



## Supply chain partnership based on revenue sharing

Satyaveer Singh Chauhan, Jean-Marie Proth

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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

## *Supply chain partnership based on revenue sharing*

Chauhan S. Satyaveer — Proth Jean-Marie

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THÈME 4

A large blue rectangle occupies the lower half of the page. Overlaid on it is a large, light grey 'R' that is partially cut off by the left edge. To the right of the 'R', the words 'apport' and 'de recherche' are written in a white, italicized serif font, stacked vertically. A horizontal grey brushstroke is positioned below the text.

*apport  
de recherche*





## Supply chain partnership based on revenue sharing

Chauhan S. Satyaveer <sup>\*†</sup>, Proth Jean-Marie <sup>‡†</sup>

Thème 4 — Simulation et optimisation  
de systèmes complexes  
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**Abstract:** The profit margin of retailers and suppliers are decreasing as more and more market players are joining the market. The customer generally switch to another brand / retailer as the price increases. Since supplier revenue depends upon the retailer's revenue; retailer's revenue indirectly depends upon the supplier's price and the customer demand; and customer demand depends upon the retail price hence it is necessary to have a business coordination to win the market. In this business partnership both partners coordinate with each other to decide the retail and wholesale price, profit margin and inventory level in stock in order to get big market share and hence higher revenue.

In this paper we present a producer-retailer partnership model based on profit sharing. We assume that customer demand depends upon the retail price and tends to zero as the price of commodity tends to infinite. We propose an approach to maximize the combined profit and sharing the profit among partners proportional to their risk. The properties of the problem are explored and an optimal algorithm based on the results is presented.

**Key-words:** Supply chain, revenue sharing and contracts

\* chauhan@loria.fr

† INRIA-SAGEP, Ile du Saulcy, Metz, 57045, France.

‡ proth@loria.fr, Tél: 0387 31 54 58 (Corresponding author)

## Partage des risques et des bénéfices dans une chaîne d'approvisionnement

**Résumé :** La marge bénéficiaire des détaillants et des grossistes se réduit de plus en plus lorsque la concurrence augmente. Le client change généralement de marque ou de fournisseur lorsque les prix augmentent. Le volume vendu décroît donc lorsque le prix augmente et, d'autre part, une augmentation des bénéfices fait croître les prix. Il faut donc trouver un moyen terme de façon à optimiser les bénéfices, puis faire en sorte que les bénéfices des partenaires soient équilibrés. Il faut donc une coordination entre grossistes et distributeurs. Dans ce papier nous présentons un modèle d'association de grossiste et détaillant basé sur le partage des bénéfices. Nous supposons que la demande du client dépend du prix de vente et décroît lorsque le prix de vente croît. Nous proposons une approche qui consiste à maximiser le bénéfice total, puis à partager le profit entre les partenaires proportionnellement à leur risque. Les propriétés du problème sont explorées et un algorithme optimal basé sur les résultats est présenté.

**Mots-clés :** Chaîne d'approvisionnement, partage, bénéfices, association

# 1 Introduction

Supply chains are considered as solutions for effectively meeting customer requirements such as low costs, high product variety, quality and shorter lead times. The key issues of supply chains are best supply chain formation and efficient coordination. The emphasis is given to reduce cost by optimising production, inventory, transportation and effective operational level monitoring. Since several partners unite to form a single virtual organization to face the time based opportunity, in the optimisation model all cost reduction issues tackled globally where objective is to maximize profit. This approach is called centralize supply chain planning when one decision maker takes the decision and all members receive revenues according to their respective sell/demand e.g. provider get a revenue on how much producer purchase the raw material. The objective of this global approach is maximizing profit based on expected customer demand by reducing operating cost. Plenty of literature is dedicated to the approach. [11], [4], [8]. For an extensive review on strategic production-distribution models and supply chain, the reader is referred to [11] and [7], [3].

But in reality and in competitive market, the customer demand varies according to the retailer price or in other words demand shift from one supply chain to other. The retailer price depends upon the purchasing price from provider, locally added extra cost due to packing, storage etc and the benefit taken by retailer per unit. When both provider and retailer are the part of supply chain and the variability of demand also depends upon the price then setting of retailer and distributor price is the one of the important issues to be considered in designing the supply chains. The second important issue closely related to the above is the sharing of profits.

