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Some results on the controllability of planar variational inequalities

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Thème 4 — Simulation et optimisation
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Abstract: This note deals with the controllability of a class of planar complementarity dynamical systems, which can also be viewed as planar variational inequalities. It is shown that the complementarity conditions influence a lot the controllability of the system.

Key-words: variational inequalities, complementarity systems, projected dynamics, unilateral dynamics, controllability.

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Résultats préliminaires sur la commandabilité des inéquations variationnelles en 2D

Résumé : Cette note concerne la commandabilité d'une classe de systèmes de complémentarité, qui peuvent être vus comme des inéquations variationnelles de dimension 2. On montre que les conditions de complémentarité influencent largement la commandabilité de ces systèmes.

Mots-clés : Inéquations variationnelles, systèmes de complémentarité, dynamique projetée, dynamique unilatérale, commandabilité.

1 Introduction

Hybrid dynamical systems constitute a very large class of systems [2]. It is consequently necessary to focus on specific subclasses to make their study possible, see e.g. [14] [15] [16] for controllability issues in piecewise-linear systems. An interesting subclass is made of so-called complementarity systems [13] [1]. Similarly to the fact that the stability of unilaterally constrained systems can significantly differ from that of their unconstrained counterpart [3] [4], it will be shown that their controllability properties can differ a lot as well. This reinforces the fact that such nonsmooth dynamical systems deserve full attention and are not a mere extension of unconstrained or bilaterally constrained systems. In this note we will restrict ourselves to a narrow class of complementarity systems, that we call planar evolution variational inequalities. These systems are also sometimes called projected dynamical systems [3] [6] and are used to model the dynamics of oligopolistic markets, spatial price equilibrium, elastic demand traffic equilibirum [3]. As illustrated at the end of the note, they can also model some electrical circuits with ideal diodes. In this note it is shown that the controllability of such systems, with piecewise continuous inputs, depend a lot on the convex set that within which the state is constrained to evolve.

2 Planar evolution variational inequalities

The linear complementarity systems (LCS) [1] we are dealing with in this study, possess the following dynamics

$$\left\{\begin{array}{l} \dot{z}_1 = z_2 + C_1^T\lambda \\ \\ \dot{z}_2 = u + C_2^T\lambda \\ \\ 0 \leq \lambda \perp Cz + d \geq 0 \end{array}\right. \tag{1}$$

where $C = (C_1\ C_2) \in \mathbb{R}^{m\times 2}$, $C_1 \in \mathbb{R}^m$ and $C_2 \in \mathbb{R}^m$ are the two columns of C, $d \in \mathbb{R}^2$, $\lambda \in \mathbb{R}^m$. The non-negativity is understood componentwise. The LCS in (1) is equivalent to the linear evolution variational inequality (VI)

$$\left\{\begin{array}{l} \langle \dot{z} + Az + Bu, v - z\rangle \geq 0,\ \forall v \in K \\ \\ z(t) \in K,\ \forall\, t \geq 0 \end{array}\right. \tag{2}$$

where $z = (z_1, z_2)^T \in \mathbb{R}^2$, $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $K = \{(z_1, z_2) \in \mathbb{R}^2 | C_1 z_1 + C_2 z_2 + d \geq 0\}$.

It is noteworthy that, seen from an LCS point of view, the controlled dynamics in (1) is rather a narrow class. However the VI formalism in (2) shows that it is not so restrictive from an application point of view, since VI are widely used in some domains of science (see [3] for market and finance applications). VI can also represent some electrical circuits with ideal diodes [4]. The LCS in (1) is a particular gradient complementarity system [7], which is in turn equivalent to so-called projected dynamical systems [3] [5] [6]. So our study could be that of controllability of particular planar projected dynamical systems. There certainly remains a big gap between this work, and obtaining similar results for general projected dynamical systems (to say nothing for general LCS). It is worthy to remark that only the state space transformations $x = Lz$ with $LL^T = I_2$, where I_2 is the 2×2 identity, preserve the variational inequality formalism in (2) [4]. Consequently the results apply to a restricted class of planar evolution VI only. Since the studies on controllability of this type of dynamical systems are rare, this paper nevertheless has some interest.

