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Effective bandwidths in a priority queueing system with leaky bucket regulated traffic sources

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Abstract: A new admission control algorithm is proposed in this paper for traffic streams controlled by a dual leaky bucket enforcing on the one hand, the peak rate with a stringent tolerance, and on the other hand, the mean rate with a given a bucket size. The proposed algorithm relies on a model, which stochastically dominates the theoretical system composed of deterministic worst case traffic sources multiplexed in a finite capacity queue. Assuming that there are N source types characterized by the traffic parameter sets $(\sigma_i, \rho_i, \pi_i)$, $i = 1, \dots, N$, this dominating model is the $M^{[X]}/M/1/K$ system, where input batch Poisson process is the superposition of N batch Poisson processes; the ith batch Poisson process is composed of batches of size σ_i arriving at rate ρ_i/σ_i. We derive effective bandwidths for both the $M^{[X]}/M/1/K$ system with heterogeneous traffic and the $M^{[X]}/M/1/K$ system including a head of line priority level. In the former case, the effective bandwidth obtained by using the $M^{[X]}/M/1/K$ system is compared with other effective bandwidths, which have been proposed earlier in the literature for ATM networks. It turns out that the proposed effective bandwidth definition leads to similar results but has the advantage of yielding admissibility regions delimited by hyper-planes. In the latter case, for the numerical values considered for the bucket sizes and the buffer capacities, the effective bandwidths, which rely on exact upper bounds, are very close to those obtained by using the so-called reduced service rate approximation. This rigorously justifies a posteriori that approximation.

Key-words: Effective bandwidths, priority queueing systems, statistical multiplexing, asymptotic estimates.

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Bande passsante équivalente d'un système avec priorité

Résumé : Un nouveau contrôle d'admission est proposé pour des trafics contrôlé par un leaky bucket en présence de flux prioritaires. L'algorithme repose sur une système majorant stochastiquement un système multiplexant des sources dans un file d'attente à capacité finie. Ce système majorant présente l'intérêt d'avoir un zone d'admission qui est définie par un hyperplan, donnant directement la notion de bande équivalente pour les sources. L'algorithme de contrôle d'admission permet d'avoir une qualité de service *garantie* pour les systèmes avec priorité, à la différence de la plupart des autres algorithmes de contrôle d'admission dans ce domaine. Une comparaison avec un algorithme heuristique de réduction de la capacité (RSR) est effectuée.

Mots-clés : Bande passante effective. Système avec priorité. Contrôle d'admission.

1 Introduction

The need for offering quality of service in broadband integrated packet networks has given rise over the past ten years to a huge amount of research in networking. One popular service model to offer quality of service consists of negotiating traffic parameters as well as quality of service objectives (e.g., packet loss probability and/or maximum transfer delays). In this model, prior to the transmission of information, a traffic contract is negotiated between the user and the network. During the transmission phase, quality of service is guaranteed as long as traffic offered by a user is conforming to the traffic contract.

In order to prevent any quality of service degradation due to traffic contract violations, the traffic parameters are controlled at network access in the so-called policing function; traffic may possibly be shaped so as to conform to the prescribed parameters. In practice, this approach has several shortcomings, in particular due to the difficulty experienced by users to assess their traffic parameters. Nevertheless, the above service model has prevailed in the standardization of ATM as well as Frame Relay networks, and more recently in the evolution of the Internet with the IntServ model and MPLS, and is in use in current service offers by network operators.

Besides traffic parameter enforcement, the network has to perform admission control. Indeed, to offer the desired quality of service, the network has to provision sufficient resources to accommodate the information flow offered by the user. In fact, admission control is a critical function for a network operator in the sense that this function has to limit the load of the network in order to guarantee quality of service while accepting sufficient traffic so as to optimize the utilization of network links. In addition, admission control is distributed over several network elements (switches or routers) with limited computation capacities. Thus, in order to offer very short response times so as to make the network as far as possible transparent, the admission control algorithm must be simple and require limited computational efforts. *Efficiency* and *simplicity* are hence two basic requirements in the design of an admission control algorithm.

The traffic parameter set, which is the most commonly used in practice, is composed of the peak rate π, the mean rate ρ and the bucket size σ. The use of these three parameters has been motivated by the simplicity of the associated control mechanism, referred to as dual leaky bucket and composed of one leaky bucket with a stringent tolerance for the peak rate π and another leaky bucket with bucket size σ for the mean rate ρ. The traffic parameters (π, σ, ρ) are the basic parameters used in network calculus (see [8, 9, 25]). In the framework of the VBR service for ATM networks, which is the main focus of the present paper, π is the peak cell rate and ρ is the sustainable cell rate; $\lfloor 1 + (\sigma - 1)\pi/(\pi - \rho) \rfloor$ is the maximum burst size.

The problem of admission control has been studied in great details within the framework of the VBR service in ATM networks. This problem precisely consists of determining the number of connections, described by the peak cell rate, the sustainable cell rate and the maximum burst size, which can be accepted in a buffer of size K so as the cell rejection probability is less than or equal to some prescribed value ε (typically, ranging form 10^{-9} to

10^{-7}). The exact model corresponding to the superposition of periodic On/Off sources in a buffer of size K has been analysed by Cidon *et al* [7] and Garcia *et al* [17].

With regard to admission control, the main problem raised by the exact resolution of the above model is in that it requires a huge computational effort. To overcome this problem, different techniques have been proposed to associate with a packet flow an equivalent bandwidth. This quantity represents the total amount of resources (buffer *and* bandwidth) needed for accommodating a packet flow characterized by the (σ, ρ, π) parameters. Since the fundamental papers by Guerin *et al* [20] and Kelly [21], different refinements have been introduced in the concept of effective bandwidth. In particular, effective bandwidths naturally appear as coefficients in the exponential decay term of the survivor probability distribution function of the occupation of a queue fed with Markovian sources (see [10, 14]).

All the theoretical investigations on effective bandwidths have led to different definitions of the effective bandwidth for a packet flow. Among all of them, one may cite the definition proposed by Elwalid, Mitra and Wentworth [13], which relies on the reduction of a buffered system to a loss model (i.e., with no buffer) used to estimate the effective bandwidth of a packet flow via refined Chernoff bounds. In that work, special effort has been devoted to bounding by an optimal hyper-plane the region of admissibility, which is in general a concave domain. This is intended to greatly simplify the admission decision; see [5] for the problem of linearizing admissibility regions. Another effective bandwidth definition has been introduced by Gibbens and Hunt [18], which is based on a generalization of the celebrated Anick-Mitra-Sondhi model [1]. See also Mandjes [24] and Duffield [6] for large deviations techniques, which can be used to define equivalent bandwidths.

The concept of effective bandwidth has been merely investigated in the framework of FIFO queues and less attention has been paid to queues with different priority levels. However, most ATM switches or even routers supporting DiffServ, which are available today on the market, implement priority schemes to segregate flows with real time constraints against flows with semantic integrity requirements. In this case, a still open problem is to develop an admission algorithm, which takes into account a time priority level.

The problem of admission control in the presence of priorities has been investigated, for instance, by Berger and Whitt [3, 4]. The commonly followed approach consists of accounting for the higher priority level by reducing the service rate seen by the lower priority level. This is the so-called reduced service rate (RSR) approximation. This approach has the merit of simplicity, but it has been observed in earlier studies (see [11] for instance), that this approximation may severely underestimate the loss probability in the lower priority queue and then, too many connections may be accepted, at the risk of not meeting the quality objectives. Besides the RSR approximation, large deviation techniques relying on refined Chernoff bounds have also been used to develop admission control algorithms; see for instance Elwalid and Mitra [15] and more recently Reisslein *et al* [26] et Shakkottai and Srikant [27].

In this paper, we investigate the concept of effective bandwidth for a queue with two priority levels. In contrary to earlier studies on the same topic, we do not search for refined bounds for the exact model consisting of the superposition of deterministic On/Off sources.

Instead, we derive an effective bandwidth for a traffic source described by its mean rate and bucket size. In particular, we relax the constraint on the peak rate, which is assumed to be infinite. It turns out that this approach, which has the merit of simplicity, leads to results, which are competitive when compared with those obtained via other admission control algorithms.

For this purpose, we introduce a system, which stochastically dominates the system composed of worst case traffic sources multiplexed in a finite capacity queue. This system is an $M^{[X]}/M/1/K$ queue, where the batch size depends on the bucket sizes of the individual sources, the Poisson input rate is the sum of the mean rates divided by the corresponding bucket sizes, and the mean service time is the service time of one packet. The crucial advantage of this system is in that the admissibility region is delimited by a hyper-plane. Moreover, a head of line priority (for real time traffic) can easily be introduced and effective bandwidths can still be derived with an admissibility region still delimited by a hyper-plane. Surprisingly, the effective bandwidths derived via exact upper bounds justify a posteriori the RSR approximation for the range of loss probabilities considered, even though the exact tail behaviors in the real system and the RSR system are different, which is in line with the observation made in [11].

The organization of this paper is as follows: The motivation for introducing the $M^{[X]}/M/1/K$ system is presented in Section 2. The analysis of this system is carried out in Section 3. The extension to a system with heterogeneous traffic is performed in Section 4. The analysis of the system with a priority level is carried out in Section 5. Comparisons with other admission control algorithms are performed in Section 6. Further considerations on admission control algorithms are presented as concluding remarks in Section 7. Some basic elements concerning the $M^{[X]}/M/1/K$ queue are recalled in Appendix and are based on the paper by Tijms and Van Ommeren [28]. Throughout this paper, we pay special attention to the case of VBR traffic in ATM networks, which is the principal target of all the calculations developed in this paper.

