Circular-arc Graph Coloring and Unrolling

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Abstract: The register periodic allocation problem is viewed as unrolling and coloring the underlying structure of circular-arc graph. The problem is to find relations between the unrolling degree and the chromatic number. For this purpose we distinguish cyclic colorings that can be found by means of the *meeting graph* and non-cyclic ones for which we prove the asymptotic property: let r be the width of the original interval family. Then the u-unrolled graph is r or $r+1$-colorable for u large enough.

Key-words: register allocation, loop unrolling, cyclic coloring, acyclic coloring, circular-arc graph

(Résumé : tsvp)

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Coloriage et déroulage de graphes d'intervalles circulaires

Résumé : Le problème de l'allocation périodique de registres est vue comme le coloriage et le déroulage de la structure sous-jacente qu'est le graphe d'intervalles circulaires. Le problème est de trouver des relations entre le degré de déroulage et le nombre chromatique. À cet effet nous distinguons les coloriages cycliques, qui peuvent être trouvés à l'aide du *meeting graph*, des coloriages acycliques, pour lesquels nous prouvons la propriété asymptotique suivante : soit r l'épaisseur maximale de la famille d'intervalles, alors le graphe déroulé u fois est coloriable avec r ou $r + 1$ couleurs pour u assez grand.

Mots-clé : allocation de registres, déroulage de boucle, coloriage cyclique, coloriage acyclique, graphe d'intervalles circulaires

1 Introduction

Circular-arc graphs are a subclass of intersection graphs. An undirected graph G is a circular-arc graph if there exists a family $\mathcal{F}$ of intervals on a circle which maps the vertices of G, and two vertices are adjacent if and only if the corresponding intervals have a nonempty intersection. Circular-arc graphs are used in several applications. Their coloring has been primarily studied by Tucker [14]. Finding the chromatic number of these graphs has been proven to be an NP-complete problem by Garey et al. [5]. In the particular case of proper circular-arc graphs, this problem has polynomial complexity [13, 12]. Other results concerning circular-arc graphs can be found in [6, 7, 8].

Our primary interest in circular-arc graph coloring was motivated by the problem of register allocation in loops encountered in programs for high performance microprocessors. In optimizing compilers, register allocation is traditionally performed by coloring (i.e. allocating on registers) the *interference graph* G. Vertices of G are the lifetimes of the program variables [1]. There is an edge between two vertices/variable lifetimes if and only if the variables are simultaneously alive at some point of the program [1]. In the case of simple loops, the interference graph is a circular-arc graph [5, 7]. An additional feature of our problem is that optimal schedule of instructions may create variable lifetimes spanning more than one iteration - therefore overlapping with themselves. This means that two consecutive instances of the variable can not be kept in the same register. A possible hardware solution is to have a *rotating register file* [2], where the reference pointer is shifted one cycle ahead at each iteration of the loop, it can be seen as a ring of registers. The minimal size needed for a rotating register file is computed in Section 2. A software solution is to unwind the loop a suitable number u of times for describing explicitly the allocation into different registers. The resulting interference graph G^u (called *u-unrolled graph*) can then be colored. Actually, graph unrolling has other effects than just rendering the graph colorable. Indeed, the chromatic number $\chi(G^u)$ of the u-unrolled graph can vary dramatically with u as is shown in Figure 4. Unwinding the loop does not disturb the schedule of instructions but it increases the size of the code which is another important factor of performance in modern microprocessors. Therefore it is important to be able to control the unrolling degree as well as the number of registers needed simultaneously.

In this paper, we first present our framework, called the *meeting graph* [4], for finding a *cyclic* coloring of the family of intervals. Circular-arc graph coloring is viewed as a decomposition of the meeting graph into circuits. Then general coloring (cyclic and acyclic) is considered and we prove the following result: Let $r = \min_{k \in I\!N^*} (\chi(G^k))$, ($r$ happens to be the *width* of the interval family) there exists a bound k_0 from which every G^k, $k \geq k_0$, is $r+1$-colorable.

As the nodes of the graph match the intervals, we use the terms interval coloring and circular-arc graph coloring interchangeably. In the figures, the circles are represented by lines where the last point is the same as the first one, therefore intervals spanning the first point are cut.

