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Rapport de recherche

Analyse de la méthode de rééquilibrage dans un système de transport urbain en libre service.

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Résumé

Ce papier s'intéresse à un type de système de transport dans lequel les véhicules sont mis à la disposition des abonnés dans des stations. Les abonnés peuvent prendre possession d'une voiture à l'aide d'une carte à puce. Ils utilisent le véhicule, puis le rendent en le déposant dans l'une quelconque des stations. A certains moments de la journée, certaines stations manquent de véhicules, alors que d'autres en sont submergées. Le rééquilibrage consiste à redistribuer les véhicules dans les stations afin de revenir à une situation dans laquelle les stations ne sont ni en rupture, ni en excès de véhicules afin de garantir un taux de service aussi élevé que possible compte tenu du nombre de véhicules disponibles dans le système. L'objectif de ce rapport est d'analyser le processus de rééquilibrage et de proposer une heuristique efficace.

Mots clefs : Transport urbain, Gestion des systèmes de transport, Méthodes de rééquilibrage

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Analysis of the balancing process in a pool of self-service cars

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Abstract

In the kind of transportation system studied in this paper, cars are placed at the disposal of subscribers (customers) in stations. Customers have access to the cars using non-contact smart cards. They can use a car for a while and return it in the same or another station. At some times of the day, either an overflow or a shortage of cars may happen at one or more stations. The balancing process consists of redistributing the cars among the stations in order to avoid overflow and shortage, that is to guaranty a service ratio that is as high as possible, taking into account the number of cars available in the system. The goal of this paper is to make a systematic analysis of the balancing process and to propose an efficient balancing heuristic algorithm.

Key words: Urban transportation, transportation system management, balancing method.

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1. Introduction

A new transportation means, which is complementary to the existing public transportation system, is currently under development in some European cities. It consists of a pool of cars (some time electric cars) distributed over a set of stations. The number of stations is usually limited (less than ten stations are currently implemented in a system). A subscriber (customer) can pick up one car in a station, use it for a while, and return it to another (or the same) station. Indeed, either an overflow or a shortage of cars may happen at one or more stations at some times of the day. The balancing process, whose study is the object of this paper, consists of redistributing the cars among the stations in order to maximize their availability to the customers, that is to maximize the service ratio. This ratio is the number of customers who find a car available when they arrive in a station divided by the number of customers who visit a station to pick up a car, per time unit.

Several studies have already been conducted around the balancing activity. Daviet and Parent (1996 a, b) proposed a platooning technique to redistribute the cars among the stations. Parent et al. (1996) described Praxitèle, a new public transportation system that was tested in France. The physical aspects of the batteries of electric vehicles charging and discharging attracted the attention of Canzler (1996). Chauvet, Hafez and Proth (1999) have studied the management of the electric car batteries. The goal of this paper was to define the optimal level of the charge, that is the minimal charge that makes a car available to customers.

Surprisingly, few studies have been devoted to the balancing methods. In our opinion, Chauvet, Hafez and Proth (1997) have proposed the most complete one at that time. It is based on the concept of favorable and unfavorable states. The favorable state is the distribution of the electric cars in the stations that guarantees that the system can reach a given (large) horizon h_f with the highest probability, assuming that no balancing action is conducted. An unfavorable state is the distribution of the electric cars in the stations that guarantees that at least one of the stations will run out of cars or will be overload before a given (small) horizon h_u with a probability greater than or equal to a given value, assuming that no balancing action is conducted. The proposed management consists in triggering a balancing action as soon as the system reaches an unfavorable state. Balancing the system consists in switching from the unfavorable state reached by the system to the favorable state

Chauvet, Haouba and Proth (1999) introduced an Unceasing Balancing Method (UBM). It requires one truck visiting continuously the stations in a given order. At each station, the truck either leaves cars when needed (assuming that the truck contains some cars), or takes cars from the station if cars are in excess (assuming that some capacity is available in the truck) or does nothing. Thus, when the truck arrives in a station, an evaluation of the state of the system is performed in order to make a decision. It should be noticed that the order the stations are visited is given. The choice of this order is an open problem.