2 Problem formulation

We consider in this paper statistical multiplexing of traffic sources, which are regulated by a dual leaky bucket. One leaky bucket controls the peak rate π of a source with a stringent tolerance. The second leaky bucket enforces the mean rate ρ of a source with a bucket size σ.

When multiplexing such (σ, ρ, π) regulated sources in a FIFO queue with finite capacity, admission control algorithms are generally developed by taking into account the so-called worst-case traffic pattern associated with a source. Such a pattern is composed of bursts of size $b = \lfloor 1 + (\sigma - 1)\pi/(\pi - \rho)$ at the peak rate, the distance between the starting times of two consecutive bursts is b/ρ. The duration of a burst is b/π and the silence period between two bursts is $b(\pi - \rho)/(\pi\rho)$.

However, it has recently been shown by Kesisidis and Konstantopolous [22] that in a fluid flow context, the periodic On/Off process is not the worst case when considering the loss probability. Such a result is in line with the counterexamples exhibited by Doshi [12], who showed that in buffered system, the worst case is not the periodic On/Off process. In the case of a single input process, the traffic pattern, which maximizes the loss probability, is in fact periodic and composed of bursts at the peak rate, followed by activity periods at the mean rate ρ and followed in turn by silent periods so as to respect the mean rate constraint. It is nevertheless easy to check that the worst case probability given in [22] is less than the loss probability obtained when batches of size σ arrive according to a Poisson process with intensity ρ/σ.

This result is true for a single input process, but it is reasonable to conjecture that it holds when considering several input processes. A rigorous proof of this result will be addressed in a forthcoming paper and relies on stochastic ordering. This leads us to replace each individual (σ, ρ, π)-regulated flow with a batch Poisson process; the batch size is equal to σ and the intensity of the Poisson process is ρ/σ. Now, when considering statistical multiplexing of different traffic sources, we are led to analyze a FIFO queue fed with a batch Poisson process, which corresponds to the superposition of the individual batch Poisson processes. Finally, taking into account the discrete nature of packets, we assume that packet service times are exponentially distributed. This is clearly not the case for ATM. However, since the exponential distribution is more variant than the deterministic distribution, the $M^{[X]}/M/1/K$ queue stochastically dominates the $M^{[X]}/D/1/K$ queue. Once again, this fact can rigorously be proved via stochastic ordering arguments [2]. Hence, by considering the $M^{[X]}/M/1/K$ queue for admission control, we derive conservative equivalent bandwidths.

Note that the $M^{[X]}/M/1/K$ system has already been considered by Gravey *et al* [19] to determine the maximal burst size in order to allow a bandwidth allocation on the basis of the mean rate only. In this paper, we do not search for this maximal value but we allow for heterogeneous burst sizes. The counterpart is that we have an over-allocation factor on the sustainable cell rate.

The real motivation for introducing the $M^{[X]}/M/1/K$ system is in that we can easily account for a priority level and carry out exact computations. When considering a priority level, we assume that the higher priority packets have preemptive resume priority over the lower priority packets. This is clearly a distortion of the real system, since the service of a packet cannot be interrupted. Nevertheless, if the size of packets is small (and this is precisely the case for ATM cells), the difference between the preemptive resume priority and the real system will not be significant. This is why we neglect this border effect in the following.

In the case of ATM VBR traffic, let us mention that the mean rate m is equal to the sustainable cell rate (SCR) of the connection, the peak rate p is equal to the peak cell rate (PCR) and b is equal to the maximum burst size (MBS).

3 Upper bounds for the loss probability in an $M^{[X]}/M/1/K$ queue

The $M^{[X]}/M/1/K$ queue has been analyzed in details by Tijms and Van Ommeren in [28] and some basic results are recalled in the Appendix. Our goal in this section is to derive simple upper bounds for the loss probability in that queueing system. These bounds will then be used to develop a new admission control algorithm in the following sections.

Throughout this section, we consider a single-server queue with capacity $K < \infty$ fed with batches of packets, which arrive according to a Poisson process with rate λ. The batch size has a general probability distribution $\{\beta_j,\, j = 1, 2, \ldots\}$ with generating function $\beta(z) = \sum_{j=1}^{\infty} \beta_j z^j$ and finite mean $\bar{\beta}$.

The system can contain only K packets, including the one in service. A batch, whose size exceeds the remaining capacity in the buffer is partially lost due to overflow. The channel can handle only one packet at a time and the service times of packets are independent random variables with a common probability distribution, which we here assume exponential, with finite mean $\bar{S}$. Letting $\rho = \bar{\beta}\bar{S}$, it is assumed that the offered load ρ is less than one.

The loss probability in the system is defined as

$$\pi_{\text{loss}}(K) = \text{the long-run fraction of packets that are lost.}$$

Let $\beta^*(z) = [1 - \beta(z)]/[(1 - z)\bar{\beta}]$ be the generating function of the distribution $\{\beta_j^* = \sum_{i>j} \beta_i/\bar{\beta},\, j = 0, 1, \ldots\}$, and let R denote the convergence radius of the power series $\beta(z)$, which is also the convergence radius of the series $\beta^*(z)$. (Note that the point $z = 1$ is a removable singularity for the generating function $\beta^*(z)$.)

In the case $\rho R \beta^*(R) > 1$, it is shown in the Appendix that there exists a unique real number $z_0 \in (1, R)$ such that

$$\rho z_0 \beta^*(z_0) = 1. \tag{1}$$

Moreover, z_0 is the unique solution with the smallest module to the equation $\rho z \beta^*(z) = 1$.

By using the results recalled in the Appendix on the $M^{[X]}/M/1/K$, due to Tijms and Van Ommeren, we derive upper bounds for the loss probability $\pi_{\text{loss}}(K)$ in the cases $\rho R \beta^*(R) > 1$ and $\rho R \beta^*(R) < 1$. At first glance, this latter condition might seem groundless for practical applications, but it will appear crucial when a higher priority queue will be added to our model.

Proposition 1 *The loss probability is such that*

$$\pi_{loss}(K) \leq \begin{cases} \beta^*(z_0) z_0^{-K} & \text{if } \rho R \beta^*(R) > 1, \\ \beta^*(R) R^{-K} & \text{if } \rho R \beta^*(R) \leq 1, \end{cases} \tag{2}$$

where z_0 is defined by equation (1).

Proof. Let $\{X_n\}$ denote the queue length distribution at batch arrival instants. X_n is defined by the recursion: $X_n = \max\{0, X_{n-1} + B_n - A_n\}$, where B_n are i.i.d. random variables with generating function $\beta(z)$ and A_n is the number of points of a Poisson process with intensity $1/\bar{S}$ in a time period exponentially distributed with mean $1/\lambda$.

By using Kingman's upper bound [23], we have for all $k \geq 0$, $\mathbb{P}\{X \geq k\} \leq Y^{-k}$ where $Y = z_0$ if $\rho R \beta^*(R) > 1$ and $Y = R$ if $\rho R \beta^*(R) \leq 1$. Since the queue length Q seen by an arriving packet in the stationary regime is such that $Q = X + B'$, where B' is a random variable with generating function $\beta^*(z)$, independent of X, we have

$$\mathbb{P}\{Q \geq K\} = \mathbb{P}\{X + B' \geq K\} = \mathbb{P}\{B' \geq K\} + \sum_{k=0}^{K-1} \mathbb{P}\{B' = k\}\mathbb{P}\{X \geq K - k\} \leq Y^{-K}\beta^*(Y).$$

Inequality (2) is obtained by using equation (23). This completes the proof. ■

4 The $M^{[X]}/M/1/K$ queue with heteregeneous traffic

4.1 The equivalent single-source batch model

Let us first modify the $M^{[X]}/M/1/K$ model studied in the previous section in order to introduce different source types. Assume that there are N source types and $x_i \in \mathbb{N}$ sources of type i, $1 \leq i \leq N$. Each source of type i generates batches of packets according to a Poisson process with rate ${}_i$; the batch size has a general probability distribution with generating function $\beta_i(z)$ and finite mean $\bar{\beta}_i$.

In the following, we assume that the batch sizes of the different source types are upper bounded by a given constant κK, where κ is "sufficiently" small (say, $\kappa \leq 10\%$). Thus, the batch size can only take values in $\{1, 2, \ldots, \kappa K\}$. This assumption is not really constraining because K take large values. Typically, in the case of ATM, $K = 3000$ cells and the maximum batch size can range up to 300 cells. Moreover, the above assumption entails that the radius of convergence is $R = \infty$.

Let $x = (x_i)_{1 \leq i \leq N}$. Obviously, our new model is equivalent to the single-source model of Section 3, where batches arrive at rate

$$= (x) = \sum_{i=1}^{N} x_{i\,i},$$

have the generating function

$$\beta(z, x) = \sum_{i=1}^{N} \frac{x_{i\,i}}{(x)} \beta_i(z),$$

and mean

$$\bar{\beta}(x) = \sum_{i=1}^{N} \frac{x_{i\,i}}{(x)} \bar{\beta}_i.$$

Let $\rho_i = {}_i\bar{\beta}_i\bar{S}$, $1 \leq i \leq N$, and

$$\rho(x) = (x)\beta(x)\bar{S} = \bar{S}\sum_{i=1}^{N} x_i\rho_i,$$

We assume that the offered load $\rho(x)$ is less than one. Finally, let $z_0(x)$ denote the solution with the smallest module to the equation $\rho(x)z\beta^*(z,x) = 1$, where $\beta^*(z,x) = [1-\beta(z,x)]/[(1-z)\bar{\beta}(x)]$.