2 Unrolling and coloring

On a circle with p points labeled from 0 to $p-1$, we consider a family $\mathcal{F}$ of intervals delimited by two points. The problem is to assign a color $c(I) \in \mathbb{N}$ to every interval I such that two overlapping intervals are assigned different colors. A specificity of our problem is that the length of the intervals may be greater than one circumference of the circle, like interval 1 in the interval family of Figure 1. Any interval I is hence defined by its starting point $s(I)$ comprised within 0 and $p-1$ and its ending point $e(I) > s(I)$, where $e(I)$ may be greater than $p-1$. I is denoted as $[s(I), e(I)[$. For instance in Figure 1, interval 3 is noted $[1, 10[$, hence it begins at point 1 and ends at point $10 \bmod 7 = 3$ on the circle, since $s(3) > 7$.

The interference graph $G(\mathcal{F}) = (\mathcal{F}, \mathcal{E})$ or conflict graph resulting from this family is constructed as follows: the vertices correspond to the intervals and the edges are the couples of intervals that overlap around the circle, like in Figure 6(a) and (c).

We now define the unrolling transformation. Let u be some positive integer. We consider the u-duplicated circle with points from 0 to $up-1$. For each interval $I = [s, e[$, we create u instances $I(k) = [s_k, e_k[$ where $s_k = (s + (k-1)p) \bmod up$ and $e_k = s_k + e - s$. The u-unrolled graph G^u is the interference graph of the resulting family: $G^u = G(\mathcal{F}^u)$ where $\mathcal{F}^u = \{I(k)/I \in \mathcal{F}$ and $1 \leq k \leq u$. The example in Figure 2 shows the interval family of Figure 1 unrolled three times. An unrolling degree equal to 1 corresponds to the original interval family, $G^1 = G$.

A coloring of G is an assignment c of some color $c(I) \in \mathbb{N}$ to each interval I such that two related vertices have different colors. $(I, J) \in \mathcal{E} \Rightarrow c(I) \neq c(J)$. We let $\chi(G)$ denote the chromatic number of G. $\chi(G)$ is the minimal number of colors needed for coloring G.

Since we allow intervals spanning more than one turn around the circle, it may happen that G is not colorable because some interval I may overlap itself, $(I, I) \in \mathcal{E}$. In that case $\chi(G)$ is undefined. However it is clear that for u greater than the maximum number of rounds of intervals the graph G becomes colorable since the size of the circle is up and the size of intervals I and instances $I(k)$, $1 \leq k \leq u$ remains constant equal to $s - e$.

A well-known lower bound on the chromatic number of circular-arc graphs is the *maximal width* r of the family defined as the maximum number of overlapping intervals at any point of the circle. It is clear that the width r^u of the u unrolled graph G^u is still r, $r = r^u$. Without loss of generality, we will assume in the remaining of the paper that each point of the circle is contained in exactly r intervals. In other words, the width is constant around the circle. This is achieved by adding fictitious one time unit intervals around the circle when necessary, like intervals 9 or 11 in Figure 1.

It has been already proven elsewhere [9, 3] that this lower bound is achievable when unrolling the graph: $\exists u, \chi(G^u) = r$. In Section 3 we characterize the degrees u for which G^u is cyclically colorable with r colors, as well as the number of colors needed for cyclically coloring G^r. In Section 4 we prove that for u large enough, $\chi(G^u) \leq r + 1$.

