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Abstract: Prediction of accurate travel time is an important aspect in real time monitoring of the traffic behavior on a city network. The dynamic traffic flows entering the network affect the behavior of the system and the free flow movement of vehicles. To study the complex system dynamics arising due to varying input flows, we begin with a single lane system. The analysis of the behavior of the system is done to derive the travel time from the characteristics of the lane, the input flow and the constraints on the output flow.

Key-words: Traffic flow , Congestion, Travel Time, System State

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Estimation du Temps de Trajet dans un systéme à une voie

Résumé : La possibilité de prévoir les temps de transport est un aspect important pour le contrôle en temps réel du trafic urbain. Les flux de véhicules qui entrent dans le réseau ont un impact sur les flux à l'intérieur du réseau. Dans ce papier, nous étudions dans le détail le cas d'une simple voie rectiligne et nous montrons comment obtenir le temps du trajet àppartir des caractéristiques de la voie rectiligne, du flux d'entrée et des contraintes sur le flux de sortie.

Mots-clés : Flux de véhicules, Congestion du trafic, Durée de transport, Etat du système.

1 Introduction

Recent developments in the area of transportation have given birth to Advanced Traveler Information Systems (ATIS) that provide historical, real time and predictive information to support travel decisions. Travelers seeking to travel from their current locations to specified destinations require best routings that minimize the travel times. The best route for any trip relies to a great extent on the accurate prediction of trip travel times.

There has been much research on the travel time prediction. In the context of prediction methodologies, various time series models [1,2,3] and artificial neural network models [4,5] have been developed. In the context of input data source, most previous studies used "indirect" travel time data [1,2,6,7,8]. The parameters of traffic data such as volume, occupancy and speed were obtained directly, while travel time was calculated as a function of these parameters. In most existing studies focused on link travel time estimation, it is generally assumed that the path travel time is the addition of travel times on the consisting links. Chen et al [9] studied the computation of travel time based on paths rather than link. In a dynamic network, the travel time computation depends not only on the geometry of the path, but also on the congestion present on the arcs of the path.

The approaches to simulate the traffic can be microscopic or macroscopic. The microscopic approach has resulted in car-following theories which study the behavior of one vehicle following another. The macroscopic approach is analogous to theories of fluid dynamics or continuum theories. Paralleled with experiments, many physical models have been proposed. In [10] for instance, the models presented are traffic stream models, queuing models, car following models and hydrodynamic models. Cellular automaton models [11] have been developed under which the microscopic following behavior is reduced to a minimal set of simple driving rules that qualitatively reproduce the dynamics of traffic flow including transitions to congestion. The hydrodynamic models [12,13] provide macroscopic definition of the traffic flow. They describe the dynamics of traffic flow by representations of traffic flows in terms of aggregate measures such as flow rate, average speed and density. The dynamic behavior of these aggregate variables is governed by variations in environment. The macroscopic approach presents a higher level of abstraction than the microscopic one.

The present study is restricted to the study of a single lane defined by its capacity, its length, the flow at the input of the lane and the output capacity. We propose the main relations that characterize the dynamics of this simple system and show how the traveling time of a car is affected by the history of the system.

The rest of the paper is organized in 5 sections. In section 2, we introduce the notations and present the problem. Section 3 is dedicated to the analysis of the system. Section 4 presents a stepwise evolution of the constant flow. In section 5, a numerical illustration is provided. The conclusion with future scope of research is presented in section 6

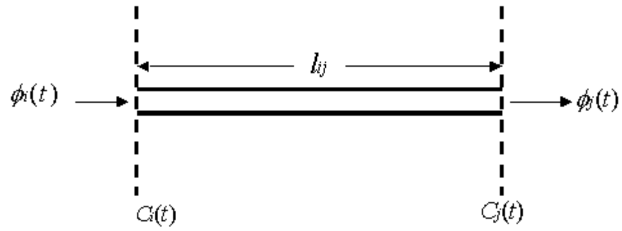

Figure 1: A single lane system

2 Problem Description

We consider a single lane with one input and one output. The flow at the input is known. Our goal is to define the time required to go from input to output, knowing the flow at the input node.

Let us consider the simple lane system depicted in Figure 1. The lane is denoted by *(i,j)* where i is the origin node and j is the destination node. The lane *(i,j)* has three attributes: its length l_{ij}, its traffic accommodation capacity c_{ij} and the number of vehicles inside the lane at time t denoted by $n_{ij}(t)$. The input flow at the entrance node i at any time t is $\phi_i(t)$ and the input capacity is $C_i(t)$. We denote by $\phi_i^*(t)$ the flow that really enters the lane when an input flow $\phi_i(t)$ arrives at the entrance of the lane. The flow exiting from the lane at output node j at any time t is $\phi_j(t)$, and the output capacity is denoted by $C_j(t)$. Also, at any time t,

$$\phi_i^*(t) \leq Min(C_i(t), c_{ij}, \phi_i(t))$$
$$\phi_j(t) \leq Min(C_j(t), c_{ij})$$

For this study:

- $\phi_i(t)$ is given as a function of time.
- $C_j(t)$ is also given as a function of time.

Note that in a network:

- $\phi_i(t)$ will be the total flow provided by the predecessors of *(i,j)*.
- $C_j(t)$ will be the constraint imposed by the successor lanes of *(i,j)*

Note also that $C_i(t)$ results from the state $(n_{ij}(t))$ and the capacity (c_{ij}) of the lane *(i,j)*. If *(i,j)* is full, that is if $n_{ij}(t) = l_{ij}c_{ij}/\nu$, then $C_i(t)$ is less than c_{ij} provided the capacities of the successor lanes is less than c_{ij}. In the future, we will insert the lane *(i,j)* to a network by means of $\phi_i(t)$, $\phi_j(t)$, $C_i(t)$, $C_j(t)$.

