Queueing Analysis of Simple FEC Schemes for IP Telephony

Eitan Altman, Chadi Barakat, Victor Manuel Ramos Ramos

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Queueing analysis of simple FEC schemes for IP Telephony

Eitan Altman — Chadi Barakat — Victor Manuel Ramos Ramos

N° 3998

September 2000

THÈME 1

Rapport de recherche

ISSN 0249-6399 ISRN INRIA/RR--3998--FR+ENG

Queueing analysis of simple FEC schemes for IP Telephony

Eitan Altman , Chadi Barakat , Victor Manuel Ramos Ramos

Thème 1 — Réseaux et systèmes
Projet Mistral

Rapport de recherche n° 3998 — September 2000 — 34 pages

Abstract: In interactive voice applications, FEC schemes are necessary for the recovery from lost packets. These schemes need to be simple with a light coding and decoding overhead so as not to impact the interactivity. The objective of this paper is to study a well known simple FEC scheme that has been proposed and implemented, in which for every packet n, some redundant information is added to some subsequent packet $n + \phi$. If packet n is lost, it will be reconstructed in case packet $n + \phi$ is well received. The quality of the reconstructed copy of packet n will depend on the amount of information of packet n carried by packet $n + \phi$. We propose a detailed queueing analysis based on a ballot theorem, and obtain simple expressions for the audio quality as a function of the amount of redundancy and its relative position with respect to the original information. For a utility function that is linear in the amount of information, the analysis shows that this FEC scheme does not scale well and any amount of added redundancy might cause a degradation in the overall quality. We then introduce and analyze other utility functions in order to obtain a quality function that is better adapted to the characteristics of this kind of applications.

Key-words: IP telephony, ballot theorem, FEC, audio quality, utility function.

Unité de recherche INRIA Sophia Antipolis
2004, route des Lucioles, B.P. 93, 06902 Sophia Antipolis Cedex (France)
Téléphone : 04 92 38 77 77 - International : +33 4 92 38 77 77 —Fax : 04 92 38 77 65 - International : +33 4 92 38 77 65

Analyse de files d'attente pour des méchanismes FEC simples en Téléphonie sur IP

Résumé : Les mécanismes de FEC sont indispensables pour l'implantation des applications de transmission de la voix en temps réel pour récupérer des pertes de paquets. Ces mécanismes doivent être simples, et le processus de codage et de décodage ne doit pas introduire beaucoup de surcharge pour ne pas affecter l'interactivité. Le but de cet article est d'étudier un mécanisme de FEC simple mais très utilisé, qui a été proposé et mis en œuvre, dans lequel pour chaque paquet n on ajoute de la redondance sur un paquet postérieur $n + \phi$. Si le paquet n est perdu, il peut être reconstruit si le paquet $n + \phi$ est bien réçu. La qualité de la copie reconstruite du paquet n dépendra de la quantité de FEC que l'on ajoute sur le paquet $n + \phi$. Nous proposons une analyse détaillée de files d'attente basée sur un théorème de *ballot*, et nous obtenons des expressions simples pour la qualité de l'audio en fonction de la quantité de redondance et en fonction de sa position par rapport à l'information originale. L'analyse montre que pour une fonction d'utilité qui est linéaire par rapport à la quantité d'information, il n'est pas recommandable d'utiliser ce mécanisme, et que n'importe quelle quantité de redondance ajoutée pourrait affecter de façon négative la qualité totale. Nous présentons et analysons d'autres fonctions d'utilité pour obtenir une fonction de qualité mieux adaptée aux caractéristiques de ce type d'applications.

Mots-clés : Téléphonie sur Internet, théorème de ballot, FEC, qualité de l'audio, fonction d'utilité.

1 Introduction

Mechanisms for recovering from packet losses can be classified as open loop mechanisms and closed loop mechanisms [17]. *Closed loop mechanisms* like ARQ (Automatic Repeat reQuest) are not adequate for real-time interactive applications since they increase considerably the end-to-end delay due to packet retransmission. *Open loop mechanisms* like FEC (Forward Error Correction) are better adapted to real-time applications given that packet losses are recovered without the need for a retransmission; some redundant information is transmitted with the basic data flow. Once a packet is lost, the receiver uses (if possible) the redundant information to reconstruct the lost information. FEC schemes are recommended whenever the end-to-end delay is large so that a retransmission deteriorates the overall quality.

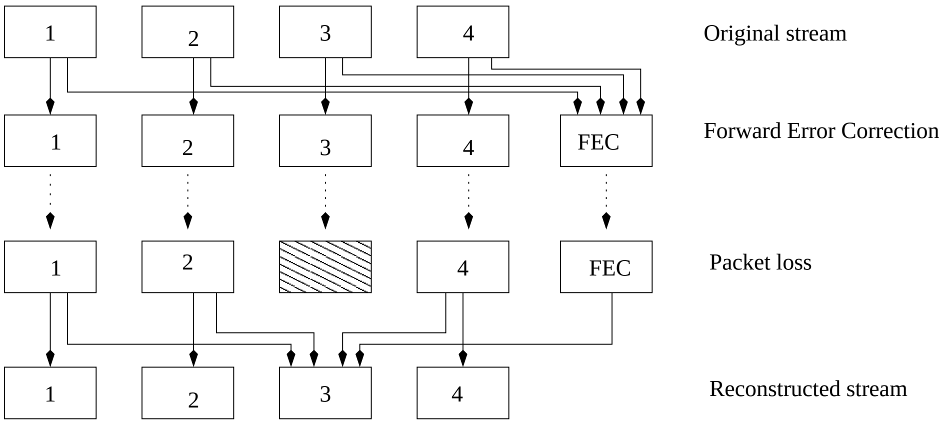

Figure 1: Media-independent FEC. The FEC packet protects a block of $n-1$ packets, it's the result of XORing the original block of packets.

1.1 Forward Error Correction

In interactive real-time communication, FEC techniques are used to repair losses of data during transmission. Perkins and Hodson [16] classify FEC techniques as *media-independent FEC*, and *media-specific FEC*.

FEC has been often used for loss recovery in audio communication tools. It is a sender-based repair mechanism. An efficient FEC scheme is one that is able to repair the most of packet losses. Now, when FEC fails to recover from a loss, applications

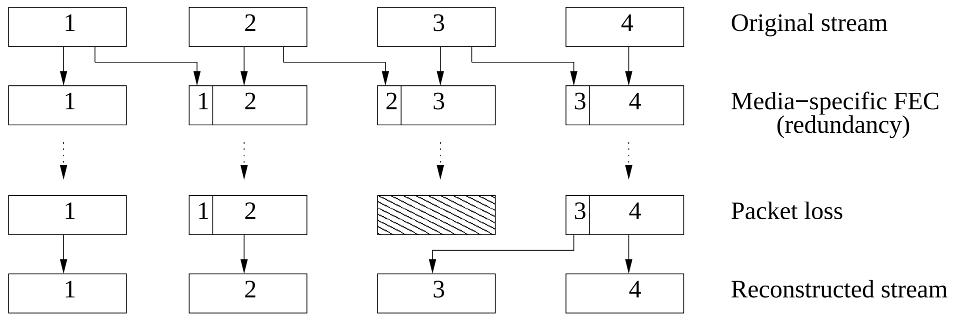

Figure 2: Media-specific FEC. The simple FEC mechanism where packet $n+1$ carries redundant information of packet n.

can resort to other receiver-based repair mechanisms like insertion, interpolation, or regeneration, using well known methods [17]. The FEC schemes proposed in the literature are often simple, so that the coding and the decoding of the redundancy can be quickly done without impacting the interactivity.

1.1.1 Media-independent FEC

Media-independent FEC is often implemented using block or algebraic codes, like parity codes and Reed-Solomon codes. These codes work by taking a codeword of k data packets to generate $n-k$ additional check packets for transmitting n packets over the network. The additional packets are intended to aid in the recovering of lost packets at the receiver.

An example of parity coding has been developed and implemented by Rosenberg [20], which consists in applying a XOR operation across a group of packets to generate the corresponding parity packets. Figure 1 depicts this scheme. One parity packet (the FEC packet) is generated after XORing $n-1$ data packets; if there is just one loss in a group of n packets, that loss is recoverable.

Perkins has called this kind of FEC mechanism *media-independent* because the FEC operation does not depend on the contents of the packets. The principal advantage of this simple scheme is its simplicity, and in case of loss the repair for that loss is obtained by a single XOR operation. The main disadvantage is the delay and the increased bandwidth imposed on the transmission. A high delay would deteriorate the interactivity of a conversation, and thus the number of protected packets impose the playout delay at the receiver in case of loss.

1.1.2 Media-specific FEC

Media-specific FEC consists basically in adding to each packet a redundant portion of the previous one. This basic principle is illustrated in Fig. 2.

In this FEC scheme we recognize the *original stream* which we intend to protect from packet losses, and the *FEC stream* that is going to be transmitted through the network. To get the FEC stream, we add to packet $n+1$ a redundant portion obtained from packet n. This redundant portion is obtained usually by coding packet n with a lower-bandwidth, lower-quality encoding technique. The choice of the encoding technique is not obvious, and that problem has been studied extensively; Bolot and Vega García [5] have proposed the use of low bit rate codecs such as LPC, and full bit rate such as GSM, and tools like FreePhone [10] and Rat [15] have implemented it. These tools generally work by implementing the FEC mechanism just described, so that if packet n is dropped in the network, it can be recovered and played out in case packet $n+1$ is correctly received just as it's illustrated in Figure 2. Thus, if the reconstruction succeeds, the lost packet is played out with a lower quality, but that would be better than to play nothing at the receiver.

The main advantage of media-specific FEC in comparison with media-independent FEC is its low latency, because with this simple scheme one has to wait just one packet in case of loss, instead of an entire block of packets as in the previous case. This low latency characteristic makes it suitable for interactive applications.

It has been proved that losses in the Internet are often bursty, and that means that if packet n is lost, there exists a nonnegligible probability that packet $n+1$ is also lost. We could solve that by spacing the redundant copy by more than one packet, but that could affect the interactivity of a session. Thus the spacing between the original packet and its redundancy is a compromise between loss recovery and interactivity.