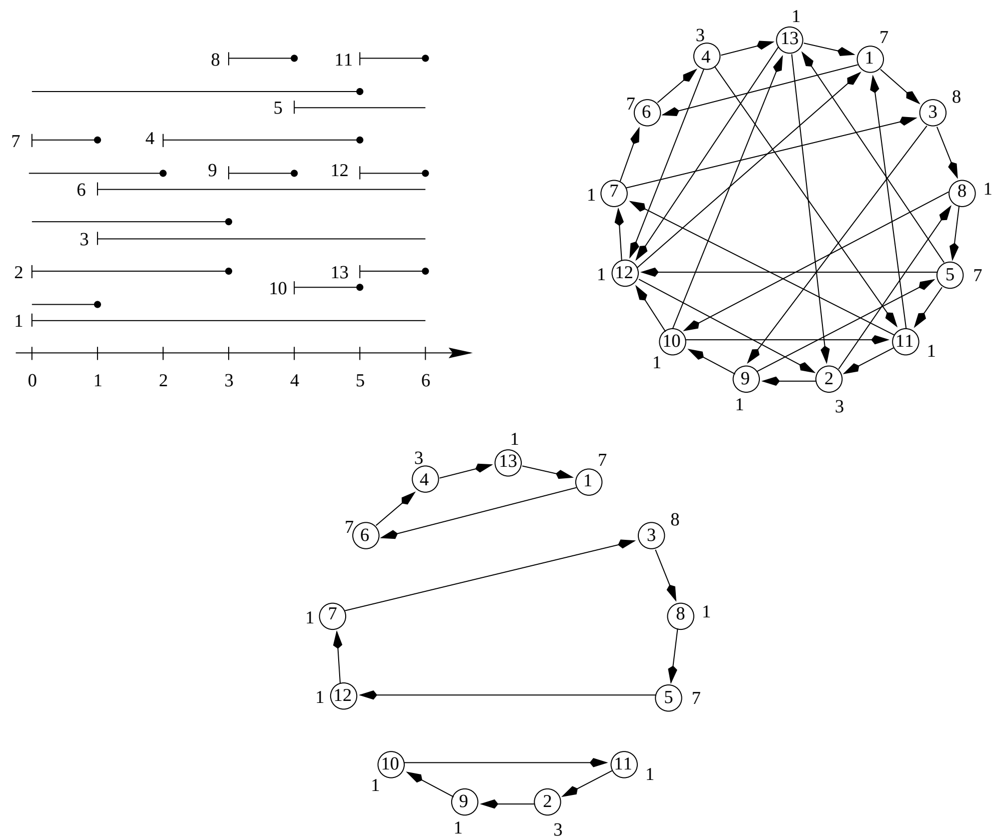

Figure 1: Interval family and meeting graph and decomposed graph

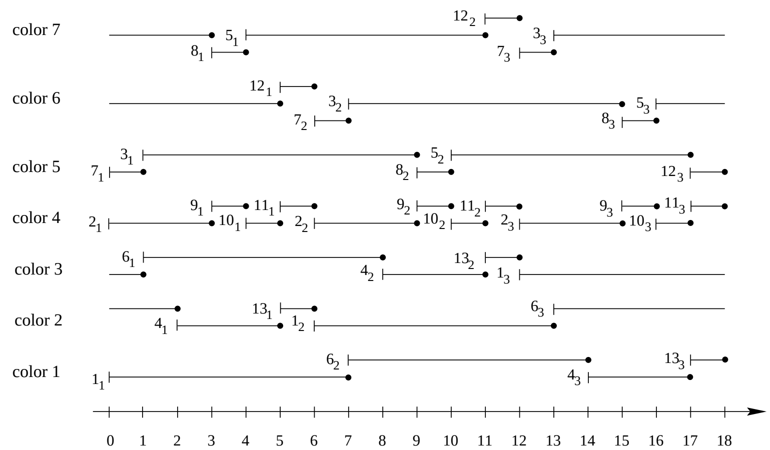

Figure 2: Unrolled and colored interval family

3 Cyclic circular-arc graph coloring

3.1 Definition

Among possible colorings, we consider first the class of *cyclic colorings* that have the following property: let R be the number of colors used, then there exists a permutation σ of $\{1, ..., R\}$ such that the entire coloring can be deduced from the coloring of the original family of intervals by: $c(I(k+1)) = \sigma(c(I(k)))$ for $1 \leq k \leq u-1$ and $c(I(1)) = \sigma(c(I(u)))$. This implies that $\sigma^u = Id$.

Cyclic colorings can be conveniently studied using the *meeting graph* of the initial family $\mathcal{F}$ of intervals. The meeting graph is a directed graph defined as follows. As in the previous section we assume that the width is constant around the circle. The nodes of the meeting graph are the intervals, and there is an arc between the nodes representing intervals I and J whenever $e(I) \bmod p = s(J) \bmod p$. Furthermore a weight is added to each node, equal to the length $s(I) - e(I)$ of each interval I. Figure 6 shows a meeting graph and a circular-arc graph built from the same interval family.

