About Folding-Unfolding Cuts and Cuts Modulo

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Abstract: We show in this note that cut elimination in deduction modulo subsumes cut elimination in deduction with the folding and unfolding rules.

Key-words: Cut elimination, deduction modulo, folding, unfolding.

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Les coupures par pliage-dépliage et les coupures modulo

Résumé : On montre que l'élimination des coupures en déduction modulo généralise l'élimination des coupures avec des règles de pliage et de dépliage.

Mots-clés : Élimination des coupures, déduction modulo, pliage, dépliage.

In first-order natural deduction, a cut is a sequence formed with an introduction rule followed by an elimination rule. This notion can be extended to deal with the axioms of some theories, e.g. type theory, set theory, the Stratified Foundations, ... Prawitz [10] proposes a rather uniform way to extend the notion of cut by first extending first-order natural deduction with two rules: *folding* and *unfolding* (called λ-*introduction* and λ-*elimination* by Prawitz) and then considering a sequence of such rules as a new form of cut (see also [2, 9, 1, 3, 8]). We have recently proposed another way to extend deduction called *deduction modulo* [5, 7] where propositions equivalent modulo a congruence are identified. Identifying propositions this way extends the notion of cut. We show in this note that deduction modulo subsumes deduction with the *folding* and *unfolding* rules.

1 Deduction with the *folding* and *unfolding* rules

In this note, we consider a fixed theory $\mathcal{T}$ formed with axioms

$$\forall x_1 \ldots \forall x_n \ (P_i \Leftrightarrow Q_i)$$

where the P_i's are atomic propositions and the Q_i's arbitrary propositions.

We say that a proposition B *folds* to an atomic proposition A (resp. that A *unfolds* to B) if $A = \theta P_i$ and $B = \theta Q_i$ for some axiom $\forall x_1 \ldots \forall x_n \ (P_i \Leftrightarrow Q_i)$ of $\mathcal{T}$ and some substitution θ. We assume that $\mathcal{T}$ is such that an atomic proposition unfolds to at most one proposition.

The axioms of $\mathcal{T}$ can be replaced by the deduction rules

$$\frac{\Gamma \vdash \theta Q_i}{\Gamma \vdash \theta P_i} \textit{folding}$$

$$\frac{\Gamma \vdash \theta P_i}{\Gamma \vdash \theta Q_i} \textit{unfolding}$$

and it is easy to check that a sequent $\mathcal{T}, \Gamma \vdash A$ can be proved in first-order natural deduction if and only if the sequent $\Gamma \vdash A$ can be proved in deduction with the *folding* and *unfolding* rules. Indeed, any instance of the *folding* and *unfolding* rules can be simulated using an axiom of $\mathcal{T}$ and the axioms of $\mathcal{T}$ can be proved in deduction with the *folding* and *unfolding* rules.

Since an atomic proposition unfolds to at most one proposition, a sequence formed with a *folding* and an *unfolding* rule has the form

$$\cfrac{\cfrac{\cfrac{\ldots}{\Gamma \vdash B}}{\Gamma \vdash A} \textit{folding}}{\Gamma \vdash B} \textit{unfolding}$$

and can be reduced to

$$\frac{\ldots}{\Gamma \vdash B}$$

Such a sequence is thus called a *folding-unfolding cut.*

Cut elimination terminates for some theories $\mathcal{T}$, but it does not for others.

2 Deduction modulo

In deduction modulo, a theory is formed with a set of axioms Γ *and a congruence* $\equiv$ on propositions. Here, the congruence is the smallest congruence identifying P_i and Q_i for each i.

The deduction rules take this congruence into account. For instance, the *modus ponens* is not stated as usual

$$\frac{\Gamma \vdash A \Rightarrow B \quad \Gamma \vdash A}{\Gamma \vdash B}$$

as the first premise need not be exactly $A \Rightarrow B$ but may be only congruent to this proposition, hence it is stated

$$\frac{\Gamma \vdash C \quad \Gamma \vdash A}{\Gamma \vdash B} \text{ if } C \equiv A \Rightarrow B$$

All the rules of intuitionistic natural deduction may be stated in a similar way (figure 1).

A *cut* in deduction modulo is, like in first-order logic, a sequence formed with a introduction and an elimination rule. Cut elimination terminates for some congruences, but it does not for others.