1.2 Audio quality

There exists a number of factors that can affect an audio conversation. First, interactivity is affected directly by the end-to-end delay [1]; second, jitter affects the fluidity of the speech; and third, packet loss affects the audio quality perceived at the destination. It is this last factor which is the main concern of our work in this paper.

[1]A session is defined *interactive* if the end-to-end delay is less than 250ms, including media coding and decoding, network transit and host buffering [16].

The measure of audio quality is not an obvious procedure, because it can be measured in different ways. In general, there exist two categories to classify the way we measure audio quality:

- **Subjective assessment**, and
- **Objective assessment**,

each of them differ in the way they conceive quality. *Subjective assessment* is often made by performing listening tests using a large number of subjects. Listeners listen to a set of audio signals without being told about their nature. Audio quality is evaluated by the listeners by giving a score that ranges between 1 to 5; 1 for bad, 5 for excellent. Then, a score is calculated by averaging the listener's score resulting in a Mean Opinion Score (MOS), giving the required audio quality measure. The ITU-T and ITU-R [26, 25] recommendations suggests scales for assessing voice transmission over telephone networks, image quality over television systems, and also for multimedia applications. Watson and Sasse [28] claim that these ITU-recommended methods for subjective quality assessment are not suitable for assessing the quality of many newer services and applications, and they describe their testing methodology.

Given that there exists this disagreement and this diversity of ways of measuring subjective audio quality, *objective assessment* is still well adapted to study the effects that certain factors have over audio quality. In objective tests, audio quality is evaluated by measuring the distortion between the decoded audio signal and the original signal. Objective tests can be made using a great diversity of measures, like SNR distortion, loss rate, etc.

Audio quality has been studied in various ways. Bolot et al. [4] have recently proposed a linear optimization method giving the best set of parameters to obtain the maximum possible (subjective) quality, and has implemented it in Freephone. Sanneck et al. [22] propose a new FEC scheme that they have called "speech property-based FEC" which adjusts the amount of added redundancy adaptively to the properties of the speech signal, and uses objective quality standardized [27] measures to evaluate audio quality. Meky and Saadawi [14] use radial basis functions neural networks for predicting speech quality.

In this paper we address the problem of audio quality under the media-specific FEC scheme depicted in Figure 2. We evaluate analytically the audio quality at the destination as a function of the parameters of the FEC scheme, of the basic audio flow and of the network. The performance of this FEC scheme has been evaluated via simulations [19, 18], and tools like Freephone and Rat have implemented it.

In [13], the authors propose to increase the offset between the original packet and its redundancy. They claim that the loss process is bursty and thus, increasing the offset could give better performance than having the redundancy placed in the packet following immediately the original; however, they didn't propose any analytic expression that permits to study the impact of this spacing on the audio quality.

We use probabilistic methods and a ballot theorem[24] to find an explicit expression for the audio quality in the case of a general offset not necessarily equal to one. We use a simple function for the audio quality proportional to the volume of data we receive. We call this a *linear* utility function. Our results show that we always lose in quality with this simple FEC scheme. Some similar negative results have already been obtained using analytical tools for more sophisticated media independent FEC schemes, see [7, 1, 11]. The fact that we always loose by adding FEC can be due to several factors: (i) the FEC is inefficient, (ii) the modeling of the dynamics is inadequate, (iii) the modeling of the utility function is inadequate. We examine these points in the two last sections of the paper. In particular, we extend the analysis to general service times, and analyze the the case of multiplexing between several input flows (the audio flow and an exogenous flow). We show that even then, adding FEC leads to poor performance, although in some cases we do observe some gain. We then show that for other utility functions as proposed by Shenker [23], adding FEC can improve substantially the performance. Although these last results justify the use of media-specific FEC for telephony over Internet, we believe that the poor performance of FEC using the linear utility function is a good indication that the media-specific FEC is inefficient, and should be replaced by more efficient ones; we discuss this issue in more detail in the concluding section.

The paper is organized as follows, in Section 2 we describe the general scenario for applications using FEC, and we define a quality function which we'll use throughout most of the rest of the paper. In Section 3 we study the simple case where packet $n+1$ carries redundant information of packet n assuming an $M/M/1/K$ queueing model. In Section 4 we solve the problem in the general case where packet $n+\phi$ carries redundant information of packet n with $\phi \geq 1$. We look in Section 5 at the quality in the case of infinite spacing $\phi \to \infty$. We analyze the case of multiplexing between several input flows (the audio flow and an exogenous flow) in Section 6. In Section 7 we propose the use of different utility functions to obtain a quality function that depends on the characteristics of the application. We present some concluding remarks in Section 8.

2 Analysis

In a large network as the Internet, a flow of packets crosses several routers before reaching the other end. Most of the losses from a flow occur in the router having the smallest available bandwidth in the chain of routers, so that one may model the whole chain of routers by one single router called "*the bottleneck*". This assumption has both theoretical and experimental [2, 6] justification. We shall use the simple $M/M/1/K$ queue to model the network and thus the loss process of audio packets. In other words, we assume that packets arrive at the bottleneck according to a Poisson process of intensity λ, and we assume that the time required to process a packet at the bottleneck is distributed according to an exponential random variable of parameter μ. The Poisson assumption on the inter-arrival times could be justified by the random delay added to packets by non routers located upstream the bottleneck. The service time represents the time between the beginning of the transmission of an audio packet on the bottleneck interface leading to the destination until the beginning of the transmission of the next packet from the same audio flow. Since the two packets may be spaced apart by a random number of packets from other applications, one may use the exponential distribution as a candidate for modeling the service time of audio packets at the bottleneck. The reason for choosing this simplistic model for the network is to be able to obtain simple mathematical formulas that give us some insights on the gain from using FEC.

Let $\rho = \lambda/\mu$ be the intensity of audio traffic. For $\rho < 1$, the loss probability of a packet in steady state is given by [12]

$$\pi(\rho) = \frac{1-\rho}{1-\rho^{K+1}}\rho^K, \tag{1}$$

and for $\rho = 1$ it is equal to

$$\pi(\rho) = \frac{1}{K+1}.$$

Now we add redundancy to each packet in a way that if a packet is lost, it can be still "partially" retrieved if the packet containing its redundancy is not lost. The redundancy is located ϕ packets apart from the original packet. It consists in a low quality copy of the original packet. Let α be the ratio of the volume of the redundant information and the volume of the original packet. α is generally less than one. Along with the possibility to retrieve the information lost in the network, we should consider the negative impacts of the addition of FEC on the loss probability. This addition may have an impact on the service times since packets require now more time to be retransmitted at the output of the bottleneck. It may also have an impact on the buffering capacity at the bottleneck since each packet now contains more bits. We shall thus propose the following two possible negative impacts of FEC, in order to study later the tradeoff between the positive and negative impacts:

- **Impact of FEC on service time.** We assume that audio packets including redundancy require now a longer service time which is exponentially distributed with parameter $\frac{\mu}{(1+\alpha)}$. This can be the case when our audio flow has an important share of the bottleneck bandwidth. If it is not the case, this assumption can hold when the exogenous traffic at the bottleneck (or at least an important part of it) is formed of audio flows that implement the same FEC scheme.

- **Impact of FEC on buffering.** The loss probability will be affected by this procedure in one of the following ways: (1) Since packets are now longer by a factor $(1+\alpha)$, we can consider that the amount of buffering is diminished by this quantity, or (2) We can assume that the queue capacity is not function of packet length, but rather of number of packets. In that case, the queue capacity is not affected by inserting FEC. The loss probability in the presence of FEC takes the form indicated in Eq. (2).

$$\pi_\rho(\alpha) = \frac{1-\rho_\alpha}{1-\rho_\alpha^{K_\alpha+1}}\rho_\alpha^{K_\alpha} \tag{2}$$

where,

$$\begin{aligned} \rho_\alpha &= \rho(1+\alpha) \\ K_\alpha &= \frac{K}{1+\alpha} \quad \text{for case (1),} \\ &= K \quad \text{for case (2).} \end{aligned} \tag{3}$$

Before we define the quality of audio received at the destination, we introduce a random variable Y_n that indicates a successful arrival of a packet at the destination or no. Then,

$$\begin{aligned} Y_n &= 0, \quad \text{if packet } n \text{ is lost, and} \\ Y_n &= 1, \quad \text{if packet } n \text{ is correctly received.} \end{aligned}$$

Let ϕ be the variable indicating the distance between the original packet and its redundancy. We make the simple assumption that the audio quality is proportional to the amount of information we received. A quality 1 indicates that we are receiving all the information (the basic audio flow). The quality we get after the reconstruction of an original packet from the redundancy is taken equal to α, where α is the ratio of redundancy volume to original packet volume. We thus define the quality function as,

$$\begin{aligned} & Q(\alpha) \\ & = P(Y_n = 1) + \alpha P(Y_n = 0) P(Y_{n+\phi} = 1 | Y_n = 0) \\ & = 1 - \pi_\rho(\alpha)(1 - \alpha P(Y_{n+\phi} = 1 | Y_n = 0)) \end{aligned} \tag{4}$$

This equation gives us the audio quality got at the destination under a FEC scheme of rate $1+\alpha$, and of distance ϕ between an original packet and its redundancy. For the case $\alpha = 0$, our definition for the quality coincides with the probability that a packet is correctly received. For the case of $\alpha = 1$, it coincides with the probability that the information in an original packet is correctly received, either because it was not lost, or because it was fully retrieved from the redundancy. One may imagine to use another quality function that the one we chose. In particular, one can use a quality which is not only a function of the amount of data correctly received but also of the coding algorithm used. Different algorithms have been used in [10, 15] for coding the original data and the redundancy. In the rest of the paper, we will use the following notation:

We ask the following question: "*How does the audio quality vary as a function of α?*" That would permit us to evaluate the benefits from such a recovery mechanism and to find the appropriate amount of redundancy α that must be added to each packet. In the next sections we find the audio quality for different values of ϕ. The only missing parameter is the probability that the redundant information on a packet is correctly received given that the packet itself is lost. This is the parameter $P(Y_{n+\phi} = 1 | Y_n = 0)$ in Eq. (4). In the following sections we put ourselves in the stationary regime and we calculate this probability.