Let us first consider a circuit C of the meeting graph. The total weight of its nodes is a multiple of the length p of the circle, say $r_C p$. We also call r_C the weight of the circuit. If we unroll the graph u times, a multiple of r_C, then we can color the intervals corresponding to the nodes of the circuit with r_C colors as follows. We cyclically assign color k to the k^{th} instance of the circuit by taking into account the copy to which each interval belongs to. The intervals assigned to color k can belong to a copy different from copy k as in Figure 3. Note that r_C is also the width of this interval sub-family.

If we consider the subgraph C of G composed of the intervals of the circuit, we obtain the following property:

Theorem 1 *$\chi(C^u) = r_C$ for each multiple u of r_C.*

Proof: The coloring described above implies that $\chi(C^u) \leq r_C$. Since r_C is also the width of C, $r_C \leq \chi(C^{r_C})$. □

In Figure 3, we can see a meeting graph composed of a circuit of weight 2. That is we can color it with 2 colors, when it is duplicated $2k$ times with $k \in I\!N^*$. This figure shows also the coloring for $k = 1$ and $k = 2$.

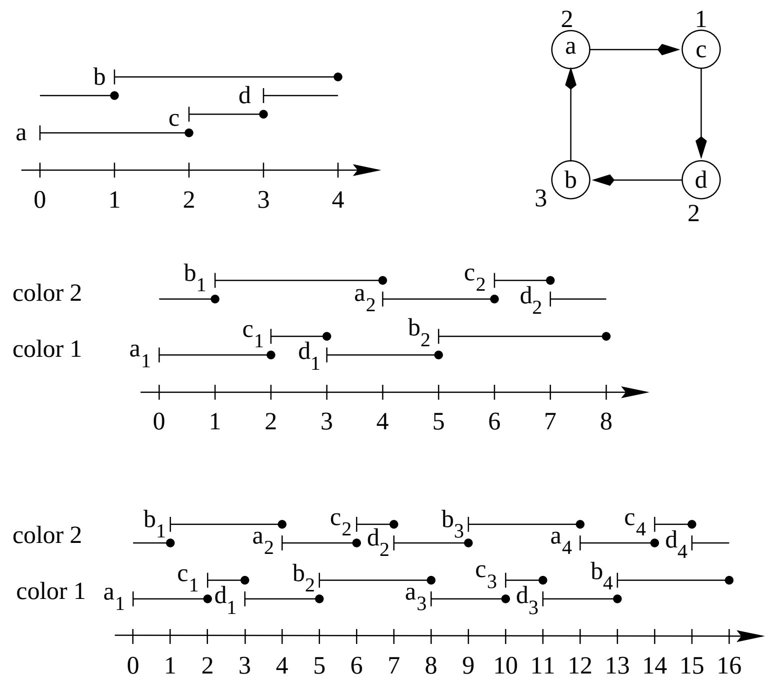

Figure 3: An interval family and its meeting graph. The colored interval family unrolled two and four times.

Now, let us assume that the meeting graph can be decomposed into a set of n circuits of respective weights $r_1, r_2, \ldots, r_n$. By coloring the intervals corresponding to the circuits as previously explained, we obtain the following result.

Theorem 2 *Let $\mathcal{F}$ be a family of intervals, the unrolled circular-arc graph G^u is cyclically r-colorable where $u = lcm(r_1, r_2, ...r_p)$, where $r_1,..,r_p$ are the weights of the circuits of the decomposed meeting graph built upon $\mathcal{F}$.*

Proof: From Theorem 1, we know that if we unroll each circuit of weight r_q a multiple of r_q, then we will need r_q colors for these intervals. So by unrolling a common multiple of all the weights of the circuits of the graph, we are sure to color with r colors. The least integer which allows this coloring is $u = lcm(r_1, \ldots, r_p)$. $\square$

3.2 Rotating register file

The existence of a decomposition is induced by specific properties of the meeting graph that are explained in [4]. One interesting property is that each connected component of the meeting graph contains at least one Hamiltonian circuit. This implies the following corollary:

Corollary 1 *If the meeting graph of G is connected, then $\chi(G^r) = r$.*

This happens to give a convenient upper bound for the unrolling degree when looking for good degrees for a given number of colors.

