta \ddot{b}_l(t) + \sum_s R_{ls}\frac{w_s^*}{\tau_s^{*2}}R_s^T D_1 \delta \dot{b}(t-\tau_{ls}^{f*})\right] \\
&\quad - \sum_s R_{ls}\frac{w_s^*}{\tau_s^{*2}}R_s^T D_1 \delta \ddot{b}(t-\tau_{ls}^{f*}) - \sum_s R_{ls}\frac{2+w_s^{*2}}{2\tau_s^{*2}}R_s^T \delta \dot{p}(t-\tau_{ls}^{f*}-\tau_s^{b*}). \qquad (94)
\end{aligned}$$

From (94), the linearized model of the AIMD model for general networks can be converted to

$$\delta \ddot{b}(t) = \mathrm{diag}(A_{2l})\delta\ddot{b}(t) + \sum_s \sum_l \Big[\alpha_{ls}\delta\dot{b}(t-\tau_{ls}^{f*}) + \beta_{ls}\delta\ddot{b}(t-\tau_{ls}^{f*}) + \gamma_{ls}\delta\dot{p}(t-\tau_{ls}^{f*}-\tau_s^{b*})\Big] \quad (95)$$

that can be rewritten as the state-space model (73).

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