Optimal Wavelength-Routed Multicasting

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Thème 1 — Réseaux et systèmes
Projet SLOOP

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Abstract: Motivated by wavelength division multiplexing in all-optical networks, we consider the problem of finding a set of paths from a fixed source to a multiset of destinations, which can be coloured by the fewest number of colours so that paths of the same colour do not share an arc. We prove that this minimum number of colours is equal to the maximum number of paths that share one arc, minimized over all sets of paths from the source to the destinations. We do this by modeling the problems as network flows in two different networks and relating the structure of their minimum cuts. The problem can be efficiently solved (paths found and coloured) using network flow techniques.

Key-words: All–optical networks, WDM routing, multicast, graphs, flows.

(Résumé : tsvp)

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Algorithme optimal de multicast tout-optique

Résumé : Ce rapport est motivé par l'étude du routage par multiplexage en longueur d'onde (en anglais *Wavelength Division Multiplexing* : *WDM*) dans les réseaux tout–optiques. Nous considérons le problème du *multicast* dans lequel un processeur fixé désire communiquer simultanément avec un certain nombre d'autres processeurs. Il s'agit de trouver dans le graphe associé des chemins reliant la source du multicast aux destinations et de colorer ces chemins, de manière à ce qu'aucun lien du réseau ne soit traversé par deux chemins de la même couleur. Nous résolvons ce problème en utilisant des techniques de flots. Il en découle un algorithme polynomial pour calculer une solution optimale.

Mots-clé : Réseaux tout–optiques, multicast, graphes, routage, flots.

1 Motivation and Definitions

Optics is emerging as a key technology in communication networks, promising very high speed local or wide area networks of the future. A single optical wavelength supports rates of gigabits per second, which in turn support multiple channels of voice, data and video [4, 5]. Multiple laser beams that are propagated over the same fiber on distinct optical wavelengths can increase this capacity even further. This is achieved through *Wavelength Division Multiplexing* (or *WDM*) [2], by partitioning the optical bandwidth into several channels and allowing the transmission of multiple data streams concurrently along the same optical fiber.

All–optical (or *single–hop* [6]) communication networks provide all source-destination pairs with end-to-end transparent channels that are identified through a wavelength and a physical path. Wavelengths being a limited resource, solutions to the problem of efficient routing and wavelengths allocation are of importance for the future development of optical technology.

The problem we consider here is motivated by *switched* networks with reconfigurable wavelength selective optical switches, without wavelength converters, where different signals may travel on the same communication link (but on different wavelengths) into a node, and then exit from it on different links, keeping their original wavelengths. See the recent survey [1] for an account of the theoretical problems and results obtained for this all–optical model.

We use the standard terminology of digraphs and flow networks [3]. A *dipath* ('directed path') in digraph $G = (V, A)$ is a sequence of nodes $P = (v_1, v_2, \ldots, v_k)$, $k \geq 1$, such that $(v_i, v_{i+1}) \in A$ for $1 \leq i \leq k-1$. For any sets S and S' in a digraph, we denote by $m(S, S')$ the number of arcs beginning in S and ending in S'. In a flow network [3], we denote by $c(u, v)$ the capacity of the arc (u, v), and by $c(S, \bar{S})$ the capacity of the cut $(S, \bar{S})$.

We are now ready to formulate our problems. Let G be a digraph. A *request* in G is an ordered pair of nodes (x, y) (corresponding to a message to be sent from x to y). An *instance* I in G is a collection (multiset) of requests (a request (x, y) may appear more than once). A *routing* R of an instance I is a collection of dipaths $R = \{P(x, y) \mid (x, y) \in I\}$, where $P(x, y)$ is a dipath from x to y.

Let G be a digraph and I an instance in G. For each routing R of I, we denote by $\vec{w}(G, I, R)$ the minimum number of wavelengths ('colours') that can be assigned to the dipaths in R, so that no two dipaths of the same wavelength share an arc. The *WDM problem* (G, I) asks for a routing R of I which minimizes $\vec{w}(G, I, R)$. We denote by $\vec{w}(G, I)$ this minimum of $\vec{w}(G, I, R)$ over all routings R for I.

Let again G be a digraph and I an instance. For each routing R of I, the *load* of an arc $\alpha \in A$ in the routing R, denoted by $\vec{\pi}(G, I, R, \alpha)$, is the number of dipaths of R containing α. The load of a routing R, denoted by $\vec{\pi}(G, I, R)$, is the maximum of $\vec{\pi}(G, I, R, \alpha)$ over all arcs $\alpha \in A$. The *congestion problem* (G, I) asks for a routing R of I which minimizes its load. We denote by $\vec{\pi}(G, I)$ this minimum of $\vec{\pi}(G, I, R)$ over all routings R for I.