In steady state the flow moves with a constant speed ν on the lane except if we are in a blocking situation. The goal is to analyze the behavior of the single lane system under varying input flows and input and output capacities and to compute the time required to join i to j.

3 System Analysis

Let us consider the single lane system depicted in Figure 1. When the input flow does not exceed the capacities of the lane at any point in time, there exists no congestion inside the system. This is the ideal situation where the input flows will reach the exit in minimum time. However, when the flow inside the system exceeds the capacity of the lane at the exit, we observe a queue buildup of waiting vehicles inside the lane. Before analyzing the system, we make the following assumptions:

- The vehicles do not pass one another. This implies that vehicles follow First In First Out (FIFO) policy while traversing the lane.
- If a vehicle is not delayed, it moves at a constant speed ν.

At time t, the number of vehicles present inside the lane can be expressed as:

$$n_{ij}(t + \triangle t) = n_{ij}(t) + (\phi_i^*(t) - \phi_j(t)) \triangle t \tag{1}$$

In the continuous model, this can be rewritten as :

$$dn_{ij}(t)/dt = \phi_i^*(t) - \phi_j(t) \tag{2}$$

In the remaining sections of this paper, t_0 represents the starting time, that is the time at which the study starts. At time t_0, we know:

- The number of cars that are in the system, $n_{ij}(t_0)$.
- The input flow $\phi_i(t_0)$. We assume that $\phi_i(t)$ is known for any $t \geq t_0$.
- The output capacity $C_j(t_0)$. We also know the evolution of the output capacity for any $t \geq t_0$.
- The input capacity $C_i(t_0)$, which is the consequence of the state of lane *(i,j)* , i.e. $n_{ij}(t_0)$ and the output capacity $C_j(t_0)$ of the lane.

Let us now analyze the single lane system under the influence of these factors:

Case 1: It is the case when the lane is not full :

$$n_{ij}(t_0) < c_{ij} l_{ij}/\nu$$

In this case:

$$C_i(t) = c_{ij} \tag{3}$$

which means that nothing prevents the cars to enter the lane assuming that $\phi_i(t) \leq c_{ij}$. Relation (3) expresses the fact that, if the lane is not full it can accept a maximum flow at the entrance.

$$\phi_j(t) = Min(n_{ij}(t)\nu/l_{ij}, C_j(t)) \tag{4}$$

Relation (4) means that the output flow is the minimum of the maximum flow that is the result of the state of the successor lanes and the flow that can be provided by the cars that are in the lane. This leads us to consider two subcases:

Case 1.1: This is the case when

$$n_{ij}(t_0)\nu/l_{ij} > C_j(t_0)$$

Under this condition the number of cars that can exit from the system per unit of time are restricted to $C_j(t_0)$ at time t_0. The output flow at time $t \in [t_0, t_1]$ is:

$$\phi_j(t) = C_j(t)$$

The number of cars inside the system at time t_1 are :

$$n_{ij}(t_1) = n_{ij}(t_0) + \int_{t_0}^{t_1} (\phi_i^*(t) - C_j(t))dt \quad (5)$$

where $n_{ij}(t_0)$ is known and t_1 is equal to the minimum of:

(i). The first time at which $C_j(t)$ becomes first greater than $n_{ij}(t)\nu/l_{ij}$.
(ii). The first time, if any, at which $n_{ij}(t)$ becomes equal to $c_{ij}l_{ij}/\nu$. It is the case when the lane becomes full.

Both (i) and (ii) depend upon the input flow $\phi_i(t)$. As long as $t \in [t_0, t_1]$

$$dn_{ij}(t)/dt = \phi_i^*(t) - C_j(t) \quad (6)$$

The solution depends on functions $\phi_i(t)$ and $C_j(t)$. In section 4, we will consider the case when these functions are stepwise constant, and we will be able to provide a solution to differential equation (6).

Case 1.2: This is the case when

$$n_{ij}(t_0)\nu/l_{ij} \leq C_j(t_0)$$

Under this condition, the cars present inside the lane can smoothly exit from the system at time t_0. The output flow for time $t \in [t_0, t_1]$ is:

$$\phi_j(t) = n_{ij}(t)\nu/l_{ij}$$

The number of cars inside the system for time $t \in [t_0, t_1]$ are given by:

$$n_{ij}(t) = n_{ij}(t_0) + \int_{t_0}^{t} (\phi_i^*(\mu) - n_{ij}(\mu)\nu/l_{ij})d\mu \quad (7)$$

where $n_{ij}(t_0)$ is known and t_1 is equal to the minimum of :

(i). The first time at which $C_j(t)$ becomes first lesser than $n_{ij}(t)\nu/l_{ij}$.
(ii). The first time, if any, at which $n_{ij}(t)$ becomes equal to $c_{ij}l_{ij}/\nu$.

