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Predicting travel time on urban networks using simulation

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Abstract: Prediction of accurate travel time plays an important role in dynamic route guidance on urban networks. The dynamic traffic flows entering the network affect the behavior of the system and the free flow movement of vehicles. To study the complex system dynamics of an urban network under varying input flows, we begin with the basic unit of the network called a lane. The analysis of the behavior of a single lane is done to derive the travel time from the characteristics of the lane, the input flow and the constraints on the output flow. The system analysis of the single lane is then applied to the whole network for studying the transfer of flows inside the network. A dynamic network model has been proposed for the network analysis. A simulation software has been developed using the dynamic network model which generates travel times on the network using the input data as the flows coming at the entrances of the system, the initial system states of the lanes and the characteristics of the network.

Key-words: Traffic flow , Traffic Simulation, Travel Time, System State

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Estimation du temps de trajet en réseaux urbains en utilisant la simulation

Résumé : La prévision du temps précis de déplacement joue un rôle important dans les guidages dynamiques en réseaux urbains. Les flux de voitures qui entrent dans le réseau affectent le comportement des véhicules. Pour étudier la dynamique d'un réseau urbain sous des flux d'entrée variables, nous étudions d'abord le comportement d'une voie rectiligne. L'analyse du comportement d'une voie simple est faite pour déduire le temps de déplacement des caractéristiques physiques de la voie, du flux d'entrée et des contraintes sur ces flux. L'analyse du système constitué d'une voie simple est alors utilisée pour étudier le comportement du réseau complet. Nous avons proposé un modèle dynamique pour l'analyse de réseau. Un logiciel de simulation a été développé. Il fournit les temps de déplacement entre chaque entrée et chaque sortie en fonction de l'état initial du système et des flux d'entrée.

Mots-clés : flux de véhicules, simulation du trafic, durée de transport, état du système

1 Introduction

Traffic flow simulation has been an effective method to assess traffic conditions for various designing alternatives of transportation facilities. It has emerged as an important evaluation tool for Intelligent Transportation System (ITS) strategies in recent years.

There has been much research on the travel time prediction. In the context of prediction methodologies, various time series models [1,2] and artificial neural network models [3,4] have been developed. In the context of input data source, most previous studies used "indirect" travel time data [1,2,5,6]. The parameters of traffic data such as volume, occupancy and speed were obtained directly, while travel time was calculated as a function of these parameters. In most of the existing studies focused on link travel time estimation, it is generally assumed that the path travel time is the addition of travel times on the consisting links. Chen et al [7] studied the computation of travel time based on paths rather than link. In a dynamic network, the travel time computation depends not only on the geometry of the path, but also on the congestion present on the arcs of the path.

The approaches to simulate the traffic [8,9] can be microscopic or macroscopic. The microscopic approach has resulted in car-following theories which study the behavior of one vehicle following another. In microscopic traffic simulation [10], each individual vehicle is tracked and the vehicle's movement in the system is determined by the characteristics of the driver, vehicle performance, and its interactions with network geometrics and surrounding vehicles. The macroscopic approach [11,12] is analogous to theories of fluid dynamics or continuum theories. In macroscopic traffic flow simulation the flow-density relationship is used to govern vehicle movement and the individual vehicles are not tracked in the model.

The present study follows the macroscopic simulation approach and is dedicated to the study of an urban network defined by the input flows at the entrances of the network and the characteristics of the network. The characteristics of the network are governed by the traffic capacity, the length, the output capacity and the initial system states of the constituent lanes. The system analysis of a single lane is done and is then applied to the whole network to observe the transfer of flows inside the network. A dynamic network model is proposed for conducting the network analysis under varying input flows and initial system states of the lanes of the network. A software has been developed using the dynamic network model to study the evolution of flows and computation of travel times inside the network.

The rest of the paper is organized in 6 sections. In section 2, we introduce the notations and present the problem. Section 3 is dedicated to the analysis of a single lane system under piecewise constant input flow and output capacity of the lane. Section 4 presents the transfer of flows inside the network. In section 5, the dynamic network model is proposed. Section 6 presents the simulation details along with a numerical illustration. The conclusion with future scope of research is presented in section 7.

2 Network Definitions

Let us represent a traffic network by a directed graph G = (N,A) where N is the set of nodes and A is the set of directed lanes. Let us denote the set of entrance nodes by E and set of destination nodes by D. Let us represent the entrance node by index e and the destination node by index d. The input flow arriving at node $e \in E$ at time t is denoted by $\phi_e(t)$.

Let us represent a node by i. The set of predecessor nodes of node i is denoted by *Pr(i)* and the set of successor nodes is denoted by *Sc(i)*. Let us denote the total number of predecessor nodes of node i by P and the total number of successor nodes by S.