Denoting by $\pi_{\text{loss}}(K,x)$ the overflow probability, our goal is to determine the admissibility region

$$\mathcal{A}_\varepsilon = \{x \in \mathbb{N}^N : \pi_{\text{loss}}(K,x) \leq \varepsilon\}$$

for some given $\varepsilon \ll 1$ (say, $\varepsilon = 10^{-7}$). For this purpose, we use the upper bounds derived in the previous sections in order to characterize the admissibility region $\mathcal{A}_\varepsilon$ and to define equivalent bandwidths.

4.2 Loss probability and effective bandwidths

In a first step, we prove the following theorem, which will allow us to define the effective bandwidth for traffic sources with a given type.

Theorem 1 *Under the condition that the batch size is less than κK, $\kappa \in (0,1)$, it is granted that the loss probability $\pi_{loss}(K,x) \leq \varepsilon$ provided that*

$$\sum_{i=1}^{N} x_i a_i(\varepsilon) \leq 1,$$

where the parameters $a_i(\varepsilon)$ are given by:

$$a_i(\varepsilon) = \rho_i(1/\varepsilon)^{1/K'}\beta_i^*((1/\varepsilon)^{1/K'}), \quad 1 \leq i \leq N. \tag{3}$$

where $K' = (1-\kappa)K$.

Proof. According to Proposition 1, it is guaranteed that the loss probability is such that $\pi_{\text{loss}}(K,x) \leq \varepsilon$ provided that

$$\beta^*(z_0(x),x)z_0(x)^{-K} \leq \varepsilon.$$

Since, for fixed x,

$$\beta^*(z,x) = \frac{(1-\beta(z,x))}{(1-z)\bar{\beta}(z,x)} = \frac{(\beta^*)'(z_1,x)}{\bar{\beta}(z,x)}$$

for some $z_1 \in (1, z_0(x))$, we have $\beta^*(z,x) \leq z_0(x)^{\kappa K}$. Hence, $\pi_{\text{loss}}(K,x) \leq \varepsilon$ if $z_0(x) \leq (1/\varepsilon)^{1/K'}$, where $K' = (1-\kappa)K$. By definition of $z_0(x)$, this is equivalent to the condition

$$\rho(x)(1/\varepsilon)^{1/K'}\beta^*((1/\varepsilon)^{1/K'},x) \leq 1.$$

Since

$$\rho(x) z\beta^*(z,x) = (x)\bar{\beta}(x)\bar{S}z\frac{1-\beta(z,x)}{(1-z)\bar{\beta}(x)} = \sum_{i=1}^{N} x_i\rho_i z\beta_i^*(z),$$

the result follows. This completes the proof. ■

Now, coming back to the problem of admission control, we consider traffic sources, which are (σ, ρ, π)-regulated. Sources of type i are characterized by a fixed set of traffic parameters $(\sigma_i, \rho_i, \pi_i)$; we assume that the bucket size σ_i is an integer (expressed in cells in the case of ATM or in bytes in the case of IP packets). As stated in Section 2, to develop an admission control algorithm, we assume that the packet flow offered a source of type i is a batch Poisson process with rate ρ_i/σ_i and a batch size equal to σ. In view of Theorem 1, we have the following result.

Theorem 2 *Consider statistical multiplexing of x_i traffic sources characterized by the traffic parameters $(\sigma_i, \rho_i, \pi_i)$, $1 \leq i \leq N$, in a buffer with capacity $K < \infty$ such that for all $i = 1, \dots, N$, $\sigma_i < \kappa K$, where $\kappa \in (0,1)$. The admissibility region $\mathcal{A}_\varepsilon$ is given by*

$$\mathcal{A}_\varepsilon = \left\{ (x_1, \dots, x_N) \in \mathbb{N}^{\mathbb{N}} : \sum_{i=1}^{N} x_i e_i(\varepsilon) \leq 1 \right\},$$

where the effective bandwidth $e_i(\varepsilon)$ is given by

$$e_i(\varepsilon) = \min\left\{ \pi_i, \rho_i (1/\varepsilon)^{1/K'} \beta_i^*((1/\varepsilon)^{1/K'}) \right\}. \tag{4}$$

with $K' = (1-\kappa)$.

The practical interest of the above result is in that the equivalent bandwidth of a source depends only upon its characteristics and the buffer capacity. There is thus no intricate relation between the traffic parameters of the different source types. Moreover, it is worth noting that the equivalent bandwidth is equal to the mean rate (expressed in cell/s or byte/s) of the traffic source multiplied by a factor $(1+\eta)$, where η is an over-allocation factor given by

$$\eta = (1/\varepsilon)^{1/K'} \frac{(1-(1/\varepsilon)^{\sigma/K'})}{(1-(1/\varepsilon)^{1/K'})\sigma} - 1. \tag{5}$$

Such properties are really in line with the definition of the equivalent bandwidth, which has to capture through a single parameter the behavior of a source when multiplexed with other sources in a queue. The over-allocation factor depends only on the ratio σ_i/K and not on the other characteristics of the traffic source. Finally, note that when the buffer capacity is such that $\sigma_i \ll \kappa K$, the equivalent bandwidth $e_i(\varepsilon)$ is closed to the mean rate of the source and then, the over-allocation factor is negligible

In actual situations, the equivalent bandwidth given by equation (4) can be used when VBR connections with a bucket size less than 300 cells are multiplexed in a queue with a capacity equal to 3000 cells while the target loss probability is $\varepsilon = 10^{-7}$. Such bucket size values are commonly encountered in VBR services currently offered by network operators. Moreover, 3000 cells are usual buffer capacities implemented for VBR traffic in ATM switches available today on the market. Figure 1 displays the over-allocation factor η as a function of the bucket size σ.

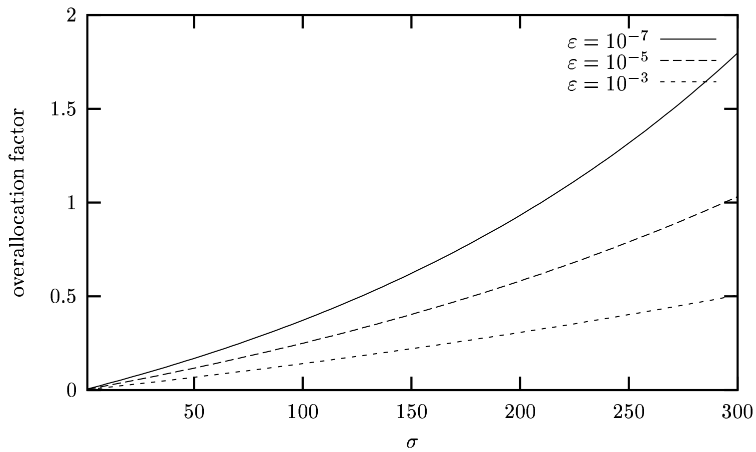

Figure 1: Over-allocation factor η as a function of the bucket size, when $\kappa = 10\%$, $K = 3000$ cells and different values of ε.

It turns out that the effective bandwidth may be as large as three times the mean rate when the batch size is equal to 300 and $\varepsilon = 10^{-7}$. While this might at first glance seem very penalizing, we shall see in Section 6 that the above effective bandwidth is nevertheless competitive when compared with other admission control algorithms. Moreover, in spite of the over-allocation factor, the key advantage of the above equivalent bandwidth definition is in that the equivalent bandwidth defines an admissibility region, which is delimited by a hyper-plane. This is a crucial advantage when comparing the above admission control algorithm with other algorithms defining concave admissibility regions, which can be delimited by hyper-planes only through intricate computations. Finally, the key feature of the proposed approach is in that a higher priority level can be easily introduced; this point is addressed in the next section.

5 Introduction of a higher priority $M/G/1$ queue

5.1 The equivalent single-source model

We now complete our multiple type source model by introducing a higher priority $M/G/1$ queue. We still assume that the batch size of the low priority packets cannot exceed a given constant κK, with $\kappa \in (0,1)$. Moreover, let $K' = (1-\kappa)K$.

Independently from the N source types considered in the previous section, high priority packets arrive at the single-server channel according to a Poisson process with rate ${}_0$. Their service times have a general probability distribution function $F_0(x)$ with Ł $F_0^*(s) = \int_0^\infty e^{-sx} dF_0(x)$, $\Re(s) \geq 0$, and finite mean $\bar{S}_0$. This new stream has preemptive resume priority over the original sources, and has a separate buffer of infinite capacity. Letting $\rho_0 = {}_0\bar{S}_0$, it is assumed that the total offered load $\rho_0 + \sum_{i=1}^N x_i \rho_i$ is less than one.

As far as the original sources are concerned, all they see is that service is interrupted during the busy periods of this $M/G/1$ queue. The time between two consecutive busy periods is exponentially distributed with parameter λ_0; the durations of the busy periods have a probability distribution function $G_0(x)$ with Ł $G_0^*(s) = \int_0^\infty e^{-sx} dG_0(x)$, $\Re(s) \geq 0$; it is well-known (see for example Feller [16], Chap. XIV.4) that G_0^* may be characterized as the smallest root of the equation

$$G_0^*(s) = F_0^*\left(s + {}_0(1 - G_0^*(s))\right); \tag{6}$$

the mean duration of a busy period is $\bar{B}_0 = -(G_0^*)'(0) = \bar{S}_0/(1-\rho_0)$.