Both (i) and (ii) depend upon the input flow $\phi_i(t)$. As long as $t \in [t_0, t_1]$

$$dn_{ij}(t)/dt = \phi_i^*(t) - n_{ij}(t)\nu/l_{ij} \tag{8}$$

We will see in section 4, that the differential equation (8) has a simple solution in the case when $\phi_i(t)$ is constant over time interval $t \in [t_0, t_1]$

Case 2: It is the case in which the single lane system is full.

$$n_{ij}(t_0) = c_{ij}l_{ij}/\nu$$

In this case:

$$\phi_j(t_0) = Min(C_j(t_0), c_{ij})$$

The maximum flow at the output is the minimum of the maximum flow $C_j(t_0)$ that is the result of the state of the successor lanes and the maximum flow c_{ij} in lane (i, j). Let us study the evolution of $\phi_j(t)$ under two cases:

Case 2.1:

This is the case when

$$n_{ij}(t_0)\nu/l_{ij} > C_j(t_0)$$

Under this condition, at time t_0, only an input flow equal to $C_j(t_0)$ can enter into the system. If the input flow is greater than $C_j(t_0)$, then at time $t \geq t_0$, the number of cars waiting at the entrance of the lane increases by $\phi_i(t) - C_j(t)$ per unit of time. The input flow entering the system cannot exceed $C_j(t_0)$. At time t_0, we can say that:

$$\begin{aligned} C_i(t_0) &= C_j(t_0) \\ \phi_j(t_0) &= C_j(t_0) \end{aligned} \tag{9}$$

as long as $n_{ij}(t) = c_{ij}l_{ij}/\nu$ remains true or as long as the input flow is greater than or equal to $C_j(t)$. If $\phi_i^*(t_0) < C_j(t_0)$, then the number of cars start decreasing and we are back to case 1. As long as $n_{ij}(t) = c_{ij}l_{ij}/\nu$ remains true, we have:

$$dn_{ij}(t)/dt = 0 \quad for \quad t \in [t_0, t_1] \tag{10}$$

where t_1 is the time at which $C_j(t)$ becomes greater than or equal to c_{ij} or $\phi_i(t)$ becomes less than $C_j(t)$. Let us consider the smallest instant ϵ such that

$$n_{ij}(t_1 + \epsilon) < c_{ij}l_{ij}/\nu$$

It can be observed from equations (9) and (10) that for time $t < t_1$, the lane is full with the input flow entering the system equal to the output flow. For time $t > t_1$, the number of cars inside the system decreases. The diffusion of cars from the system after time t_1 facilitates the movement of waiting cars at the entrance of the lane inside the system.

Case 2.2:
This is the case when

$$n_{ij}(t_0)\nu/l_{ij} \leq C_j(t_0)$$

Under this condition the cars present inside the lane start diminishing if $\phi_i(t) < c_{ij}$ since $C_j(t_0) \geq c_{ij}$ and we return to case 1.

Let us now compute the travel time to join the origin i to the destination j of a single lane system under the influence of these system parameters. Let us denote the travel time for the single lane system by θ_{ij}. The travel time to join the input to the output of the lane *(i,j)* is :

$$\int_{t_0}^{\theta_{ij}} \phi_j(t)dt = n_{ij}(t_0) \tag{11}$$

assuming that the car for which we want to compute the travel time enters at time t_0.

4 Stepwise Evolution of Constant Flow

Let us now analyze the behavior of the single lane system under discrete input flows and output capacities. In practice, the flow is checked at discrete time intervals and therefore we can assume that the flow remains constant on specific time intervals. Besides, the output and input capacities of the lane change only when either a successor lane or the lane under consideration is full. As a consequence, we can consider that the parameters of the problem remain constant on intervals $[t_0, t_1]$, $[t_1, t_2]$... etc. Let us now analyze the single lane system under the influence of discrete input flow and output capacity at constant time intervals.

4.1 Estimation of System State

Depending upon the number of cars present inside the system at time t, the input flow, the traffic accomodation capacity and the output capacity of the lane, the state of the system varies. Let us now compute the system state for the single lane system under discrete input flows and output capacities for constant time intervals in the following cases:

Case 1: It is the case when

$$n_{ij}(t_0) < c_{ij}l_{ij}/\nu$$
$$n_{ij}(t_0)\nu/l_{ij} > C_j(t_0)$$

Under this condition, the lane is not full but the number of cars present inside the system exceed the output capacity at time t_0. As a result, cars exit from the system at a rate equal to the output capacity of the lane. From equation (6), we know the following relationship holds true for this case at time $t \in [t_0, t_1]$,

$$dn_{ij}(t)/dt = \phi_i^*(t) - C_j(t)$$

Let us assume that $\phi_i(t)$ and $C_j(t)$ are constant during time $t \in [t_0, t_1]$. Thus, we denote them respectively by ϕ_i and C_j and :

$$n_{ij}(t) = (\phi_i^* - C_j)t + K \tag{12}$$

The constant K is defined using the number of cars at time $t = t_0$, that is:

$$K = n_{ij}(t_0) - t_0(\phi_i^* - C_j) \tag{13}$$

Finally, considering equations (12) and (13),

$$n_{ij}(t) = n_{ij}(t_0) + (t - t_0)(\phi_i^* - C_j) \tag{14}$$

At this point, we have to consider three cases:

Case 1.1: $\phi_i^* > C_j$

It is the case where the number of cars increases and the lane would be full at time t^* such that:

$$n_{ij}(t^*) = c_{ij} l_{ij} / \nu$$

This equation can be rewritten as:

$$n_{ij}(t_0) + (\phi_i^* - C_j)(t^* - t_0) = c_{ij} l_{ij} / \nu$$

This leads to:

$$t^* = t_0 + (c_{ij} l_{ij} / \nu - n_{ij}(t_0)) / (\phi_i^* - C_j)$$

If $t^* \geq t_1$, we have just to compute the state of the system at time t_1 and continue the study on $[t_1, t_2]$. If $t^* < t_1$, the lane is full at time t^* and, starting from the state of the system at time t^*, we continue the study on $[t^*, t_1]$.

Case 1.2: $\phi_i^* = C_j$

The state of the system remains stable on $[t_0, t_1]$. We thus continue the study on $[t_1, t_2]$, starting at t_1 from the same state as the state at time t_0.