3 Proof-terms

We use a functional notation for proofs. To each proof of a sequent $A_1, ..., A_n \vdash B$ in first-order natural deduction, we associate a proof-term whose free variables are among $\alpha_1, ..., \alpha_n$. Proofs built with the axiom rules using the axiom A_i are written α_i, proofs built with the introduction and elimination rules of the implication are written $\lambda\alpha\ \pi$ and $(\pi\ \pi')$, proofs built with the introduction and elimination rules of the conjunction are written $\langle\pi, \pi'\rangle$, $fst(\pi)$ and $snd(\pi)$, proofs built with the introduction and elimination rules of the disjunction are written $i(\pi)$, $j(\pi)$ and $(\delta\ \pi_1\ \alpha\pi_2\ \beta\pi_3)$, proofs built with the elimination rule of the contradiction are written $(botelim\ \pi)$ proofs built with the introduction and elimination rules of the universal quantifier are written $\lambda x\ \pi$ and $(\pi\ t)$ and proofs built with the introduction and elimination rules of the existential quantifier are written $\langle t, \pi\rangle$ and $(exelim\ \pi\ x\alpha\pi')$. *Reduction* on proof-terms is defined by the following rules that eliminate cuts step by step.

$$(\lambda\alpha\ \pi_1\ \pi_2) \rhd [\pi_2/\alpha]\pi_1$$

$$fst(\langle\pi_1, \pi_2\rangle) \rhd \pi_1$$

$$snd(\langle\pi_1, \pi_2\rangle) \rhd \pi_2$$

$$(\delta\ i(\pi_1)\ \alpha\pi_2\ \beta\pi_3) \rhd [\pi_1/\alpha]\pi_2$$

$$\frac{}{\Gamma \vdash B} \text{ axiom if } A \in \Gamma \text{ and } A \equiv B$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash C} \Rightarrow\text{-intro if } C \equiv (A \Rightarrow B)$$

$$\frac{\Gamma \vdash C \quad \Gamma \vdash A}{\Gamma \vdash B} \Rightarrow\text{-elim if } C \equiv (A \Rightarrow B)$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash C} \wedge\text{-intro if } C \equiv (A \wedge B)$$

$$\frac{\Gamma \vdash C}{\Gamma \vdash A} \wedge\text{-elim if } C \equiv (A \wedge B)$$

$$\frac{\Gamma \vdash C}{\Gamma \vdash B} \wedge\text{-elim if } C \equiv (A \wedge B)$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash C} \vee\text{-intro if } C \equiv (A \vee B)$$

$$\frac{\Gamma \vdash B}{\Gamma \vdash C} \vee\text{-intro if } C \equiv (A \vee B)$$

$$\frac{\Gamma \vdash D \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \vee\text{-elim if } D \equiv (A \vee B)$$

$$\frac{\Gamma \vdash B}{\Gamma \vdash A} \bot\text{-elim if } B \equiv \bot$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash B} (x, A) \ \forall\text{-intro if } B \equiv (\forall x \ A) \text{ and } x \notin FV(\Gamma)$$

$$\frac{\Gamma \vdash B}{\Gamma \vdash C} (x, A, t) \ \forall\text{-elim if } B \equiv (\forall x \ A) \text{ and } C \equiv [t/x]A$$

$$\frac{\Gamma \vdash C}{\Gamma \vdash B} (x, A, t) \ \exists\text{-intro if } B \equiv (\exists x \ A) \text{ and } C \equiv [t/x]A$$

$$\frac{\Gamma \vdash C \quad \Gamma, A \vdash B}{\Gamma \vdash B} (x, A) \ \exists\text{-elim if } C \equiv (\exists x \ A) \text{ and } x \notin FV(\Gamma B)$$

Figure 1: Natural deduction modulo

$$(\delta\ j(\pi_1)\ \alpha\pi_2\ \beta\pi_3) \rhd [\pi_1/\beta]\pi_3$$

$$(\lambda x\ \pi\ t) \rhd [t/x]\pi$$

$$(exelim\ \langle t, \pi_1\rangle\ \alpha x \pi_2) \rhd [t/x, \pi_1/\alpha]\pi_2$$

Proofs in deduction modulo, are written as in first-order natural deduction, and the proof reduction rules are the same.