Furthermore, it is easily seen that the overflow probability in the $M^{[X]}/M/1/K$ queue will remain unchanged if we contract each busy period in one point and gather all the packets arrived meanwhile from the original sources in one single batch. It is a matter of elementary calculation to obtain that the generating function of this batch size is

$$\beta_0(z,x) = G_0^*\left(\sum_{i=1}^N x_{i i}(1-\beta_i(z))\right), \tag{7}$$

and that its mean value is

$$\bar{\beta}_0(x) = \bar{B}_0 \sum_{i=1}^N x_{i i}\bar{\beta}_i.$$

This procedure is equivalent to replacing the higher priority $M/G/1$ queue with one additional source of type, say, 0, which generates batches of packets according to a Poisson process with rate ${}_0$; the batch size has a probability distribution with generating function $\beta_0(z,x)$ and finite mean $\bar{\beta}_0(x)$; the packet service times are independent random variables having the usual exponential distribution with finite mean $\bar{S}$.

Finally, our new model is equivalent to the single-source model of Section 3, where batches arrive at rate

$$(x) = {}_0 + \sum_{i=1}^N x_{i i},$$

have generating function

$$\beta(z,x) = \frac{{}_0}{(x)}\beta_0(z,x) + \sum_{i=1}^{N} \frac{x_{ii}}{(x)}\beta_i(z),$$

and mean value

$$\bar{\beta}(x) = \frac{{}_0}{(x)}\bar{\beta}_0(x) + \sum_{i=1}^{N} \frac{x_{ii}}{(x)}\bar{\beta}_i = \frac{\sum_{i=1}^{N} x_{ii}\bar{\beta}_i}{(x)(1-\rho_0)}.$$

Letting

$$\rho(x) = (x)\bar{\beta}(x)\bar{S} = \frac{\sum_{i=1}^{N} x_i\rho_i}{1-\rho_0},$$

the offered load $\rho(x)$ is less than one as expected.

5.2 Loss probability in the presence of the high priority queue

Still denoting by $\pi_{\text{loss}}(K,x)$ the overflow probability, we intend to use the upper bounds of Section 3, but we have to determine whether $\rho(x)R(x)\beta^*(R(x),x) > 1$ or not, where $R(x)$ is the radius of convergence of the series $\beta^*(z,x)$. First note that

$$\rho(x)z\beta^*(z,x) = (x)\bar{\beta}(x)\bar{S}z\frac{1-\beta(z,x)}{(1-z)\bar{\beta}(x)} = \rho_0(x)z\beta_0^*(z,x) + \sum_{i=1}^{N} x_i\rho_i z\beta_i^*(z),$$

where $\rho_0(x) = {}_0\bar{\beta}_0(x)\bar{S}$.

According to equation (7), we have

$$\rho_0(x)z\beta_0^*(z,x) = {}_0\bar{S}z\frac{1-G_0^*\left(\sum_{i=1}^{N} x_{ii}(1-\bar{\beta}_i(z))\right)}{1-z}.$$

The r.h.s. of the above equation is equal to

$${}_0\bar{B}_0\frac{1-G_0^*\left(\sum_{i=1}^{N} x_{ii}(1-\beta_i(z))\right)}{\left[\sum_{i=1}^{N} x_{ii}(1-\beta_i(z))\right]\bar{B}_0} \times \sum_{i=1}^{N} x_{ii}\bar{S}z\frac{1-\beta_i(z)}{1-z},$$

which yields

$$\rho_0(x)z\beta_0^*(z,x) = \frac{\rho_0}{1-\rho_0}\overline{G}_0^*\left(\sum_{i=1}^{N} x_{ii}(1-\beta_i(z))\right)\sum_{i=1}^{N} x_i\rho_i z\beta_i^*(z),$$

where $\overline{G}_0^*(s) = (1-G_0^*(s))/(s\bar{B}_0)$ is the Ł of the probability density function $(1-G_0(x))/\bar{B}_0$. We thus obtain

$$\rho(x)z\beta^*(z,x) = \frac{\bar{S}z}{z-1}f_0\left(\sum_{i=1}^{N} x_{ii}(\beta_i(z)-1)\right), \tag{8}$$

with

$$f_0(s) = s\left(1 + \frac{\rho_0}{1-\rho_0}\overline{G}_0^*(s)\right).$$

Note that the function f_0 only depends on the characteristics of the high priority $M/G/1$ queue.

At this stage we have to make an assumption on the tail behaviour of $F_0(x)$. Define

$$S = \sup\{s : F_0^*(-s) < \infty\} \quad \text{and} \quad T = \sup\{t : G_0^*(-s) < \infty\}. \tag{9}$$

In the following, we assume that $S > 0$ and, to avoid trivialities, that $x \neq 0$.

Since the power series $\beta_i(z)$, $1 \leq i \leq N$, have infinite convergence radii, equation (8) clearly shows that the convergence radius $R(x)$ of $\beta^*(z,x)$ (or equivalently of $\beta(z,x)$) is the unique real number $z > 1$ such that:

$$\sum_{i=1}^{N} x_{ii}(\beta_i(z) - 1) = T. \tag{10}$$

Using equation (8) again, we deduce that:

$$\rho(x)R(x)\beta^*(R(x),x) = \frac{\bar{S}R(x)}{R(x)-1}f_0(T). \tag{11}$$

Set

$$C = \frac{\bar{S}(1/\varepsilon)^{1/K'}}{(1/\varepsilon)^{1/K'}-1}f_0(T).$$

We have to consider the two cases $C > 1$ and $C < 1$ separately. We first consider the case $C < 1$, which entails that $\bar{S}f_0(T) < 1$. In the following, we formulate the results in an RSR approximation style. As mentioned in the Introduction, for the numerical applications considered in this paper, the equivalent bandwidths obtained by using the upper bounds derived in Section 3 are closed to those obtained via an RSR approximation.

Theorem 3 *Assume that* $C < 1$. $\pi_{loss}(K) \leq \varepsilon$ *if*

$$a_i(\varepsilon) = (1-\rho_0)\frac{i}{T}\left(\beta_i\left((1/\varepsilon)^{1/K}\right) - 1\right), \tag{12}$$

for $1 \leq i \leq N$, *and* $K' = (1-\kappa)K$.

Proof. Under the assumption $C < 1$, $R(x) > (1/\varepsilon)^{1/K'}$ and $\rho(x)R(x)\beta^*(R(x),x) < 1$. From Proposition 1, $\pi_{\text{loss}}(K) \leq \varepsilon$ if $R(x) \geq (1/\varepsilon)^{1/K}$. From the definition the convergence radius and equation (10), this condition is equivalent to

$$\sum_{i=1}^{N} x_{ii}\left(\beta_i\left((1/\varepsilon)^{1/K'}\right) - 1\right) \leq T.$$

This completes the proof. ■

Let us now consider the case when $C > 1$. Before stating the analogue of Theorem 3, let us prove a technical lemma.

Lemma 1 *Under the assumption $\bar{S} f_0(T) < 1$ and $(1/\varepsilon)^{1/K'} < 1/(1-\bar{S} f_0(T))$, the condition*

$$\sum_{i=1}^{N} x_{ii} \left[\beta_i \left(\frac{1}{1-\bar{S} f_0(T)} \right) - 1 \right] \leq T \tag{13}$$

implies the inequality

$$\sum_{i=1}^{N} x_{ii} \left[\beta_i \left((1/\varepsilon)^{1/K'} \right) - 1 \right] < f_0^{-1} \left((1-\varepsilon^{1/K'})/\bar{S} \right).$$

Proof. Assume that $\bar{S} f_0(T) < 1$. Under the assumption $(1/\varepsilon)^{1/K'} < 1/(1-\bar{S} f_0(T))$, we have for all $i = 1, \ldots, N$,

$$(1/\varepsilon)^{1/K'} \beta_i^*((1/\varepsilon)^{1/K'}) < \frac{1}{1-\bar{S} f_0(T)} \beta_i^* \left(\frac{1}{1-\bar{S} f_0(T)} \right),$$

since the function β_i^* is increasing. Hence,

$$\frac{\beta_i \left((1/\varepsilon)^{1/K'} \right) - 1}{\beta_i \left(1/(1-\bar{S} f_0(T)) \right) - 1} f_0(T) < \frac{(1/\varepsilon)^{1/K'} - 1}{\bar{S} (1/\varepsilon)^{1/K'}}$$

Since f_0 is convex on $[0, T]$, with $f_0(0) = 0$, we have

$$f_0 \left(\frac{\beta_i((1/\varepsilon)^{1/K'}) - 1}{\beta_i(1/(1-\bar{S} f_0(T))) - 1} T \right) \leq \frac{\beta_i((1/\varepsilon)^{1/K'}) - 1}{\beta_i(1/(1-\bar{S} f_0(T))) - 1} f_0(T) < \frac{(1/\varepsilon)^{1/K'} - 1}{\bar{S} (1/\varepsilon)^{1/K'}}.$$

Thus,

$$f_0 \left(\frac{\beta_i \left((1/\varepsilon)^{1/K'} \right) - 1}{\beta_i \left(1/(1-\bar{S} f_0(T)) \right) - 1} T \right) < \frac{(1/\varepsilon)^{1/K'} - 1}{\bar{S} (1/\varepsilon)^{1/K'}}$$

and then,

$${}_i \left[\beta_i \left((1/\varepsilon)^{1/K'} \right) - 1 \right] T < {}_i \left[\beta_i \left(\frac{1}{1-\bar{S} f_0(T)} \right) - 1 \right] f_0^{-1} \left((1-\varepsilon^{1/K'})/\bar{S} \right).$$

and the result follows. This completes the proof. ■

Theorem 4 *If $C > 1$ and $(1/\varepsilon)^{1/K'} < 1/(1-\bar{S}f_0(T))^+$, it is guaranteed that the loss probability $\pi_{loss}(K,x) \leq \varepsilon$ provided that*

$$\sum_{i=1}^{N} x_i a_i(\varepsilon) \leq 1-\rho_0,$$

where the parameters $a_i(\varepsilon)$ are given by:

$$a_i(\varepsilon) = \lambda_i(1-\rho_0)\frac{\beta_i\left((1/\varepsilon)^{1/K'}\right)-1}{f_0^{-1}\left((1-\varepsilon^{1/K'})/\bar{S}\right)} \tag{14}$$

for $1 \leq i \leq N$.