Let $i \in N$ and $j \in Sc(i)$. A lane is denoted by *(i,j)* with tail node i and head node j. Each lane *(i,j)* has three attributes: its length l_{ij}, its traffic accommodation capacity c_{ij} and the number of vehicles inside the lane at time t denoted by $n_{ij}(t)$. The input flow arriving at the entrance of the lane is denoted by $\phi_{ij}(t)$. The input flow that enters inside the lane is denoted by $\phi^e_{ij}(t)$. The input capacity of the lane is denoted by $C^e_{ij}(t)$. The proportion of flow arriving at node i that wants to be directed to node j is represented by p_{ij}. The flow exiting from the lane at output node j at any time t is $\phi^o_{ij}(t)$. The output capacity of the lane is denoted by $C^o_{ij}(t)$. The number of waiting vehicles at any $e \in E$ are represented by $w_e(t)$. In steady state the flow moves with a constant speed ν_{ij} in the lane *(i,j)*.

The goal is to analyze the behavior of the network under piecewise constant input flows and initial system states of the lane. This system analysis is then used to predict the travel time on the various paths of the network.

3 The Single Lane Flow

3.1 Assumptions

Before analyzing the system under piecewise constant input flows, we make the following assumptions:

- The vehicles do not pass one another. This implies that vehicles follow First In First Out (FIFO) policy while traversing the lane.
- If a vehicle is not delayed, it moves at a constant speed.

Let us denote t_0 as the starting time, that is the time at which the study starts. At time t_0, we know:

- The number of cars present in the lanes of the system, $n_{ij}(t_0)$.
- The number of cars $w_e(t_0)$ waiting at the node $e \in E$ of the network.
- The input flow $\phi_e(t)$ arriving at the node $e \in E$ of the network. We assume that $\phi_e(t)$ is known for any $t \geq t_0$ and is constant on $[t_0, t_1]$.
- The length l_{ij}, traffic accommodation capacity c_{ij} and the speed of vehicles ν_{ij}.

- The proportion p_{ij} of the flow that arrives at node i and is directed towards node j (if possible due to the capacity of *(i,j)*).
- The output capacity $C^o_{ij}(t_0)$, which represents the maximum flow that can leave the lane. We know the evolution of the output capacity for any $t \geq t_0$ and assume that this capacity is constant on $[t_0, t_2]$.
- The input capacity $C^e_{ij}(t_0)$, which is the consequence of the state of lane *(i,j)* , i.e. $n_{ij}(t_0)$ and the output capacity $C^o_{ij}(t_0)$ of the lane. $C^e_{ij}(t_0)$ is the maximum flow that can enter the lane.
- The time t_m, until which both the input flow $\phi_e(t)$ and the output capacity $C^o_{ij}(t_0)$ of the lane remain constant. Indeed, $t_m = Min(t_1, t_2)$

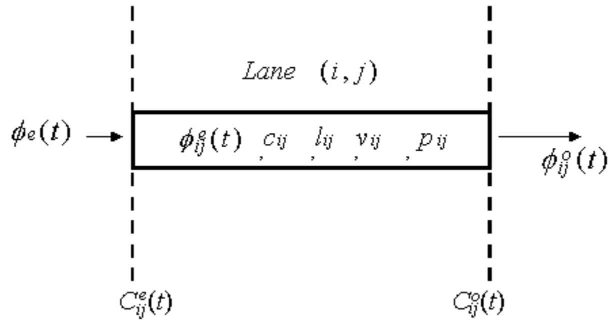

Figure 1: A Single Lane System

Let us consider the single lane system depicted in Figure 1. When the input flow does not exceeds the capacities of the lane at any point in time, there exists no congestion inside the system. This is the ideal situation where the input flows will reach the exit in minimum time. However, when the flow inside the system exceeds the capacity of the lane at the exit, we observe that the number of vehicles inside the lane increases.

Assume that the flows are constant. In this case, the number of vehicles present inside the lane at time t can be expressed as:

$$n_{ij}(t) = n_{ij}(t_0) + (\phi^*_i - \phi_j) * (t - t_0) \tag{1}$$

Let us now analyze the single lane system under the influence of discrete input flow and output capacity of the lane.

Case 1: It is the case when the lane is not full, that is $n_{ij}(t_0) < c_{ij}l_{ij}/\nu_{ij}$. In this case, the input capacity of the lane is:

$$C_{ij}^e(t_0) = c_{ij}$$

Case 1.1: This is the case when $\phi_i(t_0) \geq c_{ij}$. In this case, the flow that arrives at the entrance of the lane exceeds or is equal to the input capacity c_{ij}. Under this condition the number of cars that can enter the lane per unit of time are restricted to the input capacity of the lane.

$$\phi_{ij}^e(t_0) = C_{ij}^e(t_0)$$

The number of cars waiting at the entrance of the lane increase by $\phi_i(t_0) - C_{ij}^e(t_0)$ each unit of time. The output flow of the lane is:

$$\phi_{ij}^o(t_0) = Min\{n_{ij}(t_0)\nu_{ij}/l_{ij}, C_{ij}^o(t_0)\}$$

The travel time $\theta_{ij}(t_0)$ for the vehicles in the lane is:

$$\theta_{ij}(t_0) = n_{ij}(t_0)/Min\{n_{ij}(t_0)\nu_{ij}/l_{ij}, C_{ij}^o(t_0)\}$$

