On the Petri Net Realization of Context-Free Graphs

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Abstract: Given a finite or infinite labeled transition graph defined by a graph grammar, we show an algorithm that decides whether this graph is isomorphic to the reachable state graph of some finite unlabeled Petri net and that produces in this case a minimal net realizing the graph.

Key-words: Infinite graphs, graph grammars, Petri nets, regions, separation, polyhedral cones, semi-linear sets

(Résumé : tsvp)

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De la réalisation des graphes context-free par des réseaux de Petri

Résumé : Etant donné un graphe de transitions ètiquetées fini ou infini, défini par une grammaire de graphes, nous donnons un algorithme qui décide si ce graphe est isomorphe au graphe des marquages accessibles d'un réseau de Petri fini, dont les événements sont étiquetés injectivement, et qui calcule alors un réseau de Petri minimal le réalisant.

Mots-clé : graphes infinis, grammaires de graphes, réseaux de Petri, régions, séparation, cones polyédraux, ensembles semi-linéaires

1 Introduction

The Petri net realization problem for graphs is the question whether a directed graph labeled on arcs is isomorphic to the reachable state graph of some Petri net, with events of the net in bijection with labels of the graph. This problem, restricted to finite graphs, was shown decidable for elementary nets [ER90a] [ER90b] and for P/T-nets [BBD95] [BD96]). The decision procedures are based on regions in graphs, or projections thereof on the state graph of a net with a single place. Regions are implicit places, and synthesizing nets from graphs amounts to make them explicit. The device of regions was introduced by Ehrenfeucht and Rozenberg [ER90a]; it was adapted shortly after to fit in with P/T-nets under the step firing rule [Muk92] or the sequential firing rule [DS93]. The characterization of graphs isomorphic to state graphs of P/T-nets found in these papers applies to finite or infinite graphs. It is thus a shortcoming of our past attempts on the decision of the Petri net realization problem to cope with finite graphs only. This may be felt all the more because one can decide on the realization of formal languages by unbounded Petri nets [Da98]. The decision applies notably to deterministic context-free languages and one might be tempted to infer therefrom a decision of the Petri net realization problem for the deterministic context-free graphs [MS85]. We do not know any reduction of one decision problem to the other; still, our goal is to show a decision of the Petri net realization problem for context-free graphs. The main motivation is to explore the border of the domain within which Petri net synthesis may be helpful. Two specific areas have already been identified in this domain, with applications to asynchronous circuits [CKKLY96] and to distributing reactive automata [Cai99]). Solving the net realization problem for wider classes of graphs or languages will hopefully open new fields of application. We present below the finest version of the problem we can solve today. Deciding on net realization for context-free graphs asks for combining the methods from [BBD95] and [Da98], thus mixing linear algebra and operations on semi-linear sets, with revised techniques for computing finite sets of generating regions in graphs with countable bases of cycles. The organization of the paper is as follows.

Contents

2 Regions in Graphs

Graphs isomorphic to reachable state graphs of Petri nets may be characterized by two regional axioms adapted from Ehrenfeucht and Rozenberg's axioms for elementary transition systems. The purpose of the section is to recall this characterization, appeared with minor variations in [Muk92] and [DS93]. Let us fix the terminology.

> A graph is a transition system $G = (S, E, T)$ where S is a finite or infinite set of nodes (or states), E is a finite non-empty set of labels (or events), and $T \subseteq S \times E \times S$ is a set of labeled arcs (or transitions). An automaton is a pointed graph $G = (S, E, T, s_0)$ where s_0 is a distinguished node (the initial state). In the sequel, $s \xrightarrow{e} s'$ and $s \xrightarrow{e}$ are abbreviations for $(s, e, s') \in T$ and $\exists s' \in S \ (s \xrightarrow{e} s')$. A reachable graph is a pointed graph such that $S = \{s \mid s_0 \xrightarrow{*} s\}$ where $\xrightarrow{*}$ is the inductive and reflexive

closure of $\rightarrow = \cup \{ \stackrel{e}{\rightarrow} \mid e \in E \}$. An event-reduced graph is a reachable graph such that every event $e \in E$ labels some transition in T. We are mainly interested in event-reduced graphs. Let us adapt in consequence the definition of reachable state graphs of Petri nets.

Recall that a marked Petri net is a quadruple $N = (P, E, F, M_0)$, where P and E are respective sets of places and events, F is the flow relation, and M_0 is the initial marking. The sets P and E are disjoint, markings are maps $M : P \rightarrow \mathbb{N}$, and F is given by a map from $(P \times E) \cup (E \times P)$ to $\mathbb{N}$. The set P may be infinite unless specified otherwise, but E is always finite. An event e has concession at marking M if and only if $M(p) \geq F(p, e)$ for every place p, in which case it may be fired, resulting in a transition $M\ [\ e > M'$ such that $M(p) - F(p, e) = M'(p) - F(e, p)$ for every place p. Let $\stackrel{*}{\longrightarrow}$ be the inductive and reflexive closure of the relation $\rightarrow$ on markings such that $M \rightarrow M'$ if $M\ [\ e > M'$ for some event e. A marking M is reachable if $M_0 \stackrel{*}{\longrightarrow} M$. The reachability set of N, denoted $RS(N)$, is the set of the reachable markings. An event with concession nowhere in the reachability set may be considered fictitious and hence irrelevant. In this paper, we identify the reachable state graph of the net $N = (P, E, F, M_0)$ with the graph $N^* = (RS(N), E', T, M_0)$ such that $T = \{ M \stackrel{e}{\rightarrow} M' \mid M, M' \in RS(N) \wedge M\ [\ e > M' \}$ and $E' = \{ e \in E \mid \exists M\ \exists M'\ (M, e, M') \in T \}$. Thus, N^* is event-reduced.

We state now the central definitions of regions and separation by regions on which relies the characterization of graphs isomorphic to reachable state graphs of marked Petri nets.

Definition 2.1 (Regions) *A region of $G = (S, E, T, s_0)$ is a pair $(\sigma, {}^\bullet\eta)$ made of two maps $\sigma : S \rightarrow \mathbb{N}$ and ${}^\bullet\eta : E \rightarrow \mathbb{N}$ such that: i) $\sigma(s) \geq {}^\bullet\eta(e)$ whenever $s \stackrel{e}{\rightarrow}$ in G, and ii) there exists a map $\eta : E \rightarrow \mathbb{Z}$ such that $\eta(e) + {}^\bullet\eta(e) \geq 0$ and $\sigma(s) + \eta(e) = \sigma(s')$ whenever $(s \stackrel{e}{\rightarrow} s')$ in G. The map η is called the justifying map for $(\sigma, {}^\bullet\eta)$.*

Definition 2.2 (Separated graphs) *A graph $G = (S, E, T, s_0)$ is separated if the following axioms are satisfied for every $s, s' \in S$ and $e \in E$:*
(SSA) *$s \neq s' \Rightarrow \sigma(s) \neq \sigma(s')$ for some region $(\sigma, {}^\bullet\eta)$,*
(ESSA) *$\neg\,(s \stackrel{e}{\rightarrow})\ \Rightarrow \sigma(s) < {}^\bullet\eta(e)$ for some region $(\sigma, {}^\bullet\eta)$.*

A subset of regions containing enough elements to witness the satisfaction of both axioms is called an admissible set of regions.

Theorem 2.3 *An event-reduced graph is isomorphic to the reachable state graph of some marked Petri net if and only if it satisfies the separation axioms* **SSA** *and* **ESSA**. *An event-reduced graph may be realized by a finite Petri net if and only if it shows a finite admissible subset of regions.*

This theorem was established in [DS93]; we nevertheless state an explicit proof below, for it may help understanding the paper.

Proof: In order to show that every graph realized by a marked Petri net is separated, it suffices to observe that every place p of a net $N = (P, E, F, M_0)$ determines an induced region of N^*, such that $\sigma(M) = M(p)$ for any marking $M \in RS(N)$ and ${}^\bullet\eta(e) = F(p, e)$ for every event e. The justifying map $\eta(e) = F(e, p) - F(p, e)$ fits actually with Def. 2.1; SSA holds from the definition of markings, and $ESSA$ holds, for it expresses merely that an event with concession at M may be fired at M. The proof for the other direction is slightly more difficult. Given an event-reduced graph $G = (S, E, T, s_0)$, assumed to be separated, let P be an admissible subset of regions of G and let $N = (P, E, F, M_0)$ be the marked net derived as follows. For any region $p = (\sigma, {}^\bullet\eta)$ in the admissible set P and for every event $e \in E$, let $F(p, e) = {}^\bullet\eta(e)$ and $F(e, p) = \eta(e) + {}^\bullet\eta(e)$ where η is the justifying map for p; also let $M_0(p) = \sigma(s_0)$. In order to establish the theorem, it suffices to show that G and N^* are isomorphic graphs. By definition, $N^* = (RS(N), E, T, M_0)$, where $(M \xrightarrow{e} M') \in T$ if and only if $M \in RS(N)$ and $M\,[\ e > M'$. Define $\sim \subseteq (S \times RS(N))$ such that $s \sim M$ if and only if $\sigma(s) = M(p)$ for all regions $p = (\sigma, {}^\bullet\eta) \in P$. From this definition, relation $\sim$ is injective. From SSA, as P is an admissible set, $\sim^{-1}$ is injective. Observing that $s_0 \sim M_0$, that G and N^* are reachable, and that N^* is deterministic, it suffices to show that $\sim$ is a bisimulation. Suppose $s \sim M$. If $s \xrightarrow{e}$ in G then by definition of regions, $\sigma(s) \geq {}^\bullet\eta(e)$ for any $p = (\sigma, {}^\bullet\eta) \in P$; thus $M(p) \geq F(p, e)$ and $M\,[\,e >$ in N. Conversely, if $M\,[\,e >$ in N then by definition of the firing rule, $M(p) \geq F(p, e)$ for any $p = (\sigma, {}^\bullet\eta) \in P$; thus $\sigma(s) \geq {}^\bullet\eta(e)$. As P is an admissible subset of regions of G, it follows that $s \xrightarrow{e}$ in G. We let the reader verify that $s \sim M$, $s \xrightarrow{e} s'$ and $M \xrightarrow{e} M'$ entail $s' \sim M'$. ∎

3 Regions, Cycles and Spanning Trees

Extending to infinite graphs the approach followed in [BBD95], we will characterize regions in graphs in terms of cycles and spanning trees. The characterization thus obtained is generally not effective; it will be refined subsequently to an effective characterization of regions in context-free graphs. In the sequel, $G = (S, E, T, s_0)$ is a fixed event-reduced graph, finite or infinite.