Proof. Assume that $C > 1$. If the convergence radius $R(x)$ is such that $R(x) < 1/(1-\bar{S}f_0(T))^+$, which is equivalent to the condition

$$\sum_{i=1}^{N} x_{ii}\left[\beta_i\left(\frac{1}{(1-\bar{S}f_0(T))^+}\right)-1\right] > T, \tag{15}$$

then $\rho(x)R(x)\beta^*(R(x),x) > 1$. From Proposition 1, the loss probability $\pi_{\text{loss}}(K) \leq \varepsilon$ if $z_0 \geq (1/\varepsilon)^{1/K'}$, which is equivalent to the condition

$$\rho(x)(1/\varepsilon)^{1/K'}\beta^*((1/\varepsilon)^{1/K'},x) \leq 1.$$

In view of equation (8), this condition is equivalent to

$$f_0\left(\sum_{i=1}^{N} x_{ii}(\beta_i((1/\varepsilon)^{1/K'})-1)\right) \leq \frac{(1/\varepsilon)^{1/K'}-1}{\bar{S}(1/\varepsilon)^{1/K'}} = \frac{1-\varepsilon^{1/K'}}{\bar{S}}.$$

The function f_0 is continuous and increasing on $[0,T]$, with $f(0)=0$ and $f_0(T) > (1-\varepsilon^{1/K})/\bar{S}$ (by assumption). Hence, the above condition is equivalent to

$$\sum_{i=1}^{N} x_{ii}(\beta_i((1/\varepsilon)^{1/K'})-1) \leq f_0^{-1}\left((1-\varepsilon^{1/K'})/\bar{S}\right). \tag{16}$$

Thus, we have proved that under the assumption $R(x) < 1/(1-\bar{S}f_0(T))^+$, $\pi_{\text{loss}}(K) \leq \varepsilon$ if inequality (16) is satisfied.

Assume now that $\bar{S}f_0(T) < 1$ and $R(x) > 1/(1-\bar{S}f_0(T))$. In that case, the convergence radius is such that $\rho(x)R(x)\beta^*(R(x),x) < 1$. Since $C > 1$, then $R(x) > 1/(1-\bar{S}f_0(T)) > (1/\varepsilon)^{1/K'}$ and hence, by Proposition 1, $\pi_{\text{loss}}(K) < \varepsilon$. From the definition of the convergence radius and equation (10), the condition $R(x) > 1/(1-\bar{S}f_0(T))$ is equivalent to inequality (13). But, from Lemma 1, this latter inequality implies inequality (16). Hence, this latter inequality is a dummy condition to ensure that $\pi_{\text{loss}}(K) \leq \varepsilon$ when $C > 1$ and $R(x) > 1/(1 - \bar{S} f_0(T))$. This completes the proof. ■

Remark. When $\lambda_0 \to 0$ while the service time distribution of high priority packets remains unchanged, then $G_0 \stackrel{w}{\to} F_0$, $f_0(s) \sim s$, and $T = S$, where S is the pole with the smallest module of the Laplace transform F_0^*. Under the assumption that $C = \bar{S}T/(1 - \varepsilon^{1/K}) > 1$ and $(1/\varepsilon)^{1/K'} < 1/(1 - \bar{S}T)$, the coefficients $a_i(\varepsilon)$ given by equation (14) are similar to the coefficients $a_i(\varepsilon)$ given by equation (3). In fact, when considering the loss probability, the high priority queue is transparent to the low priority queue only when the condition $C > 1$ pertains. The single condition $\lambda_0 \to 0$ is not sufficient to ensure transparency. Loosely speaking, service times in the high priority queue should not be too long so that the loss probability in the low priority queue is not altered by the high priority queue. Not too long service times qualitatively mean that T is large enough to ensure the validity of the condition $C > 1$. When we have the strict equality $\lambda_0 = 0$, then we take $T = \infty$ because the distribution F_0 is purely artificial, and the condition $C > 1$ is satisfied. In that case, the two expressions (14) and (3) coincide.

To conclude this section, let us consider the case when all packets have exponential service times, with mean $\bar{S}_0$ (resp. $\bar{S}$) for high (resp. low) priority packets. From the definition of the variables S and T, it is a matter of simple analysis to obtain that

$$\overline{G}_0^* \left((1-\rho_0)t/\bar{S}_0\right) = \frac{2}{1 + t + \sqrt{(1+t)^2 + 4\rho_0 t/(1-\rho_0)}},$$

and

$$T = \frac{(1-\sqrt{\rho_0})^2}{\bar{S}_0}, \quad f_0(T) = \frac{1-\sqrt{\rho_0}}{\bar{S}_0}, \quad C = \frac{\bar{S}}{\bar{S}_0}\frac{1-\sqrt{\rho_0}}{1-\varepsilon^{1/K}}. \tag{17}$$

Trite calculations eventually lead to the surprisingly simple expressions:

$$\frac{f_0^{-1}(u)}{u} = 1 - \frac{\rho_0}{1 - \bar{S}_0 u}, \quad u \leq \frac{1-\sqrt{\rho_0}}{\bar{S}_0}.$$

Theorems 3 and 4 may then be reformulated as follows.

Theorem 5 *Let $K' = (1-\kappa)K$. The loss probability $\pi_{loss}(K, x) \leq \varepsilon$ if*

$$\sum_{i=1}^{N} x_i a_i(\varepsilon) \leq 1 - \rho_0,$$

where the coefficients $a_i(\varepsilon)$ are given for $1 \leq i \leq N$ by:

- *if $C < 1$,*

$$a_i(\varepsilon) = \frac{(1+\sqrt{\rho_0})}{(1-\sqrt{\rho_0})}{}_i\bar{S}_0(\beta_i((1/\varepsilon)^{1/K'}) - 1), \tag{18}$$

- *if $C > 1$,*

$$a_i(\varepsilon) = \rho_i (1/\varepsilon)^{1/K'} \beta_i^*((1/\varepsilon)^{1/K'}) \frac{\bar{S} - (1-\varepsilon^{1/K'})\bar{S}_0}{\bar{S} - \frac{1-\varepsilon^{1/K'}}{1-\rho_0}\bar{S}_0}. \tag{19}$$

5.3 Effective bandwidths

As in Section 4.2, we assume that x_i sources of type i, characterized by the traffic parameters $(\sigma_i, \rho_i, \pi_i)$, $i = 1, \dots, N$, are multiplexed into a buffer with capacity $K < \infty$. The bucket sizes σ_i, $i = 1, \dots, N$ are upper bounded by the constant κK, where $\kappa \in (0, 1)$.

Now, we assume that this buffer is served only when a high priority buffer is empty. This high priority buffer is fed with a Poisson process. This Poisson assumption is supported by the fact that in real implementations, time priority is given to real time packet flows, which are essentially with constant bit rates. The Poisson process is then a good approximation for the superposition of a large number of such real time flows.

Motivated by applications to ATM networks, we assume that all packets (high and low priority packets) have the same mean service time $\bar{S}$. In that case, the condition $C > 1$ is satisfied if $\varepsilon^{1/K'} > \sqrt{\rho_0}$. Since ε takes typical values of the form 10^{-n} for $n \in \{4, 5, .., 10\}$ and the reduced buffer capacity K' is several thousands of packets large, $\varepsilon^{1/K}$ is close to one. Then, the condition $\varepsilon^{1/K'} > \sqrt{\rho_0}$ naturally holds. For instance, for $K = 3000$, $\kappa = 10\%$, and $\varepsilon = 10^{-7}$, this condition translates into $\rho_0 < 98.8\%$, which is a very loose requirement. Hence, in view of Theorem 5, we have the following result for defining effective bandwidths associated with the (σ, ρ, π)-regulated sources.