To address the similar issue several types of tools (called contracts) discussed in the literature. For instance, in the Revenue Sharing (RS) contracts ([2]) the retailer keeps the fraction of revenue, say  $\theta < 1$ , and return the rest  $(1 - \theta)$  of the revenue to supplier. This contract can be describe by two parameters- wholesale price  $w$  and the retailer's profit quota  $\Phi$ . [6] has discussed RS contracts and extended the approach to three-stage supply chain.

The other contracts presented in literature are Backup agreement ([5]), Incentive mechanism ([10]) and allocation rules ([1]). For an extensive review of supply chain contracts reader is referred to [6]. [9] formulated the two echelon multi product problem with deterministic demand. For each product they introduce transfer price levels and select the one such as to maximize the total profit. The whole problem is formulated as mixed integer linear programming problem. The formulation is based on Nash bargaining approach.

In this paper we formulate the two-echelon supply chain coordination problem for stochastic demand.

The paper is organized in four sections. In section 2 we describe the problem and propose the properties of the problem. Section 3 presents the algorithm. Numerical example is presented in section 4 and finally section 5 is conclusion.

## 2 Problem formulation

Let a provider buy semi finished goods at  $A$  per unit and sell it to retailer at  $w_p$  per unit. Similarly, retailer sells finished goods to customers at  $w_r$  per unit. The customer demands ( $u$ ) are random and follow a given density function  $\phi(u)$ . Since market is competitive, in this model, we assume that fraction of customer demand disappear as the price of product set by retailer increases. We introduce a function  $f(w_r)$ , strictly decreasing, which gives the fraction of expected customers that are ready to pay the price  $w_r$ . In other words, the quantity sold by the retailer is  $f(w_r).u$  when the demand is  $u$ . Here we make a realistic assumption that  $f(w_r)$  is a concave and decreasing function of  $w_r$ .

As we mentioned earlier, provider and retailer have a business partnership to face the competition in which they share profit and loss according to their investment. To face the randomness of customer demand both provider and retailer stock  $I_p$  and  $I_r$  inventories respectively. Holding an inventory incur a holding cost which is  $c_p$  and  $c_r$  per unit for provider and retailer respectively. We assume that inventory can be transferred from provider to retailer within negligible time. In this agreement, retailer pay the backlogging cost  $r_r$  per unit to customer if the demand exceeds  $I_r$  and since, system has a total of  $I_p + I_r$  inventory, therefore retailer pay an extra backlogging cost on demand over  $I_p + I_r$  to customer if demand exceeds  $I_p + I_r$ . We denote this extra backlogging cost per unit by  $\Delta$  i.e.  $r_r + \Delta$  if the demand is greater than  $I_p + I_r$ . Mathematically,

$$B_{r1} = \begin{cases} 0 & \text{if } f(w_r).u \leq I_r \\ (f(w_r).u - I_r)r_r & \text{if } f(w_r).u > I_r \end{cases} \quad (1)$$

and

$$B_{r2} = \begin{cases} 0 & \text{if } f(w_r).u \leq I_p + I_r \\ (f(w_r).u - (I_p + I_r))\Delta & \text{if } f(w_r).u > I_p + I_r \end{cases} \quad (2)$$

Thus retailer pays total of  $B_r = B_{r1} + B_{r2}$  as backlogging cost.

Provider pays a backlogging cost  $r_p$  per unit if the demand goes above than  $I_p + I_r$  i.e.

$$B_p = \begin{cases} 0 & \text{if } f(w_r).u \leq I_r + I_p \\ (f(w_r).u - (I_r + I_p))r_p & \text{if } f(w_r).u > I_r + I_p \end{cases} \quad (3)$$