The speed ν_{ij}^* of the vehicles exiting from the lane is:

$$\nu_{ij}^* = l_{ij} * [Min\{n_{ij}(t_0)\nu_{ij}/l_{ij}, C_{ij}^o(t_0)\}/n_{ij}(t_0)]$$

The number of cars inside the system at time $t \in [t_0, t_m]$ are :

$$n_{ij}(t) = n_{ij}(t_0) + (\phi_{ij}^e(t_0) - \phi_{ij}^o(t_0)) * (t - t_0) \tag{2}$$

The lane would become full at time t' where,

$$t' = t_0 + [\{c_{ij}l_{ij}/\nu_{ij} - n_{ij}(t_0)\}/\{c_{ij} - Min(C_{ij}^o(t_0), n_{ij}(t_0)\nu_{ij}/l_{ij})\}] \tag{3}$$

The system state changes at time t^* where,

$$t^* = Min(t_m, t')$$

Case 1.2: This is the case when $\phi_i(t_0) < c_{ij}$. In this case, the flow that arrives at the entrance of the lane is less than the input capacity c_{ij}. Under this condition, all the cars arriving at the entrance of the lane each unit of time can enter inside the lane.

$$\phi_{ij}^e(t_0) = \phi_i(t_0)$$

The output flow from the lane is:

$$\phi_{ij}^o(t_0) = Min\{(n_{ij}(t_0)\nu_{ij}/l_{ij}), C_{ij}^o(t_0)\}$$

The number of cars inside the system at time $t \in [t_0, t_m]$ are given by:

$$n_{ij}(t) = n_{ij}(t_0) + [\{\phi_{ij}^e(t_0) - Min(n_{ij}(t_0)\nu_{ij}/l_{ij}, C_{ij}^o(t_0))\} * (t - t_0)] \tag{4}$$

Case 1.2.1: This is the case when $n_{ij}(t_0)\nu_{ij}/l_{ij} \leq C^o_{ij}(t_0)$. In this case, the flow that leaves the lane does not exceeds the output capacity $C^o_{ij}(t_0)$ of the lane. Under this condition, the output flow is given by

$$\phi^o_{ij}(t_0) = n_{ij}(t_0)\nu_{ij}/l_{ij}$$

The travel time $\theta_{ij}(t_0)$ for the vehicles in the lane is:

$$\theta_{ij}(t_0) = l_{ij}/\nu_{ij}$$

The speed ν^*_{ij} of the vehicles exiting from the lane is:

$$\nu^*_{ij} = \nu_{ij}$$

The number of cars inside the system at time $t \in [t_0, t_m]$ are given by:

$$n_{ij}(t) = n_{ij}(t_0) + [\{\phi^e_{ij}(t_0) - (n_{ij}(t_0)\nu_{ij}/l_{ij})\} * (t - t_0)]$$

The system would attain equilibrium with the output flow being equal to the input flow at time t'

$$t' = t_0 + l_{ij}/\nu_{ij}$$

The system state changes at time t^* where

$$t^* = Min(t_m, t')$$

Case 1.2.2: This is the case when $n_{ij}(t_0)\nu_{ij}/l_{ij} > C^o_{ij}(t_0)$. In this case, the flow that leaves the lane does exceeds the output capacity $C^o_{ij}(t_0)$ of the lane. Under this condition, the output flow is

$$\phi^o_{ij}(t_0) = C^o_{ij}(t_0)$$

The travel time $\theta_{ij}(t_0)$ for the vehicles in the lane is:

$$\theta_{ij}(t_0) = n_{ij}(t_0)/C^o_{ij}(t_0)$$

The speed ν^*_{ij} of the vehicles is given by:

$$\nu^*_{ij} = l_{ij} * (C^o_{ij}(t_0)/n_{ij}(t_0))$$

The number of cars inside the system at time $t \in [t_0, t_m]$ are given by:

$$n_{ij}(t) = n_{ij}(t_0) + [\{\phi^e_{ij}(t_0) - C^o_{ij}(t_0)\} * (t - t_0)]$$

Case 1.2.2.1: This is the case when $\phi_i(t_0) < C^o_{ij}(t_0)$. In this case, the flow that arrives at the entrance of the lane does not exceeds the output capacity $C^o_{ij}(t_0)$ of the lane. Under this condition, the number of vehicles inside the lane decrease and the lane would reach the equilibrium with output flow equal to the input flow at time t' where

$$t' = t_0 + [\{(\phi^e_{ij}(t_0)l_{ij}/\nu_{ij}) - n_{ij}(t_0)\}/\{\phi^e_{ij}(t_0) - C^o_{ij}(t_0)\}] \quad (5)$$