Case 1.3: $\phi_i^* < C_j$

Considering relation(14), we see that the number of cars in the lane decreases from time t_0 onwards. It can be also observed that the flow $n_{ij}(t)\nu / l_{ij}$ would reach C_j at time t^*, which is given by:

$$t^* = t_0 + (n_{ij}(t_0) - c_{ij} l_{ij} / \nu) / (C_j - \phi_i^*)$$

If $t^* \geq t_1$, we recompute the state of the system at time t_1 and continue the study on time interval $[t_1, t_2]$. If $t^* < t_1$, then $n_{ij}(t^*)\nu / l_{ij} = C_j$ at time t^* and we continue the study on $[t^*, t_1]$

Case 2: It is the case when

$$n_{ij}(t_0) < c_{ij} l_{ij}/\nu$$
$$n_{ij}(t_0)\nu/l_{ij} \le C_j(t_0)$$

Under this condition, the number of cars present inside the lane exceed neither the traffic accommodation capacity nor the output capacity of the lane at time t_0. As a result, cars face no congestion while traveling on the lane. From equation (8), we know that the following relationship is applicable for this case at time $t \ge t_0$, as long as the above conditions hold true.

$$dn_{ij}(t)/dt = \phi_i^* - n_{ij}(t)\nu/l_{ij}$$

Solving the differential equation, we obtain:

$$n_{ij}(t) = \phi_i^* l_{ij}/\nu + [n_{ij}(t_0) - \phi_i^* l_{ij}/\nu] e^{-\nu(t-t_0)/l_{ij}} \quad (15)$$

Thus, $n_{ij}(t)$ tends asymptotically to $\phi_i^* l_{ij}/\nu$ as t increases. Let us consider three cases:

Case 2.1: $n_{ij}(t_0) > \phi_i^* l_{ij}/\nu$

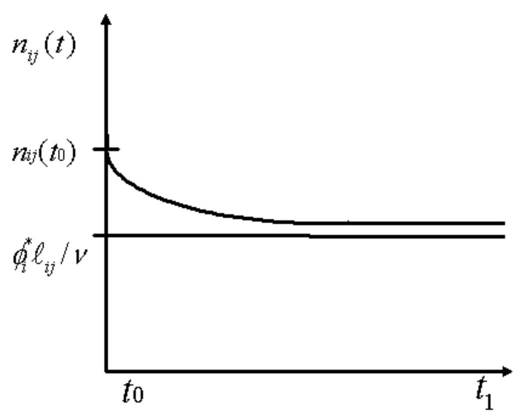

Figure 2: Evolution of $n_{ij}(t)$ when $n_{ij}(t_0) > \phi_i^* l_{ij}/\nu$

In this case, conditions given for case 2 remain true until time t_1. Thus, we compute the state for $t = t_1$ and continue the analysis on $[t_0, t_1]$. Figure 2 represents the variation of system state with time when $n_{ij}(t_0) > \phi_i^* l_{ij}/\nu$. It can be seen that the number of cars present inside the system decreases with time t.

Case 2.2: $n_{ij}(t_0) = \phi_i^* l_{ij}/\nu$

The state remains stable on $[t_0, t_1]$ and we continue the study on $[t_1, t_2]$, the state at time t_1 being the same as the state at time t_0.

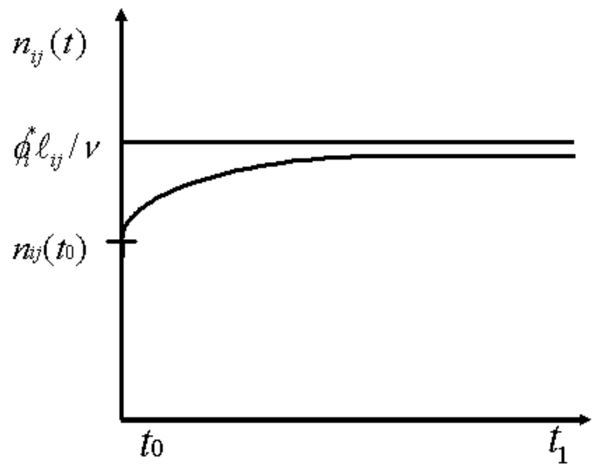

Figure 3: Evolution of $n_{ij}(t)$ when $n_{ij}(t_0) < \phi_i^* l_{ij}/\nu$

Case 2.3: $n_{ij}(t_0) < \phi_i^* l_{ij}/\nu$

This case is same as case 2.1 since $\phi_i^* \leq c_{ij}$. Figure 3 represents the variation of system state when $n_{ij}(t_0) < \phi_i^* l_{ij}/\nu$. It can be seen that the number of cars present inside the system increases with time t.

Case 3:

It is the case when

$$n_{ij}(t_0) = c_{ij} l_{ij}/\nu$$
$$n_{ij}(t_0)\nu/l_{ij} > C_j(t_0)$$

Under this condition, the single lane system is full and the number of cars present inside the lane exceed the output capacity of the lane at time t_0. As a result cars exit from the system at a rate equal to the output capacity of the lane. The behavior of the system will depend upon the input flow. Let us consider three cases:

Case 3.1: $\phi_i^* < C_j$

In this case, according to differential equation (6),

$$n_{ij}(t) = n_{ij}(t_0) + (\phi_i^* - C_j)(t - t_0)$$

Thus, $n_{ij}(t)$ decreases and we come back immediately to case 1.

Case 3.2: $\phi_i^* = C_j$

The system remains stable on $[t_0, t_1]$, and we restart the computation on $[t_1, t_2]$ at time t_1 starting from the same state as time t_0

Case 3.3: $\phi_i^* > C_j$

Only a flow C_j enters the system and we come back to case 3.2, though accumulation of cars take place at the entrance of the lane.