Note that the model is valid only if  $r_p > r_r + \Delta > r_r$ . Now, the expected backlogging cost of provider  $\overline{B_p}$  and retailer  $\overline{B_r}$  are:

$$\overline{B_p} = r_p \cdot \left\{ f(w_r) \int_{\frac{I_p + I_r}{f(w_r)}}^{+\infty} u \cdot \phi(u) \cdot du - (I_p + I_r) \int_{\frac{I_p + I_r}{f(w_r)}}^{\infty} \phi(u) du \right\} \quad (4)$$

$$\overline{B_{r1}} = r_r \cdot \left\{ f(w_r) \int_{\frac{I_r}{f(w_r)}}^{\infty} u \cdot \phi(u) \cdot du - I_r \int_{\frac{I_r}{f(w_r)}}^{\infty} \phi(u) du \right\} \quad (5)$$

$$\overline{B_{r2}} = \Delta \left\{ f(w_r) \int_{\frac{I_p + I_r}{f(w_r)}}^{\infty} u \cdot \phi(u) \cdot du - (I_p + I_r) \int_{\frac{I_p + I_r}{f(w_r)}}^{\infty} \phi(u) du \right\} \quad (6)$$

Remark: Since provider pays a backlogging cost to retailer and retailer pays a backlogging cost to customers, therefore if the demand goes above than  $I_p + I_r$  retailer receives net amount of

$$(r_p - (r_r + \Delta)) \left\{ f(w_r) \int_{\frac{I_p + I_r}{f(w_r)}}^{+\infty} u \cdot \phi(u) \cdot du - (I_p + I_r) \int_{\frac{I_p + I_r}{f(w_r)}}^{\infty} \phi(u) du \right\}$$

as compensation from provider.

Now the problem is to define the selling price  $w_p$  and  $w_r$  of provider and retailer such as to maximize their individual benefit  $b_p$  and  $b_r$  respectively. Mathematically,

$$w_p \cdot f(w_r) \cdot \bar{u} = A \cdot f(w_r) \cdot \bar{u} + c_p \cdot I_p + b_p + \bar{B}_p \quad (7)$$

$$w_r \cdot f(w_r) \cdot \bar{u} = w_p \cdot f(w_r) \cdot \bar{u} + c_r \cdot I_r + b_r + \bar{B}_r - \bar{B}_p \quad (8)$$

In the above equation we deduct  $\bar{B}_p$  since provider pays compensation to retailer if backlogging appears at provider level.

$$b_p + b_r = w_r \cdot f(w_r) \cdot \bar{u} - [A \cdot f(w_r) \cdot \bar{u} + c_p \cdot I_p + \bar{B}_p + c_r \cdot I_r + \bar{B}_r - \bar{B}_p]$$

$$b_p + b_r = w_r \cdot f(w_r) \cdot \bar{u} - [A \cdot f(w_r) \cdot \bar{u} + c_p \cdot I_p + c_r \cdot I_r + \bar{B}_r - \bar{B}_p]$$

or for simplicity,

$$b_p + b_r = B(I_p, I_r, w_r) = w_r \cdot f(w_r) \cdot \bar{u} - \overline{K(I_p, I_r, w_r)} \quad (9)$$

where  $\overline{K(I_p, I_r, w_r)}$  is

$$\begin{aligned} \overline{K(I_p, I_r, w_r)} &= A \cdot f(w_r) \cdot \bar{u} + I_p \cdot c_p + I_r \cdot c_r + \\ & r_r \left\{ f(w_r) \int_{\frac{I_r}{f(w_r)}}^{+\infty} u \cdot \phi(u) du - (I_r) \int_{\frac{I_r}{f(w_r)}}^{+\infty} \phi(u) du \right\} + \\ & (r_r + \Delta) \left\{ f(w_r) \int_{\frac{I_p + I_r}{f(w_r)}}^{+\infty} u \cdot \phi(u) du - (I_p + I_r) \int_{\frac{I_p + I_r}{f(w_r)}}^{+\infty} \phi(u) du \right\} \end{aligned} \quad (10)$$

From equation (9) we can see that  $B(I_p, I_r, w_r)$  is a function of  $w_r, I_p, I_r$ . But for a given  $w_r$  the profit can be increased by reducing holding and backlogging costs since all the costs are the function of  $I_p, I_r$ . Our objective is to maximize  $B(I_p, I_r, w_r)$  or to minimize  $\overline{K(I_p, I_r, w_r)}$  for a given  $w_r$ .

### Result 1

For a given  $w_r$   $\overline{K(I_p, I_r, w_r)}$  is a convex function of  $I_r$  and  $I_p$ .