The system state change time is equal to t^* where,

$$t^* = Min(t_m, t')$$

Case 1.2.2.2: This is the case when $\phi_i(t_0) = C^o_{ij}(t_0)$. In this case, the flow that arrives at the entrance of the lane equals the output capacity $C^o_{ij}(t_0)$ of the lane. Under this condition, the state of the system remains stable until time $t^* = t_m$. The output flow remains constant and is given by,

$$\phi^o_{ij}(t_0) = C^o_{ij}(t_0)$$

Case 1.2.2.3: This is the case when $\phi_i(t_0) > C^o_{ij}(t_0)$. In this case, the flow that arrives at the entrance of the lane exceeds the output capacity $C^o_{ij}(t_0)$ of the lane. Under this condition, the number of the vehicles inside the lane increase and the lane would be full at time t' where

$$t' = t_0 + [\{(c_{ij}l_{ij}/\nu_{ij}) - n_{ij}(t_0)\}/\{\phi^e_{ij}(t_0) - C^o_{ij}(t_0)\}] \quad (6)$$

The system state changes at time t^* where

$$t^* = Min(t_m, t')$$

Case 2: It is the case in which the single lane system is full. That is, $n_{ij}(t_0) = c_{ij}l_{ij}/\nu_{ij}$. In this case, the maximum flow $C^e_{ij}(t_0)$ that can be accepted by the lane is equal to the minimum of the traffic accommodation capacity and the output capacity of the lane.

$$C^e_{ij}(t_0) = Min\{C^o_{ij}(t_0), c_{ij}\}$$

The output flow from the lane is:

$$\phi^o_{ij}(t_0) = Min\{C^o_{ij}(t_0), c_{ij}\}$$

The travel time $\theta_{ij}(t_0)$ for the vehicles in the lane is:

$$\theta_{ij}(t_0) = n_{ij}(t_0)/Min\{C^o_{ij}(t_0), c_{ij}\}$$

The speed ν^*_{ij} of vehicles exiting from the lane is :

$$\nu^*_{ij} = l_{ij} * Min\{C^o_{ij}(t_0), c_{ij}\}/n_{ij}(t_0)$$

Case 2.1:

This is the case when $\phi_i(t_0) \geq C^e_{ij}(t_0)$. In this case, the flow that arrives at the entrance of the lane exceeds or is equal to the input capacity $C^e_{ij}(t_0)$ of the lane. The input flow entering the lane is:

$$\phi^e_{ij}(t_0) = C^e_{ij}(t_0)$$

The number of vehicles accumulating at the entrance of the lane are $\phi_i(t_0) - C^e_{ij}(t_0)$ per unit of time. The system state change time $t^* = t_m$.

Case 2.2:

This is the case when $\phi_i(t_0) < C^e_{ij}(t_0)$. In this case, the flow that arrives at the entrance of the lane does not exceeds the input capacity $C^e_{ij}(t_0)$ of the lane. Under this condition, the input capacity of the lane is:

$$C^e_{ij}(t_0) = c_{ij}$$

The input flow entering the lane is:

$$\phi^e_{ij}(t_0) = \phi_i(t_0)$$

The number of cars inside the system decreases with time and the lane would reach equilibrium with the output flow being equal to the input flow at time t' where

$$t' = t_0 + [\{(\phi^e_{ij}(t_0)l_{ij}/\nu_{ij}) - n_{ij}(t_0)\}/\{\phi^e_{ij}(t_0) - Min(c_{ij}, C^o_{ij}(t_0))\}] \quad (7)$$

The system state changes at time t^* where

$$t^* = Min(t_m, t')$$

Note: If a car is in the lane at time t_0, and if its position is given by its distance x from the entrance of the lane, then it would reach the end of the lane at time t_v:

$$t_v = t_0 + \{(l_{ij} - x)/v^*_{ij}\}$$

where v^*_{ij} is the speed.

In this case, we consider that the state of the system changes at time t^{*^*} where

$$t^{*^*} = Min(t^*, t_v)$$

and the position z of the car in the lane is

$$z = (t^{*^*} - t_0)v^*_{ij} + x$$

where $0 \leq x \leq l_{ij}$. If $x > l_{ij}$, the car is no more in the lane.

3.2 Numerical Example

Let us consider the single lane system depicted in Figure 1. The length of the lane is $l_{ij} = 10$, the traffic accommodation capacity is $c_{ij} = 20$ and the speed of the cars $\nu = 1$. The number of cars present inside the system at time t_0 are $n_{ij}(t_0) = 150$. We assume that the input flows and the output capacity of the lane remains piecewise constant on intervals $[t_0, t_1], [t_1, t_2]$, ..etc.

The piecewise constant input flow ϕ_e arriving at the entrance of the lane is presented in Table 2. The output capacity C^o_{ij} of the single lane system is given in Table 3. To trace the position of the cars inside the system, a red car is introduced at the entrance of the lane at different times(Table 3).