Remark 1 *A more general class of systems consists of choosing* $A = \begin{pmatrix} a & 1 \\ 0 & 0 \end{pmatrix}$ *for some real* a *(the second row in* A *can easily be set to* $(0,0)$ *by feedback). Then applying a back-stepping procedure [9] and a pre-feedback input* $u = \bar{u} - a^2 z_1 - a z_2$*, one can put the system under the form*

$$\begin{cases} \langle \dot{\tilde{z}} + A\tilde{z} + B\bar{u}, w - \tilde{z}\rangle \geq 0, \ \forall w \in \tilde{K}(z_1) \\ \\ \tilde{z}(t) \in \tilde{K}(z_1), \ \forall\, t \geq 0 \end{cases} \tag{3}$$

where $\tilde{z} = (z_1, \tilde{z}_2)^T$*,* $\tilde{z}_2 = z_2 - z_{2d}$*,* $z_{2d} = -az_1$*,* $\tilde{K}(z_1) = \{w | w = v - \tilde{z}, v \in K\} = \{w | Cw + d + C \begin{pmatrix} 0 \\ az_1 \end{pmatrix} \geq 0\}$*. The system in (3) is therefore a VI with state dependent convex set* $\tilde{K}(z_1)$*: the set* $\tilde{K}$ *moves as the state evolves. The well-posedness studies for such VI seem for the moment limited to Moreau's sweeping process [10, §3.3], i.e. with* $A = 0$*. The obtained system is also called a parabolic quasi-variational inequality. We post-pone the study of such dynamical controlled systems to a future time.*

The results in [4] and [8] assure that continuous solutions of (2) with locally differentiable derivative exist and are unique on $[0, +\infty)$, for all continuous and locally differentiable inputs $u(\cdot)$.

Definition 1 *The system in (1) (equivalently in (2)) is said to be* $K-$*controllable, if any state* $z_f \in K$ *can be reached from any state* $z_i \in K$*, in a finite or infinite time* T*, and with an admissible input* $u(\cdot)$*.*

Admissibility of the input means that the well-posedness conditions are respected. We do not make the difference between finite and infinite T to simplify the presentation.

The objective of this work is to prove that, under some restrictions on the convex set K, the state $x(\cdot)$ can be steered inside K (including the boundary ∂K) from any $x_0 \in K$ to any $x_1 \in K$. To begin with and to motivate the study, let us remark that in case $m = 1$ and $K = \{z | z_2 \geq -c, c < 0\}$, then surely the system is not $K-$controllable. Indeed z_1 can only move from the left to the right in the phase plane, since $\dot{z}_1 = z_2 \geq -c > 0$.

This controlled VI is accessible [11] with reachable subspaces from $(z_1(0), z_2(0))$ equal to $\{(z_1, z_2) | z_1 \geq z_1(0), z_2 \geq -c\}$, but not $K-$controllable.

Remark 2 *It is not clear how this simple uncontrollable case, could be extended to higher dimensions.*

Let us note that adding some "imaginary" state re-initialization rules on ∂K such that $K-$controllability holds, is not envisaged here since the dynamical systems in (1) or (2) are the topic of the study. However motivated by this simple example of non-controllability, one guesses that a crucial step in the study will be to prove whether or not one is able to move on ∂K in order to reach some states which are otherwise unreachable. Due to the complementarity conditions (third line in (1)) which imply that the vector field is modified when ∂K is attained, this will under certain conditions be possible.

3 Main result

The following assumption is made and supposed to hold in the sequel:

Assumption 1 *The set K has a positive measure in $\mathbb{R}^2$.*

It is easy to construct C and d such that indeed $K = \emptyset$ or it has zero measure.

