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Abstract: We introduce a subclass of Valk's self-modifying nets. The considered nets appear as statified sums of ordinary nets and they arise as a counterpart to cascade products of automata via the duality between automata and nets based on regions in automata. Nets in this class, called stratified nets, cannot exhibit circular dependences between places: inscription on flow arcs attached to a given place, depend at most on the contents of places in lower layers. Therefore, the synthesis problem has similar degrees of complexity for (ordinary) nets and for stratified nets, hence it is tractable.

Key-words: Selfmodifying Petri Nets, Cascade Product, Net Synthesis Problem.

(Résumé : tsvp)

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La composition en cascade de réseaux de Petri

Résumé : Nous introduisons une classe spécifique de réseaux de Petri automodifiants obtenus comme somme en cascades de réseaux de Petri ordinaires, correspondant par dualité à des produits en cascades au sens de Krohn et Rhodes. À la différence des réseaux automodifiants définis par Valk, les réseaux de cette classe n'ont pas de dépendence circulaires entre places ce qui autorise des algorithmes de synthèse de ces réseaux de complexité du même ordre que pour les réseaux sous-jacents.

Mots-clé : Réseaux de Petri automodifiants, composition en cascade, synthèse de réseaux.

1 Introduction

The cascade product of automata, celebrated in the algebraic theory of automata by the Krohn-Rhodes decomposition theorem for automata and semigroups [19, 13], may be carried to the theory of Petri net through the region based duality between automata and nets, parametric on the type of nets [4] [1]. Stratified sums of nets are dual to cascade products of automata, and they produce from arbitrary types classes of nets intermediate between ordinary nets (with static flow arcs) and self-modifying nets (with marking dependent flow arcs).

Self-modifying nets introduced by Valk [27, 28] are generalizations of Petri nets in which the flow relation between a place and a transition is a linear function of the marking. Techniques of linear algebra used in the study of the structural properties of Petri net can be adapted in this extended framework, in particular each transition may be associated with a matrix and the modification of the marking due to a sequence of firable transitions can be encoded by the corresponding product of matrices. Stratified nets are self-modifying nets for which an ordering on places may be chosen so that the matrices associated with transitions are triangular, which amounts to say that there are no circular dependencies between places in the specification of flow arc inscriptions: places may be partionned into *layers* so that inscription on flow arcs attached to a given place, depend at most on the contents of places in lower layers.

The purpose of this paper is to introduce the model of stratified Petri nets and to produce a polynomial algorithm for their synthesis. The synthesis problem consists in deciding whether a given graph is isomorphic to the state graph of some net, and then constructing the net. This problem was originally solved for elementary net systems using the notion of *region* [10]. The solution was then extended to Petri nets [7, 22] using the related notion of *generalized region*. Generalized regions for pure Petri nets correspond exactly to the *synchronic distances* which were introduced by C.A. Petri as a tool to measure the relative degree of freedom between sets of events in a concurrent system [7]. Such a generalized region is characterized, up to an additive constant, as an integral ponderation on the alphabet of actions whose algebraic sum vanishes

[1] for instance elementary nets, Petri nets, Petri nets with inhibitor arcs are associated with different type of nets

along any closed path of the transition system. The synthesis problem for pure Petri nets can then be solved in polynomial time using algorithms borrowed from the litterature on linear algebra over the rationals [2]. This algorithm, that was latter extended for general Petri Nets [6], is adapted in this paper for the class of stratified Petri nets. Let us recall that in contrast this method cannot be adapted for elementary nets systems for which the synthesis problem was indeed proved to be NP-complete [3]. However some boolean counterpart of the notion of generalized regions gives rise to the so-called *flip-flop nets* which are a slight extension of elementary nets that admits a polynomial time synthesis [25]. Similarly, one might define the class of stratified flip-flop nets and prove that in that case also, because of the absence of circular dependencies between places, the synthesis of stratified flip-flop nets has a polynomial time solution.