4.2 Estimation of Travel Time

Let us now compute the travel time to join the origin i to the destination j of a single lane system. The general equation for the travel time θ_{ij} in the lane *(i,j)* is:

$$\int_{t_0}^{\theta_{ij}} \phi_j(t) = n_{ij}(t_0) \tag{16}$$

When the functions are piecewise constant on $t \in [t_0, t_1], [t_1, t_2], ...$, then θ_{ij} is obtained as follows:

1. Compute the greatest integer n such that:

$$S_n = \sum_{k=1}^{n} \phi_j^k (t_k - t_{k-1}) \leq n_{ij}(t_0)$$

   Note that S_n represents the number of cars that had left the system between time t_0 and t_n. The output flow on period $[t_{k-1}, t_k]$ is denoted by ϕ_j^k. Note that we may obtain $n = 0$.

2. Compute $\triangle t$ as follows:

$$\triangle t = (n_{ij}(t_0) - S_n)/\phi_j^{n+1}$$

   The car reaches the exit of the lane at time $t_n + \triangle t$ considering that the car enters the system at time t_0.

Let us now trace the position of cars inside the single lane system under discrete input flows and output capacities. Assume that at time t_0, the cars just enter the lane. Let us consider that input flows not exceeding the traffic accomodation capacity of the lane continue to arrive at the entrance of the system during time intervals $[t_0, t_1], [t_1, t_2]$, ..etc. As a result all the cars will enter inside the lane at time $t > t_0$. Let us denote the position of the cars from the entrance of the lane at time t by $x_{ij}(t)$. The position of cars with time t is given by:

$$x_{ij}(t) = l_{ij} * (t - t_0)/\theta_{ij}$$

assuming that $x_{ij}(t)$ is positive. The cars will exit from the system at time $t = t_0 + \theta_{ij}$ where θ_{ij} represents the travel time inside the lane. Let us now consider two cases depending upon travel time θ_{ij}:

Case 1: It is the case when

$$n_{ij}(t) \leq c_{ij} l_{ij}/\nu$$
$$n_{ij}(t)\nu/l_{ij} \leq C_j$$

If the output capacity constraints of the lane are not exceeded, then $\theta_{ij} = l_{ij}/\nu$. The cars enter the system at t_0 and leave the system at time $t_0 + l_{ij}/\nu$. At time $t_0 + l_{ij}/\nu$, the position $x_{ij}(t)$ of cars is l_{ij}. We assume that the above conditions hold true until time $t_0 + \theta_{ij}$.

Case 2: It is the case when

$$n_{ij}(t) \leq c_{ij} l_{ij}/\nu$$
$$n_{ij}(t)\nu/l_{ij} > C_j$$

It is the case when the output capacity C_j is less than the flow $n_{ij}(t)l_{ij}/\nu$ that tends to leave the lane. If this conditions hold true until time $t_0 + n_{ij}(t_0)/C_j$, then a car that enters the system at time t_0 will leave the system at time $t_0 + n_{ij}(t_0)/C_j$.

Let us now summarize the results obtained from the estimation of system state and travel time under discrete input flows and output capacities in Table 1. It can be seen form Table 1 that during time $t \in [t_0, t_1]$,

Table 1: Stepwise evolution of constant flow

System State at time t_0	*Time t^* at which this kind of evolution ends*	*Evolution of $n_{ij}(t), \theta_{ij}$ and position $x_{ij}(t)$ of cars from the entrance of the lane (i,j)*
$n_{ij}(t_0) < c_{ij} l_{ij}/\nu$ $n_{ij}(t_0)\nu/l_{ij} \leq C_j(t_0)$ $C_i = c_{ij}$ $\phi_j = n_{ij}(t)\nu/l_{ij}$	$t^* = Min(t_1, t_0 + \delta)$ where $\delta = l_{ij}/\nu$	$n_{ij}(t) = \phi_i^* l_{ij}/\nu +$ $[n_{ij}(t_0) - \phi_i^* l_{ij}/\nu] e^{-\nu(t-t_0)/l_{ij}}$ $\theta_{ij} = l_{ij}/\nu$ $x_{ij}(t) = l_{ij}(t - t_0)/\theta_{ij}$ $x_{ij}(t_0) = 0$ (*Cars Enter*) $x_{ij}(t_0 + \theta_{ij}) = l_{ij}$ (*Cars Exit*)
$n_{ij}(t_0) < c_{ij} l_{ij}/\nu$ $n_{ij}(t_0)\nu/l_{ij} > C_j(t_0)$ $C_i = c_{ij}$ $\phi_j = C_j$	$t^* = Min(t_1, t_0 + \delta, t_2)$ where $\delta =$ $(c_{ij} l_{ij}/\nu - n_{ij}(t_0))/$ $(\phi_i - C_j)$ and $t_2 = t_0 + \theta_{ij}$	$n_{ij}(t) = n_{ij}(t_0) + (\phi_i^* - C_j)(t - t_0)$ $\theta_{ij} = n_{ij}(t_0)/C_j$ $x_{ij}(t) = l_{ij}(t - t_0)/\theta_{ij}$ $x_{ij}(t_0) = 0$ (*Cars Enter*) $x_{ij}(t_0 + \theta_{ij}) = l_{ij}$ (*Cars Exit*)
$n_{ij}(t_0) = c_{ij} l_{ij}/\nu$ $n_{ij}(t_0)\nu/l_{ij} > C_j(t_0)$ $C_i = C_j$ $\phi_i = C_j$ $\phi_j = C_j$	$t^* = Min(t_1, t_0 + \delta, t_2)$ where $\delta =$ $(c_{ij} l_{ij}/\nu - n_{ij}(t_0))/$ $(\phi_i - C_j)$ and $t_2 = t_0 + \theta_{ij}$	$n_{ij}(t) = n_{ij}(t_0) + (\phi_i^* - C_j)(t - t_0)$ $\theta_{ij} = n_{ij}(t_0)/C_j$ $x_{ij}(t) = l_{ij}(t - t_0)/\theta_{ij}$ $x_{ij}(t_0) = 0$ (*Cars Enter*) $x_{ij}(t_0 + \theta_{ij}) = l_{ij}$ (*Cars Exit*)