Let us denote the faces of the convex set K as D^i, such that $D^i \subseteq \{z | a_i z_1 + b_i z_2 + d_i = 0\}$. In other words the faces are segments (possibly unbounded, like in the case K is a cone, or if K is defined as a half-space), and the segments can be extended to straight lines whose equations in the plane are $a_i z_1 + b_i z_2 + d_i = 0$, $1 \leq i \leq m$. Thus $C_1 = (a_1, ..., a_m)^T$, and $C_2 = (b_1, ..., b_m)^T$, $d = (d_1, ..., d_m)^T$. Let us place ourselves in the phase plane of the system, with the two axis $(0, z_1)$ and $(0, z_2)$. Then the following is true

Lemma 1 *The system in (1) (equivalently in (2)) is $K-$controllable if and only if there is no face of K such that:*

- *there is a portion of D^i with finite negative slope on the right (resp. left) of the point $D^i \cap \{z | z_1 = 0\}$, when K is below (resp. above) D^i.*
- *D^i is vertical and above (resp. below) $\{z | z_1 = 0\}$ if K is on the right (resp. left) of D^i.*
- *D^i is horizontal and in the half-space $\{z | z_2 < 0\}$ (resp. $\{z | z_2 > 0\}$) if K is below (resp. above) D^i.*
- *$D^i = \{z | z_2 = 0\}$.*

■

An illustration of these various cases is provided in figure 1. The system is not K_1-controllable, because the only way to attain a point on the left of the vertical line (l) from a point on the right of (l), is to follow the boundary of K. However once the point A has been reached, it is impossible to move on ∂K_1 towards A'. The system can be steered on the line AA' only in the direction of B. Therefore all points in K_1 which are situated on the left of (l), cannot be attained from points in K_1 on the right of (l). The system is K_2-controllable, however adding the dashed portion to K_2, one obtains K_4 and the system is not K_4-controllable. Indeed the points in K_4 on the left of (l), can be reached only by following ∂K_4 along CD. It is possible to move along the face EC from E to C, but C can be attained only asymptotically, and it cannot be crossed. Thus points on CD cannot be reached from the right of (l), and the portion of K_4 that is on the left of (l) is unreachable from points on the right of (l). The system is K_3-controllable.

These various results are a consequence of the complementarity conditions in (1), which preclude the use of *any* control input $u(\cdot)$ on ∂K: $u(\cdot)$ has to assure that the trajectory *remains* on ∂K.

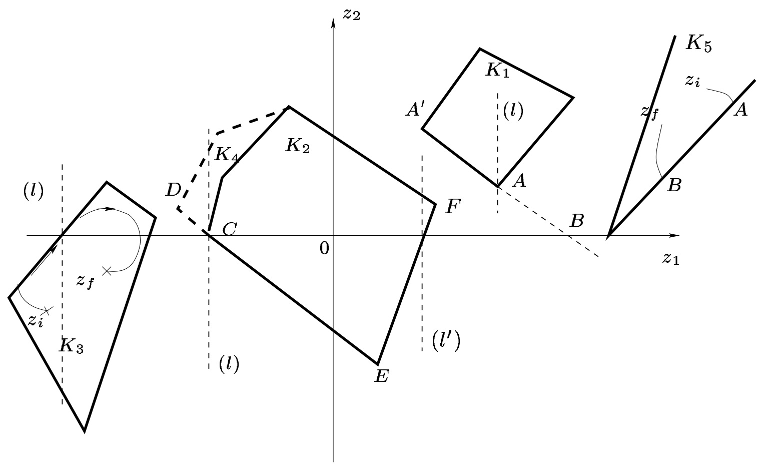

Figure 1: Examples of $K-$controllable and $K-$uncontrollable systems.

Before proving lemma 1, let us state intermediate results which characterize the motion on the boundary $\partial\Phi$.