In her Ph. D. thesis, Hafez (1999) proposed an approach that considers the management of both the charge of the batteries and the distribution of the electric cars. To disconnect the balancing problem and the recharge problem, she assume that the balancing process is perfect

when studying the recharge problem. In other words, she assume that there are always a number of cars greater or equal to the demand in each station, which does not means that all these cars are available and thus that it is possible to comply with the demands.
The problem is presented in section 2. In section 3, we present theoretical results that give some necessary conditions to reach an optimal balancing process. Section 4 is devoted to a heuristic balancing algorithm based on the results presented in the previous section. In section 5, we introduce a reactive algorithm, that is an algorithm in which the number of cars to load or unload in the stations, as well as the next station to visit, are computed according to the state of the system. Section 6 is the conclusion.

2. Problem formulation

We consider a system composed with n stations denoted by $S=\{s_1, s_2, ..., s_n\}$. The distance $d(s_i,s_j)$ between stations s_i and s_j is known and given in terms of the number of elementary periods required to reach s_j starting from s_i. An elementary period is typically 8 minutes. Thus, a distance is an integer value in the remaining of the paper.
For each station s_i, we know a set of pairs $\{v_j^i, p_j^i\}_{j=1,...,n_i}$ where v_j^i is a number of cars (positive or negative) and p_j^i is the related probability: this define the random variable V^i. In the remaining of this paper, we assume that these parameters do not depend on time. v_j^i is the difference between the number of cars that arrive in station s_i and the number of cars that leave the station during one elementary period. We denote by $m_i = \sum_{j=1}^{n_i} v_j^i p_j^i$ the mean number of cars that are added in station s_i (m_i>0) or that are removed from station s_i (m_j<0) during one elementary period. In this paper, we assume that $\sum_{i=1}^{n} m_i = 0$, which means that the system is globally balanced on each elementary period. Thus, the system is assumed to be steady. This hypothesis does not restrict the conclusions of the paper but makes them easily understandable.
We consider a horizon H and the number $q_i(\eta)$ of cars required in each station $i \in \{1, 2, \dots, n\}$ to meet customers' requirements during this period with a probability greater than or equal to a given probability η. The horizon H is a parameter of the problem and represents a number of elementary periods. The greater H, the greater the numbers q_i, i=1, 2… n, of cars required in the stations s_i where m_i<0 to meet customers' requirements during H elementary periods with a probability greater than or equal to η. We usually choose H large enough to allow the truck to visit each station at least twice. The value of η belongs typically to interval [0.7, 1].
The set $Q(\eta)=\{q_1(\eta), q_2(\eta), \dots, q_n(\eta)\}$ is the set of optimal thresholds associated with the stations. It is the set of numbers of cars that should be in the stations to have a well-balanced system. Note that if H and/or η increases, then the thresholds $q_i(\eta)$ increase and we have to increase the number of cars in the system to preserve the same service level. This point will be made clear below.
A truck is in charge of the balancing process, which is the transportation of cars from stations where the number of cars exceed the threshold towards stations where the number of car is

less than the threshold. The capacity of the truck is known and denoted by *c*. *c* is the maximal number of cars that can be transported by the truck. Since the cars used in such a self-service transportation system are mini- electric cars, *c* can usually take a value up to 15.
A customer can pick up an electric car at a station, use it and return it to another (or the same) station. When a customer does not find a car as he/her arrives in a station, we say that the station runs out of cars or that we have a shortage in this station. Indeed, the problem is to find a balancing process that reduces, and even suppress, car shortages.

3. Theoretical results

The constraints that apply to this problem are the following:

1. A pair of constraints concerns the truck. The first constraint is the capacity *c* of the truck in charge of the balancing process. The smaller *c*, the greater the number of truck moves to transport the same number of cars, which could be impossible in a period less than or equal to *H*. The second constraint that apply to the truck is its speed. The greater the speed, the faster the balancing process.
2. The number of cars in the system. If this number is too small, it will be difficult, and even impossible, to simultaneously reach the thresholds in the stations and meet customers' requirements.