The property about Hamiltonian circuits has also a nice application in computer architecture: the rotating register file (RRF) is a register file that is virtually shifted one location ahead when needed, typically at each iteration of a loop. Allocating a set of variable life-time intervals into a rotating register amounts exactly to finding a Hamiltonian circuit in the meeting graph. Hence the following strategy is used:

- if the meeting graph is connected, r registers are sufficient for the allocation on a RRF.
- if it is not connected $r+1$ registers are required, by adding one round of fictitious unit time intervals.

For example, we can allocate the interval family of Figure 1 with 7 rotating registers like this:

- interval 1: R_1, R_2
- interval 3: R_2, R_3
- interval 8: R_3
- interval 5: R_3, R_4
- interval 11: R_4
- interval 2: R_5
- interval 9: R_5
- interval 10: R_5
- interval 12: R_5
- interval 7: R_6

- interval 6: R_6, R_7
- interval 4: R_7
- interval 13: R_7

3.3 Loop unwinding

In the absence of a rotating register file, $r + 1$ may be too large an unrolling degree. So the latter result must be refined. Theorem 1 proves that there always exists u such that G^u is r-colorable. In order to find the smallest possible u, we try to decompose the meeting graph into as many circuits as possible. Decomposing amounts to fix a Hamiltonian circuit in each connected component. A valid decomposition is determined by choosing a set of non intersecting chords in the Hamiltonian circuits of the connected components. Such an heuristic is developed further in [4]. Figure 1 presents an example where a meeting graph is decomposed thanks to this heuristic. In this case $r = 7$, and we succeeded in decomposing the graph into three circuits by taking 4 chords, which is the maximum we can do. We obtain $C_1 = \{2, 9, 10, 11\}$ of weight $w(C_1) = 1$, $C_2 = \{1, 6, 4, 13\}$ of weight $w(C_2) = 3$, and $C_3 = \{7, 3, 8, 5, 12\}$ of weight $w(C_3) = 3$. So we must unroll $lcm(1, 3, 3) = 3$ times to obtain an interval family colorable with 7 colors. The coloring is shown in Figure 2. We have:

- color 1: $1_1, 6_2, 4_3, 13_3$
- color 2: $1_2, 6_3, 4_1, 13_1$
- color 3: $1_3, 6_1, 4_2, 13_2$
- color 4: $2_1, 9_1, 10_1, 11_1, 2_2, 9_2, 10_2, 11_2, 2_3, 9_3, 10_3, 11_3$
- color 5: $7_1, 3_1, 8_2, 5_2, 12_3$
- color 6: $7_2, 3_2, 8_3, 5_3, 12_1$
- color 7: $7_3, 3_3, 8_1, 5_1, 12_2$

In case no interval overlaps itself, this heuristic happens also to be useful for directly coloring the original family of intervals, and gives an improvement of the upper bound for the chromatic number of circular-arc graphs given by Tucker [14]. This new upper bound [11] is equal to $2r - n$ where n is the number of circuits which decompose the graph. Of course, the more the graph is decomposed, the better the bound.

3.4 Valid unrolling degrees

Cyclic colorings are a restricted class of colorings of unrolled circular-arc graphs. As a matter of fact, since the unrolling degree u for coloring on R colors must satisfy $\sigma^u = Id$, this implies that the degree found must be the order of some permutation on R elements.

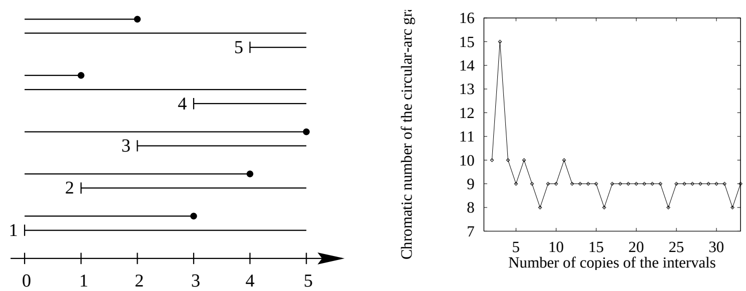

Figure 4: Interval family and chromatic number variation of the induced circular-arc graph

Hence the prime factors of u must be less than R. As a counterpart, this gives an upper bound on the minimum degree necessary for achieving an R-coloring. For instance, if $R = 8$, we know that we can find u less than 15, that is the maximal degree of a permutation on 8 elements. We are not aware of a general formula for computing the maximal degree $M(r)$ of a permutation on r elements. There is, however, the following asymptotic estimation: $M(R) = e^{(1+O(1))\sqrt{R \ln R}}$ [10].