To begin with, we recall some notations and definitions about graphs. Let $\partial^0, \partial^1 : T \to S$ and $\lambda : T \to E$ map the arcs of G to their respective source, target, and label. A 0-chain is a map from S to $\mathbb{Z}$. A 1-chain is a map from T to $\mathbb{Z}$. A 1-chain is finite if it evaluates to 0 almost everywhere. The restriction of the 1-chain c on the subset of arcs T' is the 1-chain c' such that $c'(t) = c(t)$ for $t \in T'$ and $c'(t) = 0$ for $t \in T \setminus T'$. A node occurs on the 1-chain c if it is the source or target of an arc t such that $c(t) \neq 0$. Let ∂ be the map from 1-chains to 0-chains such that, for $c : T \to \mathbb{Z}$ and $s \in S$, $\partial(c)(s) = \sum\{ c(t) \mid \partial^1(t) = s\} - \sum\{ c(t) \mid \partial^0(t) = s\}$. A cycle of G is a 1-chain c such that $\partial(c)$ is the null map. A spanning tree for G is a connected subgraph (S, E, Θ, s_0) -thus $\Theta \subseteq T$- with no cycle except the null map. The remaining arcs in $T \setminus \Theta$ are the chords. The Parikh vector of the 1-chain c is the map $\psi(c) : E \to \mathbb{Z}$ such that $\psi(c)(e) = \sum\{ c(t) \mid \lambda(t) = e\}$ for $e \in E$.

Lemma 3.1 *A map $\eta : E \to \mathbb{Z}$ justifies a region $(\sigma, {}^\bullet\eta)$ of G if and only if:*
i) $\eta(e) + {}^\bullet\eta(e) \geq 0$ for all $e \in E$, and
ii) the scalar products $\sigma \cdot \partial(c)$ and $\eta \cdot \psi(c)$ are equal for any finite 1-chain c.

Proof: By linearity, $\sigma \cdot \partial(c) = \eta \cdot \psi(c)$ for every finite 1-chain c if and only if $\sigma \cdot \partial(t) = \eta \cdot \psi(t)$ for every arc $t \in T$. Now $\sigma \cdot \partial(t) = \sigma(\partial^1(t)) - \sigma(\partial^0(t))$ and $\eta \cdot \psi(t) = \eta(\lambda(t))$, hence condition (ii) may be rewritten to $\sigma(\partial^0(t)) + \eta(\lambda(t)) = \sigma(\partial^1(t))$. This is one of the requirements for justifying maps (see Def. 2.1). The other requirement is exactly condition (i). ∎

From now on, let $G' = (S, E, \Theta, s_0)$ be a fixed spanning tree for G; and for each $s \in S$, let c_s be the (unique) 1-chain in this spanning tree such that $\partial(c_s) = s - s_0$. Thus $c_s(t) \in \{-1, 0, 1\}$ for $t \in \Theta$.

Proposition 3.2 *A map $\eta : E \to \mathbb{Z}$ justifies regions of G if and only if*
$\star$ $\eta \cdot \psi(c) = 0$ *for every cycle c, and*
$\star$ $\eta \cdot \psi(c_s)$ *is uniformly bounded from below for $s \in S$.*
When these conditions are satisfied, the possible values $\sigma(s_0)$ and the possible weights ${}^\bullet\eta(e)$ for regions $(\sigma, {}^\bullet\eta)$ justified by η are jointly characterized by the conditions:
i) $\sigma(s_0) + \eta \cdot \psi(c_s) \geq {}^\bullet\eta(e)$ *whenever* $s \xrightarrow{e}$ *in G,*
ii) $\eta(e) + {}^\bullet\eta(e) \geq 0$ *for all $e \in E$.*

Proof:
$(\Rightarrow)$ Let $(\sigma, {}^\bullet\eta)$ be a region of G, justified by a map $\eta : E \to \mathbb{Z}$. For every cycle c of G, $\partial(c) = 0$ by definition of cycles, and hence $\eta \cdot \psi(c) = 0$ by Lemma 3.1. For any node $s \in S$, $\sigma(s) - \sigma(s_0) = \eta \cdot \psi(c_s)$ by Lemma 3.1, hence $-\sigma(s_0)$ is a lower bound for $\eta \cdot \psi(c_s)$. Condition (i) may be rewritten to $(s \xrightarrow{e}) \Rightarrow \sigma(s) \geq {}^\bullet\eta(e)$, *i.e.* to condition (i) in Def. 2.1. Condition (ii) follows trivially from condition (ii) in Def. 2.1.

$(\Leftarrow)$ One can clearly choose ${}^\bullet\eta(e) \in \mathbb{N}$ for each $e \in E$ such that condition (ii) is satisfied. Assume adequate values have been fixed. As E is finite and non-empty and G is event-reduced, and considering that $\eta \cdot \psi(c_s)$ is uniformly bounded from below, $\eta \cdot \psi(c_s) - {}^\bullet\eta(e)$ reaches a minimum in $\mathbb{Z}$ when e ranges over E and s ranges over the nodes such that $s \xrightarrow{e}$ in G. Therefore, one can certainly choose $\sigma(s_0) \in \mathbb{N}$ such that condition (i) is satisfied. Assume an adequate value has been fixed for $\sigma(s_0)$. This definition extends to a unique map $\sigma : S \to \mathbb{Z}$ such that $\sigma \cdot \partial(c_s) = \eta \cdot \psi(c_s)$ for all $s \in S$. Thus, $\sigma(s) = \sigma(s_0) + \eta \cdot \psi(c_s)$. Condition (i) ensures that $\sigma(s) \geq {}^\bullet\eta(e)$ whenever $s \xrightarrow{e}$ in G, *i.e.* condition (i) in Def. 2.1. Let us now establish condition (ii) in Def. 2.1, requiring that $\sigma \cdot \partial(t) = \eta \cdot \psi(t)$ for every arc $t \in T$. We proceed in separate ways for the arcs of the spanning tree and for the chords.
Considering that an arc t of the spanning tree such that $\partial^0(t) = s$ and $\partial^1(t) = s'$ may be expressed as the difference $c_{s'} - c_s$, the relation $\sigma \cdot \partial(t) = \eta \cdot \psi(t)$ follows directly from the definition $\sigma(s) = \sigma(s_0) + \eta \cdot \psi(c_s)$ for the arcs $t \in \Theta$.
For each chord $t \in T \setminus \Theta$, there exists in G a unique cycle c_t such that $c_t(t') = 1$ for $t' = t$, $c_t(t') \in \{-1, 0, 1\}$ for $t' \in \Theta$, and $c_t(t') = 0$ elsewhere. The cycle c_t is called the *fundamental* cycle of G determined by chord t. Let c be the 1-chain of G' defined as $c = c_t - t$; thus $c + t$ is a cycle and $\partial(c) + \partial(t) = 0$. Let

$\partial^0(t) = s$ and $\partial^1(t) = s'$, hence $\partial(c) = s - s'$. As c is a chain of the spanning tree G', $\partial(c) = s - s'$ entails $c = c_s - c_{s'}$. The following relations are therefore satisfied: $\sigma \cdot \partial(t) = -\sigma \cdot \partial(c) = \sigma \cdot (\partial(c_{s'}) - \partial(c_s)) = \sigma \cdot \partial(c_{s'}) - \sigma \cdot \partial(c_s) = \eta \cdot \psi(c_{s'}) - \eta \cdot \psi(c_s) = \eta \cdot (\psi(c_{s'}) - \psi(c_s))$. Considering that $c_s - c_{s'} + t$ is a cycle, $\eta \cdot \psi(c_s - c_{s'} + t) = 0$ by the hypothesis on η. Thus $\eta \cdot (\psi(c_{s'}) - \psi(c_s)) = \eta \cdot \psi(t)$, and $\sigma \cdot \partial(t) = \eta \cdot \psi(t)$ as was to show.
In order to conclude the proof, it remains to show that $\sigma : S \to \mathbb{N}$, *i.e.* that $\sigma(s) \geq 0$ for all $s \in S$. Since any node is the source or target of some arc, this follows from (i) for source nodes s, and this follows similarly from (i) and (ii) taken jointly for target nodes s' such that $s \xrightarrow{e} s'$. ∎

The above proposition may be refined, without loss of generality, by restricting the condition on cycles to bear upon fundamental cycles c_t (definition given in the proof). It is well known that the collection of all fundamental cycles c_t forms a basis for the finite cycles of G, which means that any finite cycle c of G writes in a unique way as a linear combination $\sum_t z_t \, c_t$ where t ranges over $T \setminus \Theta$, $z_t \in \mathbb{Z}$ and $z_t = 0$ for almost every t (see e.g. [Ber70] or [GM79]). It follows that the Parikh-vectors of finite cycles are the finite linear combinations $\psi(c) = \sum_t z_t \, \psi(c_t)$. Requiring $\eta \cdot \psi(c) = 0$ for all cycles c is thus equivalent to requiring $\eta \cdot \psi(c_t) = 0$ for fundamental cycles c_t.

Summing up, it has been shown that the regions of G may be characterized from the following:
- ⋆ a map sending each node $s \in S$ to the integer vector $\psi(c_s) \in (E \to \mathbb{Z})$,
- ⋆ a map sending each chord $t \in (T \setminus \Theta)$ to the integer vector $\psi(c_t)$,
- ⋆ a map sending each node $s \in S$ to the ready-set $\Re(s) = \{e \mid s \xrightarrow{e}\}$.

We will show that such infinite data reduce to finite and computable data in the special case of context-free graphs, yielding an effective characterization of regions suitable for synthesis algorithms.

4 Context-Free Graphs

A context-free graph is a rooted graph of finite degree such that, by removing all nodes within fixed distances from the root, one obtains on the whole a finite number of types of isomorphic connected components [MS85]. Müller and Schupp have shown that context-free graphs coincide with transition graphs of pushdown automata. This result was refined by Caucal who showed that transition graphs of pushdown automata coincide in an effective way with rooted graphs of finite degree generated from deterministic graph grammars [Cau92]. We will decide on the Petri net realization of context-free graphs. A workable representation of context-free graphs is needed for this purpose. Borrowing from [Cau92], we recall below the representation obtained from uniform graph grammars.