**Proof**

Observe the Hessian matrix:

$$H(I_p, I_r) = \begin{bmatrix} \frac{\partial^2 \bar{K}}{\partial I_p^2} & \frac{\partial^2 \bar{K}}{\partial I_p \partial I_r} \\ \frac{\partial^2 \bar{K}}{\partial I_p \partial I_r} & \frac{\partial^2 \bar{K}}{\partial I_r^2} \end{bmatrix}$$

where :

$$\begin{aligned} \frac{\partial^2 \bar{K}}{\partial I_p^2} &= \frac{r_r + \Delta}{f(w_r)} \cdot \phi\left(\frac{I_p + I_r}{f(w_r)}\right) \\ \frac{\partial^2 \bar{K}}{\partial I_p \partial I_r} &= \frac{r_r + \Delta}{f(w_r)} \cdot \phi\left(\frac{I_p + I_r}{f(w_r)}\right) \end{aligned}$$

and

$$\frac{\partial^2 \bar{K}}{\partial I_r^2} = \frac{r_r + \Delta}{f(w_r)} \cdot \phi\left(\frac{I_p + I_r}{f(w_r)}\right) + \frac{r_r}{f(w_r)} \cdot \phi\left(\frac{I_r}{f(w_r)}\right)$$

which shows that all the leading principal minors of  $H(I_p, I_r)$  are nonnegative. This completes the proof.  $\blacksquare$

**Result 2**

$I_p$  and  $I_r$  being fixed,  $B(I_p, I_r, w_r)$  is a concave function of  $w_r$ .

**Proof**

Second derivative of  $B(I_p, I_r, w_r)$  w.r.t  $w_r$  gives

$$\frac{\partial^2 B}{\partial w_r^2} = 2.f'(w_r)\bar{u} + f''(w_r)\bar{u}.w_r - \frac{\partial^2 K(I_p, I_r, w_r)}{\partial w_r^2} \quad (11)$$

$$\frac{\partial K(I_p, I_r, w_r)}{\partial w_r} = A\bar{u}f'(w_r) + (r_r + \Delta) \int_{\frac{I_p + I_r}{f(w_r)}}^{\infty} u \cdot \phi(u) du + r_r \int_{\frac{I_r}{f(w_r)}}^{\infty} u \cdot \phi(u) du$$

$$\frac{\partial^2 K(I_p, I_r, w_r)}{\partial w_r^2} = A\bar{u}f''(w_r) + (r_r + \Delta) \frac{(I_p + I_r)^2}{f^3(w_r)} \phi\left(\frac{I_p + I_r}{f(w_r)}\right) + r_r \frac{(I_r)^2}{f^3(w_r)} \phi\left(\frac{I_r}{f(w_r)}\right) \quad (12)$$

from (11) and (12) we derive:

$$\frac{\partial^2 B}{\partial w_r^2} = 2.f'(w_r)\bar{u} - f''(w_r)\bar{u}[A - w_r] - \left[ \frac{(I_p + I_r)^2 \cdot (r_r + \Delta)}{f^3(w_r)} \phi\left(\frac{I_p + I_r}{f(w_r)}\right) + \frac{(I_r)^2 \cdot r_r}{f^3(w_r)} \phi\left(\frac{I_r}{f(w_r)}\right) \right] \quad (13)$$

We see from the equation (13) that

- the first term is always negative since  $f'(w_r) < 0$ ,

- the second term is always positive since for a feasible solution  $w_r$  should be greater than  $A$  which gives  $(A - w_r) < 0$  and  $f''(w_r)$  is also less than zero since  $f(w_r)$  is a concave function of  $w_r$ ,
- the third term is always positive.

Since all the positive terms are preceded by negative sign, the second derivative given by equation (13) is always negative. This completes the proof. ■

### Result 3

The problem described in section 1 has an unique solution.

#### Proof

Consider the first order derivative of  $B(I_p, I_r, w_r)$  w.r.t.  $I_p, I_r, w_r$

$$\frac{\partial B}{\partial w_r} = w_r \cdot \bar{u} \cdot f'(w_r) + \bar{u} \cdot f(w_r) - A \cdot \bar{u} \cdot f'(w_r) - (r_r + \Delta) \cdot \int_{\frac{I_p + I_r}{f(w_r)}}^{\infty} u \cdot \phi(u) du - r_r \int_{\frac{I_r}{f(w_r)}}^{\infty} u \cdot \phi(u) du \quad (14)$$

$$\frac{\partial B}{\partial I_p} = -c_p + (r_r + \Delta) \int_{\frac{I_p + I_r}{f(w_r)}}^{\infty} \phi(u) du \quad (15)$$

and

$$\frac{\partial B}{\partial I_r} = -c_r + (r_r + \Delta) \int_{\frac{I_p + I_r}{f(w_r)}}^{\infty} \phi(u) du + r_r \int_{\frac{I_r}{f(w_r)}}^{\infty} \phi(u) du \quad (16)$$