4 Acyclic circular-arc graph coloring

There are degrees for which an r-coloring exists but a cyclic r-coloring does not. In Figure 6, a 3-coloring exists but no cyclic 3-coloring. This family is unrolled 5 times, but 5 is not a multiple of 3 nor 2, which are the LCMs of the weights of the only two decompositions possible, namely $\{A, B\}$ and $\{A\}, \{B\}$.

In this section we consider general colorings and relax the property of cyclicity assumed in the previous section. General colorings have, however, a very good asymptotic property. If u is large enough, then the circular-arc graph induced by the unrolled family is always $r+1$-colorable, as exemplified in Figure 4. The interval family has a maximal width equal to 8, and when we unroll u times, with $u \geq 12$, we always get a circular-arc graph which is 8- or 9-colorable.

To prove this property, we first consider the case of one interval spanning r circumferences of the circle. Without loss of generality, we can consider that the circle has only one point labeled 0. If the graph is unrolled u times, then there are u points on the unrolled circle, labeled 0 to u-1. Also, for a given unrolling degree u, we divide u by r: $u = \lambda r + \mu$, where $0 \leq \mu < r$. Figure 5 helps to illustrate the proof.

Theorem 3 *Let $\mathcal{F}$ be the family composed of one single interval spanning r circumferences of the circle and let $\mathcal{F}$ be unrolled u times, then the u-unrolled circular-arc graph G^u is $r+1$-colorable if and only if $u \geq r$ and $\mu \leq \lambda$.*

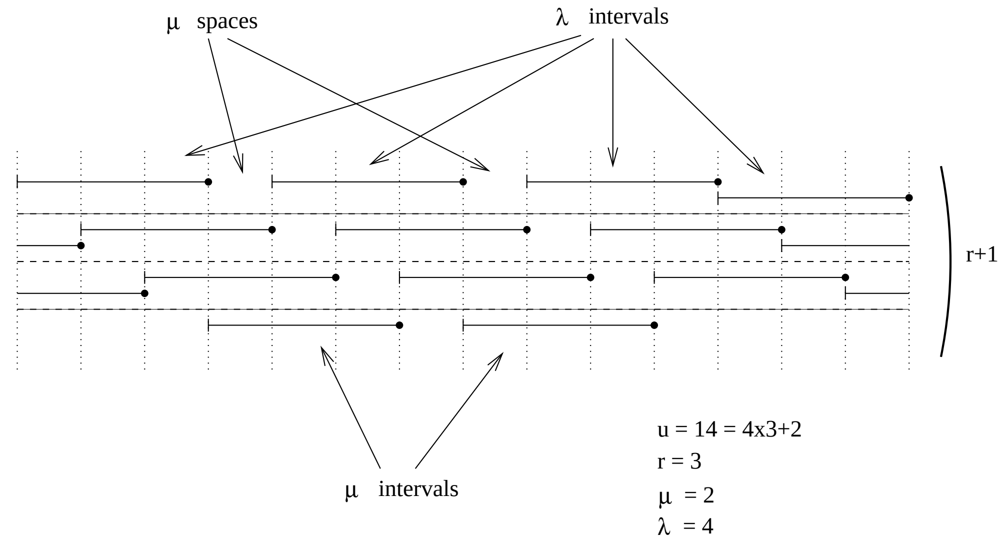

Figure 5: Intervals put on $r + 1$ layers

Proof: We first prove that if the conditions are met, then G^u is $r+1$-colorable. This is done by building the coloring explicitly. The initial interval is $I = [0, r[$. After unrolling has been performed there are u instances of interval I, $I(q)$, $1 \leq q \leq u$, where $I(q) = [q-1, r+q-1[$.

The first $\mu \times (r + 1)$ instances are cyclically allocated on the $r + 1$ colors. Then the $r \times (\lambda - \mu)$ remaining instances are allocated on the r first colors. This coloring makes sense because $\lambda \geq \mu$ and $\lambda \leq 1$ by the condition $u \geq r$.