Definition 4.1 (Deterministic graph grammar) *Let F be a finite set of non-terminal symbols with positive arities, let E be a set of terminal symbols with arity two (disjoint from F), and let X be a set of node variables (x_i). A (terminal or non-terminal) hyperarc is a word $gx_1 \dots x_{ar(g)}$ headed by a (terminal or non-terminal) symbol g with arity $ar(g)$. A hypergraph is a nonempty set of hyperarcs. A deterministic graph grammar on (F, E, X) is a set of productions $fx_1 \dots x_{ar(f)} \to H_f$, one for each $f \in F$, such that H_f is a finite hypergraph and $x_i \neq x_j$ for $i \neq j$. Variables occurring on the left of productions are bound variables. The other variables are free variables.*

In the sequel, we identify without saying a graph (S, E, T) with the associated hypergraph T on $(\emptyset, E, S)$, where each arc $s \xrightarrow{e} s'$ is represented as a hyperarc ess'. A deterministic graph grammar $\mathcal{G}$ produces from any finite hypergraph Γ_0 a series of hypergraphs Γ_n as follows. Let Γ_{n+1} be derived from Γ_n by substituting for every hyperarc $fx'_1 \dots x'_k$ matching a production $fx_1 \dots x_k \to H_f$ a hypergraph $H_f\,[x'_i/x_i]_{\,i=1\dots k}\,[x''_j/x_j]_{\,j=k+1\dots l}$ in which each free variable x_j of the production is replaced by a fresh node variable x''_j. By taking for each n the induced restriction of Γ_n on the arcs (*i.e.* the hyperarcs ex_1x_2 labeled by terminal symbols $e \in E$), one forms an increasing sequence of finite graphs G_n. The graph generated by $\mathcal{G}$ from Γ_0 is the limit $G = \cup_n G_n$ of this sequence.

Theorem 4.2 (Caucal) *Transition graphs of pushdown automata coincide with reachable graphs of finite degree generated by deterministic graph grammars; the correspondence is effective in both directions; further, one may impose on the axiom Γ_0 to be a unary hyperarc f_0x_0 such that x_0 matches the initial configuration of the automaton.*

Definition 4.3 (Uniform grammars) *A deterministic graph grammar is uniform if the following conditions on the arcs and hyperarcs of H_f are satisfied for every production $f x_1 \ldots x_k \rightarrow H_f$:*
1) each bound variable x_i occurs on some arc and it does not occur on the non-terminal hyperarcs;
2) at least one bound variable x_i occurs on each arc;
3) every free variable occurring in H_f occurs on some arc;
4) in each non-terminal hyperarc, and in each pair of distinct non-terminal hyperarcs, all the occurrences of free variables are distinct.

Proposition 4.4 (Caucal) *A deterministic graph grammar generating a nonempty connected graph of finite degree from a fixed axiom Γ_0 may be transformed to a uniform grammar generating the identical graph from Γ_0.*

One may impose w.l.o.g. on a uniform grammar $\mathcal{G}$ that distinct productions have disjoint sets of free variables, and that each non-terminal symbol $f \in F$ occurs at most once on the right of each production. A convenient presentation of the generated graph G is then obtained. Denote by $\mathcal{X}$ the set of free variables of the grammar, and let $f_0 x_0$ be the axiom. If one sets apart the root of the graph, matched by the variable x_0, each node at depth $k+1$ may be coded by a word $f_0 \ldots f_k\, x$ such that $f_{i+1} \in F$ occurs in H_{f_i} for $i < k$ and $x \in \mathcal{X}$ occurs in H_{f_k}. Thus, for any arc $s \xrightarrow{e} s'$ in G, $\{s, s'\} = \{x_0, f_0\, x\}$ or $\{\phi f x, \phi f f' x'\}$ or $\{\phi f x, \phi f x'\}$ with $x, x' \in \mathcal{X}$, $\phi \in F^*$, and $f, f' \in F$. This way of representing graphs is adopted in the next section, where all uniform grammars are supposed to conform to the above specifications. For the sake of illustration, a typical example is shown below. This example will be continued throughout the paper.

Example 4.5 *Let $\mathcal{G}$ be the uniform grammar with productions as follows (x' and x'' are bound variables, x_1 to x_9 are free variables):*
$f_0\, x' \rightarrow \{a x' x_1,\ r x' x_2,\ f_1\, x_1 x_2\}$
$f_1\, x' x'' \rightarrow \{a x' x_3,\ f_2\, x_3,\ r x' x_4,\ a x'' x_4,\ r x'' x_5,\ f_1\, x_4 x_5\}$
$f_2\, x' \rightarrow \{b x' x_6,\ e x_7 x',\ f_3\, x_6 x_7\}$
$f_3\, x' x'' \rightarrow \{c x' x'',\ b x' x_8,\ e x_9 x',\ d x_9 x'',\ f_3\, x_8 x_9\}$.
The graph G produced from $f_0\, x_0$ is depicted in Fig. 1.

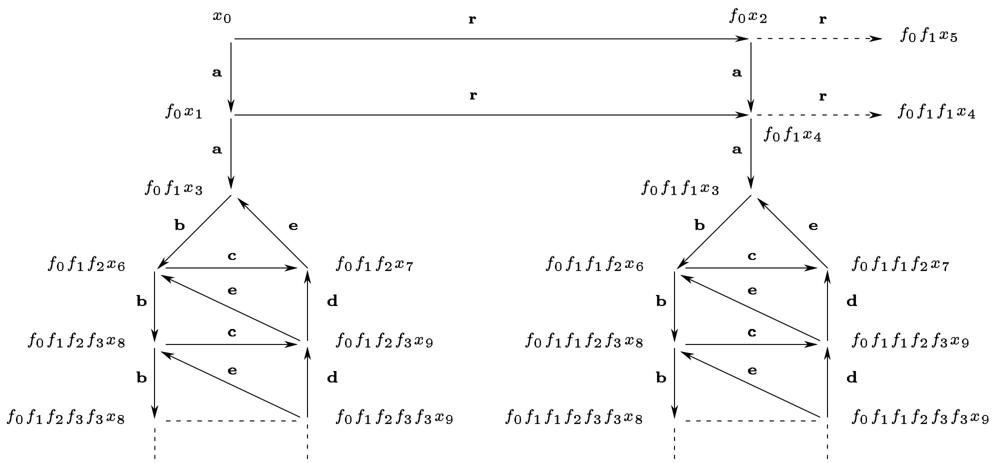

Figure 1: A context-free graph G

5 Regions of Context-Free Graphs

Relying on uniform grammars, we will refine the characterization of regions given in Section 3 to an effective characterization of the regions of a context-free graph. This amounts to show that, given a graph $G = (S, E, T, s_0)$ produced from a uniform grammar $\mathcal{G}$, and given a spanning tree $G' = (S, E, \Theta, s_0)$ produced from a simplified grammar $\mathcal{G}'$, such that every arc in Θ connects nodes at different depths in G, one can construct:
1) a finite set Γ of generators for the Parikh-vectors of cycles in G,
2) a finite automaton A such that G' may be folded onto A in such a way that the rooted paths of G' are sent bijectively to the runs of A and that all nodes $s \in S$ sent to the same state of A have the same ready-set in G.
These two items capture exactly the information we need on a context-free graph for deciding on its realization by Petri-nets.

Let $\mathcal{G}$ be a uniform grammar on (F, E, X) with productions $fx_1 \dots x_{ar(f)} \to H_f$, let $\mathcal{X} \subseteq X$ be the set of free variables of $\mathcal{G}$, and let $G = (S, E, T, s_0)$ be the graph produced by $\mathcal{G}$ from the axiom f_0x_0. Thus, a node $\phi\, x \in S$ such that $\phi = f_0 \dots f_k$ and $x \in \mathcal{X}$ is at distance $k+1$ from the root $s_0 = x_0$, and the restriction of G on the subset of nodes $s' = \phi\, \phi' x'$ with ϕ as a left factor is determined by f_k up to isomorphism of graphs. For more accuracy, let us introduce special operators on graphs.

Definition 5.1 *Given a graph $G' = (S', E', T')$ such that $S' \subseteq F^*\mathcal{X}$, and a word $\phi \in F^*$, let $G' \prec \phi$ (resp. $G' \succ \phi$) be the induced restriction of G' on the nodes $\phi' x$ such that $\phi' \leq \phi$ (resp. $\phi' \geq \phi$). Similarly, let $G'/\phi = (S'/\phi, E', T'/\phi)$ be the isomorphic copy of $G' \succ \phi$ such that $\phi\,\phi' x \in S'$ is mapped by the isomorphism to $\phi' x \in S'/\phi$; and let $\phi \cdot G' = (\phi \cdot S', E', \phi \cdot T')$ be the isomorphic copy of G' such that $\phi' x \in S'$ is mapped by the isomorphism to $\phi\,\phi' x \in \phi \cdot S'$.*

Coming back to the context-free graph $G = (S, E, T, s_0)$, one can observe that G/ϕ is determined *exactly* by the last symbol in ϕ, meaning that $G/\phi = G/\phi'$ whenever ϕ and ϕ' end with the same $f \in F$ (and there exists actually nodes $\phi\, x, \phi'\, x' \in S$). Considering the extra conditions we have posed on uniform grammars, one can observe that the ready-set of a node $s = \phi\, x$ in G is totally determined by the free variable $x \in \mathcal{X}$. The ready-set $\Re(x) = \Re(s)$ of the node $s = \phi\, x$ may be computed from the (unique) hypergraph H_f in which x occurs and the (unique) hypergraph $H_{f'}$, if it exists, such that $f' \ldots x \ldots \in H_f$.

Owing to these observations, computing from $\mathcal{G}$ a regular tree G' spanning G and folding it to a finite automaton is straightforward. One introduces for this purpose a modified grammar $\mathcal{G}'$ with productions $f x_1 \ldots x_{ar(f)} \rightarrow H'_f$ in bijective correspondence with the productions $f x_1 \ldots x_{ar(f)} \rightarrow H_f$ of $\mathcal{G}$. For each $f \in F$, H'_f is a subset of H_f, containing all the non-terminal hyperarcs and containing no terminal arc between bound variables, such that each variable $x \in \mathcal{X}$ free in H_f occurs on exactly one arc in H'_f (exy or eyx where $e \in E$ and $y \in X \backslash \mathcal{X}$). Let $G' = (S, E, \Theta, s_0)$ be the graph derived from $f_0 x_0$ using grammar $\mathcal{G}'$, then G' is a tree that spans G, every arc in Θ connects nodes at different depths in G, and G' is a regular tree since G'/ϕ is determined exactly by the last symbol in ϕ.