3 Spacing by $\phi = 1$

In this section we analyze the case where the redundant information on packet n is carried by packet $n+1$, i.e., $\phi = 1$. This mechanism is implemented e.g. in [10]. The

Table 1: Notation used in this paper.

Expression	**Definition**
$Q(\alpha)$	The audio quality.
ϕ	The offset between the original packet and the packet including its redundancy.
K_α	The size of the queue just as it's defined in Eq. (3).
X_j	The random variable which represents the number of packets in the system just before the arrival of the j-th audio packet.
Z_j	The random variable which represents the number of services between the arrivals of the $j-1$-th and the j-th audio packets[a].

[a]As is frequently done, we include in Z_j not only real services but also "potential services": these are services that occur while the system is empty; thus at the end of such a service no packet leaves.

probability that the redundancy is correctly received given that the original packet is lost, is no other than the probability that the next event after the loss of the original packet is a departure and not an arrival. This happens with probability,

$$P(Y_{n+1} = 1 | Y_n = 0) = \frac{1}{\rho(1+\alpha)+1}. \tag{5}$$

Substituting (5) in (4), we obtain

$$Q_{\phi=1}(\alpha) = 1 - \pi_\rho(\alpha)\Big(1 - \frac{\alpha}{\rho(1+\alpha)+1}\Big).$$

To study the impact of FEC on the audio quality, we plot $Q_{\phi=1}(\alpha)$ as a function of α for different values of K and ρ. In Figure 3, we show the results when the buffering capacity at the bottleneck is assumed to change with the amount of FEC, and in Fig. 4 we show the results for the case where the buffering capacity is not changed. The two cases are described in Section 2. We see that, for both cases, audio quality deteriorates when α increases (when we add more redundancy), and this deterioration becomes more important when the traffic intensity increases and

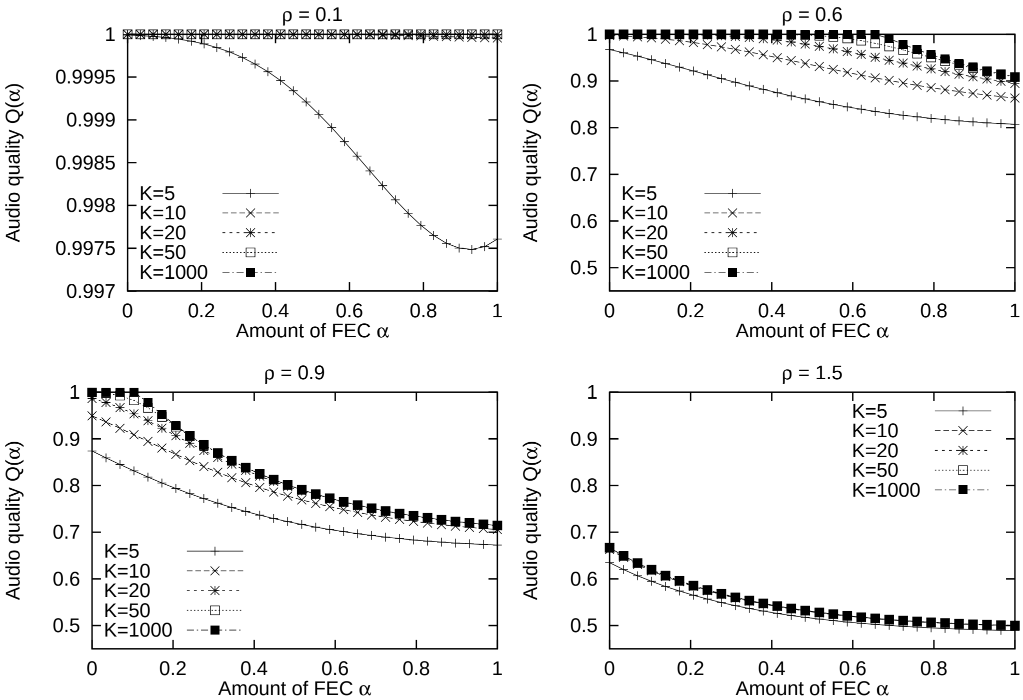

Figure 3: $\phi = 1$ and the queue capacity is changed.

when the buffer size decreases. The main interpretation of such behavior is that the loss probability of an original packet increases with α faster than the gain in quality we got from retrieving the redundant information. This should not be surprising. Indeed, even in more sophisticated schemes in which a single redundant packet is added to protect a whole block of M packets, it is known that FEC often has an overall negative effect, see [7, 1, 11]. Yet in such schemes the negative effect of adding the redundancy is smaller than our scheme, since the amount of added information per packet is smaller (since a single packet protects a whole group of M packets). Note however, that for such schemes we know that in case of light traffic the overall contribution of FEC is positive [1, 11]. This motivates us to analyze more precisely the impact of FEC in our simplistic scheme in case of light traffic.

Define the function $\Delta(\rho) = Q(1) - Q(0)$ and consider the case where the buffering capacity at the bottleneck is not affected by the amount of FEC. This is an optimistic

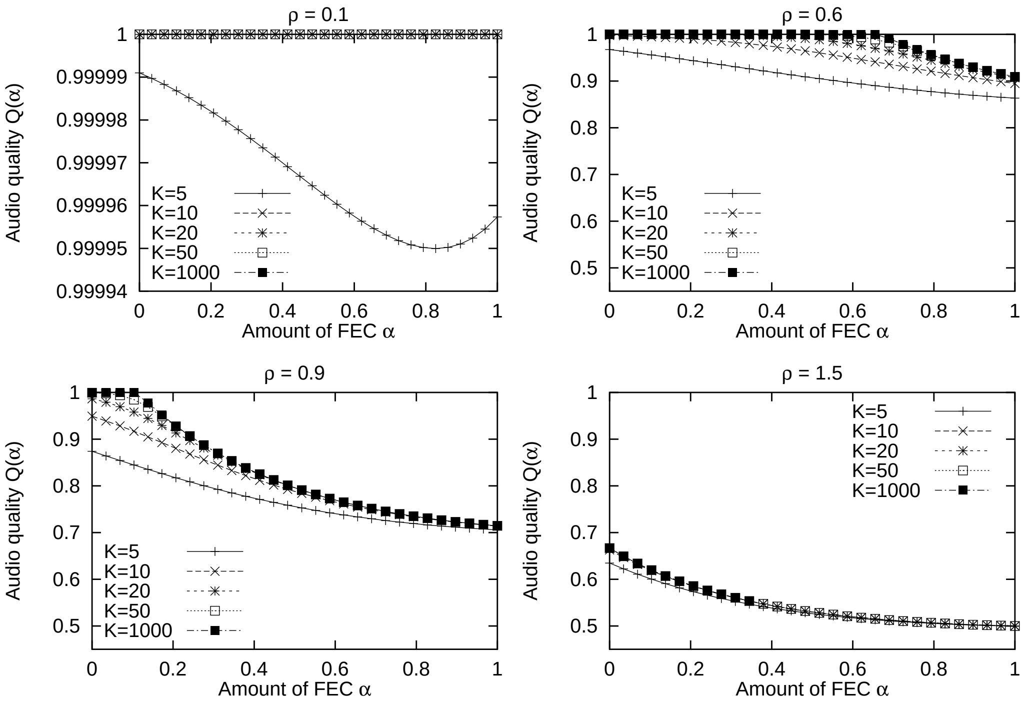

Figure 4: $\phi = 1$ and the queue capacity is not changed.

scenario where it is very probable to see the gain brought by FEC, of course if this gain exists. We have,

$$\Delta(\rho) = -2(2\rho)^{\frac{K}{2}} \left(\frac{1+\rho}{2\rho+1}\right)\left(\frac{1-2\rho}{1-(2\rho)^{\frac{K}{2}+1}}\right) + \frac{1-\rho}{1-\rho^{K+1}}\rho^K \tag{6}$$

Finding $\lim_{\rho\to 0} \Delta(\rho)$ would permit us to evaluate the audio quality for a very low traffic intensity. We took $K = 2M$ in Eq. (6) and we expanded $\Delta(\rho)$ in a Taylor series. We found that all the first coefficients of the series $c_0, c_1, ..., c_{M-1}-$th are equal to zero, and that the coefficient c_M is negative and equal to $-2(2\rho)^M$. c_i is the coefficient of ρ^i is the Taylor series of $\Delta(\rho)$ and can be calculated by

$$c_j = \frac{d}{d\rho^j}\Delta^j(\rho)_{|\rho=0}$$

Thus, for small ρ, $\Delta(\rho)$ can be written as $-2(2\rho)^M + o(\rho^M)$ and the gain from the addition of FEC can be considered as negative. With this simple FEC scheme, we lose in audio quality when adding FEC even for a very low traffic intensity. This loss in quality decreases with the increase in buffer size.

4 General case: Spacing by $\phi \geq 1$

Now, we consider the more general case where the spacing between the original packet and its redundancy is greater than 1. The idea behind this type of spacing is that losses in real networks tend to appear in bursts and thus spacing the redundancy from the original packet by more than one packet improves the probability to retrieve it in case the original packet is lost. Indeed, a packet loss means that the queue is full and then the probability of losing the next packet is higher than the steady state probability of losing a packet. The spacing gives the redundancy of a packet more chance to find a non full buffer at the bottleneck, and thus to be correctly received. We note that the phenomenon of the correlation between losses of packets was already modeled and studied in other papers: [7, 1, 11]. Measurements have also shown that most of the losses are correlated [5, 3, 21].

Here, we are interested in finding the probability that packet $n+\phi$ is lost given that packet n is also lost. This will give us $P(Y_{n+\phi} = 1 | Y_n = 0)$ which in turn gives us the expression for the audio quality (Eq. 4). Since we assume that the system is in its steady state, we can omit the index n and substitute it by zero. Let K_α denote the buffer size after the addition of FEC just as it's defined in Section 2. We have $Y_0 = 0$ which means $X_0 = K_\alpha$. We are interested in the probability that $X_\phi = K_\alpha$. For the ease of calculation we consider the case $\phi \leq K_\alpha$. We believe that this is quite enough given that a large spacing between the original packet and the redundancy leads to an important jitter and a poor interactivity.