3.1. Relaxing the number of cars in the system

Let us first assume that the number of cars in the system is very large and, thus, that this number is no more a constraint. We also assume that the *n* stations are visited in the order $s_{i_1},\ s_{i_2},\ ...,\ s_{i_n}$ and that the truck covers the circuit once during each period *H*. Result 1 gives a necessary condition on *c* for the truck to be able to perform the balancing process.

Result 1

If there exists $\{i_j,\ i_{j+1},\ ...,\ i_{j+s}\} \subset \{i_1,\ i_2,\ ...,\ i_n\}$ such that either $H\ .\sum_{k=j}^{j+s} m_{i_k} > c$ or $H.\sum_{k=j}^{j+s} m_{i_k} < -\,c$, then the capacity *c* of the truck is not great enough to perform the balancing process if the circuit $s_{i_1},\ s_{i_2},\ ...,\ s_{i_n}$ is covered once during each period *H.*

Proof:

1. Assume that $H.\sum_{k=j}^{j+s} m_{i_k} > c$. In this case, the truck will not be able to load the number $H.\sum_{k=j}^{j+s} m_{i_k}$ of cars that arrive in excess (on the average) during each period *H* when visiting the sequence of stations $s_{i_j},\ s_{i_{j+1}},\ ...,\ s_{i_{j+s}}$. As a consequence, the number of cars in this set of stations will increase, on the average, by at least $\sum_{k=j}^{j+s} m_{i_k} - c$ during each period *H*, and some other stations will permanently run out of cars.

2. If $H.\sum_{k=j}^{j+s} m_{i_k} < -c$, the truck will not be able, even if it is full of cars, to unload $-H.\sum_{k=j}^{j+s} m_{i_k}$ in stations s_{i_j}, $s_{i_{j+1}}$, ..., $s_{i_{j+s}}$ during each period h to make up the deficit of cars in these stations. Thus, it is impossible to perform the balancing process under this condition. o

Corollary 1

A necessary condition to implement an efficient balancing process is the following:

Each one of the stations s_i, $i \in \{1, 2, \ldots, n\}$ should be visited at least $\left\lceil H\frac{|m_i|}{c} \right\rceil$ times during each period H. In this formulation, $|x|$ is the absolute value of x while $\lceil x \rceil$ is the smallest integer greater than or equal to x.

Proof:

Since the maximum number of cars that can be load in the truck or unload from the truck when visiting station s_i is c, the maximum number of cars that can be load or unload during a period H will be $\left\lceil H\frac{|m_i|}{c} \right\rceil .c > H\frac{|m_i|}{c}.c = H.|m_i|$. In other words, if it is possible to visit each station s_i, $i \in \{1, 2, \ldots, n\}$at least $\left\lceil H\frac{|m_i|}{c} \right\rceil$times, then the capacity of the truck is large enough to be able to balance the excess or shortage of cars in the stations.

Indeed, this condition is not sufficient: the path followed by the truck to visit the stations and the number of cars loaded or unloaded at each station should fit with the requirement of the system

We would like to dwell on the fact that the above condition is necessary, but not sufficient. It does not guaranty that there exists an order to visit the stations such that the truck will contain enough cars to unload, or will have enough available capacity to load, the required number of cars when arriving in a station. It also does not guaranty that the stations can be visited in the required order during a period H.

3.2.Relaxing the constraints on the truck

In this subsection, we assume that the constraints on the truck are relaxed. In other words, we assume that the capacity of the truck is not bounded and that its speed is infinite. Taking into account these hypotheses, the following result is straightforward.

Result 2

We assume that each station is visited exactly once during each period H and that the constraints that apply to the truck are relaxed. We also assume that the number of cars simultaneously used by the customers is upper bounded by Y. We denote by V the random variable $V = \sum_{i=1}^{n} V^i$. This random variable V takes its values on $[\sum_{i=1}^{n} v_1^i, \sum_{i=1}^{n} v_{n_i}^i]$=[$Mi$, Ma], and the corresponding probabilities are easy to derive from those of the random variables V^i.