Table 1: Input Flow (ϕ_e) vs time

ϕ_e	*Time Interval*
12	0-20
15	20-50
20	50-70
10	70-100

Table 2: Output Capacity(C^o_{ij}) vs time

C^o_{ij}	*Time Interval*
15	0-10
10	10-30
20	30-60
15	60-100

Let us now analyze the system behavior under varying input flows and output capacities of the lane at discrete time intervals. The results for the above numerical example are presented in Table 3. It can be seen in Table 3:

- For time $t \in [0, 10], [30, 40], [50, 60], [80, 100]$, the input flow and the output flow do not exceed the capacity constraints of the lane and therefore the cars reach the exit in minimum time. The travel time for the red car entering the system at time 0, 30 and 50 is equal to 10.
- For time interval $[40, 50]$, the input and the output flow do not exceed the capacity constraints of the lane and the number of vehicles continue to remain constant inside

Table 3: Travel Time and System State

System State at time t_0	*Time* t^**, Travel Time, Arrival Time and Departure Time of Red Car*	*Evolution of* $n_{ij}(t)$
For $t \in [0,10]$ $\phi_i^* = 12, C_j = 15$ $C_i = 20, \phi_j = 15$	$t^* = 10, \theta_{ij} = 10$ *Red Car enters at t = 0* *Red Car exits at t = 10*	$n_{ij}(t) = 150 - 3 * t$ $n_{ij}(0) = 150$ $n_{ij}(10) = 120$
For $t \in [10,20]$ $\phi_i^* = 12, C_j = 10$ $C_i = 20, \phi_j = 10$	$t^* = 20, \theta_{ij} = 12$ *Red Car enters at t = 10* $x_{ij}(20) = 8.334$	$n_{ij}(t) = 120 + 2 * (t - 10)$ $n_{ij}(10) = 120$ $n_{ij}(20) = 140$
For $t \in [20,30]$ $\phi_i^* = 15, C_j = 10$ $C_i = 20, \phi_j = 10$	$t^* = 30$ $x_{ij}(20) = 8.334$ *Red Car exits at t = 22*	$n_{ij}(t) = 140 + 5 * (t - 20)$ $n_{ij}(20) = 140$ $n_{ij}(30) = 190$
For $t \in [30,40]$ $\phi_i^* = 15, C_j = 20$ $C_i = 20, \phi_j = 19$	$t^* = 40, \theta_{ij} = 10$ *Red Car enters at t = 30* *Red Car exits at t = 40*	$n_{ij}(t) = 190 - 4 * (t - 30)$ $n_{ij}(30) = 190$ $n_{ij}(40) = 150$
For $t \in [40,50]$ $\phi_i = 15, C_j = 20$ $C_i = 20, \phi_j = 15$	$t^* = 50, \theta_{ij} = 10$ *Red Car enters at t = 40* *Red Car exits at t = 50*	$n_{ij}(t) = 150$ $n_{ij}(40) = 150$ $n_{ij}(50) = 150$
For $t \in [50,60]$ $\phi_i^* = 20, C_j = 20$ $C_i = 20, \phi_j = 15$	$t^* = 60, \theta_{ij} = 10$ *Red Car enters at t = 50* *Red Car exits at t = 60*	$n_{ij}(t) = 150 + 5 * (t - 50)$ $n_{ij}(50) = 150$ $n_{ij}(60) = 200$
For $t \in [60,70]$ $\phi_i^* = 15, C_j = 15$ $C_i = 15, \phi_j = 15$	$t^* = 70, \theta_{ij} = 13.34$ *Red Car enters at t = 60* $x_{ij}(70) = 7.5$	$n_{ij}(t) = 200$ $n_{ij}(60) = 200$ $n_{ij}(70) = 200$ $w_e(70) = 50$
For $t \in [70,80]$ $\phi_i^* = 15, C_j = 15$ $C_i = 15, \phi_j = 15$	$t^* = 80$ $x_{ij}(70) = 7.5$ *Red Car exits at t = 73.34*	$n_{ij}(t) = 200$ $n_{ij}(70) = 200$ $n_{ij}(80) = 200$
For $t \in [80,100]$ $\phi_i^* = 10, C_j = 15$ $C_i = 20, \phi_j = 15$	$t^* = 100, \theta_{ij} = 13.34, t^{*^*} = 93.34$ *Red car enters at t = 80* *Red Car exits at t = 93.34*	$n_{ij}(t) = 200 - 5 * (t - 80)$ $n_{ij}(80) = 200$ $n_{ij}(100) = 100$

the lane. The lane is said to be in equilibrium during this time interval. The output flow is equal to the input flow. The travel time for the red car entering the system at time 40 is equal to 10.

