Pool of Self-Service Cars: A Balancing Method

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SYSTEME DE VOITURES EN LIBRE-SERVICE : UNE METHODE DE REEQUILIBRAGE

Fabrice CHAUVET[1], Ahmedou Ould HAOUBA[2] and Jean-Marie PROTH[1, 3]

Résumé

De nos jours, les grandes villes doivent faire face à des problèmes de transport qui sont cruciaux pour leur survie. Une des solutions qui se développe actuellement dans des pays développés est de mettre des systèmes de voitures électriques à la disposition des usagers pour leur permettre de faire leurs déplacements quotidiens. Dans ce type de système, les voitures stationnent dans des sites où leur batterie est rechargée. Un client utilise une voiture électrique pour se déplacer d'un site à l'autre. Alors, si nécessaire, le même client pourra utliser une autre voiture pour se déplacer vers un troisième site, et ainsi de suite.

Il est facile de comprendre que le centre de gravité d'un tel système change durant la journée. Ceci impose de transporter, de temps en temps, des voitures des sites ayant trop de voitures vers ceux qui n'auront plus assez de voitures dans un futur proche. Une telle action est appelée rééquilibrage. Le but de ce papier est de proposer une méthode de rééquilibrage qui consiste à utiliser un camion d'une capacité donnée. Ce camion visite les sites suivant un circuit prédéterminé et soit décharge des voitures dans des stations qui en ont besoin, soit prélève des voitures dans les sites où elles sont en excès. En fait, cette méthode analyse l'état de tout le système à l'arrivée du camion dans un site dans le but de prendre une décision.

Mots Clés

Méthode de rééquilibrage, Logistique, Voitures en libre-service, Transport urbain.

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POOL OF SELF-SERVICE CARS: A BALANCING METHOD

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Abstract

Nowadays, big cities have to face transportation problems which are crucial for their survival. One of the solutions that is currently spreading in developed countries is to put a pool of electric cars at the disposal of customers to make local journeys. In this kind of system, cars are parked in stations where their batteries are recharged. A customer uses an electric car to travel from one station to another. Then, if necessary, the same customer will later use another electric car to travel to a third station, and so on.

It is easy to understand that the gravity center of the set of cars changes during the day. This requires an action that consists in transporting cars from stations having excess of cars to stations that may run out of cars soon. Such an action is called a balancing action. The goal of this paper is to propose a balancing method that consists in using a truck of a given capacity. This truck visits the stations following a given circuit, and either leaves cars in stations when needed or takes cars from stations where they are in excess. Indeed, this method analyses the state of the whole system in order to make a decision at the arrival of the truck in a station.

Keywords

Balancing method, Logistics, Self-service cars, Urban transportation.

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1. INTRODUCTION

Putting a pool of electric cars at the disposal of customers to make local journeys is becoming increasingly popular in big cities. This kind of system leads to the following improvements in the citizens' environment:

(i) It introduces an additional flexibility in the existing transportation system by allowing customers to reach any location in the city.

(ii) It guarantees a cost that is, for each customer, exactly proportional to the use of the system. To reach this goal, a plastic card is provided to each customer applying for this service, and this card should be introduced in a slot of the electric car to drive it; this allows the system to record the exact period the car is used.

(iii) It reduces the pollution in the city, due to the use of electric engines.

(iv) It allows a 24-hour-a-day service when using modern surveillance systems.

Numerous studies have been conducted to analyze the behavior of a pool of self-service cars. Readers interested in some of these studies may refer to [4],[6] and [11] that present a model based on a polling system, or to [1] and [5] in which the system is modeled as a close network. Another paper (see [2]), proposes some studies around the optimal control of such a system in the stationary case.