More formally the color numbered k, $1 \leq k \leq r$ is assigned to the λ intervals

$$I((j-1)r + j + k - 1), j = 1, \ldots, \mu$$

$$I((j-1)r + \mu + k), j = \mu + 1, \ldots \lambda$$

. The last color $r + 1$ is assigned to the μ intervals

$$I((j-1)r + j + r), j = 1, \ldots, \mu$$

.

We prove that the coloring is valid: first, the intervals with the same color do not overlap each other because their length is r and the difference between two instance numbers of two consecutive intervals on the same color is r or $r+1$. Second, the total length covered by these intervals is u for the r first colors. As a matter of fact, the first interval assigned to the color k is $I(k)$ and starts at $s(I(k)) = k-1$ and the last one is $I((\lambda-1)r+\mu+k) = I(u-r+k)$ and ends at $e(I(u-r+k) = u+k-1$. Third, on the last color $r+1$ the first interval is $I(r+1)$ and begins at $s(I(r+1)) = r$, and the last one is $I(\mu(r+1))$ and ends at $e(I(\mu(r+1))) = \mu(r+1)+r-1$. Hence the total length is $e(I(\mu(r+1))) - s(I(r+1)) = \mu r + \mu - 1 \leq \lambda r + \mu < \lambda r + \mu = u$.

Conversely, let us assume that a valid $r+1$-coloring exists for an unrolling degree u. Since there is a coloring, this means that $u \geq r$, else each interval would overlap itself. Now for a color k, we consider the number $N(k)$ of intervals assigned color k. Since the $N(k)$ intervals do not overlap each other, their total length must be less than u. This writes: $N(k)r \leq u$ or $N(k) \leq u/r$. Since $N(k)$ is an integer, we get $N(k) \leq \lfloor u/r \rfloor$. The right hand side is nothing other than λ. By summing all these inequalities on the $r+1$ colors, we get $u \leq (r+1)\lambda$ that rewrites $\mu \leq \lambda$.

□

The condition of the latter theorem is met for every integer u so that λ is greater than $r-1$. This proves the corollary:

Corollary 2 *If $u \geq r(r-1)$ then the unrolled circular-arc graph G^u is $r+1$-colorable.*

We generalize this result by considering p intervals $I_1, I_2, \ldots, I_p$ spanning respectively $r_1, r_2, ..., r_p$ circumferences of the circle. As before, we divide u by r_q: $u = \lambda_q r_q + \mu_q$. We note $r = r_1 + r_2 + ... + r_p$. We can prove that:

Theorem 4 *If $u \geq \sum_{q=1}^{p} \mu_q(r_q+1) - p + 1$ for any $q = 1, ..., p$, then the unrolled circular-arc graph G^u is $r+1$-colorable.*

Proof: Without loss of generality we assume:

- that the intervals are not connected, else we consider each connected component like a single interval;
- that there are p points on the circle labeled 0, 1, ... $p-1$;
- that the interval I_q starts at the point $q-1$. $I_q = [q-1, r_q+q-1[$.

Then the u instances of these intervals in the u-unrolled graph are $I_q(k)$, $1 \leq q \leq p$ and $1 \leq k \leq u$, with $I_q(k) = [q-1+(k-1)p, q-1+(k-1)p+r_q p[$.

The principle is the same as before. $u = \lambda_q r_q + \mu_q$ with $0 \leq \mu \leq r_q$. We reserve r_q consecutive colors for the instances of I_q, namely colors numbered from $r_1 + \ldots + r_{q-1} + 1$ to $r_1 + \ldots + r_{q-1} + r_q$. These colors are assigned to $\lambda_q r_q$ intervals. The last color $r+1$ will contain all the remaining μ_q intervals.

To avoid the overlapping of intervals on the last color, we will perform the latter coloring process by starting from some intervals $I_q(c_q)$ instead of $I_q(1)$ like before. From now on we will note $I_q(k)$ for $I_q((k-1) \bmod u + 1)$ for avoiding complex notations. The coloring is therefore constructed as follows:

The color $r_1 + ... + r_{q-1} + k$, $1 \leq k \leq r_q$ is assigned to the λ_q intervals:

$$I_q((j-1)r_q + j + k - 1 + c_q), j = 1, \ldots, \mu_q$$

$$I_q((j-1)r_q + \mu_q + k + c_q), j = \mu_q + 1, \ldots \lambda_q$$

The color $r+1$ is assigned to the μ_q intervals

$$I_q((j-1)r_q + j + r_q + c_q), j = 1, \ldots, \mu_q$$

.