The relevance of the parameter $\vec{\pi}$ to our problem is shown by the following lemma:

Lemma 1 *$\vec{\mathtt{w}}(G,I) \geq \vec{\pi}(G,I)$ for any instance I in any digraph G.*

Proof. Indeed, to solve a given WDM problem (G,I) one has to use a number of wavelengths at least equal to the maximum number of dipaths having to share an arc. □

In this paper, we are interested in a special case of instances where the collection of requests has the form $\{(x,y) \mid y \in Y\}$ for a fixed node $x \in V$, called the *originator*, and a multiset Y of nodes in V. Such an instance is called a *multicast* (or a *one–to–many*) *instance*. The particular instance where Y is the set V, is called a *broadcast*.

2 Multicasting and Network Flows

In the case of multicasting, we can model both of the above problems (the WDM problem as well as the congestion problem) by network flows. Consider a digraph $G=(V,A)$ with a multicast instance $I=\{(x,y) \mid y \in Y\}$, with originator x. In what follows we assume that Y is a set, i.e., that each node y appears at most once in Y. Indeed, if Y is a general multiset, we can transform the problem by adding to each destination y which appears in Y $\mu(y)$ times, a set of $\mu(y)-1$ vertices of degree one, adjacent to y, each (as well as y) having multiplicity one.

We begin by modeling the congestion problem (G,I) (cf. Figure 1 (b)). Let s,t be two new vertices that will play the roles of source and sink. For every positive integer p, we define the network F_p to have the vertex set $V \cup \{s,t\}$, the arc set $\{(s,x)\} \cup A \cup (\bigcup_{y \in Y} \{(y,t)\})$, and the capacities $c(s,x)=\infty$, $c(u,v)=p$ for all $(u,v) \in A$, and $c(y,t)=1$ for all $y \in Y$.

Proposition 2 *$\vec{\pi}(G,I) \leq p$ if and only if F_p has a flow of value $|Y|$.*

Proof. From the definitions. □

For future reference we also consider the capacities of cuts in the network F_p. Suppose S is a subset of V and $\bar{S} = V \setminus S$. This gives rise to the cut $(S \cup \{s\}, \bar{S} \cup \{t\})$ in F_p. The capacity of this cut is infinite when $x \in \bar{S}$; otherwise it is $|S \cap Y| + p \cdot m(S,\bar{S})$.

According to the above proposition, $\vec{\pi}(G,I)$ is the smallest integer p such that F_p admits a flow of value $|Y|$. Combining this with the max-flow min-cut theorem of [3] we obtain

$$|S \cap Y| + \vec{\pi}(G,I).m(S,\bar{S}) \geq |Y|, \text{ for any } S \subseteq V. \quad (1)$$

Next we discuss the network flow model for the WDM problem (G,I) (cf. Figure 1 (c)). Let λ be a positive integer. For $1 \leq i \leq \lambda$, let $G_i=(V_i,A_i)$ be a copy of $G=(V,A)$, and let Y_0 be a copy of Y. For each $v \in V$, let v_i be the copy of v in V_i, and for each $y \in Y$, let y_0 the copy of y in Y_0. Let s,t be two new vertices which will be the source and sink. We define the network F'_λ to have the vertex set $\mathcal{V}=\{s,t\} \cup (\bigcup_{i=1}^{\lambda} V_i) \cup Y_0$, the arc set $\mathcal{A}=\bigcup_{i=1}^{\lambda}\{(s,x_i)\} \cup (\bigcup_{i=1}^{\lambda} A_i) \cup (\bigcup_{y \in Y} \bigcup_{i=1}^{\lambda} \{(y_i,y_0)\}) \cup (\bigcup_{y \in Y} \{(y_0,t)\})$, and except for the arcs (s,x_i) ($1 \leq i \leq \lambda$) of infinite capacity, all other arcs having capacity one.

Proposition 3 *$\vec{w}(G,I) \leq \lambda$ if and only if F'_λ has a flow of value $|Y|$.*

Proof. Suppose first that G admits dipaths from x to each $y \in Y$, which can be coloured by integers from $\{1,\dots,\lambda\}$ in such a way that no two dipaths sharing the same arc have the same color. Define the values $f'(u_i,v_i) = 1$, for each arc $(u,v) \in A$ which belongs to a dipath of color i ($1 \leq i \leq \lambda$). These values can be extended in a natural way to a flow f' of value $|Y|$ in F'_λ.

Conversely, assume that there exists a flow f' of value $|Y|$ in F'_λ. By construction of F'_λ, this implies that there is for each node $y \in Y_0$ an incoming arc with flow one. Hence for each $y \in Y_0$ there is exactly one i such that the node y_i has an incoming arc with flow one in G_i. In that G_i there must be a dipath from x_i to y_i consisting of arcs *all* having flow one, and we can assign the corresponding dipath in G the color i. Thereby are defined dipaths in G from x to each $y \in Y$, which are coloured by λ colours as required. $\square$

Theorem 4 *Let G be a digraph and I a multicast instance in G. Then $\vec{w}(G,I) = \vec{\pi}(G,I)$, and an optimal solution to the WDM problem (G,I) can be found in polynomial time.*

Proof. In view of lemma 1 and the above proposition, it will suffice to show that the network F'_λ, with $\lambda = \vec{\pi}(G,I)$, has a flow of value $|Y|$. We shall do this by showing that every cut has capacity at least $|Y|$, cf. [3].