Surprisingly, few studies have been devoted to the balancing methods. In our opinion, the most complete and realistic one has been proposed in [3]. This paper suggests to manage a pool of self-service cars using the concept of favorable state and the concept of unfavorable states. The favorable state is the distribution of the electric cars in the stations that guarantees that the system can reach a given large horizon with the highest probability, assuming that no balancing action is conducted. Only one favorable state is selected. An unfavorable state is the distribution of the electric cars in the stations that guarantees that at least one of the stations will run out of cars before a given short horizon with a large probability, assuming that no balancing action is conducted. The proposed management consists in triggering a balancing action as soon as the system reaches an unfavorable state. Balancing the system consists in switching from the unfavorable state reached by the system to the favorable state. The balancing approach proposed in [3] is twofold. It consists in computing first the moves of cars to be performed (loaded moves), and then the trajectories of the trucks used to carry the cars. The first problem is an uncapacitated transportation problem (see [7]) which can be solved using Hitchcok's algorithm (see [8]). The complexity of this algorithm is $O(K^3)$, where K is the number of stations. Starting from the loaded moves, the computation of the trajectories of the truck(s) is made using a heuristic algorithm. Unfortunately, this method becomes very complex and requires a large memory size when the random variables that generate car requirements in a given station are not independent identically distributed (i.i.d.) random variables.

The Unceasing Balancing Method (UBM) proposed hereafter uses one truck that visits the stations following a given circuit. At each station, the truck either leaves cars when needed (assuming that the truck contains some cars), or takes cars from the station if cars are in excess (assuming that some capacity is available in the truck) or does nothing. Thus, when the truck arrives at a station, an evaluation of the state of the system is performed in order to make a decision. The goal of this paper is mainly to propose an efficient balancing method. It should be noticed that the order the stations are visited is given. The choice of this order is an open problem.

In Section 2, we introduce some useful notations and formulate the problem. Section 3 is devoted to the decision making system (DMS). The evaluation of the method and some numerical examples are presented in Section 4. Section 5 is the conclusion.

2. NOTATIONS AND PROBLEM FORMULATION

2.1. Notations

We denote by K the number of stations, and by $p_t^c(i,j)$ the probability that c customers require a car during the elementary period $[t, t+\Delta t)$ at station i to go to station j, $i,j \in \{1,\ldots,K\}$. Δt is the length of one elementary period. Typically, Δt is chosen between 8 and 12 minutes, depending on the number of customers and the magnitude of the variations of the demand. The higher the number of customers or the larger the variation of the demand, the shorter Δt. The physical locations of the stations are known. We denote by $r_{i,j}$ the number of elementary periods required for a car to join station j starting from station i. N is the total number of electric cars available in the system.

C is the capacity of the truck used in the UCB (Unceasing Balancing Method). More precisely, C is the maximal number of electric cars that can be carried by the truck.

Note that, in most of the existing or forecasted systems, $K \in [5,10]$, $C \in [4,15]$ and $N \in [40,80]$.

The truck visits successively stations 1, 2, ..., K. $d_{i,i+1}$, $i = 1, \ldots, K-1$ (resp. $d_{K,1}$) is the time, expressed in terms of the number of elementary periods, required by the truck to move from station i to station i+1 (resp. from station K to station 1). The time required to load or unload the truck in the stations is included in this time.

We denote by $n_i^{i,k}$ the number of cars in station i when the truck arrives at station i for the k-th time and, more generally, by $n_j^{i,k}$ the number of cars in station j when the truck arrives at station i for the k-th time.

We also introduce $m_j^{i,k}$ which is the forecasted mean value of the number of cars in station j when the truck arrives at this station for the first time after its k-th visit to station i. Indeed:

$$m_i^{i,k} = n_i^{i,k}$$

$$m_j^{i,k} = n_j^{i,k} - D_j^{i,k} \tag{1}$$

where $D_j^{i,k}$ is the mean value of the flow of cars in station j during the period the truck moves from station i to station j after its k-th visit to station i. If $D_j^{i,k} > 0$, the number of cars that leave station j exceeds, on the average, the number of cars that arrive at j during the period mentioned. If $D_j^{i,k} < 0$, the number of cars that arrive at j exceeds the number of cars that leave this station.