- If the lane is not full and the number of cars present inside the system do not exceed the traffic accommodation capacity and the output capacity of the lane, then all the cars in the input flow can be absorbed by the system. Under this condition, the input capacity of the lane is same as the traffic accommodation capacity c_{ij}. As there exists no congestion inside the lane, the cars move with free flow speed ν and reach the exit in minimum time equal to l_{ij}/ν assuming that $t_0 + l_{ij}/\nu \leq t^*$.

- If the lane is not full and an input flow greater than the output capacity of the lane arrives at the entrance of the lane, then all the incoming flow can be absorbed by the system, but only flow equal to $C_j(t)$ can exit from the system. As a result some cars have to wait inside the lane. The cars present inside the lane take a time equal to $n_{ij}(t)/C_j$ to evacuate the system.

- If the lane is full at time t_0, then we can say that the flow that exits from the system can only enter into the system, provided the output capacity of the lane does not exceeds the traffic accommodation capacity of the lane. Under this situation, if an input flow greater than the output capacity of the lane arrives during time $t \in [t_0, t_1]$, then some cars have to wait at the entrance of the lane. If this situation continues till time $t > t_1$, we observe that the input capacity of the lane becomes constant and is equal to the output capacity of the lane. The cars present inside the lane take a time equal to θ_{ij} to exit from the system where:

$$\int_{t_0}^{\theta_{ij}} C_j(t)dt = n_{ij}(t_0)$$

assuming that $\theta_{ij} \leq t_1$.

5 Numerical Illustration

Let us consider the system depicted in Figure 1. The length of the lane is $l_{ij} = 10$, the traffic accommodation capacity is $c_{ij} = 20$ and the speed of the cars $\nu = 1$. The number of cars present inside the system at time t_0 are $n_{ij}(t_0) = 150$. To trace the position of the cars inside the system, a red car is introduced at the entrance of the lane under varying input flows and output capacities(Table 4). The piecewise constant input flow ϕ_i arriving at the origin node of the lane is presented in Table 2. The output capacity C_j of the single lane system is given in Table 3.

Table 2: Input Flow (ϕ_i) vs time

ϕ_i	*Time Interval*
12	0-20
15	20-50
20	50-70
10	70-100

Table 3: Output Capacity(C_j) vs time

C_j	*Time Interval*
15	0-10
10	10-30
20	30-60
15	60-100

5.1 Results

The results for the numerical example discussed above are presented in Table 4. The evolution of system state $n_{ij}(t)$ and travel time θ_{ij} for the single lane under the conditions provided above is computed using the formulations presented in section 4. To evaluate the system state and the state parameters, we look for two main conditions in the single lane system. The two conditions are :

- $\phi_i > Min(C_j, c_{ij})$
  In this case, the flow that enters inside the lane is $\phi_i^* = Min(C_j, c_{ij})$.
- $\phi_i \leq Min(C_j, c_{ij})$
  In this case, the flow that enters inside the lane is $\phi_i^* = \phi_i$.

If $\phi_i > Min(C_j, c_{ij})$, then we compute the state of the system using equation (14), otherwise the system state is derived from equation (15). The travel time for the cars is computed using equation(16).

5.1.1 System Analysis

Let us now analyze the results depicted in Table 4 for each time $t \in [t_0, t_1]$. It can be seen that:

- For time $t \in [0, 10], [30, 60], [77, 100]$, the input flow does not exceeds the accommodation capacity and the output capacity of the lane and therefore the red car entering the system at time 0, 50 and 77 reaches the exit in minimum time equal to 10.
- For time $t \in [10, 20], [20, 29.79], [60, 63.6788], [70, 76.9]$, the input flow does not exceeds the accommodation capacity but is greater than the output capacity of the lane. As a result, the flow exits at a rate equal to the output capacity of the lane and the time taken by the cars to travel the system is greater than the minimum travel time for the lane. The travel time for the red car is greater than 10 when it enters the system at time 10 and 60 respectively.

- For time $t \in [29.79, 30], [63.6788, 70]$, the lane is full and therefore an input flow equal to the output capacity of the lane can only be accepted by the system. If the flow arriving at the entrance of the lane continues to exceed C_j, then the input capacity of the lane becomes equal to the output capacity of the lane and the cars take maximum time to travel on the lane. Note that at this point of time the input flow entering the system is equal to the output flow. The red car takes maximum time = 20 to traverse the lane when it enters the system at time 29.79.

5.1.2 Graphical Analysis

Let us consider Figure 4 representing the variation of system parameters $n_{ij}(t)$ with time t. It can be seen that the number of cars present inside the lane vary in the range (100-200). From Figure 4 , we observe that:

- During time $t \in [0, 10], [30, 40], [70, 83.34]$, the number of cars present inside the system decrease with time t. For time $t \in [50, 60]$, the number of cars present inside the system increases with time t.
- During time $t \in [10, 20], [20, 29.79], [60, 63.6788]$, the number of cars present inside the system increases linearly with time t.
- During time $t \in [29.79, 30], [63.6788, 70]$, the number of cars present inside the system are maximum and equal to the traffic accomodation capacity of the lane. The numbers of cars inside the lane remain constant during these time interval.
- During time $t \in [41, 50], [83.35, 100]$, the number of cars present inside the system remain constant and the output flow is equal to the input flow since neither the traffic accomodation capacity nor the output capacity constraints are exceeded.