Now assume  $s = (I_p^*, I_r^*, w_r^*)$  is an optimal solution. According to the condition of optimality all the first order derivative w.r.t all the variables at  $s$  must vanish. Applying the above condition to all the first order derivative given by (14), (15) and (16) at  $s$  we have: from equation (16)

$$(r_r + \Delta) \int_{\frac{I_p^* + I_r^*}{f(w_r^*)}}^{\infty} \phi(u) du + r_r \int_{\frac{I_r^*}{f(w_r^*)}}^{\infty} \phi(u) du = c_r$$

substituting the above value in relation (14) we get

$$w_r^* \cdot \bar{u} \cdot f'(w_r^*) + \bar{u} \cdot f(w_r^*) - A \cdot \bar{u} \cdot f'(w_r^*) = c_r \quad (17)$$

Equation (17) shows that  $w_r^*$  has an unique value at the maximum.

Similarly, from relation (15) we find

$$(r_r + \Delta) \int_{\frac{I_p^* + I_r^*}{f(w_r^*)}}^{\infty} \phi(u) du = c_p \quad (18)$$

which shows that the sum  $I_p^* + I_r^*$  is always constant at the optimum for a given density function, holding cost and  $w_r$ . Now substituting

$$(r_r + \Delta) \int_{\frac{I_p^* + I_r^*}{f(w_r^*)}}^{\infty} \phi(u) du = c_p$$

in equation (16) we get

$$r_r \int_{\frac{I_r^*}{f(w_r^*)}}^{\infty} \phi(u) du = c_p + c_r \quad (19)$$

which shows that  $I_r^*$  also has an unique value for a given  $w_r$ . Since,  $w_r^*$  is unique therefore  $I_r$  too. Furthermore, according to relation (18)  $I_p^* + I_r^*$  is unique hence  $I_p^*$  is also unique for a given  $w_r$ . Above shows that at we have for a given parameters we have a unique optimum. This completes the proof. ■

### 3 Solution approach

In our approach, first we maximize the combined profit  $B(I_p, I_r, w_r)$  and then share the profit among partners. The algorithm and sharing mechanism is presented in the following sections.

#### 3.1 Algorithm

1. Set  $w_r = A$ .  
*Above is just a starting value, indeed this solution is not feasible.*
2. Applying gradient method to minimize the function  $\overline{K(I_p, I_r, w_r)}$  keeping  $w_r$  fixed. We will get an optimal pair  $(I_p^*, I_r^*)$  for given  $w_r$ .
3. Apply the gradient method to improve the function  $B(I_p^*, I_r^*, w_r)$  keeping  $I_p^*$  and  $I_r^*$  fixed. We obtained here  $w_r^*$
4. If the criterion  $B(I_p, I_r, w_r)$  is not improving, Stop. Else set  $w_r = w_r^*$  and go to 2.

According to the result 3, this solution is optimal.

#### 3.2 Profit sharing

In the previous section we present an approach to maximize the combined profit  $B(I_p, I_r, w_r)$  of provider and retailer. In this section we share the profit between the partners with respect to their investment.

$$\frac{b_p}{\text{provider's investment}} = \frac{b_r}{\text{retailer's investment}}$$

$$\frac{b_p}{w_p \cdot f(w_r) \cdot \bar{u} - b_p} = \frac{b_r}{w_r \cdot f(w_r) \cdot \bar{u} - b_r}$$

or

$$\frac{b_p}{A \cdot f(w_r) \cdot \bar{u} + c_p \cdot I_p + \bar{B}_p} = \frac{B(I_p, I_r, w_r) - b_p}{A \cdot f(w_r) \cdot \bar{u} + c_p \cdot I_p + b_p + c_r \cdot I_r + B_r}$$