As mentioned in relation (1), we are interested in evaluating $D_j^{i,k}$. It is the mean value of the demand made to station j minus the number of cars which arrive at station j during the time the truck moves from i to j after visiting i for the k-th time.

According to the notations introduced before:

$$D_j^{i,k} = \sum_{t=M_1^{i,k}}^{M_2^{i,k}} \left\{ \sum_{h=1,h\neq j}^{K} \sum_{v=0}^{+\infty} v\, p_t^v(j,h) - \sum_{h=1,h\neq j}^{K} \sum_{v=0}^{+\infty} v\, p_{t-r_{h,j}}^v(h,j) \right\} \qquad (2)$$

where $M_1^{i,k}$ is the first elementary period that follows the k-th visit of the truck to station i and $M_2^{i,k}$ is the elementary period that precedes the next visit to station j. The first term of the second side of the equality is the mean value of the number of cars which leave station j during the period under consideration, while the second term is the mean value of the number of cars which arrive at station j during the same period.

The value computed by relation (1) is only an evaluation of the mean value of the demand since, when writing this equality, it is assumed that a station always contains enough cars to meet the demand. This is only true when the service ratio, defined hereafter, is close to 1. Nevertheless, since the goal of the balancing activity is to optimize this ratio, it will be close to 1 if the capacity of the truck is correctly dimensioned.

2.2. Problem formulation

When the truck arrives at station $i \in \{1,2,\ldots,K\}$, the decision to be made is either to leave cars in the station, or to take cars from the station, or to take no action. This decision should take into account the following information:

(i) the number of cars in station i when the truck arrives at this station,

(ii) the average number of cars that will be in the other stations at the next visit of the truck,

(iii) the number of cars required in each station j during the next cycle which starts from this station after the k-th visit to station i. We call "cycle starting from station $j \in \{1,2,\ldots,K\}$" the sequence j, j+1, ..., K, 1, 2, ..., j-1 of stations visited by the truck.

Information (i) is known when the truck arrives at station i while information (ii) and (iii) should be forecasted.

When the truck arrives at station i for the k-th time, we know or we compute:

- the number $n_i^{i,k}$ of cars in station i,
- the evaluation of $m_j^{i,k}$ for $j \neq i$, using relations (1) and (2),
- the average number of cars required in each station $j \in \{1,2,\ldots,K\}$ during the next cycle time starting from j after the k-th visit of the truck to station i.

We assume that the observation of the system started when the truck arrived at station 1. In this case:

If $j \geq i$, the average number of cars required in station j during the next cycle time starting from j after the k-th visit of the truck to station i is :

$$F_j^k = \sum_{t=M_1^{j,k}}^{M_2^{j,k}} \left\{ \sum_{h=1,h\neq j}^{K} \sum_{v=0}^{+\infty} v\, p_t^v(j,h) - \sum_{h=1,h\neq j}^{K} \sum_{v=0}^{+\infty} v\, p_{t-r_{h,j}}^v(h,j) \right\} \tag{3-1}$$

If $j < i$:

$$F_j^{k+1} = \sum_{t=M_1^{j,k+1}}^{M_2^{j,k+1}} \left\{ \sum_{h=1,h\neq j}^{K} \sum_{v=0}^{+\infty} v\, p_t^v(j,h) - \sum_{h=1,h\neq j}^{K} \sum_{v=0}^{+\infty} v\, p_{t-r_{h,j}}^v(h,j) \right\} \tag{3-2}$$

where $M_1^{j,k}$ is the first elementary period that follows the k-th visit of the truck to station j and $M_2^{j,k}$ is the elementary period that precedes the (k+1)-th visit to station j. Note that $j - 1 = K$ if $j = 1$. Note also that these formulae are similar to formula (2). The only difference is that station j replace station i as a reference.

We denote by c_j^k, $j = 1,2,\ldots,K$, $k \in \{0,1,2,\ldots,C\}$, the number of cars in the truck when it arrives at station j for the k-th time.

The decision to be made when the truck arrives at station i for the k-th time is based on dynamic programming.