In this case, and assuming that we are in steady state:

1. The mean number of customers that cannot find a car when they arrive in a station is:

$$r = \sum_{a=0}^{Y-1} \sum_{u>a} (u-a) \Pr(A=a).\Pr(V=u), \tag{1}$$

where A is the random variable representing the total number of cars in the stations ($A \le Y$).

2. The probabilities $\Pr(A=a)$ are the solution of the linear system:

$$\Pr(A=a) = \sum_{i=g}^{a-1} \{\Pr(V=a-i).\Pr(A \le Y-a+i) + \Pr(V > a-i).\Pr(A = Y-a+i)\}.\Pr(A=i\}$$
$$+ \sum_{i=a}^{e} \{\Pr(V=a-i).\Pr(A \ge i-a) + \Pr(V < a-i).\Pr(A = i-a)\}.\Pr(A=i\} \tag{2}$$

where $g = Max(0, a-Ma)$ and $e = Min(Y, a-Mi)$ and a=1, 2, …, Y,

and:

$$\sum_{a=0}^{Y} \Pr(A=a) = 1 \tag{3}$$

When the constraints on the truck are not relaxed, the mean value r is a lower bound of the mean value of the number of customers that cannot find a car when they arrive in a station.

Proof:

Relation (1) is straightforward in steady state.

Let U be the random variable that represents the number of cars that are actually taken from the stations (U<0) or returned in the stations (U>0) during one elementary period. Since the constraints on the truck are relaxed, a car can be transported from a station to another one instantaneously. In other words, the position of the cars in the stations does not influence their use. It is why we can consider that the cars in the stations are at the disposal of anyone of the customers.

If the number of cars in the stations or in use would not be bounded, we would have U=V but, in general:

$$\Pr(U=u) = \Pr(V=u).\Pr(A \ge -u) + \Pr(V<u).\Pr(A=-u) \text{ if } -Mi \le u \le 0 \tag{4}$$

and:

$$\Pr(U=u) = \Pr(V=u).\Pr(A \le Y-u) + \Pr(V>u).\Pr(A=Y-u) \text{ if } 0 < u \le Ma \tag{5}$$

We consider two consecutive elementary periods denoted by t and t+1. We also denote by A_t and A_{t+1} the random variables representing the total number of cars in the stations during the elementary periods t and t+1 respectively.

Relation (6) is straightforward:

$$\Pr(A_{t+1}=a) = \sum_{i=g}^{e} \Pr(A_{t+1}=a / A_t = i).\Pr(A_t = i) \tag{6}$$

This relation can be rewritten as:

$$\Pr(A_{t+1}=a) = \sum_{i=g}^{e} \Pr(U=a-i).\Pr(A_t = i)$$

If we remember that we are in steady state, we finally obtain:

$$\Pr(A=a) = \sum_{i=g}^{e} \Pr(U=a-i).\Pr(A=i) \tag{7}$$

The bounds g and e are derived from the following inequalities:

- 0≤i≤Y since equality A=i is included in (7).

- $Mi \leq a\text{-}i \leq Ma$ since equality $U=a\text{-}i$ is included in (7). This leads to $a\text{-}Ma \leq i \leq a\text{-}Mi$.
Finally Max(0,a-Ma)≤i≤Min(Y,a-Mi), or $g \leq i \leq e$.
By replacing Pr(U=a-i) in relation (7) by (4) and (5) where $u=a\text{-}i$, we obtain relation (2).
Relation (3) is straightforward.
If we reintroduce the constraints on the truck, r becomes the lower bound of the mean value of the number of customers that cannot find a car when they arrive in a station.