Theorem 6 *Consider that traffic sources of type i, characterized by the traffic parameter sets $(\sigma_i, \rho_i, \pi_i)$, $i = 1, \dots, N$, are statistically multiplexed into a buffer with capacity $K < \infty$ in the presence of time priority buffer fed with a Poisson packet stream offering a load ρ_0. Under the assumptions that*

1. *all packets have the same mean service $\bar{S}$,*
2. *the bucket sizes $\sigma_i \leq \kappa K$ for $i = 1, \dots, N$ and some $\kappa \in (0, 1)$,*
3. *$\sqrt{\rho_0} < \varepsilon^{1/K'}$,*

the admissibility region $\mathcal{A}^p_\varepsilon$ is given by

$$\mathcal{A}^p_\varepsilon = \left\{ (x_1, \dots, x_N) \in \mathbb{N}^{\mathbb{N}} : \sum_{i=1}^{N} x_i e^p_i(\varepsilon) \leq 1 - \rho_0 \right\},$$

where the effective bandwidths coefficients $e^p_i(\varepsilon)$ are given by

$$e^p_i(\varepsilon) = \min \left\{ \pi_i, \frac{1 - \rho_0}{1 - (1/\varepsilon)^{1/K'} \rho_0} \rho_i (1/\varepsilon)^{1/K'} \beta^*_i((1/\varepsilon)^{1/K'}) \right\}. \tag{20}$$

Since for typical values of K' and ε, $(1/\varepsilon)^{1/K'}$ is very close to one, we see from the above result that $e^p_i(\varepsilon)/(1 - \rho_0)$ is equivalent to the effective bandwidth $e^r_i(\varepsilon)$ obtained by using the RSR approximation, namely by replacing $\bar{S}$ with $\bar{S}/(1 - \rho_0)$ in the expression of $e_i(\varepsilon)$ defined by equation (4). Hence, the admissibility regions $\mathcal{A}^p_\varepsilon$ defined in Theorem 6 and $\mathcal{A}^r_\varepsilon$ corresponding to the effective bandwidths $e^r_i(\varepsilon)$ are almost equal.

As a consequence, for the batch model proposed in this paper for computing effective bandwidths, the RSR approximation can safely be used when all packets have the same mean service time, which is the case in ATM networks. In more general situations, as long as $\varepsilon^{1/K'} \sim 1$ and service times of low and high priority packets are exponentially distributed, the RSR approximation still pertains, even if high and low priority packets do not have the same mean service times.

To conclude this section, let us mention that in general, the RSR approximation underestimates the asymptotic behavior of the low priority queue. Indeed, from Theorem 8, the tail behavior of the low priority queue is determined by the solution z_0 to the equation:

$$\frac{\bar{S}z}{z-1} f_0 \left(\sum_{i=1}^{N} x_i \lambda_i (\beta_i(z) - 1) \right) = 1.$$

But, since $f_0(s) < s/(1-\rho_0)$, we have

$$\frac{z_0}{1-\rho_0} \sum_{i=1}^{N} x_i \rho_i \beta_i^*(z_0) < 1,$$

and hence, $z_0 < z_0^r$, where z_0^r is the coefficient, which governed the tail behavior of the RSR system and which is solution to the equation

$$\frac{z_0}{1-\rho_0} \sum_{i=1}^{N} x_i \rho_i \beta_i^*(z_0) = 1.$$

This observation is in line with the results of the paper by Delas *et al* [11], which exhibits situations, where the RSR approximation may underestimate by several orders of magnitude the asymptotic behavior of the low priority queue. Surprisingly, this effect disappears, when all packets have exponentially distributed service times with the same mean.

This latter phenomenon can easily be checked when the batch size is equal to 1. In that case, we have $z_0 = 1/(\rho_0 + \rho_1)$ and $z_0^r = (1-\rho_0)/\rho_1$. It is easily checked that $z_0^r - z_0 = \rho_0(1-\rho)/(\rho_1 \rho) > 0$ and that by using Theorems 8 and 9, the RSR approximation underestimates the tail behavior of the low priority queue and the loss probability. If we assume that $\pi_{\text{loss}}(K) \sim 1/z_0^K$, then the admission criterion reads, for the real system, $\rho_0 + \rho_1 \leq \varepsilon^{1/K}$, while for the RSR system, $\rho_1(1/\varepsilon)^{1/K} + \rho_0 \leq 1$. We then check that the first criterion is more stringent than the second one, but if $\varepsilon = 10^{-n}$ with $n \ll K$, then both criteria are very close one to each other.

6 Comparison with other admission control algorithms

In this section, we compare the admission control algorithm based on Theorem 2 with other admission control algorithms, which have been so far proposed in the literature for ATM networks. Thus, we consider VBR connections of different types $i = 1, \ldots, N$. Connections of type i are characterized by a peak cell rate PCR_i, a sustainable cell rate SCR_i (both PCR_i and SCR_i are expressed in cells per second) and a maximum burst size MBS_i (expressed in cells). This maximum burst size is related to the bucket size σ_i of the Leaky Bucket used to control the sustainable cell rate as:

$$\sigma_i = 1 + (MBS_i - 1)\left(1 - \frac{SCR_i}{PCR_i}\right).$$

We assume that the above VBR connections are statistically multiplexed in a buffer with size K and that for all $i = 1, \ldots, N$, $\sigma_i \leq \kappa K$ for some $\kappa \in (0,1)$. Finally, let $K' = (1-\kappa)K$.

First of all, assuming that $\nu \times \sigma_i \ll 1$ where $\nu = n \log 10/K'$, we have

$$a_i(\varepsilon) = SCR_i\left(1 + (\sigma_i + 1)\frac{\nu}{2}\right).$$

where $a_i(\varepsilon)$ is defined by equation (3). We hence deduce that under the condition $\nu\sigma_i \ll 1$, the equivalent bandwidth given by equation (3), although conservative, is not far from the ideal value equal to SCR_i. (This value is ideal in the sense that it corresponds to a resource allocation based on the mean rate only, without introducing any over-allocation factor.)

When the above condition is not satisfied, the over-allocation factor may be significant. For instance, when $K = 3000$, $\kappa = 10\%$, $\sigma = 150$, $n = 7$, we have

$$a_i(\varepsilon) = 1.62 \times SCR_i.$$

This means that an over-allocation factor of 62% has to be added to the sustainable cell rate. The over-allocation factor is equal to 37% when $\sigma_i = 100$. Such over-allocation factors are reasonable when the sustainable is small (say, a few hundreds of Kbit/s). The over-allocation factor may however be a potential drawback for high bit rate connections. Yet, it is worth noting that (straightforward) multiplexing is intended to connections with a mean rate, which is likely small when compared to the link capacity.

The admission control algorithm developed by Gravey *et al* [19] puts a uniform upper-bound on MBS values in order to allow a resource allocation based on the SCR. Roughly speaking, with that admission control algorithm, when connections are multiplexed in a FIFO queue with capacity K, then their MBS value should be less than or equal to $K/k_\varepsilon(\rho)$, where $k_\varepsilon(\rho)$ is the capacity needed in a finite capacity $M/D/1$ queue in order to obtain a rejection probability less than ε when the load is equal to ρ. For instance, in the case when $K = 3000$ cells, $\rho = 0.85\%$, and $\varepsilon = 10^{-7}$, this admission control limits the MBS value to $K/30 = 100$ cells since then $\kappa_\varepsilon(\rho) = 30$. Note that since the load is limited to 85%, this is equivalent to introduce an over-allocation of 18% with respect to the sustainable cell rate value. In reality, the MBS values admissible by using the equivalent bandwidth defined in Theorem 2 are larger than the limit imposed by the algorithm of Gravey *et al*]. The counterpart is however that the over-allocation factor due to the equivalent bandwidth is greater.

The admission control developed by Elwalid, Mitra and Wentworth [13] defines a deterministic equivalent bandwidth $\tilde{e}_{0,i}$ as follows:

$$\tilde{e}_{0,i} = \begin{cases} \frac{PCR_i}{1+\frac{PCR_i}{C}\frac{K}{MBS_i}} & \text{if} \quad MBS_i\frac{C}{K} > \frac{SCR_i}{1-\frac{SCR_i}{PCR_i}} \\ SCR_i & \text{otherwise} \end{cases} \tag{21}$$

Numerical evidence shows that the deterministic equivalent bandwidth $\tilde{e}_{0,i}$ is much larger than the equivalent bandwidth $a_i(\varepsilon)$ given by equation (3). Then, a "statistical" equivalent bandwidth is defined as follows. Assuming that the cell rejection probability must be less than ε and that there are x_j sources of type j, the parameter s^*, which maximizes the function

$$s \to F(s) \stackrel{def}{=} sC - \sum_{j=1}^{N} x_j \log\left(1 - w_j + w_j e^{s e_{0,j}}\right),$$

where $w_j = SCR_j/e_{0,j}$, has to be determined To know whether a configuration $(x_1, \ldots, x_N)$ is admissible, one has to check that $F(s^*) \geq \log(1/\varepsilon)$, where s^* corresponds to the configuration $(x_1, \ldots, x_N)$ considered.

The admissibility region defined by such a procedure is in general concave and hence, to simplify the admission criterion, an "optimal" tangent hyper-plane, which maximizes the admissible load, has to be determined. Once this hyper-plane, which intersects the boundary of the admissibility region at point $(x_1^*, \ldots, x_N^*)$, is known, the effective bandwidth is defined by $\tilde{e}_i = c/x_j^*$, where c is the service rate of the buffer.

Some comparisons between the over-allocation factors $\tilde{\eta}_i = \tilde{e}_i/SCR_i$ and η obtained when using the equivalent bandwidths $\tilde{e}_i$ and $a_i(\varepsilon)$, respectively are reported in Tables 1, 2 and 3. (The data are taken from the paper by Elwalid *et al* [13]). It turns out that the admission control developed by Elwalid *et al* performs better when the peak to mean ratio is rather small and the bucket size is large. However, when the peak to mean ratio is "sufficiently" large (say, greater than 10) and bucket sizes are "sufficiently" small when compared with the buffer capacity, then the admission control designed in this paper leads to smaller over-allocation factors.

	SCR_i	PCR_i	σ_i	$\tilde{e}_i$	$\tilde{\eta}_i$	η
type 1	0.15	6.0	24	0.24	60%	9%
type 2	0.15	1.25	220	0.27	80%	135%

Table 1: Comparison between the over-allocation factors $\tilde{\eta}_i$ and η - buffer service rate $C = 150$, $\varepsilon = 10^{-9}$, $K = 3333$, and $\kappa = 10\%$.