At this point in time, the number of cars transported by the truck when it arrives at station i for the k-th time is $c_i^k \in \{0,1,\ldots,C\}$, and the decision to be made can be either to unload $z_i^k \in \{0,1,\ldots,c_i^k\}$ cars from the truck, or to pick up $-z_i^k \in \{1,2,\ldots, g_i^k\}$ from station i, where

$g_i^k = \mathrm{Min}(C - c_i^k, n_i^{i,k})$, $n_i^{i,k}$ being the number of cars in station i when the truck arrives at this station for the k-th time and C - c_i^k being the remaining capacity in the truck at the same time. Finally, $z_i^k \in \{-g_i^k, -g_i^k + 1, \ldots, 0, 1, \ldots, c_i^k\}$.

Thus, the average number of cars in station i after the next cycle that starts from i after the k-th visit to i is:

$$y_i^k = n_i^{i,k} + z_i^k - F_i^k \tag{4}$$

Note that i - 1 = K if i = 1.

The number of cars in the truck when it arrives at station i+1 after its k-th visit to i is:

$$c_{i+1}^k = c_i^k - z_i^k \tag{5}$$

Note that, when i = K, relation (5) should be rewritten as:

$$c_1^{k+1} = c_K^k - z_K^k$$

assuming that the observation of the system started with the truck arriving at station 1.

Similarly, for $j \in \{1,2,\ldots,K\}$ and $j \neq i$, the forecasted average number of cars in station j after the next cycle that starts from j after the k-th visit to j is:

$$y_j^k = m_j^{i,k} + z_j^k - F_j^k \quad \text{if } j > i \tag{6-1}$$

and:

$$y_j^{k+1} = m_j^{i,k} + z_j^{k+1} - F_j^{k+1} \quad \text{if } j < i \tag{6-2}$$

where:

- $m_j^{i,k}$ is given by relation (1),
- z_j^k is the number of cars unloaded from the truck in station j if $z_j^k \geq 0$ or picked up by the truck from station j if $z_j^k < 0$ at the k-th visit of the truck to station j.

Indeed:

$$z_j^k \in \{-g_j^k, -g_j^k + 1, \ldots, 0, 1, \ldots, c_j^k\}$$

where:

$$g_j^k = \mathrm{Min}(C - c_j^k, m_j^{i,k}) \tag{7}$$

and:

$$c_j^k = c_i^k - \sum_{s=i}^{j} z_s^k \quad \text{if } j > i \tag{8}$$

or:

$$c_j^{k+1} = c_i^k - \sum_{s=i}^{K} z_s^k - \sum_{s=1}^{j} z_s^{k+1} \quad \text{if } j < i \tag{9}$$

When the truck arrives at station i, we compute the sequence $z_i^k, z_{i+1}^k, \ldots, z_K^k, z_1^{k+1}, \ldots, z_{i-1}^{k+1}$ which leads to the minimal value of:

$$C(i, k, Z_i^k) = \sum_{j=i}^{K} (y_j^k)^2 + \sum_{j=1}^{i-1} (y_j^{k+1})^2$$

where $Z_i^k = \{z_i^k, z_{i+1}^k, \ldots, z_{i-1}^{k+1}\}$. The choice of this criterion requires some explanation. The total number of cars in the system is much greater than the capacity of the truck. As a consequence, it is unavoidable to run out of cars some times in some stations, at least for a limited period (Δ for instance). Thus, pushing to zero the forcasted numbers of cars which will be in the stations after the next cycle guaranties that none of the stations will be favored at the detriment of others.

Only z_i^k is applied, i.e. is used to decide if cars should be loaded in the truck or unloaded from the truck, and to decide the number of cars which are concerned. The same decision process is applied when the truck arrives at the next station, and so on. In other words, the decision making system works on a rolling horizon basis.