4. Algorithm 1: a fixed circuit is assigned to the truck

In this algorithm, we assign a value to H and we choose η in the interval [0.7,1]. The value of H should be large enough to allow the truck to visit each station once during this period. In other words, we should be able to find at least one circuit that can be covered in at most H elementary circuits. Let L be the set of circuits that meet this constraint. We then compute the subset $L_1 \subset L$ of circuits that does not meet the constraint of result 1, that is the subset of circuits that meet the necessary condition on the capacity of the truck.
The best circuit will be selected among the circuits of L_1 by simulation: it is the one that leads to the smallest percentage of customers who do not find a car when they arrive in a station.
Formally, algorithm 1 can be presented as follows.

Algorithm 1:

1. Generate a circuit at random. Let H be the number of elementary periods during which we want to cover this circuit once.
2. Compute the set L of circuits that can be covered in let than H elementary circuits.
3. We set $L_1=\emptyset$.
4. For each circuit $Z=\{s_{i_1}, s_{i_2}, ..., s_{i_n}\} \in L$, we check if there exists $\{i_j,\ i_{j+1},\ ...,\ i_{j+s}\} \subset \{i_1,\ i_2,\ ...,\ i_n\}$ such that either $H.\sum_{k=j}^{j+s} m_{i_k} > c$ or $H.\sum_{k=j}^{j+s} m_{i_k} < -c$. If not, we set $L_1 = L_1 \cup \{Z\}$.
5. If $L_1=\emptyset$, we set $H=H+1$ and we go to 2.
6. Simulation (*This simulation applies to each one of the circuits of* L_1).
   6.1. We introduce the capacity c of the truck, the number cu of cars that are in use and the number ct of cars transported by the truck.
   6.2. We choose η in the interval [0.7,1] and derive $Q(\eta)=\{q_1(\eta), q_2(\eta), \ldots, q_n(\eta)\}$ from the probability rules of the random variables V^i, i=1,2,…,n.

As we can see, the total number *of cars in the system is* $cu+ct+\sum_{i=1}^{n} q_i(\eta)$.

6.3. Introduce *Ns* that is the minimal number of elementary periods covered by the simulation. In fact, the simulation will cover $Ns1 = \lceil Ns/H \rceil . H$ elementary periods.

6.4. For i=1 to n, set $mv(i)=q_i(\eta)$. *Initialization of the number of cars in the truck.*

6.5. For i=1 to n, set $w(i)$=0. This step initializes the delays in the stations.

6.6. For i=1 to *Ns*1, do:

6.1.1. Compute the actual number of cars $a(j)$ that enter or leave each station j during the elementary period i. To obtain these values, we generate at random the quantities $b(j)$, for j=1,2,…,n, using the probability distributions of the V^j random variables. Then:

a. If $b(j)<0$, set $a(j)= b(j)$ if $-b(j)< mv(j)$ and $a(j)=-mv(j)$ if $-b(j)\geq mv(j)$.

b. If $b(j)\geq 0$, set $a(j)= b(j)$ if $b(j)<$cu and $a(j)=cu$ if $b(j)\geq$cu.

6.1.2. For j=1,…,n do:

a. If $b(j)<0$, set $w(j)= w(j)- b(j)+ a(j)$. *The delays are adjusted at this step.*

b. If $b(j)<0$, set $mv(j)= mv(j)+a(j)$. *The number of cars in each station is adjusted at this level.*

c. If $b(j)\geq 0$, set $cu=cu-a(j)$.

6.1.3. If the end of the elementary period i is the station k of the circuit, then:

a. If $mv(k)<q_k(\eta)$ then, if $ct>q_k(\eta)-mv(k)$ then $x(k)=q_k(\eta)-mv(k)$ else $x(k)$=ct.

b. If $mv(k)\geq q_k(\eta)$ then, if $c-ct>mv(k)-q_k(\eta)$ then $x(k)=-(mv(k)-q_k(\eta))$ else $x(k)=-(c-ct)$.

c. Set $mv(k)= mv(k)+x(k)$.

d. Set ct=ct-x(k).

6.7.Computation of the mean value of the unsatisfied demands:

For i=1 to n do $w(i)=w(i)$/Ns1

6.8. Computation of the ratio e_l of the number of unsatisfied demands over the total number of demands for the circuit $l \in L_1$.