- For time $t \in [10, 20], [20, 30]$, the input flow does not exceeds the input capacity but the output flow is greater than the output capacity of the lane. As a result, the flow exits at a rate equal to the output capacity of the lane and the number of cars inside the lane increases. The time taken by the cars to travel the system is greater than the minimum travel time for the lane. The travel time for the red car exceeds 10 when it enters the system at time 10 and 60 respectively.
- For time $t \in [60, 70], [70, 80]$, the lane is full and therefore an input flow equal to the output capacity of the lane can only be accepted by the system. The output flow is equal to the output capacity of the lane and the cars take maximum time to travel on the lane. The red car entering the system at time 60 and 80 take maximum time equal to 13.34 to traverse the lane.

4 Flow Transfer in a Network

Let us consider Figure 2. The goal is to define how flows arriving at node k are transferred in arcs *(k,s)* where $s \in Sc(k)$.

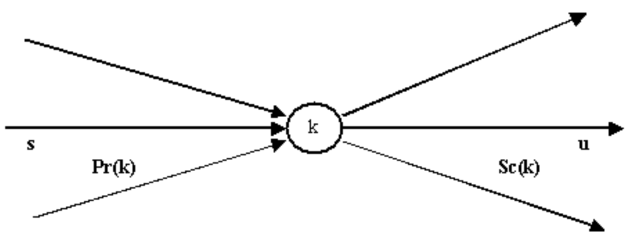

Figure 2: Flow Transfer at Node k

We assume that the maximum capacity $C^e_{ku}(t)$ of arcs *(k,u)* at time *t*, where $u \in Sc(k)$ are known. Usually, $C^e_{ku}(t) = c_{ku}$, but we may have $C^e_{ku}(t) < c_{ku}$ when arc *(k,u)* is full at time *t* and the capacity of the successors of *(k,u)* are less than c_{ku}. The way the maximum capacities influence each other will be developed hereafter.

For $s \in Pr(k)$, we define

$$C^o_{sk}(t) = \phi^o_{sk}(t) * q \tag{8}$$

where

$$q = Min_{u \in Sc(k)}[\{(C^e_{ku}/p_{ku})/ \sum_{s \in Pr(k)} \phi^o_{sk}(t)\}, 1]$$

Formula (8) provides the output capacity of arc *(s,k)* or, in other words the flows delivered by *(s,k)* and that will be transferred to the arcs *(k,u)* where $u \in Sc(k)$. Finally, the flow that will enter *(k,u)* is :

$$\phi^e_{ku}(t) = [\sum_{s \in Pr(k)} C^o_{sk}(t)] * p_{ku}$$

Note:
If $k \in E$, the flow that arrives at node k is an input flow denoted by $\phi_k(t)$. It is like we had one predecessor to k, say s, and that $c_{sk} = +\infty$. Under this hypothesis, the previous formulae apply and become:

$$C^o_{sk}(t) = Min_{u \in Sc(k)}[(C^e_{ku}(t)/p_{ku}), \phi_k(t)]$$

and

$$\phi^e_{ku}(t) = [Min_{u \in Sc(k)}\{(C^e_{ku}(t)/p_{ku}), \phi_k(t)\}] * p_{ku}$$

5 The Dynamic Network Model

The Dynamic Network Model describes the system behavior and the evolution of the flows inside the network under piecewise constant input flows at the entrance nodes and the initial system state of the lanes of the network. The evolution of a network after time t_0 depends on:

- The state of the network at time t_0.
- The input flows $\phi_e(t), e \in E$, for time $t \in [t_0, +\infty)$.

Remember that the input flows are supposed to be piecewise constant. It means that $\phi_e(t)$ will remain constant on time intervals $[t_0, t_1], [t_1, t_2]$.. etc.

5.1 Network Dynamics

The dynamics of the network is obtained by computing the dynamics of each one of the lane of the network. But, such a computation requires the output capacity of the lane under consideration and this output capacity depends on the successors of the lane. Thus, the computation of the next state of the network is twofold:

- Computation of the output capacity of each arc.
- Search of the times at which the lanes will change their state. Denote these times by t^*_{ij}, where $(i,j) \in A$. Then $t^* = Min_{(i,j) \in A}\{t^*_{ij}\}$ is the time at which the new state of the network is computed.

5.2 Computation of the output capacities of arcs

This computation is made backwards. Let $(s,k) \in A$. If $k \in D$, then $C^o_{sk} = c_{sk}$. The output capacity of the lane (s,k) is equal to the traffic accommodation capacity of the lane.

Now, assume that $k \notin D$. Since, the computation is made backward, we know the output capacities of all the arcs *(k,u)*, $k \in$ *Sc(k)*. We also know the state of the lane at time t_0, being the time at which the last state of the network is known. The output capacity of *(s,k)* is obtained by applying formula (8) with the input capacities $C^e_{ku}(t)$ of arcs (k,u) which are given by:

$$C^e_{ku}(t) = \begin{cases} c_{ku} & \text{if (k,u) is not full} \\ C^o_{ku}(t) & \text{if (k,u) is full} \end{cases} \tag{9}$$

5.3 The Algorithm

Finally, the algorithm can be summarized as follows:

1. Enter the state of the network at time t_0.
2. Compute the output capacity of each arc of the network (backward) using formulae (8-9).
3. Compute t^*_{ij} for each arc $(i,j) \in A$. These computation are based on the single lane analysis(See section 2.2).
4. Compute $t^* = Min_{(i,j) \in A}\{t^*_{ij}\}$
5. Compute the state of each arc at time t^*.
6. Set $t_0 = t^*$
7. goto Step 2. Indeed, we stop the computation at a time given by the user.