Proposition 1 *Let us consider the system in (1) or (2), with* $C_1 = a \in I\!R$, $C_2 = b \in I\!R$, $d = c \in I\!R$. *Let us define the coordinate change* $\begin{cases} x_1 = bz_1 - az_2 + \frac{bc}{a} \\ x_2 = az_1 + bz_2 + c \end{cases}$. *We denote as* P *the intersection between the line* $az_1 + bz_2 + c = 0$ *and the* z_1*-axis, i.e.* P *is the origin of the new frame* (x_1, x_2) *and the constraint boundary is* $\{x | x_2 = 0\}$. *Then*

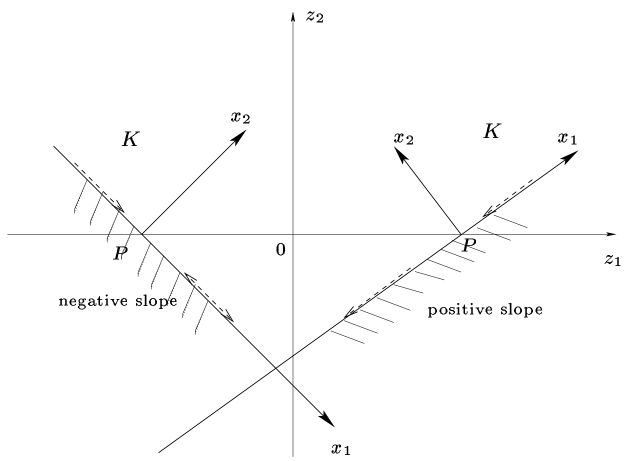

Figure 2: The new coordinate frame.

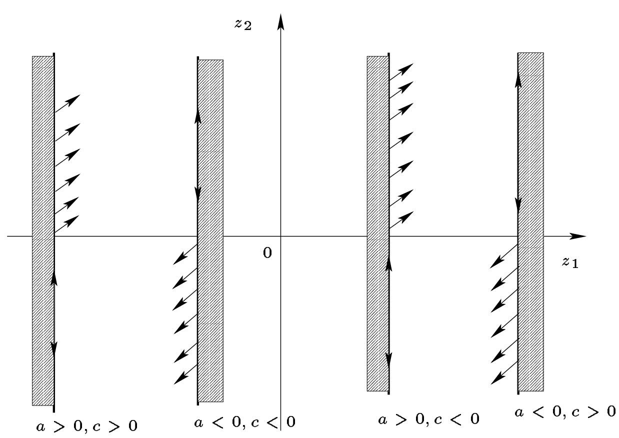

Figure 3: Trajectories on $\partial\Phi$ (vertical faces).

- *(positive slope) If* $-\frac{a}{b} > 0$, $b \neq 0$, *any point* x_{1f} *on the constraint can be attained from any point* $x_{1i} \geq x_{1f}$.
- *(negative slope) If* $-\frac{a}{b} < 0$, $b \neq 0$, *then any point* x_{1f} *on the constraint can be attained from any point* $x_{1i} \leq x_{1f}$, *and any point* x_{1f} *on the constraint can be attained from any point* $x_{1i} \geq x_{1f}$ *only on the axis* $x_1 \in [P, +\infty)$. *Moreover the point* P *can be attained from any* $x_{1i} > 0$ *only asymptotically.*

If $a = 0$ *then the boundary is a horizontal line* $z_2 = -\frac{c}{b}$ *and*

- *if* ($b > 0$ *and* $c < 0$) *or* ($b < 0$ *and* $c > 0$), *trajectories move from the left to the right,*
- *if* ($b > 0$ *and* $c > 0$) *or* ($b < 0$ *and* $c < 0$), *trajectories move from the right to the left,*
- *if* $c = 0$ *then the system remains stuck on* ∂K *at the contacting point.*