In order to obtain an explicit expression for the probability $P(X_\phi = K_\alpha | X_0 = K_\alpha)$, we first provide an explicit sample-path expression for the event of loss of the packet carrying the redundancy, given that the original packet itself was lost.

Theorem 1 *Let $X_0 = K_\alpha$ and $1 \leq \phi \leq K_\alpha$. then:*
Packet ϕ is not lost if and only if

$$X_\phi < K_\alpha \Leftrightarrow \begin{cases} Z_\phi - 1 & \geq 0 \\ & or \\ Z_\phi + Z_{\phi-1} - 2 & \geq 0 \\ & or \\ & \vdots \\ & or \\ Z_\phi + Z_{\phi-1} + \cdots + Z_1 - \phi & \geq 0 \end{cases}$$

or equivalently, packet ϕ is lost if and only if

$$X_\phi = K_\alpha \Leftrightarrow \begin{cases} Z_\phi - 1 & < 0 \\ & and \\ Z_\phi + Z_{\phi-1} - 2 & < 0 \\ & and \\ & \vdots \\ & and \\ Z_\phi + Z_{\phi-1} + \cdots + Z_1 - \phi & < 0 \end{cases} \tag{7}$$

Proof. We can express the number of packets that the $i+1$-th audio packet will find in the queue upon arrival as follows:

$$X_{i+1} = \Big((X_i + 1) \wedge K_\alpha - Z_{i+1}\Big) \vee 0 \quad \forall i \geq 0, \tag{8}$$

where $\wedge$ and $\vee$ are respectively the minimum and maximum operators. The rest of the proof goes in three steps that are summarized in Lemma 1, Lemma 2 and Corollary 1 below.

■

Now, we define

$$\tilde{X}_{i+1} \triangleq (\tilde{X}_i + 1) \wedge K_\alpha - Z_{i+1} \tag{9}$$

This new variable corresponds to the number of packets that would be found in the queue upon the arrival of packet $i+1$ if the queue size could become negative. We next show that it can be used as a lower bound for X_{i+1}.

Lemma 1 *If $\tilde{X}_0 \leq X_0$ then $\tilde{X}_i \leq X_i \quad \forall i \geq 0$.*

Proof. We proceed in the proof by induction. This relation is valid for $i = 0$. Suppose that it is valid for $i \geq 0$. We show that it is valid for $i+1$,

$$\begin{aligned}
\tilde{X}_{i+1} &\leq \Big((\tilde{X}_i + 1) \wedge K_\alpha - Z_{i+1}\Big) \vee 0 \\
&\leq \Big((X_i + 1) \wedge K_\alpha - Z_{i+1}\Big) \vee 0 \\
&= X_{i+1}
\end{aligned}$$

■

Lemma 2 *Let $\tilde{X}_0 = K_\alpha$, then $\tilde{X}_i = K_\alpha - \max_{1 \leq l \leq i} \sum_{j=l}^{i} (Z_j - 1) - 1 \quad \forall i \geq 0$.*

Proof.

$$\begin{aligned}
\tilde{X}_0 &= K_\alpha \\
\tilde{X}_1 &= K_\alpha - Z_1 \\
\tilde{X}_2 &= (K_\alpha - Z_1 + 1) \wedge K_\alpha - Z_2 \\
\tilde{X}_3 &= \Big((K_\alpha - Z_1 + 1) \wedge K_\alpha - Z_2 + 1\Big) \wedge K_\alpha \\
&\quad - Z_3 \\
&= (K_\alpha - Z_1 - Z_2 + 2) \wedge (K_\alpha - Z_2 + 1) \wedge K_\alpha \\
&\quad - Z_3 \\
&= K_\alpha - (Z_1 + Z_2 - 2) \vee (Z_2 - 1) \vee 0 - Z_3 \\
&\quad \vdots \\
\tilde{X}_i &= K_\alpha - \max_{1 \leq l < i} \Big\{0, \sum_{j=l}^{i-1} (Z_j - 1)\Big\} - Z_i \\
&= K_\alpha - \max_{1 \leq l < i} \Big\{0, \sum_{j=l}^{i-1} (Z_j - 1)\Big\} - Z_i \\
\Rightarrow \tilde{X}_i &= K_\alpha - \max_{1 \leq l \leq i} \Big\{\sum_{j=l}^{i} (Z_j - 1)\Big\} - 1.
\end{aligned}$$

■

Corollary 1 *Expression (7) holds if $X_0 = K_\alpha$ and $\phi \leq K_\alpha$.*

Proof. The right hand side in (7) is no other than $\max_{1\le l\le\phi}\Big\{\sum_{j=l}^{\phi}(Z_j-1)\Big\}$. Suppose first that $\max_{1\le l\le\phi}\Big\{\sum_{j=l}^{\phi}(Z_j-1)\Big\}<0$. Using Lemma 2 then Lemma 1, we have $\tilde{X}_\phi \ge K_\alpha$ which gives $X_\phi \ge K_\alpha$. Thus, $X_\phi = K_\alpha$.

Now, we need to show that if $X_\phi = K_\alpha$, we get $\max_{1\le l\le\phi}\Big\{\sum_{j=l}^{\phi}(Z_j-1)\Big\}<0$. We define:

$$\phi^* = \min\Big\{\phi \quad | \quad \tilde{X}_\phi < X_\phi\Big\} \tag{10}$$

According with (10), we distinguish between the two following cases:

- $\phi^* > \phi$, and
- $\phi^* \le \phi$.

Consider the first case. Using the definition of ϕ^* and Lemma 2, we write: $\phi^* > \phi \quad \Rightarrow \quad \tilde{X}_\phi = X_\phi \Rightarrow \tilde{X}_\phi = K_\alpha \Rightarrow \max_{1\le l\le\phi}\Big\{\sum_{j=l}^{\phi}(Z_j-1)\Big\}<0$.

Now, suppose that $\phi^* \le \phi$, thus $\tilde{X}_{\phi^*} < 0$ and $X_{\phi^*} = 0$. We write,

$$X_\phi \le X_{\phi^*} + (\phi - \phi^*) = (\phi - \phi^*) < \phi \le K_\alpha,$$

if there were no service. Thus, we get in this case $X_\phi < K_\alpha$ which is in contradiction with our assumption that $X_\phi = K_\alpha$. The case $\phi^* \le \phi$ does not appear if ϕ is chosen less or equal to the buffering capacity. We conclude that for $X_\phi = K_\alpha$, we have $\max_{1\le l\le\phi}\Big\{\sum_{j=l}^{\phi}(Z_j-1)\Big\}<0$.

■

According to Ballot's Theorem [24] (see the Appendix in Section 8 for details), we have for $k < \phi$:

Lemma 3

$$P\Big\{\max_{1\le l\le\phi}\Big\{\sum_{j=l}^{\phi}(Z_j-1)\Big\}<0 \,|\, \sum_{l=1}^{\phi} Z_l = k\Big\} = 1 - \frac{k}{\phi}$$

Let A^ϕ be the event that $X_\phi = K_\alpha$ given that $X_0 = K_\alpha$. We sometimes write A^ϕ to stress the dependence on ϕ. We conclude from Theorem 1 that if packet 0 is lost, i.e. if packet 0 finds K_α packets in the system, then

$$A^{\phi} = \left\{ \max_{1 \le l \le \phi} \left\{ \sum_{j=l}^{\phi} (Z_j - 1) \right\} < 0 \right\}$$

Then, we can represent the probability that packet $n+\phi$ is lost given that packet n is lost as

$$P(Y_{n+\phi} = 0 | Y_n = 0) = P(A^{\phi})$$

$$= \sum_{k=0}^{\phi-1} P(A^{\phi} | Z_1 + \cdots + Z_{\phi} = k) P(Z_1 + \cdots + Z_{\phi} = k) \tag{11}$$

Once this probability is calculated, the audio quality can be directly derived using (4).

Theorem 2 *Consider $1 \le \phi \le K_{\alpha}$ and let $\rho_{\alpha} = \rho(1+\alpha)$. Given that packet n is lost, the probability that packet $n+\phi$ is also lost is given by*

$$P(A^{\phi}) = \sum_{k=0}^{\phi-1} \left(1 - \frac{k}{\phi}\right) \left(\frac{\rho_{\alpha}}{\rho_{\alpha}+1}\right)^{\phi} \left(\frac{1}{\rho_{\alpha}+1}\right)^{k} \binom{\phi+k-1}{\phi-1} \tag{12}$$

The quality function can be calculated by substituting $P(A^{\phi})$ in (4). Note that $P(Y_{n+\phi} = 1 | Y_n = 0) = 1 - P(A^{\phi})$.

Proof. The second right hand term of (11) must be solved by combinatorial reasoning. For that purpose, we define the vector $\vec{Z}$ to be:

$$\vec{Z} = \begin{pmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_{\phi} \end{pmatrix} \tag{13}$$

where $\sum_{l=1}^{\phi} Z_l = k$, and we define S be the set of the different sets that $\vec{Z}$ may acquire: $S = \{\vec{Z}\}$. We must sum over all the possible trajectories:

$$
\begin{aligned}
& P(\sum_{l=1}^{\phi} Z_l = k) \\
&= \sum_S P(Z_1 = z_1)P(Z_2 = z_2)\cdots P(Z_\phi = z_\phi) \\
&= \sum_S \Big(\frac{\lambda}{\lambda+\mu_\alpha}\Big)^{\phi}\Big(\frac{\mu_\alpha}{\lambda+\mu_\alpha}\Big)^{k} \\
&= \Big(\frac{\lambda}{\lambda+\mu_\alpha}\Big)^{\phi}\Big(\frac{\mu_\alpha}{\lambda+\mu_\alpha}\Big)^{k} \mathrm{C}_{k+\phi-1}^{\phi-1} \qquad (14)
\end{aligned}
$$

We define μ_α as being equal to $\mu/(1+\alpha)$. It's easy to see that the combinatorial part of (14) holds. To do that, we can see the problem to be the number of distinguishable arrangements of k indistinguishable objects (the packet audio departures from the bottleneck) in ϕ interarrival intervals, just as it's depicted in Fig. 5.