6 Simulation

Using the dynamic network model, we developed a simulation software for conducting the network analysis and the computation of travel times on the network. The inputs for the simulation software are the input flows at the entrances of the network, the initial system states of the lane, the characteristics of the lane and the test cars (red cars) for tracking the various paths of the network. The output generated from the software are the minimum travel time paths for each origin-destination pair of the network. The simulation software has been developed in C++.

To compute the travel time for the paths from an origin to all destinations of the network, the simulation software uses a test car (red car). A red car is introduced at the entrance node of the network at time t_0. We assume that this red car while traveling on the network explodes into as many red cars as the successor arcs of the node until the destinations have

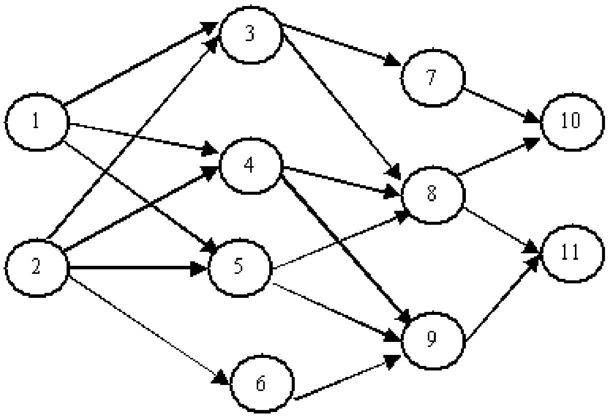

Figure 3: A multiple origin-destination network

been reached. The time at which red cars appear at the destination nodes is noted. This gives us the total number of paths generated between the origin and the destinations of the network. To compute the shortest path between the origin and each destination, the arrival time of the red car that appears first at each of the destination node is used. The red cars that exit first at each of the destination nodes are the ones that have taken the minimum traveling time paths to traverse the network. In case of one origin and n destinations, the simulation software generates *1*n* shortest traveling time paths for the network.

To generate the shortest paths from all origins to all destinations of the network, test cars are introduced at each of the entrances of the network and the first time at which the test cars from all the origins appear at each destination are recorded. If the number of origin is m and the number of destination is n, then the simulation software generates $m*n$ shortest traveling time paths for the network.

6.1 Numerical Example

Let us consider the multiple lane system depicted in Figure 3. The piecewise constant input flow ϕ_e arriving at each of the entrances of the network are presented in Table 4. The constant network parameters and the initial system state of the various lanes of the network are presented in Table 5. To trace the position of the cars inside the system, a red car is introduced at each of the entrances of the network at time 0.

Table 4: Network Parameters at time t_o

Arc Id (i,j)	p_{ij}	ν_{ij}	c_{ij}	l_{ij}	$n_{ij}(t_0)$
(1,3)	0.4	1	50	20	800
(1,4)	0.3	1	20	10	100
(1,5)	0.3	1	30	10	200
(2,3)	0.2	1	20	20	100
(2,4)	0.4	1	20	10	100
(2,5)	0.2	1	40	20	400
(2,6)	0.2	1	30	10	400
(3,7)	0.5	1	20	10	120
(3,8)	0.5	1	20	10	100
(4,8)	0.4	1	20	20	400
(4,9)	0.6	1	10	20	100
(5,8)	0.3	1	30	10	200
(5,9)	0.7	1	40	20	400
(6,9)	1.0	1	20	20	100
(7,10)	1.0	1	30	20	400
(8,10)	0.5	1	20	20	100
(8,11)	0.5	1	30	10	200
(9,11)	1.0	1	20	20	100

Table 5: Input Flow $\phi_e(t)$ vs Time

Origin Node	*Time Interval*	*Input Flow* $\phi_e(t)$
1	0-100	50
2	0-100	50

Table 6: Shortest Paths

O-D Pair	*Red Car Arrival Time*	*Red Car Departure Time*	*Shortest Path*	*Constituent Lanes*	*Red Car Lane Arrival Time*	*Red Car Lane Departure Time*
1-10	0	42.2858	1-5-8-10	(1,5) (5,8) (8,10)	0 10 22.2858	10 22.2858 42.2858
1-11	0	32.2858	1-5-8-11	(1,5) (5,8) (8,11)	0 10 22.2858	10 22.2858 32.2858
2-10	0	51.125	2-5-8-10	(2,5) (5,8) (8,10)	0 20 31.125	20 31.125 51.125
2-11	0	41.125	2-5-8-11	(2,5) (5,8) (8,11)	0 20 31.125	20 31.125 41.125