Example 5.2 (continued) *Let $\mathcal{G}'$ be the grammar with the productions:*
$f_0\, x' \rightarrow \{ax'x_1,\, rx'x_2,\, f_1\, x_1 x_2\}$
$f_1\, x'x'' \rightarrow \{ax'x_3,\, f_2\, x_3,\, ax''x_4,\, rx''x_5,\, f_1\, x_4 x_5\}$
$f_2\, x' \rightarrow \{bx'x_6,\, ex_7 x',\, f_3\, x_6 x_7\}$

$f_3\,x'x'' \to \{bx'x_8,\, dx_9x'',\, f_3\,x_8x_9\}$.
The tree G' produced from $f_0\,x_0$ is depicted in Fig. 2.

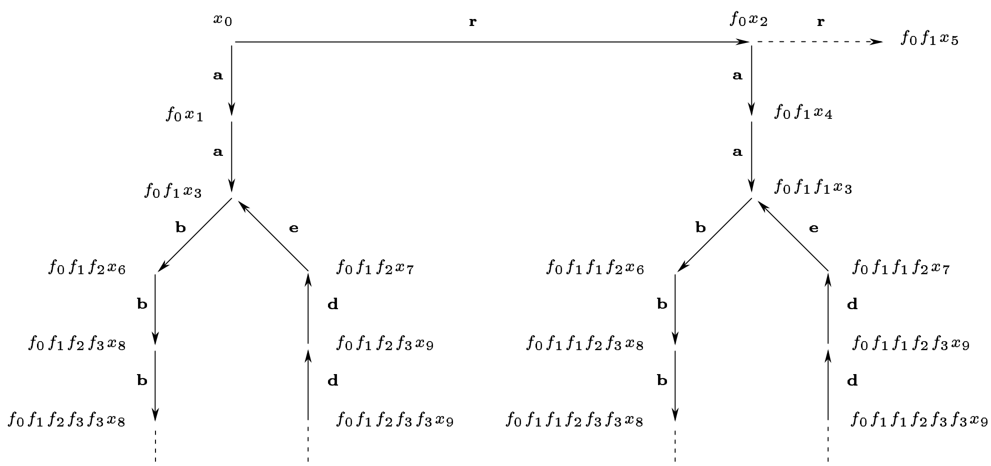

Figure 2: A spanning tree G'

All nodes $\phi\,x$ that end with the same variable x are roots of isomorphic subtrees, because x determines the last symbol f_k in ϕ. Therefore, G' may be folded to a finite automaton with set of states $\{x_0\} \cup \mathcal{X}$. Let $A = (\{x_0\} \cup \mathcal{X}, E \cup E^-, \tau, x_0)$ where $E^- = \{-e \mid e \in E\}$ and τ is the set of transitions as follows (with $x, x' \in \mathcal{X}$ and $e \in E$):
$\star$ let $x_0 \xrightarrow{e} x'$ (resp. $x_0 \xrightarrow{-e} x'$) be a transition of A if x_0 is bound to y in H_{f_0} and eyx' (resp. $ex'y$) is an arc in H'_{f_0};
$\star$ let $x \xrightarrow{e} x'$ (resp. $x \xrightarrow{-e} x'$) in A if some hyperarc $f \ldots x \ldots$ occurs in $\mathcal{G}$ such that x is bound to y in H_f and eyx' (resp. $ex'y$) is an arc in H'_f.
The regular tree G' may be folded to A by mapping the arc $\phi\,x \xrightarrow{e} \phi'\,x'$ to the transition $x \xrightarrow{e} x'$ or to the transition $x \xrightarrow{-e} x'$ according to $\phi' = \phi\,f$ or $\phi = \phi'\,f$. Since the ready-set of $s = \phi\,x$ is totally determined by x, all nodes $\phi\,x$ sent to a common state x have the same ready-set in G.

Example 5.3 (continued) *The spanning tree $\mathcal{G}'$ from Fig. 2 folds to the finite automaton A depicted in Fig. 3, with the following ready sets relative to the original graph $\mathcal{G}$ from Fig. 1:* $\Re(x_0) = \Re(x_1) = \Re(x_2) = \Re(x_4) = \Re(x_5) = \{a, r\}$, $\Re(x_3) = \{b\}$, $\Re(x_6) = \Re(x_8) = \{b, c\}$, $\Re(x_7) = \{e\}$, $\Re(x_9) = \{d, e\}$.

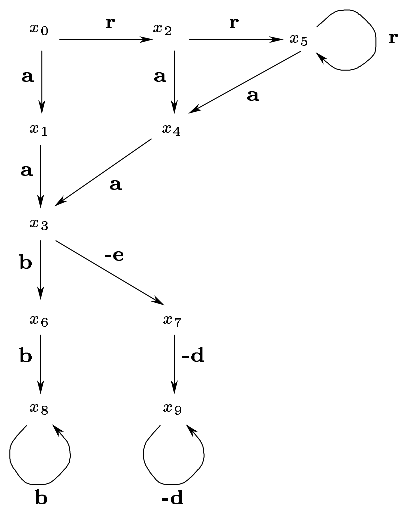

Figure 3: The folding of G'

It remains to show that rooted paths of G' are sent bijectively to runs of A. If two different paths were sent to the same run of A, there would exist in G' terminal arcs ess' and ess'' (or $es's$ and $es''s$) such that $s = \phi f x$, $s' = \phi f f' x'$, and $s'' = \phi f f'' x'$ with $f' \neq f''$. Since distinct productions have disjoint sets of free variables, this case is not possible: if x' occurs both in $H'_{f'}$ and in $H'_{f''}$ then necessarily $f' = f''$. Rooted paths of G' are therefore mapped injectively to runs of A. In order to show that the mapping is surjective, we suppose for contradiction that some rooted path in G' with target node $\phi f x$ cannot be extended by any arc $\phi f x \stackrel{e}{\rightarrow} \phi f f' x'$ (resp. reversed arc $\phi f x \stackrel{e}{\leftarrow} \phi f f' x'$) even though $x \stackrel{e}{\rightarrow} x'$ (resp. $x \stackrel{-e}{\rightarrow} x'$) is a transition of A. As x occurs in H_f and $x \stackrel{e}{\rightarrow} x'$ (resp. $x \stackrel{-e}{\rightarrow} x'$), there must exist a hyperarc $f' \ldots x \ldots \in H_f$ such that x is bound to y in $H_{f'}$ and eyx' (resp. $ex'y$) is an arc in $H'_{f'}$. Thus, $\phi f x \stackrel{e}{\rightarrow} \phi f f' x'$ (resp. $\phi f x \stackrel{e}{\leftarrow} \phi f f' x'$) is an arc in G'. It follows by an induction on the length of runs that every run of A is the image of a rooted path in G'. Therefore, the automaton A fulfils all the requirements.

Let us now start considering cycles and their Parikh-vectors. We saw that the Parikh-vector of a finite cycle c of G may always be written as a linear combination $\psi(c) = \sum_t z_t \, \psi(c_t)$, where c_t is the fundamental cycle determined by the chord t and $z_t = 0$ for almost all chords. A fundamental cycle c_t may be identified, up to the orientation fixed by the chord t, with the subgraph of G with the set of arcs $C_t = \{t' \mid t' \in T \wedge (c_t(t') = 1 \vee c_t(t') = -1)\}$. This identification is often used in the sequel without explicit mentionning. If one abstracts also from the directions of the arcs, there remains three possible forms for fundamental cycles c_t, shown in Fig. 4. In order to obtain a finite set of

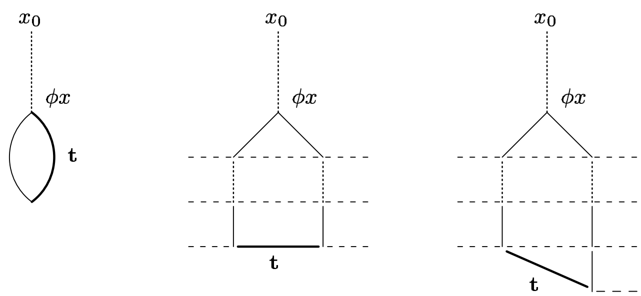

Figure 4: The possible forms of a fundamental cycle c_t

generators for Parikh-vectors of arbitrary cycles, we must further decompose fundamental cycles c_t as shown in Fig. 5. The idea is to cut fundamental cycles into slices delimited by similar pairs of nodes at constant depth, where the similarity type of the node ϕx is x.

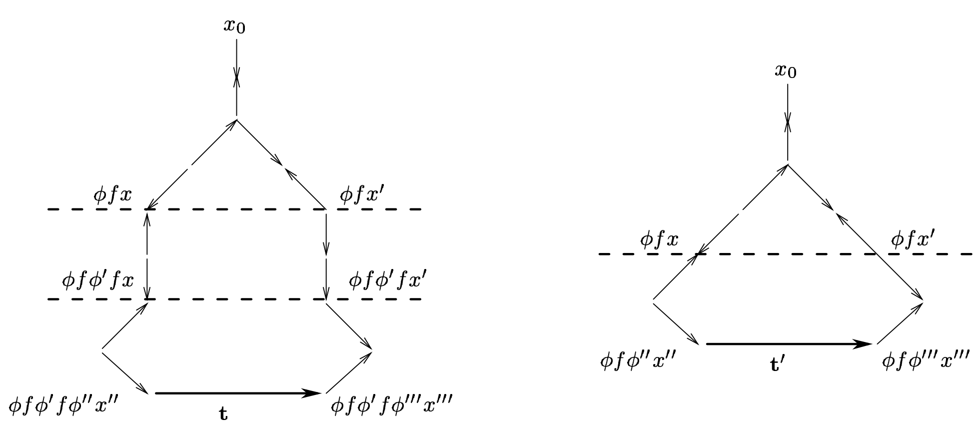

Figure 5: A slice in a fundamental cycle

Definition 5.4 *A cycle c_t with two pairs of distinct nodes $(\phi f x, \phi f x')$ and $(\phi f \phi' f x, \phi f \phi' f x')$ is said to be reducible. The restriction of c_t on the subset of arcs with source and target in the subset of nodes $\{\phi'' x'' \mid \phi f \leq \phi'' \leq \phi f \phi' f\}$ is a slice of c_t (hence slices are 1-chains like cycles). A slice is irreducible if no proper restriction of this slice is a slice.*