Stratified nets (as well as the more general self-modifying nets) may be defined for arbitrary type of nets; however in this paper we shall restrict ourself to the class of stratified (pure) Petri nets which are introduced in section (2). A polynomial solution for the synthesis of stratified Petri nets is described in section (3). Section (4) gives some concluding remarks and hints for further research.

2 Stratified Petri Nets

If K is a ring we let $K^{n,m}$ stand for the set of $n \times m$ matrices with entries in K, I_n is the identity matrix in $K^{n,n}$, $0_{n,m}$ is the matrix of $K^{n,m}$ all of whose entries are 0, a vector $X \in K^n$ is identified with a column matrix $X \in K^{n,1}$.

Definition 1 *A self-modifying net of dimension n, with m parameters is a structure $N = (E, \mu)$ consisting of a set of events E, and a linear representation $\mu : E \to \mathbb{Z}^{n,m+n}$ such that for every event e, $\mu(e) = \begin{pmatrix} \Pi(e) & \Lambda(e) \end{pmatrix}$ where $\Pi(e) \in \mathbb{Z}^{n,m}$ and $\Lambda(e) \in \mathbb{Z}^{n,n}$. A marking is any vector $M \in \mathbb{N}^n$, a parametrization (or a mode of operation) is a vector $P \in \mathbb{N}^m$, and the marking graph of the net is the transition relation $\mathbf{mg}(N)$ whose states $S = \mathbb{N}^n$ are markings, whose actions $A = E \times \mathbb{N}^m$ are events with a mode of operation, and whose transition relation $[\ > \subseteq S \times A \times S$ is given by*

$$M[e{>}_P M' \quad \Leftrightarrow \quad M' = \Pi(e) \cdot P + \Lambda(e) \cdot M \tag{1}$$

The net is said to be ordinary when the $\Lambda(e)$ are identity matrices and it is said to be stratified when the $\Lambda(e)$ are triangular matrices whose entries on the diagonal are 1's:

$$\Lambda(e)(i,i) = 1 \quad \textit{and} \quad i < j \Rightarrow \Lambda(e)(i,j) = 0$$

We let $\mathbf{PNets}(m,n)$, $\mathbf{st\text{-}PNets}(m,n)$ and $\mathbf{sm\text{-}PNets}(m,n)$ denote respectively the sets of ordinary, stratified and self-modifying Petri nets of dimension n, with m parameters.

$$\mathbf{PNets}(m,n) \subseteq \mathbf{st\text{-}PNets}(m,n) \subseteq \mathbf{sm\text{-}PNets}(m,n)$$

The marking graph $\mathbf{mg}(N)$ of a net may also be presented as a family of transition systems $N^*(P) \subseteq I\!N^{m+n} \times E \times I\!N^{m+n}$ indexed by parametrizations $P \in I\!N^m$, where

$$\left(\begin{smallmatrix} P \\ M \end{smallmatrix}\right) \xrightarrow{e} \left(\begin{smallmatrix} P \\ M' \end{smallmatrix}\right) \quad \text{in} \quad N^*(P) \quad \Leftrightarrow \quad M[e\!>_P M'$$

When there is only one parameter whose value is 1, i.e. $P = [1]$, we shall abbreviate $N^*(P)$ to N^* and $M[e\!>_P M'$ to $M[e\!> M'$.

2.1 Some Classes of Self-Modifying Nets

As it is usually necessary to have the transition relation be given by an affine transformation we shall use the canonical embedding of an affine space into a linear space, i.e. we add a *fictituous* parameter whose value is always given by the unit element 1. Technically $m = 1 + p$ where p is the number of actual parameters; these parameters will be represented by variables, called the *parametric places* of the net: x_0, x_1, ... x_p where x_0 corresponds to the fictituous parameter. The other places x_{p+1}, ... x_{p+n} of the net, where n is its dimension, are termed *internal places*. If there is no actual parameter ($p = 0$) then the transition relation is parameter-free:

$$M[e\!> M' \quad \Leftrightarrow \quad M' = \Pi(e) + \Lambda(e) \cdot M \tag{2}$$