3. THE DECISION MAKING PROCESS

At the beginning of the process, we know $n_j^{i,k}$, c_i^k, as well as the probabilities of the demands that allow to evaluate $D_j^{i,k}$ for j = 1,2,...,K (see relation (2)), F_j^k for j = i, i+1, ..., K (see relation

(3-1)) and F_j^{k+1} for j = 1, 2, ..., i-1 (see relation (3-2)).

The algorithm is twofold.

a. Forward computation

The cost when arriving at station i with a truck containing c_i^k cars is equal to zero.

α. We compute $g_i^k = \mathrm{Min}\{C - c_i^k, n_i^{i,k}\}$ and we set $E_i = \{-g_i^k, -g_i^k + 1, \ldots, 0, 1, \ldots, c_i^k\}$. The cost associated with the first decision, if the decision is $x \in E_i$, is:

$$W(i,a) = \left(a - F_i^{i,k}\right)^2 \quad \text{for } a = n_i^{i,k} + x$$

We then set:

u(i,a) = x: this is the decision made if the result is "a" cars in station i,

cc(i,a) = c_i^k - x, which is the number of cars in the truck when it leaves station i.

For $x \in \{0,1,\ldots,C\} \backslash E_i$, we set:

$W(i,a) = +\infty$

$u(i,a) = 0$

$cc(i,a) = c_i^k$

β. We consider the case j > i. This only applies if i < K. We compute:

$$W(j,a) = \underset{x \in \{-C,-C+1,\ldots,C\}}{\mathrm{Min}} \left\{ W(j-1, a-x) + \left(a - F_j^k\right)^2 \right\} \tag{10}$$

with

$$W(j-1, a-x) = +\infty$$

$$\text{when } x \notin \left\{-\mathrm{Min}\left(C - cc(j-1, a-x), a-x\right), \ldots, cc(j-1, a-x)\right\} \tag{11}$$

We set:

$$u(j,a) = x^* \tag{12}$$

where x* is the value of x that leads to the minimum in (10), and:

$$cc(j,a) = cc(j-1, a-x^*) - x^* \tag{13}$$

γ. When $j < i$ and $j > 1$, relation (10) should be rewritten as:

$$W(j,a) = \underset{x \in \{-C,\ldots,+C\}}{\text{Min}} \left\{ W(j-1, a-x) + \left(a - F_j^{k+1}\right)^2 \right\} \tag{14}$$

under condition (11).

Equalities (12) and (13) still hold.

When $j = 1$, relation (14) becomes:

$$W(1,a) = \underset{x \in \{-C,\ldots,+C\}}{\text{Min}} \left\{ W(K, a-x) + \left(a - F_1^{k+1}\right)^2 \right\}$$

b. Backward computation

Knowing W(i-1,a) for a = 0,1,...,C (or W(K,a) for a = 0,1,...,C if i = 1), we first compute a* such that:

$$W(i-1, a^*) = \underset{a \in \{0,1,\ldots,C\}}{\text{Min}} W(i-1, a)$$

Then, for j = i-1, ..., 1, K, ..., i, we compute:

a* = a* - u(j,a*)

The decision to be made is given by:

u(i,a*)

4. NUMERICAL APPLICATION

We consider the case of four stations. This simple case is presented to illustrate our approach. The distances between the stations are given in Table 1. These distances are given in terms of elementary periods. For instance, a car needs 3 elementary periods to join station 3 starting from station 1, or 30 minutes if one elementary period is equal to 10 minutes.

In Table 2, one three-dimension vector is associated with each box. The first (resp. second, third) element of the vector is the probability that zero (resp. one, two) car(s) is (are) required by customer(s) to go from the initial station (each initial station corresponds to a row of the table) to the destination station (each destination station corresponds to a column). We assume that these probabilities do not depend on time.

Table 1: Distance between stations (the $r_{i,j}$ values)

	1	2	3	4
1	0	2	3	2
2	2	0	2	3
3	3	2	0	2
4	2	3	2	0

The truck visits successively stations 1, 2, 3, 4, 1, 2, ... The distances (in terms of elementary periods) to be covered by the truck are the same as the distances to be covered by the cars, that is 2 elementary periods between two consecutive stations.