Lemma 5.5 *Let $(\phi f x, \phi f x')$ and $(\phi f \phi' f x, \phi f \phi' f x')$ be two pairs of distinct nodes on a fundamental cycle c_t, then the set of arcs $(C_t \prec \phi f) \cup \phi f \cdot (C_t / \phi f \phi' f)$ is a fundamental cycle.*

Proof: As $G/\phi f = G/\phi f \phi' f$, the considered set of arcs is a cycle of G. Let $\partial^0(t) = \phi f \phi' f \phi'' x''$ and $\partial^1(t) = \phi f \phi' f \phi''' x'''$. As $G/\phi f = G/\phi f \phi' f$, this cycle must contain an arc t' such that $\partial^0(t') = \phi f \phi'' x''$, $\partial^1(t') = \phi f \phi''' x'''$, and $\lambda(t') = \lambda(t)$. As $G'/\phi f = G'/\phi f \phi' f$, t' is a chord of G and the considered cycle contains no other chord of G, hence it is a fundamental cycle. ∎

Lemma 5.6 *Parikh-vectors of slices of fundamental cycles are Parikh-vectors of cycles.*

Proof: Let $(\phi f x, \phi f x')$ and $(\phi f \phi' f x, \phi f \phi' f x')$ be two pairs of distinct nodes on a fundamental cycle c_t. Let c be the slice of c_t delimited by these nodes (thus $\partial(c) = \phi f x' - \phi f \phi' f x' + \phi f \phi' f x - \phi f x$ or the opposite). As $G/\phi f = G/\phi f \phi' f$, the set of arcs $(C_t \succ \phi f) \cup \phi f \cdot (C_t / \phi f \phi' f)$ is a cycle of G. The Parikh-vector of c is equal to the Parikh-vector of this cycle. ∎

Proposition 5.7 *Parikh-vectors of cycles are generated by Parikh-vectors of irreducible cycles and irreducible slices of fundamental cycles.*

Proof: Parikh-vectors of cycles are linear combinations $\sum_t z_t \psi(c_t)$ of Parikh-vectors of fundamental cycles. By Lemma 5.5 and its proof, $\psi(c_t)$ may be expressed as a finite sum $\psi(c_{t'}) + \sum_i \psi(c_i)$ where $c_{t'}$ is irreducible and the c_i's are irreducible slices of c_t. ∎

At this stage, one may observe that an irreducible cycle c_t has at most $2 \times h$ nodes with $h = (1 + |F| \times |\mathcal{X}|^2)$. Considering that two cycles c_t and $c_{t'}$ such that $c_t = \phi \cdot c_{t'}$ have the same Parikh-vector (because C_t and $C_{t'}$ are isomorphic labelled graphs), Parikh-vectors of irreducible cycles form a finite set. This finite set may actually be computed. For this purpose, it suffices to proceed as follows for each possible axiom $f \ldots x \ldots$ with $f \in F$ and $x \in \mathcal{X}$: derive from the axiom a graph G_h of bounded depth h using grammar $\mathcal{G}$; derive from the axiom a spanning tree G'_h for G_h using grammar $\mathcal{G}'$; list the irreducible cycles of G_h with respect to G'_h. This takes finite time since there are finitely many axioms to consider. One may proceed in a similar way for irreducible slices of fundamental cycles, but replacing h by $3 \times h$. Gathering all the Parikh-vectors listed at either stage, one obtains a finite set $\{\gamma_1, \ldots, \gamma_n\} \subset (E \to \mathbb{Z})$, such that every Parikh-vector of a cycle writes as a linear combination $\sum_j (z_j\, \gamma_j)$ with $z_j \in \mathbb{Z}$. These integral vectors may not be linearly independent in $E \to \mathbb{Q}$. If not, Gaussian elimination may be used to extract from this set a maximal subset of linearly independent vectors $\Gamma = \{\gamma_1, \ldots, \gamma_p\}$. In the sequel, vectors $\gamma \in (E \to \mathbb{Z})$ are written as formal sums $\gamma = \sum_e \gamma(e) \cdot e$ where e ranges over E.

Example 5.8 (continued) *The irreducible cycles of* G *(see Fig. 1) w.r.t. the spanning tree* G' *(see Fig. 2) are depicted in Fig. 6. The respective Parikh-vectors are from left to right:* 0, $b+c+e$, $b+d$, $2b+c+d+e$, *and* $2b+2d$. *There is one irreducible slice, determined by nodes with similarity types* x_8 *and* x_9, *with Parikh-vector* $b+d$. *After eliminating the linear dependences, one obtains* $\Gamma = \{b+c+e,\, b+d\}$.

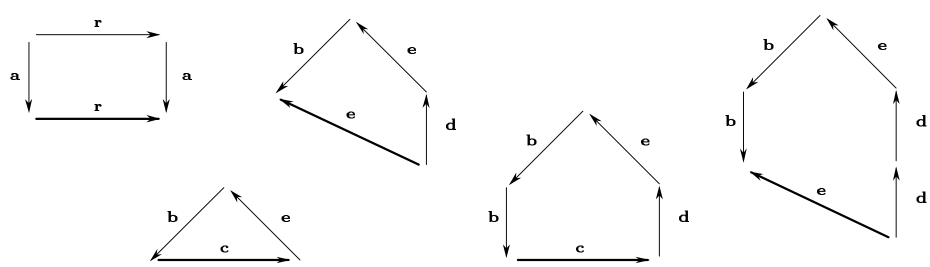

Figure 6: The irreducible cycles of G w.r.t. G'

We are in a position to state a simplified form of the net realization problem for context-free graphs, using the results from sections 2 and 3 and the special data elaborated in this section. Starting from an event-reduced graph $G = (S, E, T, s_0)$ defined by a graph grammar, we have obtained:

1) a finite set of vectors $\Gamma = \{\gamma_1, \ldots, \gamma_p\}$ generating Parikh-vectors of cycles of G up to an integer multiplication,
2) a finite automaton $A = (X, E \cup E^-, \tau, x_0)$ with runs in bijection with the rooted paths of a tree G' spanning G, such that $\psi(c_s) = \varepsilon_1 + \ldots + \varepsilon_n$ for a path c_s from s_0 to s with associated run $\rho(c_s) = x_0 \xrightarrow{\varepsilon_1} x_1 \ldots \xrightarrow{\varepsilon_n} x_n$,
3) a map $\Re : X \to \mathcal{P}(E)$ such that $s \xrightarrow{e}$ in $G \Leftrightarrow e \in \Re(x_n)$ where x_n is the target state of the run $\rho(c_s)$.
Using these data, a specialized version of Prop. 3.2 may be stated for context-free graphs. Let ρ range over $Runs(A)$, the set of runs of A, with typical run $\rho = x_0 \xrightarrow{\varepsilon_1} x_1 \ldots \xrightarrow{\varepsilon_n} x_n$. Extending the notation, let $\partial^1(\rho) = x_n$ and $\psi(\rho) = \varepsilon_1 + \ldots + \varepsilon_n$.

Proposition 5.9 *A map $\eta : E \to \mathbb{Z}$ justifies regions of G if and only if*
$\star$ *$\eta \cdot \gamma = 0$ for every $\gamma \in \Gamma$, and*
$\star$ *$\eta \cdot \psi(\rho)$ is uniformly bounded from below for $\rho \in Runs(A)$.*
When these conditions are satisfied, the possible values $\sigma(s_0)$ and the possible weights ${}^\bullet\eta(e)$ for regions $(\sigma, {}^\bullet\eta)$ justified by η are characterized by:
i) $\sigma(s_0) + \eta \cdot \psi(\rho) \geq {}^\bullet\eta(e)$ *whenever* $e \in \Re(\partial^1(\rho))$,
ii) $\eta(e) + {}^\bullet\eta(e) \geq 0$ *for all* $e \in E$.

Theorem 2.3 may finally be specialized as follows, relying on lemma 3.1.

Theorem 5.10 *G is isomorphic to the reachable state graph of a finite Petri net if and only if there exists a finite set of regions $(\sigma_i, {}^\bullet\eta_i)$, justified by corresponding maps η_i, such that the following axioms hold for all runs $\rho, \rho' \in Runs(A)$ and for every event $e \in E$:*
(SSA) $\rho \neq \rho' \Rightarrow \eta_i \cdot \psi(\rho) \neq \eta_i \cdot \psi(\rho')$ *for some* i,
(ESSA) $e \notin \Re(\partial^1(\rho)) \Rightarrow \sigma_i(s_0) + \eta_i \cdot \psi(\rho) < {}^\bullet\eta_i(e)$ *for some* i.

6 The Polyhedral Cone of Regions

Postponing the search of decision procedures for states separation and for event-state separation, we refine Prop. 5.9 by showing that regions are all integral vectors of a polyhedral cone in the rational vector space. The generators of this cone may be computed from Γ and A. In order to prepare the computation, we decompose runs of A into direct runs and round trips.

Definition 6.1 *A run ρ is direct if no state x is visited twice. A round trip is a sequence of transitions $x'_0 \xrightarrow{\varepsilon_1} x'_1 \dots \xrightarrow{\varepsilon_n} x'_n$ (where possibly $x_0 \neq x'_0$) such that $x'_0 = x'_n$ and $(x'_i = x'_j \wedge i < j) \Rightarrow (i = 0 \wedge j = n)$.*

Direct runs and round trips form respective finite sets $\Delta(A) = \{\delta_1, \dots, \delta_q\}$ and $\Omega(A) = \{\omega_1, \dots, \omega_r\}$. Each finite run may be decomposed into a direct run and a finite multiset of round trips. Thus, the Parikh-vector of a run $\rho \in Runs(A)$ may be written as a finite sum $\psi(\rho) = \psi(\delta) + \sum_j n_j \psi(\omega_j)$ where $\delta \in \Delta(A)$ and $n_j \in I\!N$ for $1 \leq j \leq r$. Conversely, there exists for each round trip $\omega \in \Omega(A)$ some direct run $\delta \in \Delta(A)$ such that $\delta\,\omega^n \in Runs(A)$ for all $n \in I\!N$. The following proposition should therefore be clear.