A parameter-free ordinary net is just a pure Petri net (or a *vector addition system* [18]) and has its transition system given by:

$$M[e\!> M' \quad \Leftrightarrow \quad M' = M + \Pi(e) \tag{3}$$

Finally if there is only one event in E, the net reduces to a pair of matrices $\Pi \in \mathbb{Z}^{n,m}$ and $\Lambda \in \mathbb{Z}^{n,n}$ and its marking graph is the transition relation $\rightarrow \subseteq \mathbb{N}^n \times \mathbb{N}^m \times \mathbb{N}^n$ given by

$$M \stackrel{P}{\rightarrow} M' \quad \Leftrightarrow \quad M' = \Pi \cdot P + \Lambda \cdot M \tag{4}$$

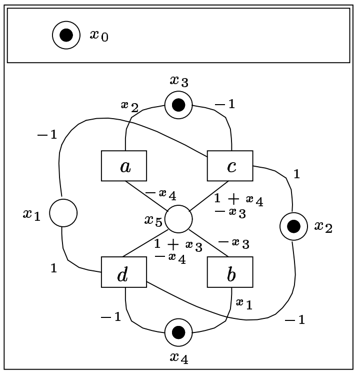
The net is then a *linear sequential machine* [16].

2.2 Net Pictorial Representation

We shall use the following pictorial representation for nets. Places are represented by circles and events $e \in E$ are represented by boxes. We call *extended marking* a vector $\tilde{M} \in \mathbb{N}^{m+n}$ whose first entry (corresponding to the fictituous parameter) has value 1. $\tilde{M} = \left(\begin{smallmatrix} P \\ M \end{smallmatrix} \right)$ where $P \in \mathbb{N}^m$ is a parametrization and $M \in \mathbb{N}^n$ is a marking. An extended marking $\tilde{M}$ is represented by indicating in each place the value in $\mathbb{N}$ given by the corresponding entry in $\tilde{M}$. Since the place corresponding to the fictituous parameter x_0 has always value 1 it is usually not represented. If $\tilde{M} = \left(\begin{smallmatrix} P \\ M \end{smallmatrix} \right)$ is an extended marking, one has

$$M[e\!>_P M' \quad \Leftrightarrow \quad M' = M + \Delta(e) \cdot \left(\begin{smallmatrix} P \\ M \end{smallmatrix} \right) \tag{5}$$

where $\Delta(e) = \mu(e) - \left(\begin{matrix} 0_m & I_n \end{matrix} \right)$, called the *incidence matrix*, records the modification of markings due to the occurrence (*firing*) of event e. The i^{th} line of $\Delta(e)$ which is a vector of $\mathbb{Z}^{m+n}$ is represented by the formal sum $\Delta(x_i, e) = \sum_{j=0}^{p+n} \Delta(e)(i,j) \cdot x_j$; since the variable x_0 always holds the value 1 we shall replace the summand $\Delta(e)(i,0) \cdot x_0$ by the constant $\Delta(e)(i,0)$. We now complete the pictorial representation of the net by drawing an arc between the place x_i and the event e labelled by the formal sum $\Delta(x_i, e)$ when this expression is not zero. Figure (1) represents a stratified Petri nets with its linear representation and part of its marking graph. Equation (5) describes the token game of selfmodifying nets which behave locally at each extended marking $\tilde{M} = \left(\begin{smallmatrix} P \\ M \end{smallmatrix} \right)$ like a vector addition system (compare with equation (3) where $\Pi(e)$ is given by $\Delta(e) \cdot \left(\begin{smallmatrix} P \\ M \end{smallmatrix} \right)$). Figure (2) displays the vector addition systems (i.e. pure Petri nets) to which the selfmodifying net of Fig. (1) evaluate in the respective extended markings.