A numerical example

We applied algorithm 1 to a case with 5 stations. The system works in steady state. The random behavior of each station is given by a list of pairs. The first element of each pair is an integer that is negative or positive. If this integer is negative, it means that the number of cars that are returned in the station during one elementary period minus the number of cars that are required by the customers during the same period is negative. In this case, the tendency of the station is to get empty. If the first element of a pair is positive, the tendency of the station is to get full. The second number of each pair is the corresponding probability. Indeed, if the first element of a pair is negative, its opposite value may be greater than the number of cars in the station: this means that some demands remain unsatisfied. Similarly, if the first element of a pair is positive and greater than the number of cars in use, it means that fewer cars than expected will enter the station: this balances the unsatisfied demands. For simplicity, we denote by i a station s_i in the remainder of this paper.

In the case considered in this example, the data are the following:

Station 1: (-2; 0.2), (-1; 0.1), (0; 0.3), (1; 0.3), (2; 0.1)

Station 2: (-2; 0.4), (-1; 0.2), (0; 0.2), (1; 0.2)

Station 3: (-1; 0.3), (1; 0.4), (2; 0.3)

Station 4: (-2; 0.1), (-1; 0.1), (0; 0.2), (1; 0.5), (2; 0.1)

Station 5: (-1; 0.4), (0; 0.5), (1; 0.1)

The distances between the stations are given in table 1. These distances are the number of elementary periods the truck requires to move from one station to another.

We assume that the truck is empty at the beginning of the simulation. We propose some results with H=12 and 8 and η=0.9. Each simulation covers 30,000 elementary periods.

The results are proposed in tables 2.a and 2.b. For each run, we selected the circuit that led to the lowest ratio e_l and we provided the circuit, the ratio e_l and the mean value of unsatisfied demands per elementary period for each station (in the order 1, 2, 3, 4 and 5).

For each pair (H,η), the parameters of the runs are the capacity of the truck and the number of cars in use at the beginning of the simulation. This number allows changing the total number of cars in the system.

Table 1: Distances between stations.

Stations ⇒ Stations ⇓	1	2	3	4	5
1	0	2	1	3	4
2	2	0	2	3	2
3	1	2	0	4	2
4	3	3	4	0	1
5	4	2	2	1	0

Table 2.a concerns the case H=12 and η=0.9. The computation leads to $q_1(0.9)=6$, $q_2(0.9)=15$, $q_3(0.9)=1$, $q_4(0.9)=1$ and $q_5(0.9)=6$. In fact, $q_3(0.9)$ and $q_4(0.9)$ are equal to zero, but we assign systematically 1 in this case. As we can see, the minimum total number of cars in the system should be equal to 29 in this case. It explains why the column "total number of cars in the system=25" remains empty.

We then consider the case H=8 and η=0.9. The computation leads to $q_1(0.9)=5$, $q_2(0.9)=11$, $q_3(0.9)=1$, $q_4(0.9)=1$ and $q_5(0.9)=5$. The results are given in table 2.b. As we can see, the minimum total number of cars in the system should be equal to 24 in this case. Note that the truck is much more active when H=8 than when H=12, which explains why the results are better in table 2.b than in table 2.a for the same number of cars and the same capacity of the truck.

Table 2.a.: Case H=12 and η=0.9

Total number of cars in the system $\Rightarrow$ $\Downarrow$ Capacity of the truck		25	40	55
10	Circuit		3; 5; 4; 2; 1; 3	2; 3; 5; 4; 1; 2
	Ratio e_l (in %)		10.9	9.4
	Unsatisfied dem. (in %)		22.5; 14; 2.6; 7; 6.1	14.3; 14.2; 2.1; 3.4; 7.3
15	Circuit		2; 3; 5; 4; 1; 2	2; 3; 4; 5; 1; 2
	Ratio e_l (in %)		9.6	8.6
	Unsatisfied dem. (in %)		20.7; 8; 5.1; 12.5; 7	20.2; 9.3; 4.9; 1; 6.2