Proposition 6.2 *A map $\eta : E \to Z\!\!Z$ justifies regions of G if and only if*
$\star$ $\eta \cdot \gamma = 0$ for every $\gamma \in \Gamma$, and
$\star$ $\eta \cdot \psi(\omega) \geq 0$ for every $\omega \in \Omega(A)$.
When these conditions are satisfied, the possible values $\sigma(s_0)$ and the possible weights ${}^\bullet\eta(e)$ for regions $(\sigma, {}^\bullet\eta)$ justified by η are characterized by:
i) $\sigma(s_0) + \eta \cdot \psi(\delta) \geq {}^\bullet\eta(e)$ for $\delta \in \Delta(A)$ and $e \in \Re\,(\partial^{\,1}(\delta))$,
ii) $\eta(e) + {}^\bullet\eta(e) \geq 0$ for all $e \in E$.

A region $(\sigma, {}^\bullet\eta)$ with justifying map η may be represented as an integral vector $(\eta\,; {}^\bullet\eta\,; \iota)$ with $\iota = \sigma(s_0)$. Let $\bar\eta$ and ${}^\bullet\bar\eta$ range over rational vectors in the vector space $E \to Q\!\!\!\!I$, and let $\bar\iota$ range over $Q\!\!\!\!I$. By Prop. 6.2, a vector $(\eta\,; {}^\bullet\eta\,; \iota)$ represents a region if and only if it is an integral solution of the finite linear system with equations and inequations as follows:

$\bar\eta \cdot \gamma = 0$ (for each $\gamma \in \Gamma$),
$\bar\eta \cdot \psi(\omega) \geq 0$ (for each $\omega \in \Omega(A)$),
$\bar\iota + \bar\eta \cdot \psi(\delta) - {}^\bullet\bar\eta(e) \geq 0$ (for each $\delta \in \Delta(A)$ and for each $e \in \Re\,(\partial^{\,1}(\delta))$,
$\bar\eta(e) + {}^\bullet\bar\eta(e) \geq 0$ and ${}^\bullet\bar\eta(e) \geq 0$ (for each $e \in E$), $\bar\iota \geq 0$.

These linear homogeneous equations and inequations define a polyhedral cone in the rational vector space. By the Farkas-Minkowski-Weyl theorem, this cone is finitely generated (see [Sch86] p.85-87). A set of generating vectors may actually be computed by Chernikova's algorithm [Che65]. Thus, the solutions of the system are all non-negative linear combinations

$(\bar\eta\,; {}^\bullet\bar\eta\,; \bar\iota) = \sum_{i=1}^m q_i \times (\bar\eta_i\,; {}^\bullet\bar\eta_i\,; \bar\iota_i)$, with coefficients $q_i \in \mathbb{Q}^+$ of a finite set of rational vectors $(\bar{\eta}_i\,;{}^\bullet\bar{\eta}_i\,;\bar{\iota}_i)$ computable from Γ, $\Delta(A)$, $\Re$ and $\Omega(A)$. The generated cone does not change under the multiplication of generators by positive numbers, hence one may assume w.l.o.g. that the generating set is a family of integral vectors

$P = \{(\eta_i\,;{}^\bullet\eta_i\,;\iota_i) \mid 1 \le i \le m\}$.

The regions of G are then all integral vectors in the rational cone $cone(P)$ generated by P. Generators $(\eta_i\,;{}^\bullet\eta_i\,;\iota_i)$ will play a crucial role for deciding upon separation: since every region in $cone(P)$ is a non-negative linear combination of the regions in P, the separation axioms are satisfied if and only if each instance of these axioms is satisfied by some region in P.

It is important to remark that two regions of G which are represented by two opposite vectors must both be equal to the trivial region represented by the null vector (this follows from Def. 2.1 using the hypothesis that G is event-reduced). The cone of regions is thus a pointed cone; hence the generating set P must coincide with the set of all extremal rays, and it is defined uniquely (up to scalar multiplication of vectors by positive integers). Regions in the set P deserve therefore to be designated as the *canonical* regions of G. The Petri net synthesized from all canonical regions may be designated likewise as the canonical Petri net synthesized from G.

Example 6.3 (continued) *In our example, where $E = \{a, b, c, d, e, r\}$, the cone of regions is defined by the reduced set of equations and inequations:*

$\eta(b) + \eta(c) + \eta(e) = 0,\ \eta(b) + \eta(d) = 0,$
$\eta(b) \ge 0,\ \eta(d) \le 0,\ \eta(r) \ge 0,$
$\iota - {}^\bullet\eta(a) \ge 0,\ \iota - {}^\bullet\eta(r) \ge 0,$
$\iota + \eta(a) - {}^\bullet\eta(a) \ge 0,\ \iota + \eta(a) - {}^\bullet\eta(r) \ge 0,$
$\iota + 2\eta(a) - {}^\bullet\eta(b) \ge 0,$
$\iota + 2\eta(a) + \eta(b) - {}^\bullet\eta(c) \ge 0,$
$\iota + 2\eta(a) - \eta(e) - {}^\bullet\eta(e) \ge 0,$
$\iota + 2\eta(a) - \eta(e) - \eta(d) - {}^\bullet\eta(d) \ge 0,$
$\eta(\varepsilon) + {}^\bullet\eta(\varepsilon) \ge 0$ *and* ${}^\bullet\eta(\varepsilon) \ge 0$ *for* $\varepsilon \in E,$
$\iota \ge 0.$

This cone has 79 rays, which were computed with the help of the Polyhedral

Table 1: Five extremal rays

	a	b	c	d	e	r	$^\bullet a$	$^\bullet b$	$^\bullet c$	$^\bullet d$	$^\bullet e$	$^\bullet r$	ι
p_1	-1	0	0	0	0	0	1	0	0	0	0	1	2
p_2	0	0	0	0	0	1	0	0	0	0	0	0	0
p_3	1	0	-2	0	2	0	0	2	2	0	0	0	0
p_4	0	1	-1	-1	0	0	0	0	1	1	0	0	0
p_5	0	0	1	0	-1	0	0	0	0	1	1	0	0

Library [Wil93]. Among these extremal rays are the 5 rays listed in table 1. The family of all extremal rays defines a marked Petri net with 79 places. The subnet of the canonical net with places p_1 to p_5 is depicted in Fig. 7. The reader

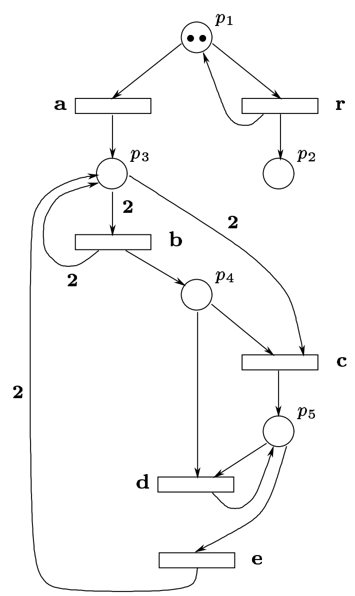

Figure 7: A synthesized net N

may verify that the reachable state graph of this subnet, shown in Fig. 8, is in fact isomorphic to the context-free graph G. The situation is similar for any larger subnet of the canonical net.

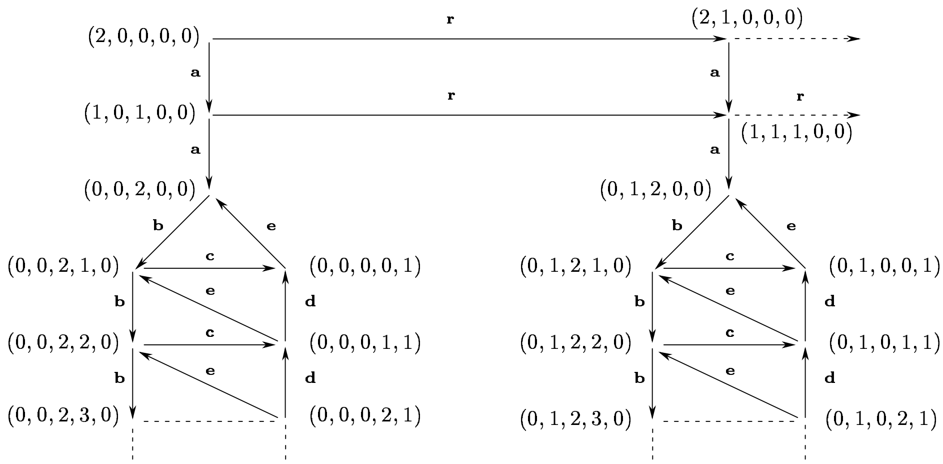

Figure 8: The reachable state graph of N

7 Deciding on Event-State Separation

From Theo. 5.10 and Prop. 6.2, the event-state separation axiom is satisfied in the context-free graph G if and only if, for every event $e \in E$ and for every run ρ of the automaton $A = (X, E \cup E^-, \tau, x_0)$ leading to a state x such that $e \notin \Re(x)$, there exists in $cone(P)$, or yet equivalently in P, some region $(\eta\,; {}^\bullet\eta\,; \iota)$ such that:

$$\iota + \eta \cdot \psi(\rho) - {}^\bullet\eta(e) < 0 \tag{1}$$

Since X is finite, we can proceed separately with subsets of runs ρ with different target states $x = \partial^1(\rho)$. So let x be a fixed state in X. The set of Parikh-vectors $\psi(\rho)$ of runs $\rho \in Runs(A)$ such that $x = \partial^1(\rho)$ is equal to the set of Parikh-vectors $\psi(w)$ of words w accepted by the automaton $A_x = (X, E \cup E^-, \tau, x_0, x)$ with x as the unique accepting state. Since A_x is a finite automaton, this set is a semi-linear subset of $E \to \mathbb{Z}$ and one can effectively derive from A_x a semi-linear expression of this subset, let

$$B_x = \cup_{i=1}^{p} (u_i + V_i^*)$$

where u_i is a vector in $(E \to \mathbb{Z})$ and V_i is a finite set of vectors in $(E \to \mathbb{Z})$.

> We recall that the linear set $u + V^*$, where $V = \{v_1, \ldots, v_q\}$, is the set of all vectors $u + \sum_{j=1}^{q} n_j v_j$ with coefficients $n_j \in \mathbb{N}$. The construction of semi-linear expressions from finite automata on free commutative monoids is explained in ([Par61],[Par66]). One may also consult [Har78]. The construction may be extended to finite automata on arbitrary commutative monoids. The correspondence between rational subsets and semi-linear subsets is indeed effective in any commutative monoid, and in particular in $(E \to \mathbb{Z})$ ([ES69], [Reu89]) .