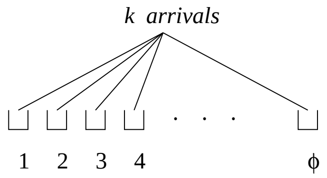

Figure 5: Model to solve the combinatorial part.

Using (14) we get finally,

$$
P(A^{\phi}) = \sum_{k=0}^{\phi-1}\Big(1-\frac{k}{\phi}\Big)\Big(\frac{\lambda}{\lambda+\mu_\alpha}\Big)^{\phi}\Big(\frac{\mu_\alpha}{\lambda+\mu_\alpha}\Big)^{k}\binom{\phi+k-1}{\phi-1} \qquad (15)
$$

which yields (12) in terms of $\rho_\alpha = \rho(1+\alpha) = \lambda/\mu_\alpha$. The quality function can be obtained by substituting (12) in (4). The value of $\pi_\alpha(\rho)$ is given in (2) and (2). ∎

We trace now plots of the audio quality as given by (4) and (12) for different values of K_α, ϕ and ρ. Figure (6) depicts the behavior of $Q(\alpha)$ when the buffering capacity at the bottleneck is assumed to be divided by a factor $(1+\alpha)$, and Figure (7) depicts this behavior when the buffering capacity is not changed.

We notice that, just as in the case of $\phi = 1$, we always lose in quality when we increase the amount of FEC even if we consider a large spacing. But, we notice also

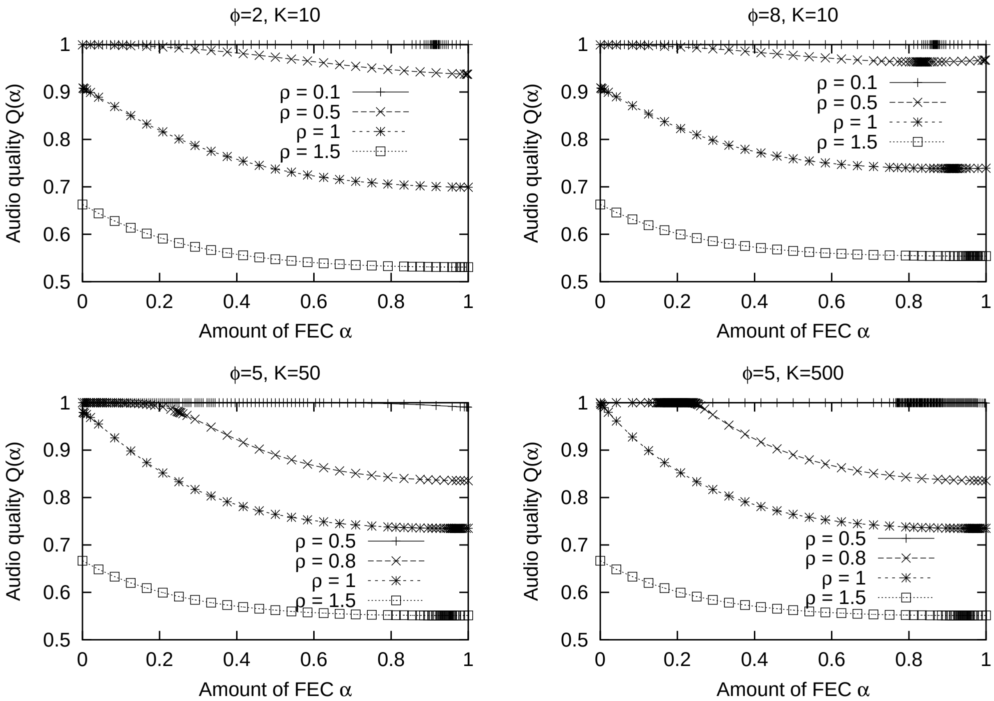

Figure 6: Quality behavior in the presence of FEC and spacing $1 < \phi < K_\alpha$ assuming thatqueue size is changed.

that for a given amount of FEC, the quality improves when spacing the redundancy from the original packet. This is the result of an improvement in the probability to retrieve the redundancy given that the original packet is lost. This monotonicity property holds, in fact, for any value of ϕ (not just for $\phi \le K_\alpha$). We show this theoretically in the next section.

4.1 Monotone increase of the quality with the spacing

The probability of loss of a packet n does not depend on ϕ. It thus remains to check the behavior of $P(X_{n+\phi} = K_\alpha | X_n = K_\alpha)$ as a function of ϕ in order to decide on the quality variation (Eq. 4). The quality is a decreasing function of this probability. For $\phi \le K_\alpha$, the latter probability is equal to $P(A^\phi)$, and the monotonicity property

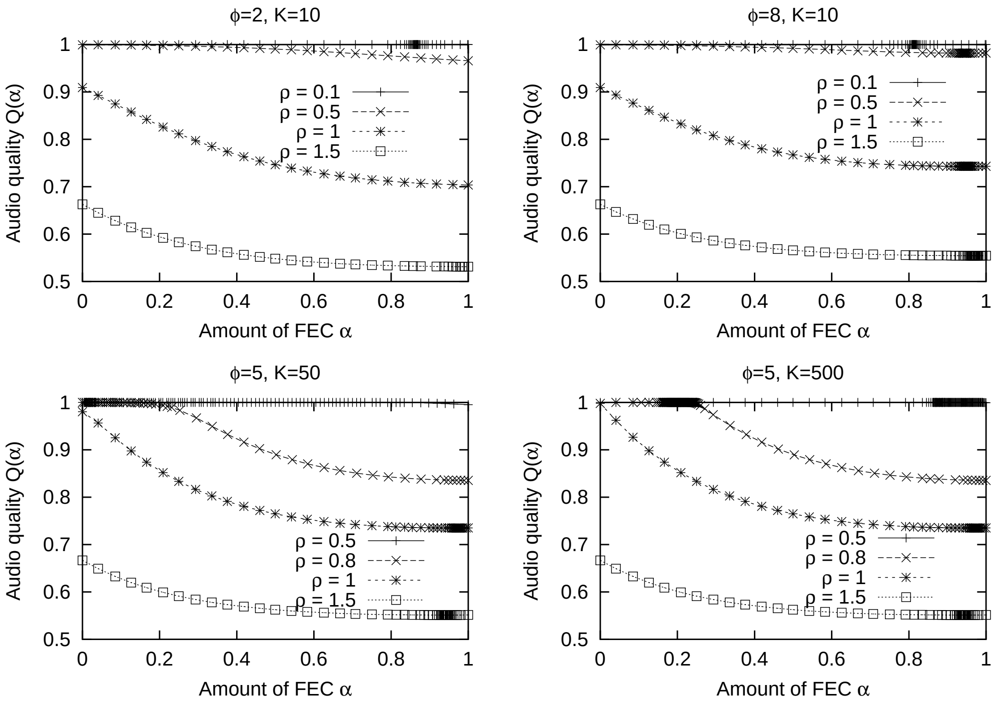

Figure 7: Quality behavior in the presence of FEC and spacing $1 < \phi < K_\alpha$ assuming that queue size is not changed.

can be seen directly from the fact that A^ϕ is a monotone decreasing set (since it requires for more summands to be smaller than zero, as ϕ increases, see Eq. (7)).

Now, to see that $P(X_{n+\phi} = K_\alpha | X_n = K_\alpha)$ is monotone decreasing for any ϕ, we observe (8), which holds for any $i > 0$, and note that X_{i+1} is monotone increasing in X_i. Thus by iteration, we get that X_ϕ is monotone increasing in X_0. Now using this monotonicity, we have

$$\begin{aligned} P(X_{\phi+1} = K_\alpha | X_0 = K_\alpha) &= P(X_\phi = K_\alpha | X_{-1} = K_\alpha) \\ &= \sum_{i=0}^{K_\alpha} P(X_\phi = K_\alpha | X_0 = i, X_{-1} = K_\alpha) \times \\ &\quad P(X_0 = i | X_{-1} = K_\alpha) \end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=0}^{K_\alpha} P(X_\phi = K_\alpha | X_0 = i) P(X_0 = i | X_{-1} = K_\alpha) \\
&\leq \sum_{i=0}^{K_\alpha} P(X_\phi = K_\alpha | X_0 = K_\alpha) P(X_0 = i | X_{-1} = K_\alpha) \\
&= P(X_\phi = K_\alpha | X_0 = K_\alpha)
\end{aligned}
$$

5 Limiting case: Spacing $\phi \to \infty$

The case of large ϕ is not of interest in interactive applications, since it means unacceptable delay. However, since we have found that the quality of the audio with FEC improves as the spacing grows, it is natural to study the limit ($\phi \to \infty$) in order to get an upper bound. Indeed, if we see that in this limiting case we do not improve the quality, it means that we lose in adding FEC according to our simple scheme for any finite ϕ.

When $\phi \to \infty$, Eq. (4) becomes:

$$Q_{\phi \to \infty}(\alpha) = 1 - \pi_\rho(\alpha) + \alpha \pi_\rho(\alpha)(1 - \pi_\rho(\alpha)) \tag{16}$$

We plot (16) in Figure 8 as a function of the amount of FEC for different values of K_α and ρ. We see well how, although we are in the most optimistic case, we lose in quality when adding FEC. That suggests that this class of FEC mechanisms are not adequate for real time transmission because it never improves the quality perceived at the receiver.

6 Multiplexing between several flows

Now, we analyze the case when several input flows arrive to the bottleneck, an audio flow and an exogenous flow which represent the superposition of all other flows. We further consider here the case of general independent service time distribution. We consider two scenarios for our analysis. In the first, we assume that the distribution of all packets of all flows is the same. This means that the more we add redundancy to a packet, the less it contains useful information. Thus, with a ratio of redundancy of α, the quality a packet n if it is well received is only $1 - \alpha$, under our linear utility assumption. If the packet is lost and reconstructed, then its utility is α.