Table 2.b.: Case H=8 and η=0.9

Total number of cars in the system $\Rightarrow$ $\Downarrow$ Capacity of the truck		25	40	55
10	Circuit	3; 5; 4; 2; 1; 3	2; 3; 5; 4; 1; 2	2; 3; 5; 4; 1; 2
	Ratio e_l (in %)	12.7	8.8	6.3
	Unsatisfied dem. (in %)	26.2; 12.9; 7.9; 19.9; 5.1	19.2; 7.8; 4.8; 10.9; 5.4	12.9; 5.9; 3.3; 5.1; 4.9
15	Circuit	3; 5; 4; 2; 1; 3	2; 3; 5; 4; 1; 2	2; 3; 5; 4; 1; 2
	Ratio e_l (in %)	11.9	9.4	6.6
	Unsatisfied dem. (in %)	25.8; 9.9; 8.7; 22.2; 4.4	22.3; 6.6; 6.2; 15.3; 4.8	15.2; 4.9; 3.9; 7.4; 4.3

If we make some simulations with H=7 and η=0.9, we obtain results that are worst than the ones of table 2.b.. The reason is that the number of elementary periods required to cover the most favorable circuits (in the sense that the truck visits alternatively stations that require and station that provide cars on the average) is greater than or equal to 8. Thus, they are not considered in the case H=7.

5. Algorithm 2: The reactive algorithm

In this section, we present an algorithm that computes the next station to be visited by the truck each time it enter a station. This computation is based on the state of the whole system.

The user provides a horizon H. As in algorithm 1, H is used to compute the number of cars that should be available in each station to provide cars to customers during H elementary periods with a probability greater than or equal to a given value η. These numbers are the thresholds associated with the stations. The threshold associated with station $i \in \{1,\ldots,n\}$ is denoted by $q_i(\eta)$. We use the same notations as the ones of section 4. In addition, we denote by cmi and cma two values such that $0 \le cmi \le cma \le c$. These value are respectively the minimum and the maximum threshold associated to the truck. The truck can unload cars in a station only if it contains more than cmi cars, that is if $ct > cmi$. It will choose as the next station the one in which it is possible to load cars in the truck.

When the truck is in a station i, the number $mv(k)$ of cars in each station k is known. We thus are able to compute the number $h(k)$ of elementary periods over which station k can meet customers' requirements with a probability greater than or equal to η using only the $mv(k)$ cars available.

We denote by i_1 a station such that $h(i_1) = \min_{k \in \{1,\ldots,n\}} h(k)$. Station i_1 is the station that contains the number of cars that can meet customers' requirements over the shortest period.

We also compute i_2 such that $h(i_2) = \max_{k \in E} h(k)$, where $E = \{k / k \in (1,\ldots,n)$ and $mv(k) \ge q_k(\eta)\}$. If several stations lead to this maximum, we select for i_2 the one that maximize the difference $mv(k))- q_k(\eta)$.

If the truck contains more than *cma* cars, then i_1 is the next station it will visit, otherwise he will visit i_2. Formally, algorithm 2 is as follows. The introduction of the data is missing.

Algorithm 2:

1. Compute the thresholds $q_k(\eta)$, k=1,…,n, using the probabilities distributions of the random variables V^k.
2. Introduce the number N of iterations
3. For i=1 to N do

3.1. Compute the actual number of cars $a(j)$ that enter or leave each station j during the elementary period i. To obtain these values, we generate at random the quantities $b(j)$, for j=1,2,…,n, using the probability distributions of the V^j random variables. Then:

a. If $b(j)<0$, set $a(j)= b(j)$ if $-b(j)< mv(j)$ and $a(j)=-mv(j)$ if $-b(j)\geq mv(j)$.

b. If $b(j)\geq 0$, set $a(j)= b(j)$ if $b(j)<$cu and $a(j)=cu$ if $b(j)\geq$cu.

3.2.For j=1,…,n do:

If $b(j)<0$, set $w(j)= w(j)- b(j)+ a(j)$. *The delays are adjusted at this step.*

If $b(j)<0$, set $mv(j)= mv(j)+a(j)$. *The number of cars in each station is adjusted at this level.*

If $b(j)\geq 0$, set $cu=cu-a(j)$.