6.2 Results

The results for the above numerical example obtained from the simulation software are presented in Table 6. It can be seen in Table 6:

- For O-D pair (1-10), the shortest path is 1-5-8-10 and the travel time is 42.2858. The red cars face no congestion while traveling on the lanes(1,5) and (5,10) and therefore they traverse these lanes in minimum time. The travel time on lane (5,8) is greater than the minimum travel time since the output capacity constraints of the lane (5,8) are violated during time interval [14.1386, 22.2858] . As a result, the red car travels at a speed lesser than 1 on lane (5,8) during this time interval.
- For O-D pair (1-11), the shortest path is 1-5-8-11 and the travel time is 32.2858.The red cars face no congestion while traveling on the lanes(1,5) and (5,10) and therefore they traverse these lane in minimum time. The travel time on lane (5,8) is greater than the minimum travel timesince the output capacity constraints of the lane (5,8) are violated during time interval [14.1386, 22.2858] . As a result, the red car travels at a speed less than 0.9 on lane (5,8) during this time interval.
- For O-D pair (2-10), the shortest path is 2-5-8-10 and the travel time is 51.125. The red cars face no congestion while traveling on the lanes(1,5) and (5,10) and therefore they traverse these lane in minimum time. The travel time on lane (5,8) is greater than the minimum travel time since the output capacity constraints of the lane (5,8)

Table 7: Red Car position $x_{ij}(t)$ vs Time t^*

Lanes Used	*Time* t^*	*Red Car Position*	*Red Car Speed*
(1,5)	8.33333	8.33333	1
	8.59337	8.59337	1
	10	10	1
(5,8)	14.1386	3.14574	0.760106
	15.0839	3.91233	0.81088
	15.4074	4.17803	0.821469
	19.5229	7.56351	0.822614
	22.2858	10	0.881861
(8,10)	29.5229	7.23711	1
	39.5229	17.2371	1
	42.2858	20	1
(8,11)	29.5229	7.23711	1
	32.2858	10	1
(2,5)	8.33333	8.33333	1
	8.59337	8.59337	1
	14.1386	14.1386	1
	15.0839	15.0839	1
	15.4074	15.4074	1
	19.5229	19.5229	1
	20	20	1
(5,8)	29.5229	8.39788	0.881861
	31.125	10	1
(8,10)	51.125	20	1
(8,11)	41.125	10	1

are violated during time interval [29.5229, 31.125]. As a result, the red car travels at a speed lesser than the free flow speed on lane (5,8) during this time interval.

- For O-D pair (2-11), the shortest path is 2-5-8-11 and the travel time is 41.125. The red cars face no congestion while traveling on the lanes(1,5) and (5,10) and therefore they traverse these lanes in minimum time. The travel time on lane (5,8) is greater than the minimum travel time since the output capacity constraints of the lane (5,8) are violated during time interval [29.5229, 31.125]. As a result, the red car travels at a speed less than 0.9 on lane (5,8) during this time interval.

Table 7 presents the position of red cars at various time t^*. It can be seen in Table 7 that red car travels with free flow speed ($= 1$) on lanes (1,5), (2,5), (8,10), (8,11). The speed of the red car is equal to 1 on lane (1,5) during time interval [0,10], on lane (2,5) during time interval [0,20], on lane (8,10) during time intervals [29.5229, 42.2858] and [29.5229, 31.125] and on lane (8,10) during time intervals [29.5229, 32.4858] and [29.5229, 41.125]. The speed of the red car is less than 1 on lane (5,8) during time intervals [14.1386, 22.2858] and [29.5229, 31.125] since $\{(n_{58}(t_0)\nu_{58}/_{58}) > C^o_{58}\}$ for these time intervals. As a result, only an output flow equal to C^o_{58} can exit from the lane and the speed ν^*_{58} of the cars inside the lane decrease.

7 Conclusion

In this paper, we proposed a dynamic network model for network analysis and travel time computation on an urban network under varying input flows and system states of the network. The system analysis of the network is done using the system analysis of the single lane defined by its capacity, its length, the flow at the input of the lane and the output capacity. The transfer of flows inside the network is followed to trace the evolution of flows inside the system and to compute the path travel times. During our study, we found that the travel times on the network depend on the input flows ϕ_e at the entrances $e \in E$, the lane parameters $(n_{ij}(t_0), C^e_{ij}, C^o_{ij}, l_{ij}, p_{ij}, c_{ij})$ and the speed of the cars (ν^*_{ij}) on the network.

In the study developed in this paper, we performed the network analysis under piecewise constant input flows. The objective was twofold:

- define the main relations that characterize the dynamics of the system.
- define the travel time on the network with regard to input flows and to show how the traveling time of a car is affected by the history of the system.

The next step of our work concerns the prediction of shortest path on a large network under varying input flows and system states of the network.

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