Starting from the data of Table 2, we obtain the results presented in Table 3. For each station, this table provides:

- the average difference between the cars that leave the station and the cars that arrive in the station during one elementary period. If this number is greater than zero, the average number of cars that leave the station is greater than the average number of cars that arrive in this station;
- the standard deviations of the previous figures

On one cycle, the average flows are eight times the above figures.

The goal is to maximize the service level ρ which is defined as:

$$\rho = \frac{\text{Number of demands that are met}}{\text{Total number of demands}}$$

It is assumed that if it is impossible to meet the demand of a customer, then the customer leaves the system.

Table 2: Probabilities of the demands (the $p_t^c(i,j)$ probabilities)

	1	2	3	4
1	x	$\begin{bmatrix}0.5\\0.5\\0\end{bmatrix}$	$\begin{bmatrix}0.4\\0.4\\0.2\end{bmatrix}$	$\begin{bmatrix}0.6\\0.3\\0.1\end{bmatrix}$
2	$\begin{bmatrix}0.7\\0.3\\0\end{bmatrix}$	x	$\begin{bmatrix}0.6\\0.4\\0\end{bmatrix}$	$\begin{bmatrix}0.6\\0.2\\0.2\end{bmatrix}$
3	$\begin{bmatrix}0.4\\0.3\\0.3\end{bmatrix}$	$\begin{bmatrix}0.4\\0.3\\0.3\end{bmatrix}$	x	$\begin{bmatrix}0.6\\0.1\\0.3\end{bmatrix}$
4	$\begin{bmatrix}0.7\\0.3\\0\end{bmatrix}$	$\begin{bmatrix}0.6\\0.4\\0\end{bmatrix}$	$\begin{bmatrix}0.7\\0.2\\0.1\end{bmatrix}$	x

Table 3: Flows of cars

Station	Average flow	Standard deviation
1	0.3	0.0081
2	- 0.5	0.0625
3	0.9	0.6561
4	- 0.7	0.2401

The service level ρ depends on two parameters, that is the number of cars in the system and the capacity of the truck. A large number of experiments have been conducted. We always made the following observation: if the number of cars in the systems remains constant, and if the capacity of the truck increases, the service level increases to some truck capacity, and then decreases slightly.

Table 4: Optimal solutions

Number of cars in the system	Optimal truck capacity	Optimal service ratio
10	4	43.5
15	4	60.1
20	4	72.0
25	4	80.1
30	6	86.0
35	6	90.6
40	8	94.1
45	10	96.5
50	10	98.1
55	10	99.0
60	12	99.5
65	12	99.7
70	14	99.9

In Table 4, we present the results of the tests made with the previous data. These results have been obtained by simulation: for each value of the number of cars in the system, we increased the capacity of the truck until the service ratio started decreasing, and we kept the best service ratio.

The following remarks have been derived from the simulation of numerous examples where it was assumed that the probabilities of car requirements are independent from the time:

1. When the number of cars in the system is fixed, there exists an optimal capacity of the truck, that is a capacity which guarantees the maximal service level. If the capacity is greater than the optimal capacity, the truck "absorbs" some cars which are no more available for the customers.
2. The optimal capacity of the truck increases when the number of cars in the system increases.
3. The optimal service level, that is the one obtained when the capacity of the truck is optimal, increases with the number of cars in the system.

5. CONCLUSION

In this paper, we studied the efficiency of the so-called "Unceasing Balancing Method". The example presented is rather realistic, since up to two cars can be required in a system to go to another station during one elementary period (which is equal to 8 to 12 minutes).

We need at least 50 cars to reach an acceptable service level. It is exactly the number of cars planed for the first implementation in France, which is at work since the beginning of January 1998 at Saint-Quentin en Yvelines, near Paris.

The balancing method proposed in this paper is acceptable and could be improved by testing the different circuits of the truck. We will continue the research by studying another balancing method where the truck selects the next station to be visited each time it leaves a station.

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