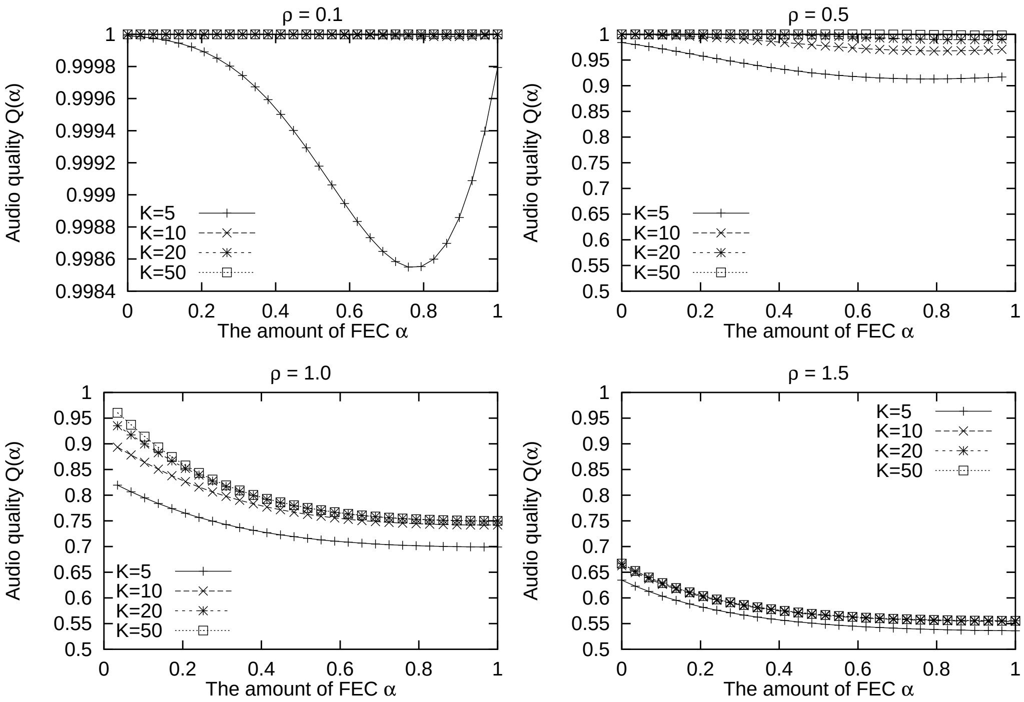

Figure 8: Quality behavior in the presence of FEC and spacing $\phi \to \infty$.

6.1 Case 1: $M + M/G/1/K$ queue with the same service time distribution

In this case we have two independent Poisson input flows, λ_1 and λ_2, representing the audio flow and the exogenous traffic (which represnts all other flows) respectively.

Define $A = \lambda_2/\lambda_1$. After a little adjustment, we obtain the following expression for the loss probability: Obviously the loss probability which equals to the steady state probability $\pi(K)$ that the queue is full (due to the PASTA property), does not depend on α (the quantity of FEC added to audio packets) because it does not affect the audio packet size.

One can again show that the quality is increasing with the spacing ϕ (where ϕ is the distance between the original packet and the packet containing its redundancy). Thus, by showing that we lose by adding FEC for $\phi \to \infty$, we conclude that we lose

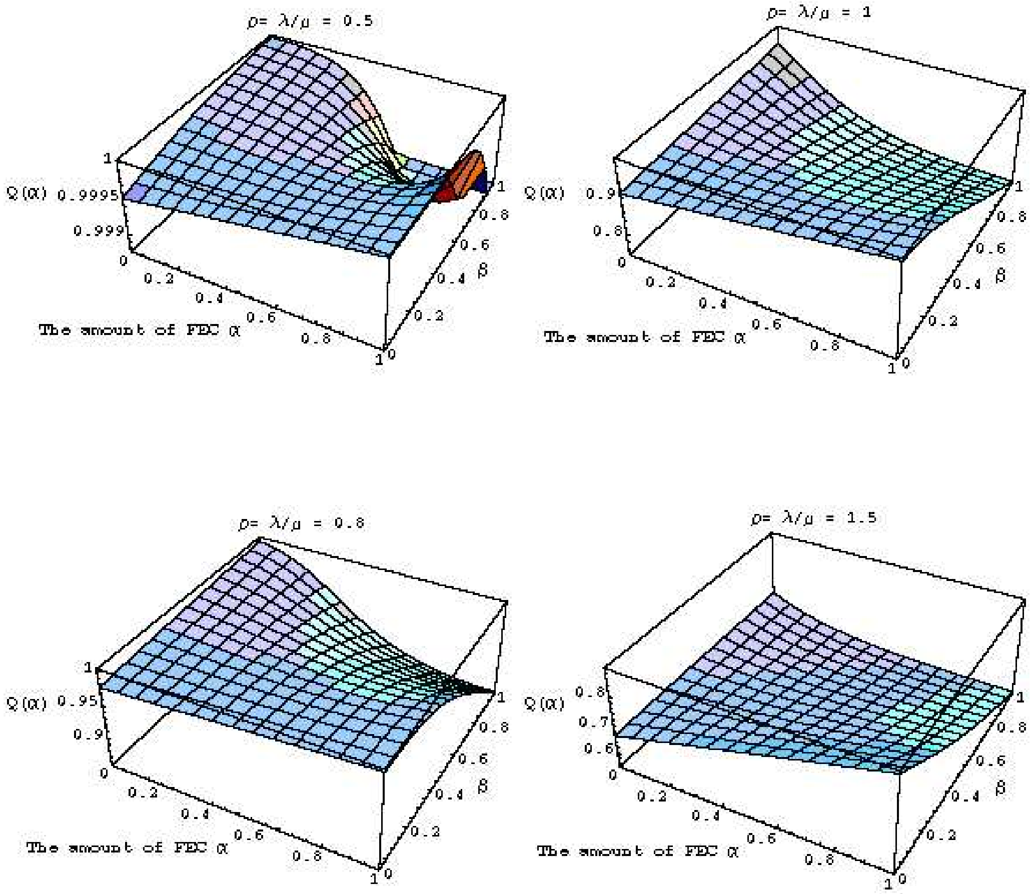

Figure 9: Audio quality for an $M/G/1/K$ queue with two flows: the audio flow and the exogenous flow. β represents the probability that an arriving packet belongs to the audio flow.

by adding FEC for any spacing. Because our assumption, we have now a new quality function given by:

$$\begin{aligned} Q(\alpha)_{|\phi\to\infty} &= (1-\alpha)(1-\pi(K)) + \alpha\pi(K)(1-\pi(K)) \\ &= (1-\pi(K))[1-\alpha(1-\pi(K))] \end{aligned}$$

We see that this quality function is always decreasing α, meaning that we're also losing quality in this case.

6.2 Case 2: $M + M/G/1/K$ queue with service time that depends on the FEC

We consider now a queue of capacity K where K is independent of the packet size. An audio flow crosses this queue together with packets from other flows. Packets from all flows arrive to the queue according to a Poisson process of intensity λ. Let β denote the probability that an arriving packet belongs to the audio flow. We consider a secnario in which the size of the audio packet increases with the amount FEC, whereas the distribution of the size of exogenous packets remains unchange. Packets from the exogenous traffic have a service rate exponentially distributed with an intensity equal to μ packets/second. Audio packets have a deterministic service rate of $\frac{\mu_0}{1+\alpha}$. The factor α represents the quantity of FEC we're adding.

The resulting system is an $M/G/1/K$ queueing system. We focus here on the audio quality when the redundancy is placed very far from the original data. Again, the quality we get in this case is better than that we obtain for any other spacing strategy. Now, in Eq. (4) the stationary drop probability is given by [8]:

$$\pi_\alpha = \frac{1 + (\rho_\alpha - 1)f}{1 + \rho_\alpha f},$$

with ρ_α the total system load,

$$\rho_\alpha = \beta\frac{\lambda(1+\alpha)}{\mu_0} + (1-\beta)\frac{\lambda}{\mu}$$

and f a factor equal to the $K-2$-th coefficient of the Taylor series of the complex function $1/(B^*(\lambda - \lambda s) - s)$. $B^*(s)$ is the LST of the service time of the overall system. We can write,

$$B^*(s) = \beta e^{-s(1+\alpha)/\mu_0} + (1-\beta)\frac{\mu}{\mu+s}$$

Once $B^*(s)$ is found, f can be calculated by

$$f = \frac{1}{(K-2)!} \frac{d^{K-2}}{ds^{K-2}} \Big(\frac{1}{B^*(\lambda - \lambda s) - s} \Big)_{|s=0}$$

We solve numerically for the quality function in order to see if it is possible to gain in performance when there is a certain amount of traffic that doesn't implement the FEC scheme. Note that in the case of exponential distribution, it follows from our results in previous sections that if all the traffic crossing the queue implements FEC, the quality deteriorates when we add redundancy.

We trace plots for $Q(\alpha)$ in Figure 9 for, for different values of λ/μ with the following assumptions:

$$\begin{aligned} \mu_0 &= \mu, \\ \rho &= \lambda/\mu \\ \lambda &= 10000\, packs/sec \\ K &= 10 \\ \beta &= [0, 1] \\ \alpha &= [0, 1] \end{aligned}$$

We see well that when β reduces and α approaches to one we start obtaining a gain . This motivates the next section that presents an approach proposed by Shenker [23] for real-time applications, in which the use of a utility function is recommended. Although Shenker didn't propose any explicit expression for the utility function, an approach shows that the use of this kind of functions must be implemented just as is shown in next section.

7 Use of different utility functions on FEC schemes

The negative results that we have obtained were for a linear utility function, which measures the *amount of information* that is well received. However, as shown in [23], different applications may have different utility functions, which are typically nonlinear. If a packet which is reconstructed from a proportion of α of additional redundant information, has a higher relative value to the application than α, we may naturally expect the FEC to have better performance than for the linear unitlity.