Since E is finite and B_x is a finite union of linear sets, we can proceed separately with each event $e \in E$ and with each linear subset $u_i + V_i^*$ of B_x. Let e be a fixed event in E, such that $e \notin \Re(x)$, and let $W = u + V^*$ be a fixed linear subset of B_x, with $V = \{v_1, \ldots, v_q\}$. We should decide whether there exists for each tuple $(n_1, \ldots, n_q)$ of non-negative integers some canonical region $(\eta\,; {}^\bullet\eta\,; \iota)$ in P such that:

$$\iota + \eta \cdot u + \sum_{j=1}^{q} n_j(\eta \cdot v_j) - {}^\bullet\eta(e) < 0 \qquad (2)$$

Claim 7.1 *If there exists for each tuple of non-negative integers $(n_1, \ldots, n_q)$ some region $(\eta\,; {}^\bullet\eta\,; \iota) \in P$ such that inequation 2 is satisfied, then there exists a region $(\eta\,; {}^\bullet\eta\,; \iota) \in P$ such that inequation 2 is satisfied for all $(n_1, \ldots, n_q)$.*

This claim is based on the following lemma and corollary.

Lemma 7.2 *$\eta \cdot v_j \geq 0$ for any region $(\eta\,; {}^\bullet\eta\,; \iota) \in P$ and for every $v_j \in V$.*

Proof: Suppose otherwise. By definition of A_x and B_x, for every $n \in \mathbb{N}$, $u + nv_j$ is the Parikh-vector of some run $\rho_n \in Runs(A)$. Thus, $\eta \cdot \psi(\rho)$ is not uniformly bounded from below for $\rho \in Runs(A)$, contradicting Prop. 5.9. ∎

Corollary 7.3 *Let $(\eta\,; {}^\bullet\eta\,; \iota)$ be a region in P such that inequation 2 holds for $(n_1, \ldots, n_q)$, then it holds for all $(n'_1, \ldots, n'_q)$ such that $n'_j \leq n_j$ for all j.*

Proof of the claim: Let $P' \subseteq P$ be a subset of canonical regions such that inequation 2 may be satisfied for all tuples $(n_1, \ldots, n_q)$ by regions in P', and let $p = (\eta\,; {}^\bullet\eta\,; \iota) \in P'$. We show that if P' contains at least two regions, the supposed property of P' holds for a strictly smaller set P''. As P is finite, the claim follows by induction on $|P'|$. We proceed by case analysis.

If p does not contribute at all to the satisfaction of inequation 2, the property assumed for P' must hold for $P'' = P' \backslash \{p\}$.

If all instances of inequation 2 are satisfied by p, the property holds for $P'' = \{p\}$.

In the remaining cases, there must exist some tuple $(n_1, \ldots, n_q)$ and some $k \in \{1, \ldots, q\}$ such that:
$\iota + \eta \cdot u + \sum_{j=1}^{q} n_j(\eta \cdot v_j) - {}^\bullet\eta(e) < 0$, and
$\iota + \eta \cdot u + \sum_{j=1}^{q} n_j(\eta \cdot v_j) + \eta \cdot v_k - {}^\bullet\eta(e) \geq 0$.
Thus necessarily $\eta \cdot v_k > 0$. As a consequence, whenever
$\iota + \eta \cdot u + \sum_{j=1}^{q} n'_j(\eta \cdot v_j) - {}^\bullet\eta(e) < 0$
for some tuple $(n'_1, \ldots, n'_q)$, there exists $m \in I\!N$ such that
$\iota + \eta \cdot u + \sum_{j=1}^{q} n'_j(\eta \cdot v_j) + m(\eta \cdot v_k) - {}^\bullet\eta(e) \geq 0$.
Now consider the tuple $(n'_1, \ldots, n'_{k-1}, n'_k + m, n'_{k+1}, \ldots, n'_q)$.
From the assumption on P', there exists $p' = (\eta' ; {}^\bullet\eta' ; \iota') \in P'$ such that:
$\iota' + \eta' \cdot u + \sum_{j=1}^{q} n'_j(\eta' \cdot v_j) + m(\eta' \cdot v_k) - {}^\bullet\eta'(e) < 0$.
Therefore, by lemma 7.2 and corollary 7.3:
$\iota' + \eta' \cdot u + \sum_{j=1}^{q} n'_j(\eta' \cdot v_j) - {}^\bullet\eta'(e) < 0$,
and the supposed property of P' holds for $P'' = P' \backslash \{p\}$. ∎

Proposition 7.4 *A region $(\eta ; {}^\bullet\eta ; \iota) \in P$ solves all instances of the inequation 2 if and only if $\iota + \eta \cdot u - {}^\bullet\eta(e) < 0$ and $\eta \cdot v_j = 0$ for every $j \in \{1, \ldots, q\}$.*

Proof: Straighforward from lemma 7.2. ∎

It follows from claim 7.1 that one may decide whether event-state separation holds in G by checking canonical regions in the finite set P against the conditions of Prop. 7.4 for each $x \in X$ and for each linear subset $u + V^*$ of $B_x = \cup_{i=1}^{p} (u_i + V_i^*)$. Altogether, we have established the following.

Theorem 7.5 *Given a context-free graph G, one may compute a finite set of canonical regions P such that the event-state separation axiom* **ESSA** *is valid with respect to regions of G if and only if it is valid with respect to regions in P, and one may decide on the latter.*

Example 7.6 (continued) *Let $x = x_9$, then $B_x = (2,0,0,-1,-1,0) + \{(0,0,0,0,0,0,1),(0,0,0,-1,0,0)\}^*$ where events in $E = \{a,b,c,d,e,r\}$ are enumerated in this order. We search for a canonical region $(\eta\,;{}^\bullet\eta\,;\iota)$ in P such that: $\iota + 2\eta(a) - \eta(d) - \eta(e) - {}^\bullet\eta(c) < 0$, $\eta(d) = 0$ and $\eta(r) = 0$. These conditions hold for p_3 in the table, hence event-state separation holds in G for the event c at all nodes ending with $x = x_9$.*

8 Deciding on States Separation

From Theo. 5.10 and Prop. 6.2, the states separation axiom is satisfied in G if and only if, for any two different runs $\rho, \rho' \in Runs(A)$, there exists in $cone(P)$, or equivalently in P, some region $(\eta\,;{}^\bullet\eta\,;\iota)$ such that:

$$\iota + \eta \cdot \psi(\rho) \neq \iota + \eta \cdot \psi(\rho') \tag{3}$$

As a preliminary to deciding on states separation, we construct from $A = (X, E \cup E^-, \tau, x_0)$ a finite automaton $A^{\bowtie}$ that recognizes all differences $\psi(\rho) - \psi(\rho')$ between Parikh-vectors of different runs $\rho, \rho' \in Runs(A)$.

The set of states of $A^{\bowtie}$ is $X \times X \times \{0,1,2,3\}$. Each state (x, x', i) represents the respective states x and x' of two copies of A run independently and produces a comparison between their respective runs ρ and ρ' as follows: $i = 0$ if $\rho = \rho'$, 1 if ρ is shorter than ρ', 2 if ρ' is shorter than ρ, and 3 if $\rho \neq \rho'$ and both runs have identical length.

The alphabet of $A^{\bowtie}$ is the subset of vectors $\Psi \in (E \to \mathbb{Z})$ such that the absolute values $|\Psi(e)|$ of their entries add up to a sum bounded by 2. Each vector Ψ measures the difference added to $\psi(\rho) - \psi(\rho')$ by firing at most one transition in each of the two copies of A. For convenience, these vectors are denoted below as 0, ε, or $\varepsilon_1 - \varepsilon_2$ (where $\varepsilon, \varepsilon_1, \varepsilon_2 \in E \cup E^-$ and $e = -(-e)$ for all $e \in E$). Thus for instance, $(-e_1) - (-e_2)$ denotes the vector Ψ such that $\Psi(e_1) = -1$, $\Psi(e_2) = 1$, and $\Psi(e) = 0$ for the remaining events.

The initial state of $A^{\bowtie}$ is $(x_0, x_0, 0)$, meaning that both copies of A are in the initial state. The accepting states of $A^{\bowtie}$ are all states (x, x', i) with $i \neq 0$, meaning that the runs of the two copies of A have already diverged (possibly by stopping one run and continuing with the other).

The transitions of $A^{\bowtie}$ derive from the transitions of A as follows. For all transitions $(x \xrightarrow{\varepsilon} x')$, $(x_1 \xrightarrow{\varepsilon_1} x_1')$, $(x_2 \xrightarrow{\varepsilon_2} x_2')$ in τ, and for each state $x'' \in X$,

Table 2: Transitions of $A^{\bowtie}$

$(x, x, 0)$	$\xrightarrow{0}$	$(x', x', 0)$	
$(x, x, 0)$	$\xrightarrow{\varepsilon}$	$(x', x, 1)$	
$(x_1, x'', 1)$	$\xrightarrow{\varepsilon_1}$	$(x'_1, x'', 1)$	
$(x, x, 0)$	$\xrightarrow{-\varepsilon}$	$(x, x', 2)$	
$(x'', x_2, 2)$	$\xrightarrow{-\varepsilon_2}$	$(x'', x'_2, 2)$	
$(x_1, x_2, 0)$	$\xrightarrow{\varepsilon_1 - \varepsilon_2}$	$(x'_1, x'_2, 3)$	*if* $\varepsilon_1 \neq \varepsilon_2$ *or* $x'_1 \neq x'_2$
$(x_1, x_2, 3)$	$\xrightarrow{\varepsilon_1 - \varepsilon_2}$	$(x'_1, x'_2, 3)$	
$(x_1, x'', 3)$	$\xrightarrow{\varepsilon_1}$	$(x'_1, x'', 1)$	
$(x'', x_2, 3)$	$\xrightarrow{-\varepsilon_2}$	$(x'', x'_2, 2)$	

let the transitions described in table 2 be transitions of $A^{\bowtie}$.

As a finite automaton, $A^{\bowtie}$ recognizes a semi-linear subset of $(E \rightarrow \mathbb{Z})$, let

$$B^{\bowtie} = \cup_{i=1}^{p} (u_i + V_i^*)$$

This semi-linear expression and the finite set P of the canonical regions are adequate data for deciding on states separation. The decision is cut in two stages. One decides first from the data $B^{\bowtie}$ whether any two different runs of A have different Parikh-vectors. If this is the case one decides next from P and $B^{\bowtie}$ whether any two Parikh-vectors of different runs $\psi(\rho)$ and $\psi(\rho')$ are separated by some canonical region $(\eta\,; {}^{\bullet}\eta\,; \iota) \in P$ such that $\eta \cdot \psi(\rho) \neq \eta \cdot \psi(\rho')$. Both stages of the decision rely on the effectiveness of the boolean operations on semi-linear subsets of $(E \rightarrow \mathbb{N})$ and $(E \rightarrow \mathbb{Z})$. The reminder below may be skipped by readers familiar with this subject.