Let us consider Figure 5 representing the variation of system parameters $\phi_i(t)$ and $\phi_j(t)$ with time t. It can be seen that the input flow lies in the range of (10 - 20) and the output flow lies in the range of (10-18.16). Let us now analyse Figure 5 for various system parameters. It can be seen that:

- For time $t \in [0, 10], [30, 40], [77, 83.34]$, the input flow does not exceeds either the accomodation capacity or the output capacity of the lane. Under this situation, the output flow is decided by the minimum of the number of cars present inside the system and the output capacity of the lane. During these time intervals, it can be seen that the output flow is less than the output capacity of the lane.
- For time $t \in [10, 20], [20, 29.79], [60, 63.6788], [70, 76.9]$, the input flow does not exceeds the accomodation capacity but is greater than the output capacity of the lane. As a result, the output flow exits at a rate equal to the output capacity of the lane. During these time intervals, it can be seen that the output flow is equal to the output capacity of the lane.

Table 4: Travel Time and System State

System State at time t_0	*Time t^*, Travel Time, Arrival Time and Departure Time of Red Car*	*Evolution of $n_{ij}(t)$*
For $t \in [0,10]$ $\phi_i^* = 12, C_j = 15$ $C_i = 20, \phi_j = 13.1$	$t^* = 10, \theta_{ij} = 10$ *Red Car enters at t = 0* *Red Car exits at t = 10*	$n_{ij}(t) = 120 + 30e^{-t/10}$ $n_{ij}(0) = 150$ $n_{ij}(10) = 131.06$
For $t \in [10,20]$ $\phi_i^* = 12, C_j = 10$ $C_i = 20, \phi_j = 10$	$t^* = 20, \theta_{ij} = 15.1036$ *Red Car enters at t = 10* $x_{ij}(20) = 6.6209$	$n_{ij}(t) = 2t + 100 + 30e^{-1}$ $n_{ij}(10) = 131.06$ $n_{ij}(20) = 151.036$
For $t \in [20,29.79]$ $\phi_i^* = 15, C_j = 10$ $C_i = 20, \phi_j = 10$	$t^* = 25.1036$ $x_{ij}(20) = 6.6209$ *Red Car exits at t = 25.1036*	$n_{ij}(t) = 5t + 40 + 30e^{-1}$ $n_{ij}(20) = 151.036$ $n_{ij}(29.79) = 200$
For $t \in [29.79,30]$ $\phi_i^* = 10, C_j = 10$ $C_i = 10, \phi_j = 10$	$t^* = 30, \theta_{ij} = 20$ *Red Car enters at t = 29.79* $x_{ij}(30) = 0.105$	$n_{ij}(t) = 200$ $n_{ij}(29.79) = 200$ $n_{ij}(30) = 200$
For $t \in [30,40]$ $\phi_i^* = 15, C_j = 20$ $C_i = 20, \phi_j = 16.84$	$t^* = 40$ $x_{ij}(30) = 0.105$ $x_{ij}(40) = 5.105$	$n_{ij}(t) = 150 + 50e^{-t/10+3}$ $n_{ij}(30) = 200$ $n_{ij}(40) = 168.394$
For $t \in [41,50]$ $\phi_i = 15, C_j = 20$ $C_i = 20, \phi_j = 15$	$t^* = 49.79$ $x_{ij}(41) = 5.605$ *Red Car exits at t = 49.79*	$n_{ij}(t) = 150$ $n_{ij}(41) = 150$ $n_{ij}(50) = 150$
For $t \in [50,60]$ $\phi_i^* = 20, C_j = 20$ $C_i = 20, \phi_j = 18.16$	$t^* = 60, \theta_{ij} = 10$ *Red Car enters at t = 50* *Red Car exits at t = 60*	$n_{ij}(t) = 200 - 50e^{-t/10+5}$ $n_{ij}(50) = 150$ $n_{ij}(60) = 181.606$
For $t \in [60,63.6788]$ $\phi_i^* = 20, C_j = 15$ $C_i = 20, \phi_j = 15$	$t^* = 63.6788, \theta_{ij} = 13.34$ *Red Car enters at t = 60* $x_{ij}(63.6788) = 2.7577$	$n_{ij}(t) = 5t - (100 + 50e^{-1})$ $n_{ij}(60) = 181.606$ $n_{ij}(63.6788) = 200$
For $t \in [63.6788,70]$ $\phi_i^* = 15, C_j = 15$ $C_i = 15, \phi_j = 15$	$t^* = 70$ $x_{ij}(63.6788) = 2.7577$ $x_{ij}(70) = 7.49625$	$n_{ij}(t) = 200$ $n_{ij}(63.6788) = 200$ $n_{ij}(70) = 200$
For $t \in [70,76.9]$ $\phi_i^* = 10, C_j = 15$ $C_i = 20, \phi_j = 15$	$t^* = 73.34$ $x_{ij}(70) = 7.49625$ *Red Car exits at t = 73.34*	$n_{ij}(t) = 100(1 + e^{-t/10+7})$ $n_{ij}(70) = 200$ $n_{ij}(76.9) = 150.158$
For $t \in [77,83.34]$ $\phi_i^* = 10, C_j = 15$ $C_i = 20, \phi_j = 12.63$	$t^* = 83.34, \theta_{ij} = 10$ *Red Car enters at t = 77* $x_{ij}(83.34) = 6.34$	$n_{ij}(t) = 100(1 + e^{-t/10+7})$ $n_{ij}(77) = 149.659$ $n_{ij}(83.34) = 126.342$
For $t \in [83.35,100]$ $\phi_i^* = 10, C_j = 15$ $C_i = 20, \phi_j = 10$	$t^* = 87$ $x_{ij}(83.35) = 6.35$ *Red Car exits at t = 87*	$n_{ij}(t) = 100$ $n_{ij}(83.35) = 100$ $n_{ij}(100) = 100$