Proofs built with the *folding* rule are written $\pi \uparrow$ and proofs built with the *unfolding* rule are written $\pi \downarrow$. The corresponding reduction rule is

$$\pi \uparrow\downarrow \rhd \pi$$

4 Translations

As already said, a sequent $\Gamma \vdash A$ is provable in deduction with the *folding* and *unfolding* rules if and only if the sequent $\mathcal{T}, \Gamma \vdash A$ is provable in first-order natural deduction. The sequent $\Gamma \vdash A$ is also provable in deduction modulo if and only if the sequent $\mathcal{T}, \Gamma \vdash A$ is provable in first-order natural deduction. Hence the sequent $\Gamma \vdash A$ is provable in deduction with the *folding* and *unfolding* rules if and only if it is provable in deduction modulo.

This can also be proved directly. If π is a proof of $\Gamma \vdash A$ in deduction with the *folding* and *unfolding* rules, then the proof π^- obtained by erasing the folding and unfolding steps in π yields a proof in deduction modulo. The converse is a little bit more difficult. Indeed, if the proposition B unfolds to B' then the proposition $A \wedge B$ and $A \wedge B'$ are congruent and hence the sequent $A \wedge B \vdash A \wedge B'$ has a trivial proof in deduction modulo, using only the *axiom* rule. But the *unfolding* rule does not apply to the proposition $A \wedge B$, but only to the proposition B. Hence the proof of $A \wedge B'$ must be written $\langle fst(\alpha), snd(\alpha) \downarrow\rangle$ where the variable α is η-expanded so that the *unfolding* rule can apply to the proposition B. This justifies the need of the following lemma.

Lemma 4.1 *If the proposition A converts to B in one step (we write this $A \leftrightarrow^1 B$), then there are proofs of $A \Rightarrow B$ and $B \Rightarrow A$ in deduction with the* folding *and* unfolding *rules. These proofs are called* conversion steps.

Proof. By induction over the structure of A.

- If A is atomic, then A unfolds to B and we take the proofs $\lambda\alpha\ \alpha \downarrow$ and $\lambda\alpha\ \alpha \uparrow$.
- If $A = A_1 \Rightarrow A_2$ then $B = B_1 \Rightarrow B_2$. We have either $A_1 = B_1$ and $A_2 \leftrightarrow^1 B_2$ or $A_1 \leftrightarrow^1 B_1$ and $A_2 = B_2$. By induction hypothesis we have in both cases proofs ρ_1 and ρ'_1 of $A_1 \Rightarrow B_1$ and $B_1 \Rightarrow A_1$ and ρ_2 and ρ'_2 of $A_2 \Rightarrow B_2$ and $B_2 \Rightarrow A_2$. We take $\lambda\alpha\ \lambda\beta\ (\rho_2\ (\alpha\ (\rho'_1\ \beta)))$ and $\lambda\alpha\ \lambda\beta\ (\rho'_2\ (\alpha\ (\rho_1\ \beta)))$.
- If $A = A_1 \wedge A_2$ then $B = B_1 \wedge B_2$. We have either $A_1 = B_1$ and $A_2 \leftrightarrow^1 B_2$ or $A_1 \leftrightarrow^1 B_1$ and $A_2 = B_2$. By induction hypothesis we have in both cases proofs ρ_1 and ρ'_1 of $A_1 \Rightarrow B_1$ and $B_1 \Rightarrow A_1$ and ρ_2 and ρ'_2 of $A_2 \Rightarrow B_2$ and $B_2 \Rightarrow A_2$. We take $\lambda\alpha\ \langle(\rho_1\ fst(\alpha)), (\rho_2\ snd(\alpha))\rangle$ and $\lambda\alpha\ \langle(\rho'_1\ fst(\alpha)), (\rho'_2\ snd(\alpha))\rangle$.

- If $A = A_1 \vee A_2$ then $B = B_1 \vee B_2$. We have either $A_1 = B_1$ and $A_2 \leftrightarrow^1 B_2$ or $A_1 \leftrightarrow^1 B_1$ and $A_2 = B_2$. By induction hypothesis we have in both cases proofs ρ_1 and ρ'_1 of $A_1 \Rightarrow B_1$ and $B_1 \Rightarrow A_1$ and ρ_2 and ρ'_2 of $A_2 \Rightarrow B_2$ and $B_2 \Rightarrow A_2$. We take $\lambda\alpha\ (\delta\ \alpha\ \beta i(\rho_1 \beta)\ \gamma j(\rho_2\ \gamma))$ and $\lambda\alpha\ (\delta\ \alpha\ \beta i(\rho'_1 \beta)\ \gamma j(\rho'_2\ \gamma))$.