> Ginsburg and Spanier proved in [GS64] that semi-linear subsets of $(E \rightarrow \mathbb{N})$ form an effective boolean algebra, which means that their intersection and complementation are computable. The same authors gave in [GS66] an effective correspondence between semi-linear subsets and Presburger subsets, i.e. subsets of $(E \rightarrow \mathbb{N})$ definable in Presburger's arithmetic. The algebraic results established in [GS64] were extended soon after by Eilenberg and Schützenberger who proved that semi-linear subsets form a boolean algebra in any finitely generated commutative

monoid [ES69]. This covers the case of $(E \to \mathbb{Z})$, but the constructions given in [ES69] are not immediately effective.

An effective construction of the intersection of semi-linear subsets of $(E \to \mathbb{Z})$ is given in [Reu89], relying on a lemma from [GS64] strengthened in [ES69]. The import of the lemma is as follows: given a homomorphism of monoids $\phi : (E_1 \to \mathbb{N}) \to (E_2 \to \mathbb{Z})$ and a vector $w \in (E_2 \to \mathbb{Z})$, the inverse image $\phi^{-1}(w)$ of w under ϕ is a semi-linear subset of $(E_1 \to \mathbb{N})$ and it is effectively computable. From this lemma follows an important corollary (used in the sequel): the set of solutions in $(E \to \mathbb{N})$ of a system of linear inequations with coefficients in $\mathbb{Z}$ is semi-linear and it can effectively be computed [Reu89]. The set of non-negative solutions $(n_1, \ldots, n_k)$ of an inequation $z_1 n_1 + \ldots + z_k n_k \geq 0$ is actually semi-linear, and such is by the lemma the set of non-negative solutions $(n_1, \ldots, n_k)$ of an equation $z_1 n_1 + \ldots + z_k n_k = z$.

Although we do not need complementation in $(E \to \mathbb{Z})$, one may prove easily that semi-linear subsets of $(E \to \mathbb{Z})$ form an effective boolean algebra from the similar property of semi-linear subsets of $(E \to \mathbb{N})$. The idea is to decompose each semi-linear subset $W \subseteq (E \to \mathbb{Z})$ into a finite union of subsets
$W = \cup \{ \zeta * W_\zeta \mid \zeta : E \to \{-1, +1\} \}$
such that $W_\zeta = (\zeta * W) \cap (E \to \mathbb{N})$
with $\zeta * W = \{ \zeta * w \mid w \in W \}$
and $(\zeta * w)(e) = \zeta(e) \times w(e)$ for $w \in W$ and $e \in E$.
It is easily seen that each set $\zeta * W$ is a semi-linear subset of $(E \to \mathbb{Z})$, hence W_ζ is also (as an intersection of semi-linear sets).
From the definition, $W_\zeta \subseteq (E \to \mathbb{N})$, hence each W_ζ is a semi-linear subset of $(E \to \mathbb{N})$.
As a consequence, its complement $\mathcal{C}W_\zeta$ with respect to $(E \to \mathbb{N})$ is semi-linear and it may be computed.
Now observe that the complement of $\zeta * W_\zeta$ with respect to $(E \to \mathbb{Z})$ may be expressed as $(\zeta * \mathcal{C}W_\zeta) \cup V_\zeta$,
where V_ζ is the set of vectors
$v : E \to \mathbb{Z}$ such that $(\zeta * v)(e) \leq -1$ for some $e \in E$.
It is not difficult to see that V_ζ is semi-linear.

We come back to the decision of states separation. The first stage is straightforward since the singleton set $\{0\}$ is a semi-linear subset of $(E \to \mathbb{Z})$ and the intersection of semi-linear subsets is effective. Thus it suffices to compute a semi-linear expression of $B^{\bowtie} \cap \{0\}$ and to check that it differs from the null expression (the union of the empty family of linear subsets) to decide that two different runs $\rho, \rho' \in Runs(A)$ have always different Parikh-vectors. Since condition 3 cannot be satisfied when $\psi(\rho) = \psi(\rho')$, states separation cannot be valid in G if the decision produces a negative answer.

Assuming that $\psi(\rho) \neq \psi(\rho')$ for any two different runs $\rho, \rho' \in Runs(A)$, we enter now the second stage. Recall that $B^{\bowtie}$ denotes the set of differences $\psi(\rho) - \psi(\rho')$ between Parikh-vectors of different runs $\rho, \rho' \in Runs(A)$. One has to decide whether there exists for each vector $w \in B^{\bowtie}$ some canonical region $(\eta\,; {}^{\bullet}\eta\,; \iota) \in P$ such that $\eta \cdot w \neq 0$. Seeing that $B^{\bowtie}$ is a finite union of linear subsets, one can deal separately with each linear subexpression.

Let $W = u + V^*$ be a linear subexpression of $B^{\bowtie}$, with $V = \{v_1, \ldots, v_q\}$. One has to decide whether there exists for each tuple $(n_1, \ldots, n_q)$ of non-negative integers some canonical region $(\eta_i\,; {}^{\bullet}\eta_i\,; \iota_i)$ in P such that:

$$\eta_i \cdot u + \sum_{j=1}^{q} n_j(\eta_i \cdot v_j) \neq 0 \tag{4}$$

For each canonical region in the set $P = \{(\eta_i\,; {}^{\bullet}\eta_i\,; \iota_i) \mid 1 \leq i \leq m\}$, let $U_i = U_i^+ \cup U_i^-$ where:

$$U_i^+ = \{(n_1, \ldots, n_q) \mid \eta_i \cdot u + \sum_{j=1}^{q} n_j(\eta_i \cdot v_j) \geq 1\}$$

$$U_i^- = \{(n_1, \ldots, n_q) \mid \eta_i \cdot u + \sum_{j=1}^{q} n_j(\eta_i \cdot v_j) \leq -1\}$$

Thus U_i is a semi-linear subset of $\mathbb{N}^q$ (as a union of two semi-linear sets). Since semi-linear subsets of $\mathbb{N}^q$ form an effective boolean algebra, one may compute a semi-linear expression of the set $\mathbb{N}^q \setminus \cup_{i=1}^{q} U_i$. Checking that this expression differs from the null expression allows to decide that condition 4 may actually be satisfied for all $(n_1, \ldots, n_q)$. Event-state separation is valid in G if and only if the answer to this question is positive for all linear subexpressions $u + V^*$ of $B^{\bowtie} = \cup_{k=1}^{p} (u_k + V_k^*)$. Altogether, we have established the following.

Theorem 8.1 *Given a context-free graph G, one may compute a finite set of canonical regions P such that the states separation axiom* **SSA** *is valid with respect to regions of G if and only if it is valid with respect to regions in P, and one may decide on the latter.*

Example 8.2 (continued) *In our running example, $B^{\bowtie} = \psi(L) \cup (-\psi)(L)$ where $L \in Rat((E \cup E^{-})^*)$ is the language defined by the regular expression*

$$(r + r^{-1})^* \, (r + r^{-1} + (a + a^2 + b) \, b^* + (a^{-1} + a^{-2} + b^*) \, ed^* + dd^*) \ .$$

By developping this expression, one obtains 9 ($\times 2$) linear subsets of $B^{\bowtie}$. The summand $L_1 = (r + r^{-1})^* \, a^{-2} ed^*$ produces for instance the linear set

$$\psi(L_1) = (-2,0,0,0,1,0) + \{(0,0,0,1,0,0),(0,0,0,0,0,1),(0,0,0,0,0,-1)\}^*$$

One has therefore to decide whether there exists for each tuple (n_1, n_2, n_3) of non-negative integers some canonical region $p = (\eta \, ; {}^\bullet\eta \, ; \iota)$ in P such that:

$$-2\eta(a) + \eta(e) + n_1 \times \eta(e) + n_2 \times \eta(r) - n_3 \times \eta(r) \neq 0 \ .$$

All instances of this relation are satisfied when selecting $p = p_1$ (or $p = p_5$) from table 1.

9 Eliminating Redundancy

From theorems 2.3, 5.10, 7.5 and 8.1, one may decide on the Petri net realization problem for the class of event-reduced context-free graphs, as our main purpose was to show. However, we have not taken care of producing irredundant Petri net realizations of context-free graphs, as may be remarked from our example. Actually, the decision method outlined in the last two sections contents itself with checking that the whole set of canonical regions of the given graph is admissible. This weakness can be remedied by eliminating redundant regions from $P = \{p_1, \ldots, p_m\}$ according to an iterative process. At the initialization of the process, let $P_0 = P$. At the i^{th} step in the iteration, one checks whether $P_{i-1} \backslash \{p_i\}$ is an admissible subset of regions, following the procedure detailed in sections 7 and 8. If this is the case one sets $P_i = P_{i-1} \backslash \{p_i\}$ for the next step in the iteration, else one sets $P_i = P_{i-1}$. The process halts with the result P_m. From Theo. 5.10 and Prop. 6.2, the Petri net synthesized from the subset of regions P_m is a minimal net realization of G: no proper subnet of this net is a realization of G. Our final result is therefore the following.

Theorem 9.1 *Given an event-reduced context-free graph, one may decide whether it can be realized by some (finite) Petri net, and one may compute in this case an irredundant net realization of the graph.*

10 Open questions

One can decide on the Petri net realization problem for context-free graphs. *Could one decide whether the reachable state graph of a net is context-free* ?

The state graph of a Petri net is a synchronized product of state graphs of one-place subnets, and these are context-free. *Could one decide upon the net realization of a synchronized product of context-free graphs* ? (a question by Caucal)

One can decide on Petri net realization for context-free graphs and for deterministic context-free languages. *Could one decide on Petri net realization for incomplete specifications combining assertions on behaviours and assertions on states* ?

We designated the Petri nets which derive from the canonical regions of a context-free graph as canonical nets. *Do these nets enjoy special properties, and could their construction be turned into a functor* ?

Extending the definition of regions and the construction of canonical nets in order to take weighted inhibitor arcs into account is straightforward. *Which decision problems can be solved in this variant framework* ?

Answering some of these questions will be the goal of further research.

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