$$\begin{aligned}
& b_p(A.f(w_r).\bar{u} + c_p.I_p + b_p + c_r.I_r + \bar{B}_r) \\
& = B(I_p, I_r, w_r) * (A.f(w_r).\bar{u} + c_p.I_p + \bar{B}_p) - b_p(A.f(w_r).\bar{u} + c_p.I_p + \bar{B}_p) \\
& b_p^2 + b_p(2A.f(w_r).\bar{u} + 2c_p.I_p + c_r.I_r + \bar{B}_r + \bar{B}_p - B(I_p, I_r, w_r).(A.f(w_r).\bar{u} + c_p.I_p + \bar{B}_p) \\
& b_p = \frac{-Z + \sqrt{Z^2 + 4 * T}}{2} \tag{20}
\end{aligned}$$

where

$$\begin{aligned}
Z &= 2.f(w_r).\bar{u} + 2c_p.I_p + 2.\bar{B}_p + c_r.I_r + \bar{B}_r \\
T &= (A.f(w_r).\bar{u} + c_p.I_p + \bar{B}_p) * B(I_p, I_r, w_r) \text{ and}
\end{aligned}$$

$$b_r = B(I_p, I_r, w_r) - b_p.$$

## 4 Numerical illustration

In this section we present two examples. We use the following concave function, which gives in fraction the remaining demand.

$$f(w_r) = 1 - \frac{w_r^2}{k} \quad \text{where } w_k^2 \leq k$$

In the first example we illustrate how the solution evolve iteration by iteration for the following parameters:

Probability density

$$\phi(u) = \left\{ \begin{array}{ll} 1/(400) & \text{if } 100 \leq u \leq 500 \\ 0 & \text{otherwise} \end{array} \right\}$$

$$\begin{aligned}
c_r &= 1.0 \quad c_p = 0.5 \\
r_r &= 5.0 \quad r_p = 6.50 \\
\Delta &= 1.0 \quad A = 1.0 \\
k &= 500
\end{aligned}$$

The result is summarised in table 1.

In the above example we started exploring solution with initial value of  $w_r = 25$ , which leads us to infeasible solution in the first iteration. In the second iteration  $w_r$  rectifies itself, automatically, if there exists a feasible solution.

In the second example we show how the function  $f(w_r)$  affect the individual profits with different value of  $k$ .

From the above table we can see that as the demand is getting stable ( $f(w_r)$  curve becomes more and more flat) the selling price and the profit is increasing. By giving  $k = \infty$  i.e. making customer demand independent of  $w_r$ , the problem becomes unbounded i.e.  $w_r$  has no upper limit.

Table 1: Evolution of solution

Iteration No.	$w_r(1)$	$B(I_p, I_r, w_r)$ obtained for fix $w_r(1)$	$I_p$	$I_r$	new optimum $w_r$ for a given $I_p, I_r$ of previous column
1.	1.0	-480.704	6.35	459.219	12.5329 (Infeasible)
2.	12.5329	1931.29	4.4	315.41	12.8601
3.	12.8601	1936.5	4	307.91	12.9595
4.	12.9595	1937	4.2	305.509	12.9899
5.	12.9899	1937.04	3.8	304.909	13.0026
6.	13.0026	1937.05	4.1	304.509	13.0049
7.	13.0049	1937.05	4.1	304.409	13.0049

Table 2: Evolution of solution w.r.t  $f$ 

$k$	$w_r$	$B(I_p, I_r, w_r)$	$b_p$	$b_r$
100	6.26994	598.582	136.475	462.107
200	8.52749	1042.66	218.772	823.892
600	14.1751	2170.43	390.321	1780.11
1000	18.0642	2951.34	489.894	2461.45
5000	39.3991	7246.07	908.112	6337.96

## 5 Conclusion

This paper present the problem of supplier-retailer relationship based on profit sharing. The model is realistic and suits to competitive market scenario where customers are more price conscious and can divert to another brand as variation in price takes place. The assumption that fraction of demand disappears with increase of price and the rule follows a concave behaviour is also realistic as demand becomes zero and never increases with price (like convex function). We prove that the profit function is concave with respect to individual variables and only a unique optimum profit exists. An algorithm is proposed based on the properties. We share profit proportional to the investment of partners but the approach can be coupled with other profit sharing contracts.

Finally, the model can be easily extended to multi echelon supply chains and multi retailer chain.

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