- Since A is reducible, it cannot be $\bot$

- If $A = \forall x\ A_1$ then $B = \forall x\ B_1$ and we have $A_1 \leftrightarrow^1 B_1$. By induction hypothesis we have proofs ρ_1 and ρ'_1 of $A_1 \Rightarrow B_1$ and $B_1 \Rightarrow A_1$. We take $\lambda\alpha\ \lambda x\ (\rho_1\ (\alpha\ x))$ and $\lambda\alpha\ \lambda x\ (\rho'_1\ (\alpha\ x))$.

- If $A = \exists x\ A_1$ then $B = \exists x\ B_1$ and we have $A_1 \leftrightarrow^1 B_1$. By induction hypothesis we have proofs ρ_1 and ρ'_1 of $A_1 \Rightarrow B_1$ and $B_1 \Rightarrow A_1$. We take $\lambda\alpha\ (exelim\ \alpha\ x\beta\langle x, (\rho_1\ \beta)\rangle)$ and $\lambda\alpha\ (exelim\ \alpha\ x\beta\langle x, (\rho'_1\ \beta)\rangle)$.

□

Corollary 4.1 *If $A \equiv A'$ and π is a proof of $\Gamma \vdash A$, then there is a proof of $\Gamma \vdash A'$ of the form $(\rho_n\ ...\ (\rho_1\ \pi)...)$ where $\rho_1, ..., \rho_n$ are conversion steps. Such a proof is called a* transformation *of π.*

Proposition 4.1 *The propositions provable in deduction with the* folding *and* unfolding *rules and the propositions provable in deduction modulo are the same.*

Proof. Erasing the folding and unfolding steps transforms any proof π in deduction with the *folding* and *unfolding* rules into a proof π^- in deduction modulo. Conversselly, if a sequent has a proof π in deduction modulo, then we can build a proof π^+ in deduction with the *folding* and *unfolding* rules inserting sequences of conversion steps when needed. □

5 Folding-unfolding cuts and cuts modulo

We now prove that cut elimination terminates in deduction with the *folding* and *unfolding* rules if and only if cut elimination terminates in deduction modulo.

Lemma 5.1 *Let A and A' be two propositions such that $A \equiv A'$ and A and A' are either two implications, two conjunctions, two disjunctions, two contradictions, two universal quantifications or two existential quantifications. Let π be a proof of $\Gamma \vdash A$ and π' a proof of $\Gamma \vdash A'$ that is a transformation of π.*

If π is an introduction, then π' reduces to an introduction π'' and the subproofs of π'' are transformations of subproofs of π.

Proof. By induction on the length of the transformation from π to π'. The result is obvious if $\pi' = \pi$. Otherwise, the proof π' has the form $(\rho_{n+1}\ (\rho_n ... (\rho_1 \pi)...))$. The proof $(\rho_n ... (\rho_1 \pi)...)$ is a proof of a proposition A'' that is either atomic or has the same head symbol as A.

If A'' is atomic then $n \neq 0$ and $(\rho_{n-1}...(\rho_1 \pi)...)$ is a proof of A', $\rho_n = \lambda\alpha\ \alpha \uparrow$ and $\rho_{n+1} = \lambda\alpha\ \alpha \downarrow$. By induction hypothesis, $(\rho_{n-1}...(\rho_1 \pi)...)$ reduces to an introduction π'' and the subproofs of π'' are transformations of the subproofs of π. The proof π' reduces to $\pi'' \uparrow\downarrow$ and then to π''.

Otherwise A'' has the same head symbol as A. By induction hypothesis, the proof $(\rho_n...(\rho_1 \pi)...)$ reduces to $\lambda\alpha\ \pi''_1$ (resp. $\langle \pi''_1, \pi''_2 \rangle$, $i(\pi''_1)$, $j(\pi''_1)$, $\lambda x\ \pi''_1$, $\langle t, \pi''_1 \rangle$), the subproofs are transformations of the subproofs of π and ρ_{n+1} has the form

$$\lambda\alpha\ \lambda\beta\ (\tau_2\ (\alpha\ (\tau'_1\ \beta)))$$

(resp.

$$\lambda\alpha\ \langle (\tau_1\ fst(\alpha)), (\tau_2\ snd(\alpha)) \rangle$$

$$\lambda\alpha\ (\delta\ \alpha\ \beta i(\tau_1 \beta)\ \gamma j(\tau_2\ \gamma))$$

$$\lambda\alpha\ (\delta\ \alpha\ \beta i(\tau_1 \beta)\ \gamma j(\tau_2\ \gamma))$$

$$\lambda\alpha\ \lambda x\ (\tau_1\ (\alpha\ x))$$

$$\lambda\alpha\ (exelim\ \alpha\ x\beta\langle x, (\tau_1\ \beta)\rangle)$$

). Thus, the proof π' reduces to an introduction π'' and the subproofs of π'' are transformations of subproofs of π. □