If $b = 0$ *then the boundary is a vertical line* $z_1 = -\frac{c}{a}$ *and*

- *if* ($a > 0$ *and* $c > 0$) *or* ($a > 0$ *and* $c < 0$) *then the system is controllable in the set* $\{z|z_2 < 0\}$ *and any trajectory initialized in the set* $\{z|z_2 \geq 0\}$ *detaches from* ∂K,
- *if* ($a < 0$ *and* $c < 0$) *or* ($a < 0$ *and* $c > 0$) *then the system is controllable in the set* $\{z|z_2 > 0\}$ *and any trajectory initialized in the set* $\{z|z_2 \leq 0\}$ *detaches from* ∂K.

■

Let us note that the case $a = b = 0$ is meaningless since the system is no longer constrained, hence it is not treated in proposition 1. The new coordinate frame (x_1, x_2) is depicted on figure 2. We note that the two depicted cases can be rotated to obtain the admissible domain below the boundary. The axis (P, x_2) points inside the admissible set K. The dashed arrows on ∂K indicate the directions in which trajectories can be controlled on ∂K. Due to the complementarity conditions, it follows that in some regions of ∂K, trajectories are restricted to move in a single direction (otherwise they leave ∂K). The cases when the boundary is vertical, is depicted in figure 3.

Proof of proposition 1: It is simple to calculate that on $\{x|x_2 = 0\}$ the system has the dynamics

$$\left\{ \begin{array}{l} \dot{x}_1 = -\frac{ab}{a^2+b^2}x_1 - au \\ \\ -\frac{a^2}{a^2+b^2}x_1 + bu \leq 0 \end{array} \right. \tag{4}$$

If $a \neq 0$ then the feedback $u = -\frac{1}{a}\left(v + \frac{ab}{a^2+b^2}x_1\right)$ yields

$$\left\{ \begin{array}{l} \dot{x}_1 = v \\ \\ -x_1 - \frac{b}{a}v \leq 0 \end{array} \right. \tag{5}$$

where v is the new input. We notice that if $-x_1 - \frac{b}{a}v = 0$ then the system "grazes" ∂K. If $-\frac{a}{b} > 0$, $b \neq 0$, then necessarily $v \leq -\frac{a}{b}x_1$, and v can be chosen < 0 so that x_1 can be made to decrease while staying on ∂K. If $-\frac{a}{b} < 0$, $b \neq 0$, then necessarily $v \geq -\frac{a}{b}x_1$. If $x_1 < 0$ then $v > 0$, so on $(-\infty, 0)$, x_1 can only increase. On $(0, +\infty) \ni x_1$, one can choose $v = -\frac{a}{b}x_1$ so that P is attained only asymptotically from any $x_{1i} > 0$.

Now if $a = 0$, the dynamics on ∂K is given by

$$\begin{cases} \dot{z}_1 = -\frac{c}{b} \\ z_2 = -\frac{c}{b} \\ u + b\lambda = 0 \text{ and } \lambda \geq 0 \ \Rightarrow bu \leq 0 \end{cases} \tag{6}$$

The result easily follows. If $b = 0$ the dynamics on ∂K is given by

$$\begin{cases} z_1 = -\frac{c}{a} \\ \dot{z}_2 = u \\ z_2 + a\lambda = 0 \text{ and } \lambda \geq 0 \ \Rightarrow az_2 \leq 0 \end{cases} \tag{7}$$

The result is a direct consequence of (7). ■

Proof of lemma 1: The proof uses the results of proposition 1. To start with, let us consider once again the set K_1 on figure 1. Then from proposition 1 it follows that trajectories can only be forced to move from A' to A, but not the reverse. Consider now K_2. Then trajectories can be controlled from E to C, though C is rechable in infinite time only. Assume that C is just below the axis $\{z | z_1 = 0\}$. It follows from proposition 1 that ∂K_2 can be tracked clock-wise by applying some suitable switching input. Thus, the points on the right of the vertical line (l') can be steered to anywhere in K_2 by first moving on FE. One may say that the dynamics is suitably modified on the boundary FE so that z_1 can decrease in the first quadrant. In the same way the system is K_5-controllable and K_3-controllable, but it is not K_4-controllable (the states on the left of the line (l) cannot be reached from anywhere in K_2). The system is K_5-controllable since as illustrated a state z_f that cannot be attained from z_i via a trajectory which remains in $K \setminus \partial K$, can be attained via a path z_iABz_f.