Table 2 gives some possible utility functions that could serve our needs, and which are similar in their form to utility functions that are proposed in [23]. Figure 10 shows

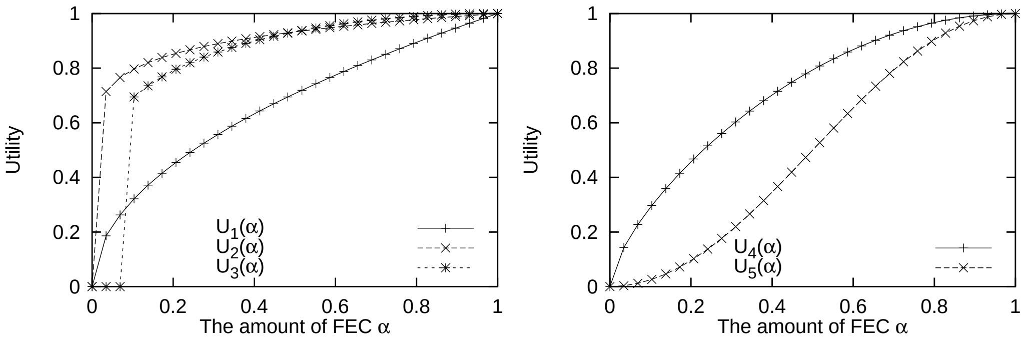

Figure 10: Possible utility functions for rate adaptive applications.

plots for them. The function $esc(\cdot)$ is equal to 1 if its argument is greater than or equal to 0, and zero otherwise.

Table 2: Different possible utility functions.

Function	**Expression**
$U_0(\alpha)$	α
$U_1(\alpha)$	$\sqrt{\alpha}$
$U_2(\alpha)$	$\alpha^{\frac{1}{10}}$
$U_3(\alpha)$	$esc(\alpha - x)\left(\frac{1-\cos\pi\alpha}{2}\right)^{\frac{1}{10}}$
$U_4(\alpha)$	$(\frac{1-\cos\pi\alpha}{2})^{\frac{1}{3}}$

We'll study two cases: $\phi = 1$, and $\phi \to \infty$ to see if we can establish a theoretical limit for the audio quality in case we obtain a gain. We're assumming in this section an $M/M/1/K$ model. Using a utility function, the quality function becomes now:

$$
\begin{aligned}
Q(\alpha)_{|\phi=1} &= (1-\pi_\rho(\alpha))U(1) + \frac{\pi_\rho(\alpha)}{\rho_\alpha + 1}U(\alpha) \qquad \text{for } \phi = 1, \text{ and} \\
Q(\alpha)_{\phi\to\infty} &= \Big(1-\pi_\rho(\alpha)\Big)U(1) + \Big(\pi_\rho(\alpha)(1-\pi_\rho(\alpha))\Big)U(\alpha) \\
&= \Big(1-\pi_\rho(\alpha)\Big)\Big(1+U(\alpha)\pi_\rho(\alpha)\Big) \qquad \text{for} \phi \to \infty.
\end{aligned}
$$

Figure 11 shows the behavior of the $Q(\alpha)_{|\phi=1}$ when we use some of the utility functions of Table 2, and compares with the case when we don't use it. Figure 12 shows the same but for function $Q(\alpha)_{\phi\to\infty}$.

We consider naturally utility functions that are equal to 0 at $\alpha = 0$. Clearly, for any given α, the gain by adding FEC (with ratio of α) is *increasing* in the utility. Hence the best quality that can be obtained is for a utility function that approaches the function $esc(\cdot)$ (and for spacing tending to infinity, as discussed previously). In that case we have indeed:

$$\begin{aligned} Q(\alpha)_{|\phi\to\infty} &= 1-\pi_\rho(\alpha)+\pi_\rho(\alpha)(1-\pi_\rho(\alpha)) \\ &= \Big(1-\pi_\rho(\alpha)\Big)+\pi_\rho(\alpha)\Big(1-\pi_\rho(\alpha)\Big) \\ &= \Big(1-\pi_\rho(\alpha)\Big)\Big(1+\pi_\rho(\alpha)\Big), \end{aligned}$$

This gives an improvement of a factor of

$$(1+\pi_\rho(\alpha))\frac{1-\pi_\rho(\alpha)}{1-\pi_\rho(0)}$$

with respect to the case of no FEC. We note that the above equation also holds for the case of an M/G/1 queue, as well as for an M+M/G/1 queue (only the expression for $\pi_\rho(\alpha)$ changes). We thus conclude the following:

Lemma 4 *For an M/G/1/∞ queue, as well as for an M+M/G/1/∞ queue, the audio quality cannot improve by more than twice by adding FEC, for any utility function. We can approach this bound by adding a small amount of FEC if the utility function is close to a step function, if the steady-state loss probability is large, and if spacing is large.*

Figures 11 and 12 depict the gain we can obtain in comparison with the case $U_0(\alpha) = \alpha$ and $U_1(\alpha) = \sqrt{\alpha}$. We see a little jump in the neighborhood of $\alpha = 0$, showing the effect of the utility function $U_2(\alpha) = \alpha^{\frac{1}{10}}$.

8 Conclusions

We have studied the effect that FEC schemes similar to that used in [10] have on audio quality. We consider the different spacing strategies $\phi = 1$, $1 \le \phi \le K_\alpha$,