Let us conclude this section by considering the admission control algorithm developed by Gibbens and Hunt [18], who have used a fluid model with heterogeneous On/Off sources to define the equivalent bandwidth $\hat{e}_i$ of a connection as follows:

$$\hat{e}_i = \frac{\zeta PCR_i + m_i + l_i - \sqrt{(\zeta PCR_i + m_i - l_i)^2 + 4 l_i m_i}}{2\zeta}, \tag{22}$$

	SCR_i	PCR_i	σ_i	$\tilde{e}_i$	$\tilde{\eta}_i$	η
type 1	0.15	6.0	24	0.28	86%	16%
type 2	0.15	1.25	220	0.32	113%	369%

Table 2: Comparison between the over-allocation factors $\tilde{\eta}_i$ and η - buffer service rate $C = 90$, $\varepsilon = 10^{-9}$, $K = 2000$, and $\kappa = 11\%$.

	SCR_i	PCR_i	σ_i	$\tilde{e}_i$	$\tilde{\eta}_i$	η
type 1	0.5	3	83	0.83	66%	22%
type 2	1.0	5	80	1.55	55%	21%

Table 3: Comparison between the over-allocation factors $\tilde{\eta}_i$ and η - buffer service rate $C = 150$, $\varepsilon = 10^{-9}$, $K = 5000$, and $\kappa = 10\%$.

where

$$m_i = \frac{SCR_i}{y_i(MBS_i - 1)}, \quad l_i = \frac{SCR_i}{MBS_i(1-y_i) + y_i},$$

and $\zeta = \log(\varepsilon)/K$ and $y_i = SCR_i/PCR_i$. It is easily checked that $\hat{e}_i = (1+\hat{\eta}_i)SCR_i$, where the overallocation factor $\hat{\eta}_i$ is given by

$$\hat{\eta}_i = \frac{1}{2\zeta}\left(\frac{\zeta}{y_i} + \frac{MBS_i}{y_i(MBS_i-1)(MBS_i(1-y_i)+y_i)}\right.$$
$$\left. -\sqrt{\left(\frac{\zeta}{y_i} + \frac{MBS_i - 2y_i(MBS_i-1)}{y_i(MBS_i-1)(MBS_i(1-y_i)+y_i)}\right)^2 + \frac{4}{y_i(MBS_i-1)(MBS_i(1-y_i)+y_i)}}\right) - 1.$$

The over-allocation factor $\hat{\eta}_i$ is compared, as a function of the bucket size σ, with the over-allocation factor η defined by equation (5) for different values of the mean to peak rate ratio y_i, when $K = 3000$ and $\kappa = 10\%$. It turns out that Gibbens and Hunt admission algorithm performs slightly better when the ratio y_i is relatively large (e.g., $y_i = 0.5$). But, our admission control becomes better when y decreases, i.e., when the peak rate is much larger than the sustainable rate. Moreover, we have rigorously proved that the RSR approximation holds for our batch model. The same should be done for the admission control by Gibbens and Hunt. Finally, that algorithm relies on an asymptotic estimate of the probability that the number of packets in the queue exceeds a high threshold when the buffer capacity is infinite. This is not exactly the same as the loss probability in the finite buffer case. Our algorithm is based upon an exact estimate of the loss probability when the buffer capacity is equal to K.

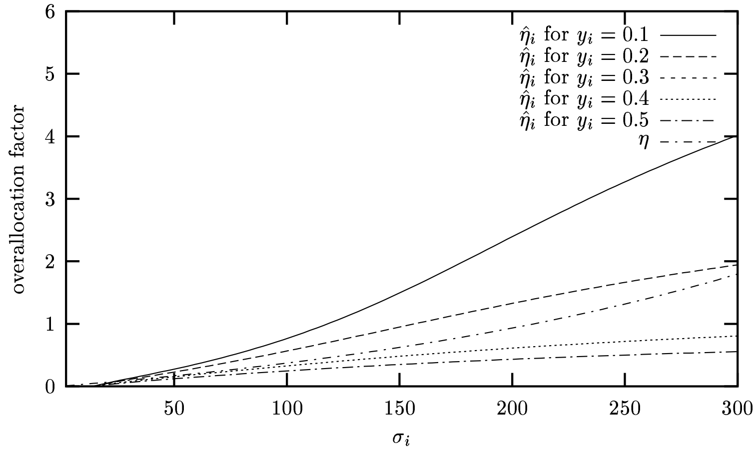

Figure 2: Comparison between η and $\hat{\eta}_i$ as a function of the bucket size σ and different values of the mean to peak rate ratio y – $K = 3000$, $\kappa = 10\%$.

7 Conclusion

We have introduced in this paper the $M^{[X]}/M/1/K$ queue to get a stochastic upper bound on the rejection probability in a queue fed with the superposition of (σ, ρ, π)-regulated traffic sources. This model is then used to develop an admission control algorithm for such traffic sources. In a pure FIFO case, it is possible to define an equivalent bandwidth for a source. This equivalent bandwidth is proportional to the mean rate and the coefficient of proportionality depends only on the ratio of the bucket size σ to the buffer capacity. It has been checked that the results given by the above equivalent bandwidth are compatible with those obtained when using other admission control algorithms published earlier in the literature. The key advantage of the proposed approach is in that the admissibility region is delimited by a hyper-plane.

In a second step, we have introduced a head of line priority level in the original system. The remarkable property of this new system is that it is still possible to define an equivalent bandwidth for (σ, ρ, π)-regulated traffic sources and the admissibility is till delimited by a hyper-plane. The key feature of the equivalent bandwidth is that it fully captures the impact of the high priority queue on the low priority queue. Moreover, the equivalent bandwidth obtained for low priority flow is very close to that obtained by using the RSR approximation. Our results are derived by using exact upper-bounds and thus justify a posteriori the RSR approximation.

To develop an admission control in a complete network, it is necessary to understand how traffic parameters are altered when a flow traverses a queue, especially in the presence oh head of line priorities. Indeed, in such a case, several bursts of a stream could be aggregated and a new maximum burst size should be computed. This point will be addressed in further studies.

A The $M^{[X]}/M/1/K$ queue

In this section, on the basis of the paper by Tijms and Van Ommeren [28], we review some basic results about the $M^{[X]}/M/1/K$ queue. Throughout this Appendix, we use the notation introduced in Section 3.

A.1 Overflow probability

Let A be the generating function of the number of packets arriving during a service time, defined by

$$A(z) = \int_0^\infty e^{\lambda t(\beta(z)-1)} F(dt),$$

where F is the exponential distribution with mean $\bar{S}$. Straightforward manipulations show that $A(z)$ is equal to

$$A(z) = \frac{1}{1+\lambda \bar{S}(1-\beta(z))}.$$

The main result concerning the buffer occupation is the following theorem, which is a theorem obtained by Tijms and Van Ommeren [*ibid.*] specialised to the case when service times are exponentially distributed.

Theorem 7 (Tijms and Van Ommeren [28]) *For any K,*

$$\pi_{loss}(K) = \frac{(1-\rho)\sum_{j\geq K} q_j(\infty)}{1-\rho\sum_{j\geq K} q_j(\infty)}, \tag{23}$$

where $\{q_j(\infty),\ j = 0,\ 1,\ \ldots\}$ is the distribution of the buffer occupancy when the buffer capacity is infinite; the associated generating function is given by

$$Q(z) \stackrel{def}{=} (1-\rho)\frac{(1-\beta(z))A(z)}{\bar{\beta}(A(z)-z)} = \frac{(1-\rho)\beta^*(z)}{1-\rho z\beta^*(z)}, \quad |z| \leq 1, \tag{24}$$

Proof. Equation (23) is a rewriting of equation (1) in [28]. Equation (24) is a rewriting of equations (5) and (4) in [28] in the case of exponential service times. ■

A.2 Asymptotics

By Theorem 7, an asymptotically exponential expansion for the overflow probability $\pi_{\text{loss}}(K)$ is obtained when the tail probabilities $q_j(\infty)$ exhibit a geometric behavior for large enough

j. These tail probabilities will decrease geometrically fast only when the batch-size distribution has no extremely long tails, which is the case in the applications considered in this paper. Therefore, we make the following assumption, which is consistent with the fact that the batch size takes only a finite number of values: we assume that the radius of convergence R of the power series $\beta(z) = \sum_{j=1}^{\infty} \beta_j z^j$ is larger than one, and $\rho R \beta^*(R) > 1$. Note that the convergence radius of the power series $\beta^*(z) = \sum_{j=1}^{\infty} \beta_j^* z^j$ is equal to R since $\beta^*(z) = [1 - \beta(z)]/[(1-z)\bar{\beta}]$. (The point $z = 1$ is a removable singularity.)

By using the fact that the function $z \to \rho z \beta^*(z)$ is a (strictly) increasing continuous function, which takes the value $\rho < 1$ at $z = 1$ and which is such that $\rho R \beta^*(R) > 1$, it is easily checked that equation (1) has a unique real solution. The asymptotic behavior of the queue length is then described as follows.

Theorem 8 (Tijms and Van Ommeren [28]) *Assuming that $R > 1$ and $\rho R \beta^*(R) > 1$,*

$$q_j(\infty) \sim \frac{(1-\rho)\beta^*(z_0)}{\rho\left(\beta^*(z_0) + z_0 {\beta^*}'(z_0)\right)} z_0^{-j-1}, \tag{25}$$

when j tends to ∞, where z_0 is defined by equation (1).