The proof of lemma 1 is done by observing that under the stated conditions, and from proposition 1, then any point in K can be steered by $u(\cdot)$ to any other point in K. Indeed if a state z_f cannot be attained from z_i via a trajectory in $K \setminus \partial K$, then a portion of path can be tracked on ∂K. For instance from the results of proposition 1, one sees that the boundary of the domain K_3 on figure 2 can be tracked clockwise. Consequently any point z_f on the right of the line (l) can be attained from any point z_i on the left of (l). There has to be a portion of the trajectory that evolves on ∂K to reach z_f from z_i. The same reasoning can be done for K_5. The conditions of lemma 1 are sufficient but can also be seen to be necessary, for if one of them fails then there exists couples of states in K which cannot be joined by a controlled trajectory.

We note that in general the input signal $u(\cdot)$ is piecewise continuous.

Remark 3 *Let us consider the case $K = \{z|g(z) \geq 0\}$ with $g(\cdot)$ a continuous function (possibly differentiable almost everywhere only, i.e. ∂K may have an infinity of corners). It does not suffice to approximate the convex set K by some polyhedron $\bar{K} \subseteq K$, and applying lemma 1, to conclude about the $\bar{K}-$controllability of the system. Indeed the dynamics is modified on ∂K, not on $\partial \bar{K}$. So it is necessary to examine the system* on *the boundary of K to conclude about controllability.*

Remark 4 *The systems considered in [14] [15] and in this note are not the same. Discrepancies are: they consider systems that evolve in polytopes K (i.e. bounded convex sets), and the dynamics on the various faces of the polytope are not specified as it is for complementarity systems (which, especially, restricts the choice of inputs on ∂K). For instance [15, propositions 4.1, 4.2] consider systems with a fixed controlled dynamics $\dot{z} = f(z,u)$, and investigates whether or not there exists a feedback input $u(z)$ such that $z(\cdot)$ can be steered in the polytope. The conditions are essentially on the orientation of the vector field $f(z,u)$ with respect to the polytope faces normal vectors. An interesting field of research would be to link the algorithmic part of [15] and the reachability of complementarity systems, injecting the specificity of complementarity systems in the study.*

4 An example

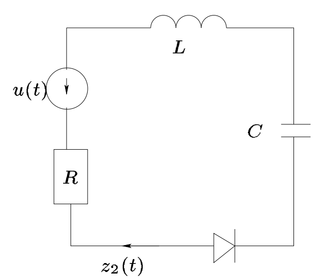

Figure 4: A simple electrical circuit.

Let us consider the simple electrical circuit in figure 4, where R is a resistor, L is an inductor, C is a capacitor, and the diode is supposed ideal. Its dynamics is given by

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -\frac{R}{L} z_2 + \frac{u(t)}{L} - \frac{1}{LC} z_1 - \frac{1}{L}\lambda \\ 0 \le \lambda \perp -z_2 \ge 0 \end{cases} \tag{8}$$

where $z_2(\cdot)$ is the current across the circuit and $-\lambda$ is the voltage of the diode, $u(\cdot)$ is a voltage control. One sees that this system is not controllable by simple application of lemma 1. One may transform the system in (8) into the canonical form in (1), by applying a pre-feedback $u(z_1, z_2) = Lv(t) + \frac{R}{L} z_2 + \frac{1}{LC} z_1$. In fact the state $z_1(\cdot)$ can only decrease, or be controlled to a constant value on $\partial\Phi$. This is intuitively sound since it corresponds to having the capacitor loaded with a non-positive current at all times.