On the first r colors the coloring is valid because of the same arguments as in the last proof. The case of the last color $r+1$ remains. We set $c_1 = 0$ and we construct c_q by recurrence on q.

The last instance of I_q colored with $r+1$ is $I_q(\mu_q r_q + \mu_q + c_q)$ that ends at point $e_q = e(I_q(\mu_q r_q + \mu_q + c_q)) = q - 1 + p(\mu_q r_q + \mu_q + c_q - 1) + r_q p$.

The first instance of I_{q+1} colored with $r+1$ is $I_{q+1}(1 + r_{q+1} + c_{q+1})$ that starts at point $s_{q+1} = q + (r_{q+1} + c_{q+1})p$. Now we choose c_{q+1} such that $s_{q+1} = e_q + 1$. The total length of intervals on the $r+1$ color is:

$$L = e_p - s_1 = \sum_{q=1}^{p} e_q - s_q + \sum_{q=2}^{p} s_q - e_{q-1} = \sum_{q=1}^{p} p(\mu_q r_q + \mu_q - 1) + p - 1$$

This length must be less than the length up of the unrolled circle. This gives the condition of the theorem. $\square$

To prove the general case, we now partition the meeting graph into its connected components. Let p be the number of connected components. In each component, we exhibit a Hamiltonian circuit. Let r_q be the number of circumferences spanned by the intervals of the Hamiltonian circuit of the q^{th} component. Then we consider the Hamiltonian circuit as a single interval, and we color it as in the previous case. This leads to the following corollary:

Corollary 3 *If $u \geq \sum_{q=1}^{p} \mu_q(r_q+1) - p + 1$ for any $q = 1, ..., p$, then the unrolled circular-arc graph G^u is $r+1$-colorable. Hence $r \leq \chi(G^u) \leq r+1$.*

5 Conclusion

We have defined unrolled circular-arc graphs and identified properties of cyclic and acyclic colorings of these graphs. We believe that this can open a new field in the theory of the circular-arc graphs. Cyclic coloring of circular-arc graphs has very interesting properties that have practical consequences in the domain of register allocation for loops on high performance processors. Some work remains to be done for refining the bounds on the valid degrees given in this paper. Controlling the unrolling degree as well as the chromatic number is of major importance in optimizing compilers. We are not aware of any other domain in operational research where minimizing the unrolling degree is such of importance.

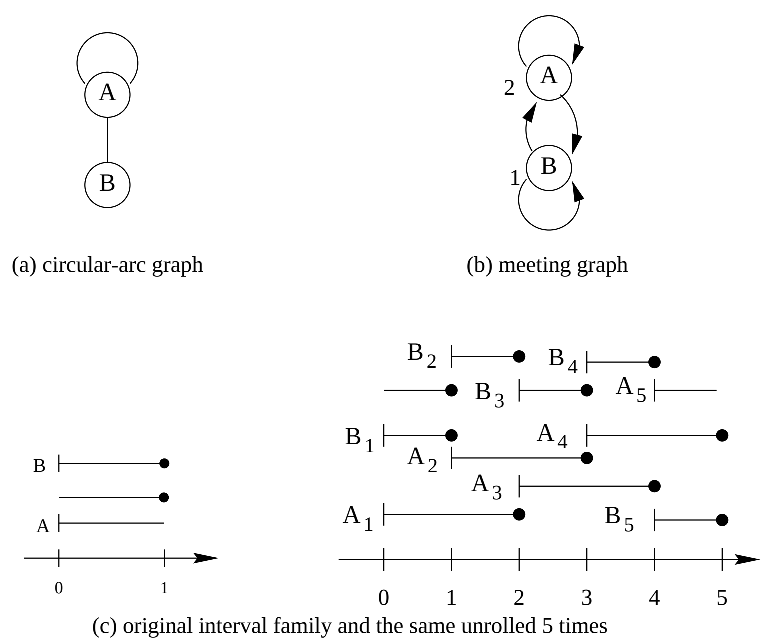

Figure 6: Interval family and the same unrolled 5 times

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We would like to thank Elena Stöhr who pointed out the asymptotic bound on $M(r)$ that we give at the end of Section 3.

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