Consider a cut $(S \cup \{s\}, \bar{S} \cup \{t\})$ in F'_λ, and let $S_i = S \cap V_i$ and $\bar{S}_i = \bar{S} \cap V_i$, for $1 \leq i \leq \lambda$, and let $S_0 = S \cap Y_0$ and $\bar{S}_0 = \bar{S} \cap Y_0$. Clearly, we may assume that each $x_i \in S_i, 1 \leq i \leq \lambda$, otherwise the capacity of the cut will be infinite. Then the capacity is

$$c(S \cup \{s\}, \bar{S} \cup \{t\}) = \sum_{i \geq 1} m(S_i, \bar{S}_i) + \sum_{i \geq 1} m(S_i, \bar{S}_0) + |S_0|$$

For each $y \in Y$, let $S_y = \{y_i \mid i \geq 1,\ \ y_i \in S\}$. If Y_i denotes the copy of Y in V_i, then $\sum_{i \geq 1} |S_i \cap Y_i| = \sum_{y \in Y_0} |S_y|$. Note also that $\sum_{i \geq 1} m(S_i, \bar{S}_0) = \sum_{y \in Y_0} m(S_y, \bar{S}_0)$, and that $m(S_y, \bar{S}_0)$ is 0 if $y \in S_0$, and is $|S_y|$ if $y \in \bar{S}_0$. For every $y \in Y$, we have $|S_y| \leq \lambda$, and hence $\sum_{y \in S_0} |S_y| \leq \lambda |S_0|$, i.e., $|S_0| \geq \frac{1}{\lambda} \sum_{y \in S_0} |S_y|$. Summarizing, we obtain:

$$\begin{aligned} c(S \cup \{s\}, \bar{S} \cup \{t\}) \quad &\geq \quad \sum_{i \geq 1} m(S_i, \bar{S}_i) + \sum_{y \in \bar{S}_0} |S_y| + \frac{1}{\lambda} \sum_{y \in S_0} |S_y| \\ &\geq \quad \sum_{i \geq 1} m(S_i, \bar{S}_i) + \frac{1}{\lambda} \sum_{y \in Y_0} |S_y| \\ &\geq \quad \sum_{i \geq 1} \left(m(S_i, \bar{S}_i) + \frac{1}{\lambda} |S_i \cap Y_i| \right) \end{aligned}$$

Recall that we set $\lambda = \vec{\pi}(G,I)$. By the inequality (1), we have $|S_i \cap Y_i| + \lambda . m(S_i, \bar{S}_i) \geq |Y|$ for each $i \geq 1$, and thus $c(S \cup \{s\}, \bar{S} \cup \{t\}) \geq |Y|$.

Our proof implies, in an obvious way, a flow-based algorithm to compute $\vec{w}(G,I)$ (and solve the WDM problem (G,I)) in polynomial time, for any multicast instance I in any digraph G. (Of course, the same applies to $\vec{\pi}(G,I)$ and the congestion problem.) $\square$

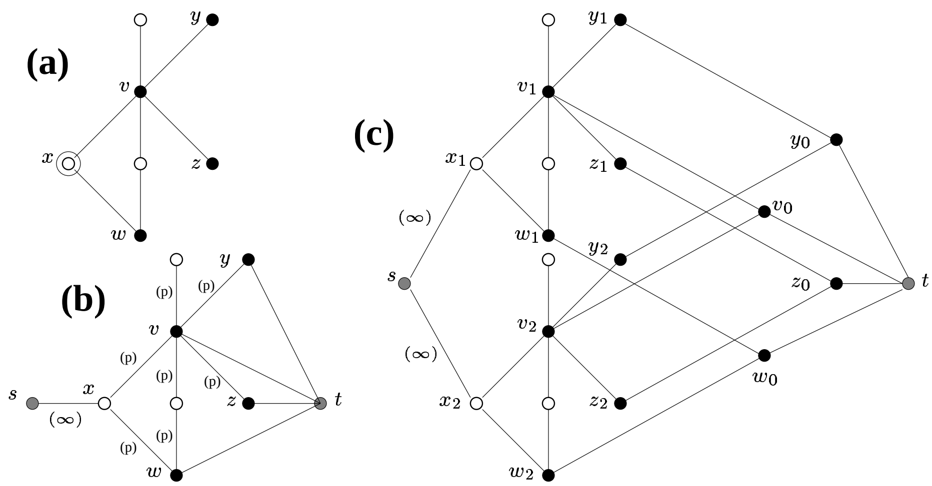

Figure 1: (a) A digraph G with originator x and where $Y = \{v, w, y, z\}$. (b) The network F_p. (c) The network F'_2. All capacities not marked are equal to one. All arc orientations are omitted.

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