5 Conclusion

In this note we have proposed a characterisation of the controllability properties of planar evolution variational inequalities with control input. These systems are a subclass of complementarity dynamical systems. They are nonsmooth and nonlinear. The material in this note relies heavily on the properties of the system on the boundary of the constraint set and on the behaviour of the trajectories of planar systems in their phase plane. Consequently an extension of this work should rely on the analytical tools in [12] that characterize the control capabilities of a system, on the boundary of its admissible domain. The class of systems that is considered is a narrow class of complementarity dynamical systems. However the results show that the controllabilty of complementarity dynamical systems differs significantly from that of unconstrained systems.

References

[1] W.P.M.H. Heemels, J.M. Schumacher, S. Weiland, 2000 "Linear complementarity systems", SIAM J. Applied Math., vol.60, no 4, pp.1234-1269.

[2] A. van der Schaft, H. Schumacher, 2000 *An Introduction to Hybrid Dynamical Systems*, Springer, LNCIS 251, London.

[3] A. Nagurney, D. Zhang, 1996 *Projected Dynamical Systems and Variational Inequalities with Applications*, Kluwer's International Series in Operations research and Management Science, Boston.

[4] D. Goeleven, B. Brogliato, 2002 "Stability and unstability matrices for linear evolution variational inequalities", submitted.

[5] D. Zhang, A. Nagurney 1995 “On the stability of projected dynamical systems”, J. of Optimization Theory and Applications, vol.85, no 1, pp.97-124.

[6] P. Dupuis, A. Nagurney, 1993 “Dynamical systems and variational inequalities”, Annals of Operations Research, vol.44, pp.9-42.

[7] W.P.M.H. Heemels, J.M. Schumacher, S. Weiland, 2000 “Projected dynamical systems in a complementary formalsim”, Operations Research Letters, vol.27, pp.83-91.

[8] D. Goeleven, M. Motreanu, V. Motreanu, 2002 “On the stability of stationary solutions of evolution variational inequalities”, to appear in Advances in Nonlinear Variational Inequalities.

[9] H.K. Khalil, 1996 *Nonlinear Systems*, 2nd edition, Prentice-Hall.

[10] M. Kunze, M. Monteiro Marques, 2000 “An introduction to Moreau’s sweeping process”, in *Impacts in Mechanical Systems*, Springer Lecture Notes in Physics LNP 551, B. Brogliato (Ed.), pp.1-60.

[11] H. Nijmeijer, A. J. van der Shaft, 1990 *Nonlinear Dynamical Control Systems*, Springer Verlag.

[12] A.A. ten Dam, E. Dwarshuis, J.C. Willems, 1997 "The contact problem for linear continuous-time dynamical systems: a geometric approach", IEEE Transactions on Automatic Control, vol.42, no 4, pp.458-472, April.

[13] B. Brogliato, 2003 “Some perspectives on the analysis and control of complementarity dynamical systems”, to appear in IEEE Transactions on Automatic Control, vol.48, no 6.

[14] L.C.G.J.M. Habets, J.H. van Schuppen, 2001 “A controllability result for piecewise-linear hybrid systems”, Proc. of the European Control Conference ECC01, Porto, Portugal (CD Rom).

[15] L.C.G.J.M. Habets, J.H. van Schuppen, 2001 “Control of piecewise-linear hybrid systems on simplices and rectangles”, Proc. Workshop Hybrid Systems – Computation and Control, Rome, Italy, 28-30 March; Lecture Notes in Computer Science, Springer Verlag, Berlin, pp.261-274.

[16] A. Bemporad, G. Ferari-Trecate, M. Morari, 2000 “Observability and controllability of piecewise affine and hybrid systems”, IEEE Transactions on Automatic Control, vol.45, no 10, pp.1854-1864, October.

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