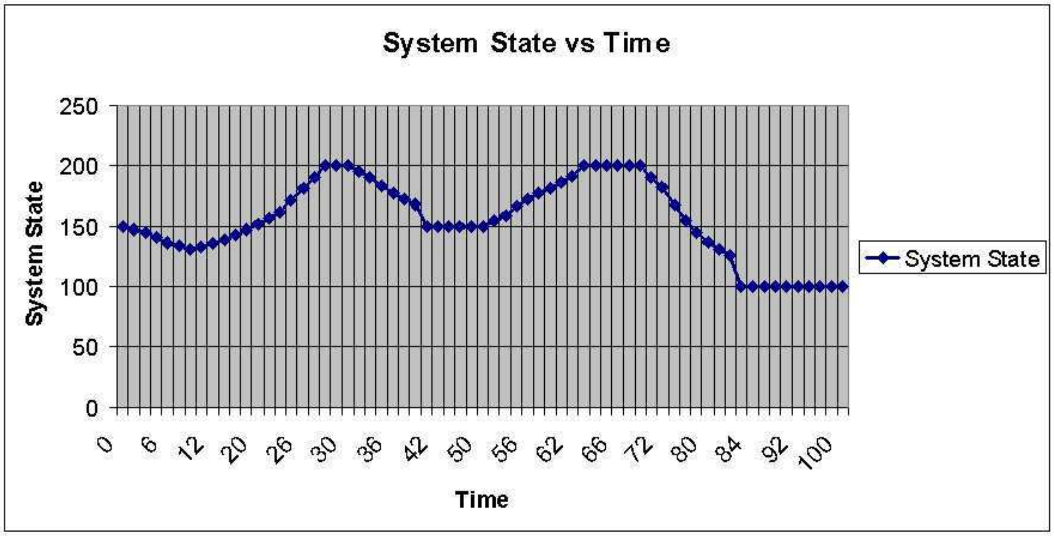

Figure 4: System State vs Time

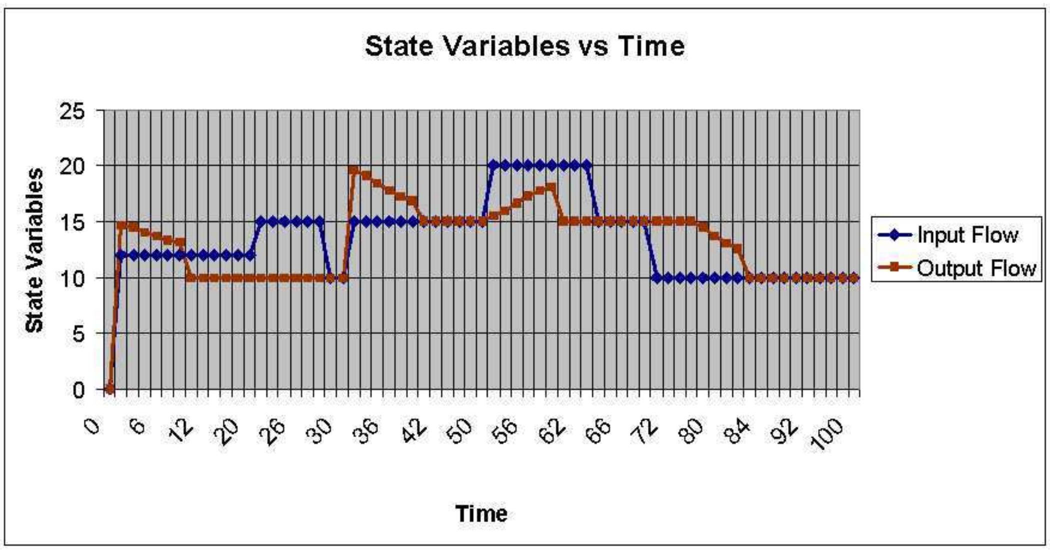

Figure 5: State Variables vs Time

- For time $t \in [29.79, 30], [63.6788, 70]$, the lane is full to its maximum capacity with $n_{ij}(t) = 200$. During these time intervals, the input flow and output flow are equal and are same as the output capacity of the lane.

- For time $t \in [41, 50], [83.35, 100]$, the lane is not full and neither the traffic accomodation capacity nor the output capacity constraints are exceeded. As a result, the flow exiting the system is same as the flow entering the system.

6 Conclusion

In this paper, we proposed a macroscopic approach for the computation of travel time on a single lane system under varying input flows. The present study is restricted to the study of a single lane defined by its capacity, its length, the flow at the input of the lane and the output capacity. We analyzed the behavior of the single lane system under these parameters to compute the time required to join the input to the output. From the system analysis, we found that the times required to join the origin to the extremity of each lane depends on the number of cars $n_{ij}(t)$ and the output capacity $C_j(t)$ of the lane at time t, the length l_{ij} of the lane and the free flow speed ν of the cars present inside the lane. The exact formulations for the travel times can be approached in Section 4.

In the study developed in this paper, we started from a single lane and performed a stepwise evolution of flows inside the system. The objective was twofold:

- define the travel time for the single lane system with regard to input flows and to show how the traveling time of a car is affected by the history of the system.
- define the main relations that characterize the dynamics of the single lane system.

The next step of our work concerns the application of single lane system to a large network.

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