3.3.If at the end of the elementary period i the truck arrives in station j then:

3.2.1. If $z(j)=-1$ then, if $mv(j)<q_j(\eta)$ and $ct>cmi$ then:

Set $x=\text{Min}(ct, q_j(\eta)-mv(j))$

Set $mv(j)= mv(j)+x$

Set $ct=ct-x$

z(j)=-1 means that the station where the truck arrives requires additional cars. This value was computed when the car visited the previous station.

3.2.2. If $z(j)=1$ then, if $mv(j)>q_j(\eta)$ and $ct<cma$ then:

Set $x=\mathrm{Min}(cma-ct, mv(j)-q_j(\eta))$

Set $mv(j)= mv(j)-x$

gggSet $ct=ct+x$

z(j)=1 means that the station where the truck arrives can provide cars to the truck. This value was computed when the car visited the previous station.

3.3.3. Based on the number $mv(k)$ of cars that are in station k, derive $h(k)$ from the probabilities distributions of the random variables V^k for $k=1,\ldots,n$.

3.3.4. Compute i_1 such that $h(i_1)=\min\limits_{k\in\{1,\ldots,n\}} h(k)$.

3.3.5. Compute i_2 such that $h(i_2)=\max\limits_{k\in E} h(k)$, where $E=\{k/k\in(1,\ldots,n)$ and $mv(k)\geq q_k(\eta)\}$. If several stations lead to this maximum, we select for i_2 the one that maximize the difference $mv(k))- q_k(\eta)$.

3.3.6. If $ct>cmi$ set $z(i_1)=-1$. The next station to be visited is station i_1.

3.3.7. If $ct<cma$ set $z(i_2)=1$. The next station to be visited is station i_2.

4. Computation of the mean value of the unsatisfied demands:

For $i=1$ to n do $w(i)=w(i)/\mathrm{Ns1}$

5. Computation of the ratio e_l of the number of unsatisfied demands over the total number of demands for the circuit $l\in L_1$.

A numerical example

The data are the same as those of the previous example. We introduce two more variables, that is *cmi* and *cma*. The results are presented in table 3.

Table 3: Case H=8 and η=0.9

Total number of cars in the system $\Rightarrow$ / $\Downarrow$ Capacity of the truck		25	40	55
10	*cmi* and *cma*	2; 8	2; 8	2; 8
	Ratio e_l (in %)	11.6	4.1	2.6
	Unsatisfied dem. (in %)	13.6; 4.7; 18.6; 18.1; 2.8	4.6; 1.8; 6.2; 6.6; 1.2	2.9; 1.2; 1.7; 4.4; 0.8
15	*cmi* and *cma*	2; 6	2; 6	2; 6
	Ratio e_l (in %)	10.1	4	2.3
	Unsatisfied dem. (in %)	12.3; 4.1; 15.9; 15.9; 2.4	4.7; 1.8; 6.1; 6.5; 1	2.6; 1; 3.3; 4.1; 0.7

For the same number of cars and the same capacity of the truck, algorithm 2 performs better than algorithm 1.

6. Conclusion

The first part of the paper analyzes two cases. In the first one, only the constraints on the truck are considered, and we assume that the number of cars in the system is not limited. Assuming that the truck covers a given circuit once during a given period, a necessary condition for the truck to be able to balance the system is proposed. In the second case, the constraints on the truck are relaxed, and only the total number of cars in the system limits its efficiency. We showed how this situation could lead to a lower bound of the probability that a station runs out of cars.

The study was made in steady state. This hypothesis was adequate to show how the characteristics of the truck on one hand, and the total number of cars available in the system on the other hand, influence the efficiency of the first balancing approach. It was also enough to show that the reactive algorithm is much better than the algorithm in which the truck is forced to cover always the same circuit.

In transient state, it is obvious that the reactive method would be much easier to adapt than the first one. The only difficulty would be the evaluation of the thresholds associated with the stations: this will be the next step of the research.

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