Proof. Since the generating function $\beta(z)$ is convergent for $|z| < R$, it follows that the right hand side of equation (24) is analytic in the domain $|z| < R$, possibly except at the zeros of the denominator $z - A(z)$, which are also the roots of the equation $1 - \rho z \beta^*(z) = 0$.

It is easily checked that $z - A(z)$ has no zeros in the domain $\{z : |z| < z_0\}$ since $|A(z)| \leq A(|z|) < |z|$ for $|z| < z_0$. Moreover, the real number z_0 is the unique zero of the function $z - A(z)$ on the circle $|z| = z_0$. To show this, we prove that if $z_1 \in \{z : |z| = z_0\}$ is solution to the equation $z - A(z) = 0$, then z_1 is real and necessarily $z_1 = z_0$. Since $|\beta(z_1)| \leq \beta(|z_1|) = \beta(z_0)$, we have $\Re(\beta(z_1)) \leq \beta(z_0)$. Moreover,

$$z_0 = |A(z_1)| \leq \int_0^\infty e^{\lambda(\Re(\beta(z_1))-1)t} dF(t) \leq \int_0^\infty e^{\lambda(\beta(z_0)-1)t} dF(t) = z_0$$

and then $\Re(\beta(z_1)) = \beta(z_0)$, which implies that $\Im(\beta(z_1)) = 0$. This entails that $\beta(z_1)$ is a real number and that $A(z_1)$ as well as z_1 are also real numbers, and hence that $z_1 = z_0$.

Note finally that z_0 is a zero with multiplicity one since $A'(z_0) > 1$ by the strict convexity of the function A.

From the above analysis, there exists a positive real number R_1 with $z_0 < R_1 < R$ such that the function $z \to 1 - \rho z \beta^*(z)$ has no zeros in the domain $|z| \leq R_1$ except at the point $z = z_0$. Consequently, we can write equation (24) as $Q(z) = H(z)/(z - z_0)$ for some analytic function $H(z)$ in $|z| \leq R_1$ with $H(z_0) \neq 0$. Using the Taylor expansion $H(z) = H(z_0) + (z - z_0)U(z)$, we next find that $Q(z)$ is of the form

$$Q(z) = \frac{a_{-1}}{z_0 - z} + U(z) \tag{26}$$

for $|z| \leq R_1$, $z \neq z_0$, where $U(z)$ is some analytic function in the domain $|z| \leq R_1$ and the residue a_{-1} is given by

$$a_{-1} = \lim_{z \to z_0} (z_0 - z) Q(z) = \frac{(1-\rho)\beta^*(z_0)}{\rho[\beta^*(z_0) + z_0 \beta^{*\prime}(z_0)]}.$$

Since $U(z)$ is analytic for $|z| \leq R_1$, a Taylor series expansion $U(z) = \sum_{j=0}^{\infty} u_j z^j$ holds for $|z| \leq R_1$. The power series $\sum_{j=0}^{\infty} u_j z^j$ is convergent for $z = R_1$ and so $u_j R_1^j$ is bounded in j. Since $Q(z) = \sum_{j=0}^{\infty} q_j(\infty) z^j$ for $|z| < z_0$, we obtain from the series expansion of the right hand side of equation (26) that

$$q_j(\infty) = \frac{(1-\rho)\beta^*(z_0)}{\rho\left(\beta^*(z_0) + z_0 \beta^{*\prime}(z_0)\right)} z_0^{-j-1} + O(R_1^{-j})$$

for all $j \geq 0$.

Using the fact that $R_1 > z_0$, we finally get the desired result. This completes the proof. ■

As an immediate consequence of the Theorems 7 and 8, the loss probability $\pi_{\text{loss}}(K)$ has the following asymptotic representation.

Theorem 9 (Tijms and Van Ommeren [28]) *Assuming that $R > 1$ and $\rho R \beta^*(R) > 1$,*

$$\pi_{loss}(K) \sim \frac{(1-\rho)^2}{\rho^2 (z_0 - 1)\left(\beta^*(z_0) + z_0 \beta^{*\prime}(z_0)\right)} z_0^{-K-1}.$$

as K tends to ∞.

Proof. Use the fact that

$$z_0 \beta^*(z_0) = 1/\rho$$

in equation (25) and the definition of $\pi_{\text{loss}}(K)$ given by equation (23), and the proof is done. ■

References

[1] D. Anick, D. Mitra, and M. Sondhi. Stochastic theory of a data-handling system with multiple sources. *Bell Sys. Tech. J.*, 61(8):1872–1894, 1982.

[2] F. Baccelli and P. Brémaud. *Elements of queueing theory*, volume 26 of *Applications of Mathematics*. Springer Verlag, New York, 1976.

[3] A.W. Berger and W. Whitt. Effective bandwidths with priorities. *IEEE/ACM Trans. on Networking*, 6(4):447–460, 1998.

[4] A.W. Berger and W. Whitt. Extending the effective bandwidth concept to networks with priority classes. *IEEE Communications Magazine*, 36(8):78–84, 1998.

[5] S. Borst and D. Mitra. Asymptotically achievable performance in ATM networks. *Adv. Appl. Prob.*, 30(2):568–585, 1998.

[6] D.D. Botvich and N.G. Duffield. Large deviations, the shape of the loss curve, and economies of scale in large multiplexers. *Queueing Systems*, 20:299–320, 1995.

[7] I. Cidon, R. Guérin, I. Kessler, and A. Khamisy. Analysis of a statistical multiplexer with generalized periodic sources. *Queueing Systems*, 20:139–169, 1995.

[8] R.L. Cruz. A calculus of network delay, Part I : network elements in isolation. *IEEE Trans. Information Theory 37(3)*, January 1991.

[9] R.L. Cruz. A calculus of network delay, Part II : network analysis. *IEEE Trans. Information Theory 37(3)*, January 1991.

[10] G. de Veciana, G. Kesidis, and J. Walrand. Resource management in wide area networks using effective bandwidth. *IEEE JSAC*, 13:1081–1090, 1995.

[11] S. Delas, R. R. Mazumdar, and C. Rosenberg. Cell loss asymptotics in buffers handling a large number of sources with HOL service priorities. In *Proc. of INFOCOM'99*, New York, 1999. Longer version submitted to Queueing Systems.

[12] B. T. Doshi. Deterministic rule based traffic descriptors for broadband ISDN : worst case behavior and connection acceptance control. In *Proc. Globecom'93*, December 1993.

[13] A. Elwalid, D. Mitra, and R.H. Wentworth. A new approach for allocating buffers and bandwidth to heterogeneous regulated traffic in an ATM node. *IEEE J. Sel. Area Commun.*, 13(6):1115–1127, August 1995.

[14] A.I. Elwalid and D. Mitra. Effective bandwidth of general markovian traffic sources and admission control of high speed networks. *IEEE/ACM Trans. Networking*, 1(3):329–343, 1993.

[15] A.I. Elwalid and D. Mitra. Analysis, approximations and admission control of a multi-service multiplexing system with priorities. In *Proc. Infocom'95*, pages 463–472, 1995.

[16] W. Feller. *An introduction to probability theory and its applications*, volume II. John Wiley & Sons Ltd, 1971.

[17] J. Garcia, J.M. Barceló, and O. Casals. An exact model for the multiplexing of worst case traffic sources. In *Proc. Sixth Int. Conf. on Data Comm. Syst. and their Performance*, Istanbul, October 1995. IFIP.

[18] R. Gibbens and P.J. Hunt. Effective bandwidthss for the multi-type UAS channel. *Queueing Systems*, pages 17–28, 1991.

[19] A. Gravey, J. Boyer, K. Sevilla, and J. Mignault. Resource allocation for worst case traffic in ATM networks. *Performance Evaluation*, 1-2:19–43, 1997.

[20] R. Guérin, H. Ahmadi, and M. Nagshineh. Equivalent capacity and its application to bandwidth allocation in high speed networks. *IEEE J. Sel. Areas Commun.*, 6:968–981, 1991.

[21] F. Kelly. Effective bandwidths at multi-type queues. *Queueing Systems*, 10:5–15, 1991.

[22] G. Kesidis and T. Konstantopoulos. Worst case performance of a buffer with independent shaped arrival processes. *IEEE Comm. Lett.*, 4(1):26–28, 2000.

[23] J.F.C. Kingman. Inequalities in the theory of queues. *Journal of the Royal Statistical Society B*, 32:102–110, 1970.

[24] K. Kumaran and M. Mandjes. Multiplexing regulated traffic streams: design and performance. In *Proc. IEEE Infocom'2001*, Tel Aviv, 2001.

[25] J.Y. Le Boudec. An efficient solution method for Markov models of ATM links with loss priorities. *IEEE JSAC, Special Issue on Teletraffic Analysis of Communications Systems*, pages 408–417, April 1991.

[26] M. Reisslein, K.W. Ross, and Srini Rajagopal. Guaranteeing statistical QoS to regulated traffic: the single node case. In *Proc. IEEE Infocom'99*, pages 1060–1071, New York, mar 1999.

[27] S. Shakkottai and R. Srikant. Many sources delay asymptotics with applications to priority queues. *Queieng Systems*, 39:183–200, 2001.

[28] H. Timjs and J.V. Ommeren. Asymptotic analysis for buffer behavior in communication systems. *Probability in the Engineering and Informational Sciences*, 3:1–12, 1989.

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