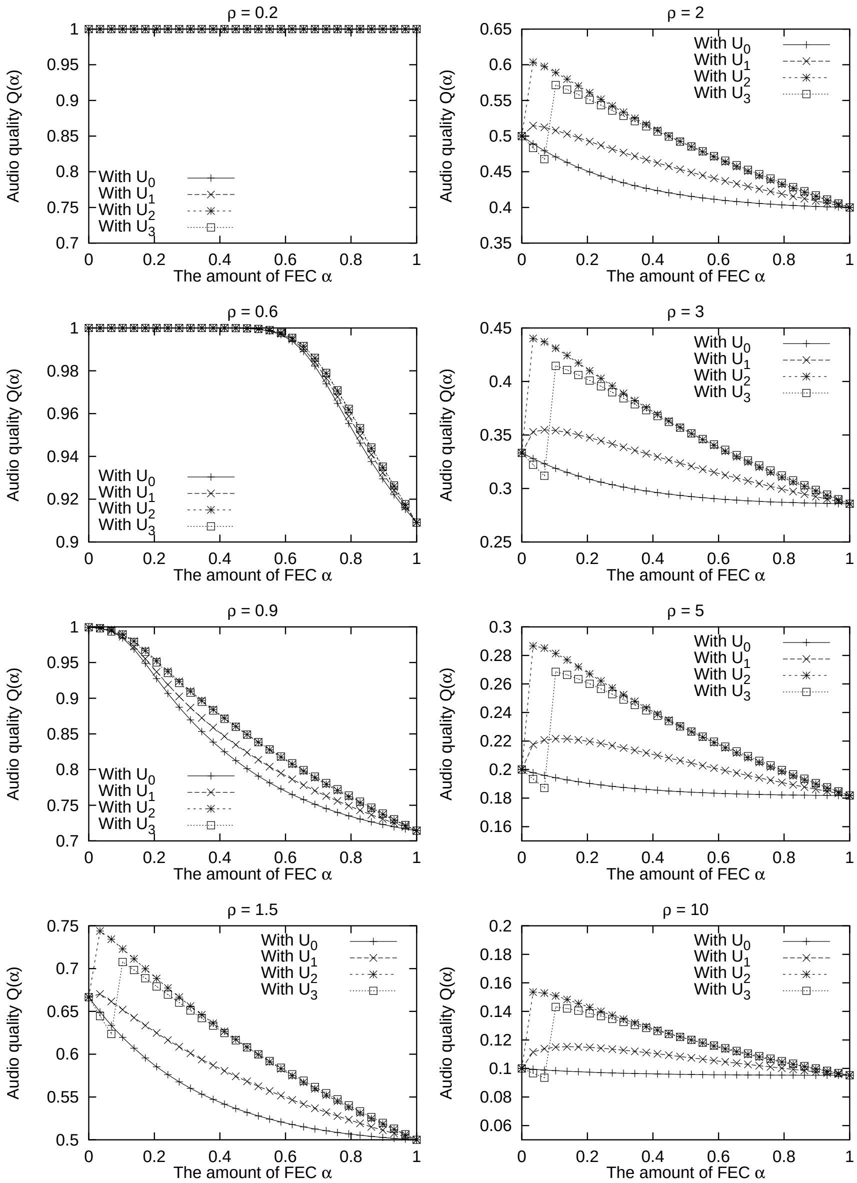

Figure 11: Case $\phi = 1$ and $K = 50$

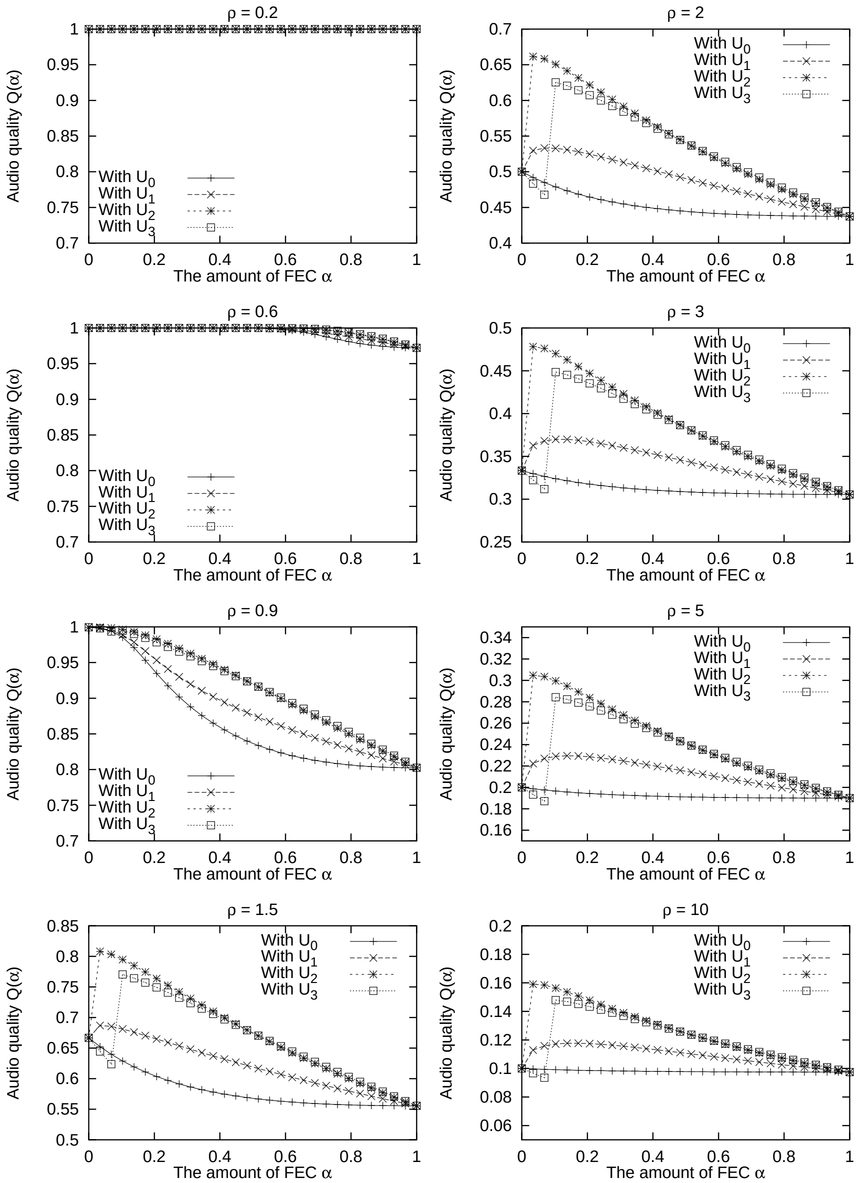

Figure 12: Case $\phi \to \infty$ and $K = 50$

and $\phi \to \infty$. Our simplistic $M/M/1/K$ queue model, as well as other more general models (that include general service times and multiplexing with other sources) show that audio quality deteriorates when applying this kind of FEC mechanism. It is therefore desirable to study other FEC methods that can provide a better quality. Recently, Ratton [9] found that media-independent FEC techniques using parity bits ([17]) perform better than media-specific FEC[5, 3, 17].

We can provide an intuitive explanation to the reason that the simplistic FEC studied here does not perform well. In this approach, each added unit of information protects just one unit of information that can be retrieved. We can define this as a protection gain of one unit. Other more sophisticated approaches allow a single unit of FEC to protect many packets (e.g. Reed Solomon coding that allows to retrieved up to n lost packets in any block of m packets to which n redundant packets are added). The protection gain of such more sophisticated mechanisms can thus be much higher. Even a simple XOR-based FEC, such as suggested in [9], has a high protection gain. We note however that the applicability of more sophisticated FEC mechanisms is still limited by delay constraints. Moreover, we note that even FEC mechanisms with high protection gain can suffer from deterioration of quality with respect to the case of no FEC, as was established in [1, 11].

We would like to give some comments on the validity of our results. Our analytical results are valid if our model for the network and the assumptions we made are correct. We believe that, due to traffic multiplexing, the $M/M/1/K$ model for the bottleneck is justified. But, this may not be sufficient since audio packets may be lost due to a transient congestion in another router. If this case is common, the FEC scheme may present better performance given that the total probability of the loss of packets does not increase so fast with the amount of FEC. We assume here that the loss probability of packets in non-congested routers does not depend on α.

We further identified a slight improvement in the performance when adding FEC, in the case that there is multiplexing with packets of other sessions which do not use FEC. Finally, we saw that using a utility function that grows fast, in such a way that for a small quantity of FEC we obtain a high audio quality, could give positive results when adding FEC. We have shown a theoretical limit in gain by adding FEC, i.e., the quality can be at most doubled by this kind of FEC mechanism, and we can approach this bound by adding a small amount of FEC if the utility function is close to a step function, if spacing is large, and if loss probabilities are high.

A Ballot theorems

In this appendix, we cite the ballot theorem that we have used to solve the problem for case $1 \leq \phi \leq K_\alpha$. The reader is referred to [24] for details.

Theorem 3 *Suppose that an urn contains n cards marked with nonnegative integers $k_1, k_2, \ldots, k_n$, respectively, where $k_1 + k_2 + \cdots + k_n = k \leq n$. All the n cards are drawn without replacement from the urn. Denote by ν_r, $r = 1, 2, \ldots, n$, the number of the card drawn at the rth drawing. Then,*

$$P\{\nu_1 + \cdots + \nu_r < r \quad \textit{for} \quad r = 1, \ldots, n\} = 1 - \frac{k}{n}, \tag{17}$$

provided that all possible results are equally probable.

References

[1] E. Altman and A. Jean-Marie. Loss probabilities for messages with redundant packets feeding a finite buffer. *IEEE Journal of Selected Areas in Communications*, 16(5):779–787, 1998.

[2] J.-C. Bolot. End-to-end delay and loss behavior in the Internet. In *Proc. ACM Sigcomm '93*, pages 289–298, San Francisco, CA, Sept. 1993.

[3] J.-C. Bolot, H. Crépin, and A. V. García. Analysis of audio packet loss in the Internet. *NOSSDAV*, 1995.

[4] J.-C. Bolot, S. Fosse-Parisis, and D. Towsley. Adaptive FEC-based error control for interactive audio in the Internet. *Proc. IEEE Infocom*, 1999.

[5] J.-C. Bolot and A. V. García. Control mechanisms for packet audio in the Internet. *Proc. IEEE INFOCOM*, 1996.

[6] O. J. Boxma. Sojourn times in cyclic queues – the influence of the slowest server. In *Computer Performance and Reliability*, pages 13–24. Elsevier Science Publishers B.V. (North-Holland), 1988.

[7] I. Cidon, A. Khamisy, and M. Sidi. Analysis of packet loss process in high-speed networks. *IEEE Transactions on Information Theory*, IT-39(1):98–108, January 1993.

[8] J. W. Cohen. *The Single Server Queue*. North-Holland, 1969.

[9] D. R. Figueiredo and E. de Souza e Silva. Efficient mechanisms for recovering voice packets in the Internet. *Globecom*, 1999.

[10] A. V. García and S. Fosse-Parisis. FreePhone audio tool. http://www-sop.inria.fr/rodeo/fphone/. High-Speed Networking Group, INRIA Sophia Antipolis.

[11] O. A. Hellal, E. Altman, A. Jean-Marie, and I. Kurkova. On loss probabilities in presence of redundant packets and several traffic sources. *Performance Evaluation*, 36-37:486–518, 1999.

[12] L. Kleinrock. *Queuing systems*. John Wiley, New York, 1976.

[13] I. Kouvelas, O. Hodson, V. Hardman, and J. Crowcroft. Redundancy control in real-time Internet audio conferencing. *Proc. of AVSPN*, 1997.

[14] M. M. Meky and T. N. Saadawi. Prediction of speech quality using radial basis functions neural networks. *Proceedings of the 2nd IEEE Symposium on Computers and Communications*, 1997.

[15] T. Mice Project. RAT: Robust Audio Tool. http://www-mice.cs.ucl.ac.uk/multimedia/software/rat/. Multimedia Integrated Conferencing for European Researchers, University College London.

[16] C. Perkins and O. Hodson. Options for repair streaming media. Request for Comments 2354, June 1998.

[17] C. Perkins, O. Hodson, and V. Hardman. A survey of packet loss recovery for streaming audio. *IEEE Network*, 1998.

[18] M. Podolsky. *Transmission of Real-time Multimedia Over the Internet*. PhD thesis, University of California, Berkeley, 1999.

[19] M. Podolsky, C. Romer, and S. McCanne. Simulation of FEC-based error control for packet audio on the Internet. *Proc. IEEE INFOCOM*, 1998.

[20] J. Rosenberg. Reliability enhancements to NeVoT. December 1996.

[21] M. R. Salamatian. *Transmission Multimédia Fiable Sur Internet*. PhD thesis, Université Paris Sud UFR Scientifique d'Orsay, December 2000.

[22] H. Sanneck and N. T. L. Le. Speech property-based FEC for Internet telephony applications. *Proceedings of the SPIE/ACM SIGMM Multimedia Computer and Networking Conference*, January 2000.

[23] S. Shenker. Fundamental design issues for the future Internet. Presentation at the IETF Meeting, July 1993.

[24] L. Takács. *Combinatorial Methods in the Theory of Stochastic Processes*. John Wiley and Sons, 1967.

[25] I. T. Union. Methodology for the subjective assessment of the quality of television pictures. ITU-R Recommendation BT.500-7.

[26] I. T. Union. Methods for subjective determination of transmission quality. ITU-T Recommendation P.800.

[27] I. T. Union. Objective quality measurement of telephone-band (300-3400 hz) speech codecs. ITU-T Recommendation P.861, February 1998.

[28] A. Watson and M. A. Sasse. Measuring perceived quality of speech and video in multimedia conferencing applications. *Proceedings of ACM Multimedia*, pages 55–60, September 1998.

Unité de recherche INRIA Sophia Antipolis
2004, route des Lucioles - B.P. 93 - 06902 Sophia Antipolis Cedex (France)

Unité de recherche INRIA Lorraine : Technopôle de Nancy-Brabois - Campus scientifique
615, rue du Jardin Botanique - B.P. 101 - 54602 Villers lès Nancy Cedex (France)
Unité de recherche INRIA Rennes : IRISA, Campus universitaire de Beaulieu - 35042 Rennes Cedex (France)
Unité de recherche INRIA Rhône-Alpes : 655, avenue de l'Europe - 38330 Montbonnot St Martin (France)
Unité de recherche INRIA Rocquencourt : Domaine de Voluceau - Rocquencourt - B.P. 105 - 78153 Le Chesnay Cedex (France)

Éditeur
INRIA - Domaine de Voluceau - Rocquencourt, B.P. 105 - 78153 Le Chesnay Cedex (France)
http://www.inria.fr
ISSN 0249-6399