Proposition 5.1 *Cut elimination terminates in deduction with* folding *and* unfolding *rule if and only of cut elimination modulo terminates.*

Proof. Assume that cut elimination modulo terminates and consider a cut elimination sequence $\pi_1, \pi_2, ...$ in deduction with the *folding* and *unfolding* rules. Each π_{i+1} is obtained from π_i either by reducing a logical cut or a folding-unfolding cut and only a finite number folding-unfolding cut reductions can be performed consecutively (as the size of proofs reduces when we reduce such a cut). When π_{i+1} is obtained by reducing a logical cut in π_i then π^-_{i+1} is obtained by reducing a cut in π^-_i and when π_{i+1} is obtained by reducing a folding-unfolding cut in π_i then $\pi^-_{i+1} = \pi^-_i$. Hence in the sequence $\pi^-_1, \pi^-_2, ...$ in deduction modulo each proof is either obtained by reducing a cut in the previous, or is equal to the previous and only a finite number of consecutive proofs can be equal. As cut elimination terminates in deduction modulo, this sequence is finite and so is the sequence $\pi_1, \pi_2, ...$

Conversely, assume that cut elimination terminates in deduction with the *folding* and *unfolding* rules and consider a proof-reduction sequence $\pi_1, \pi_2, ...$ in deduction modulo. The proof π_1 contains a redex. In π^+_1 a conversion steps may have been inserted between the introduction rule and the elimination rule of this redex. But, by lemma 5.1, reducing the conversion steps applied to the introduction permutes the introduction and the conversion steps. Thus, π^+_1 reduces to π^+_2 in at least one step. Hence the sequence $\pi^+_1, \pi^+_2, ...$ is finite and so is the sequence $\pi_1, \pi_2, ...$ □

6 Comparing deduction modulo and deduction with the *folding* and *unfolding* rules

In [7] we have shown that a theory modulo had the cut elimination property if it had some kind of many-valued model (whose truth values are sets of proofs) called a *pre-model* and we have shown that large classes of theories modulo had the cut elimination property. As cut elimination is equivalent in deduction with the *folding* and *unfolding* rules and in deduction modulo, these tools can be adapted to prove cut elimination in deduction with the *folding* and *unfolding* rules.

Alternatively, theories usually presented in deduction with the *folding* and *unfolding* rules can equivalently be presented in deduction modulo.

Proofs in deduction modulo are more compact than proofs in deduction with the *folding* and *unfolding* rules. First, because the folding and unfolding steps are left implicit in deduction modulo, but also because in deduction with the *folding* and *unfolding* rules, as show above, proofs need to be η-expanded so that the *folding* and *unfolding* rules may be applied. This η-expansion could be avoided if we extended the *folding* and *unfolding* rules to a *conversion* rule

$$\frac{\Gamma \vdash A}{\Gamma \vdash B}\ \textit{conversion}\ \text{if}\ A \equiv B$$

with arbitrary propositions A and B. But, then lemma 5.1 would not hold anymore and a conversion step inserted between a introduction and an elimination could block a cut, as in the proof (with $B \equiv B'$)

$$\cfrac{\cfrac{\cfrac{A, B \vdash A \quad A, B \vdash B}{A, B \vdash A \wedge B}\ \wedge\text{-intro}}{A, B \vdash A \wedge B'}\ \textit{conversion}}{A, B \vdash A}\ \wedge\text{-elim}$$

We would then need to extend the notion of cut and define a cut as a sequence formed with an introduction rule, a sequence of conversion rules and an elimination rule [4], i.e. essentially as a cut modulo.

At last, deduction modulo is more general than deduction with the *folding* and *unfolding* rules, as it does not require that an atomic proposition unfolds to at most one proposition. This permits in particular to include, besides equivalences between propositions such as $x \times y = 0 \equiv x = 0 \vee y = 0$, equivalence between terms such as $x + 0 \equiv x$. Then a proposition such as $(x+0) \times (y+0) = 0$ may be equivalent to many propositions such as $x \times (y+0) = 0$, $(x+0) \times y = 0$, $x+0 = 0 \vee y+0 = 0$, $x = 0 \vee y = 0$, ... while this is not possible in deduction with the *folding* and *unfolding* rules.

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