$$\mu(a) = \begin{array}{c|c|ccccc} & x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline x_1 & 0 & 1 & 0 & 0 & 0 & 0 \\ x_2 & 0 & 0 & 1 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 1 & 1 & 0 & 0 \\ x_4 & 0 & 0 & 0 & 0 & 1 & 0 \\ x_5 & 0 & 0 & 0 & 0 & -1 & 1 \end{array}$$

$$\mu(b) = \begin{array}{c|c|ccccc} & x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline x_1 & 0 & 1 & 0 & 0 & 0 & 0 \\ x_2 & 0 & 0 & 1 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 0 & 1 & 0 & 0 \\ x_4 & 0 & 1 & 0 & 0 & 1 & 0 \\ x_5 & 0 & 0 & 0 & -1 & 0 & 1 \end{array}$$

$$\mu(c) = \begin{array}{c|c|ccccc} & x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline x_1 & -1 & 1 & 0 & 0 & 0 & 0 \\ x_2 & 1 & 0 & 1 & 0 & 0 & 0 \\ x_3 & -1 & 0 & 0 & 1 & 0 & 0 \\ x_4 & 0 & 0 & 0 & 0 & 1 & 0 \\ x_5 & 1 & 0 & 0 & -1 & 1 & 1 \end{array}$$

$$\mu(d) = \begin{array}{c|c|ccccc} & x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline x_1 & 1 & 1 & 0 & 0 & 0 & 0 \\ x_2 & -1 & 0 & 1 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 0 & 1 & 0 & 0 \\ x_4 & -1 & 0 & 0 & 0 & 1 & 0 \\ x_5 & 1 & 0 & 0 & 1 & -1 & 1 \end{array}$$

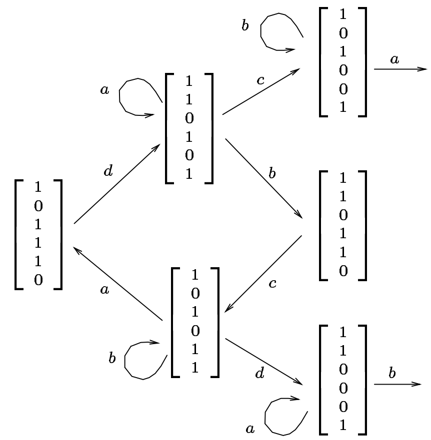


Figure 1: a stratified Petri net and part of its marking graph N^*

2.3 Cascade Composition

Definition 2 *Let* $N_1 = (E, \mu_1) \in$ **sm-PNets**(m, n) *and* $N_2 = (E, \mu_2) \in$ **sm-PNets**$(m+n, n')$ *their cascade composition* $N_1 \circ N_2$ *is the net* $N = (E, \mu) \in$ **sm-PNets**$(m, n+n')$ *where* $\mu(e) = \left(\dfrac{\mu_1(e) \mid 0_{n,n'}}{\mu_2(e)} \right)$ *for every event* $e \in E$,

Observation 3 *The cascade composition of self-modifying Petri nets is associative and the family of stratified Petri nets is the closure of the family of ordinary Petri nets by the cascade composition.*

We also observe that $\left(\begin{smallmatrix} M_1 \\ M_2 \end{smallmatrix} \right) [e\!>_P \left(\begin{smallmatrix} M_1' \\ M_2' \end{smallmatrix} \right)$ in $N_1 \circ N_2$ if and anly if $M_1 [e\!>_P M_1'$ in N_1 and $M_2 [e\!>_{\left(\begin{smallmatrix} P \\ M_1 \end{smallmatrix} \right)} M_2'$ in N_2; i.e. $\mathbf{mg}(N_1 \circ N_2) = \mathbf{mg}(N_1) \circ \mathbf{mg}(N_2)$ where the composition appearing at the right-hand side of the equality is the cascade composition of transition systems whose definition we recall:

Definition 4 *The cascade composition of transition systems* $T_1 \subseteq S_1 \times E \times S_1$ *and* $T_2 \subseteq S_2 \times (E \times S_1) \times S_2$ *is the transition system* $T \subseteq S \times E \times S$ *where* $S = S_1 \times S_2$ *and* $(s_1, s_2) \xrightarrow{e} (s_1', s_2') \in T \;\Leftrightarrow\; (s_1 \xrightarrow{e} s_1' \in T_1 \;\wedge\; s_2 \xrightarrow{(e,s_1)} s_2' \in T_2)$

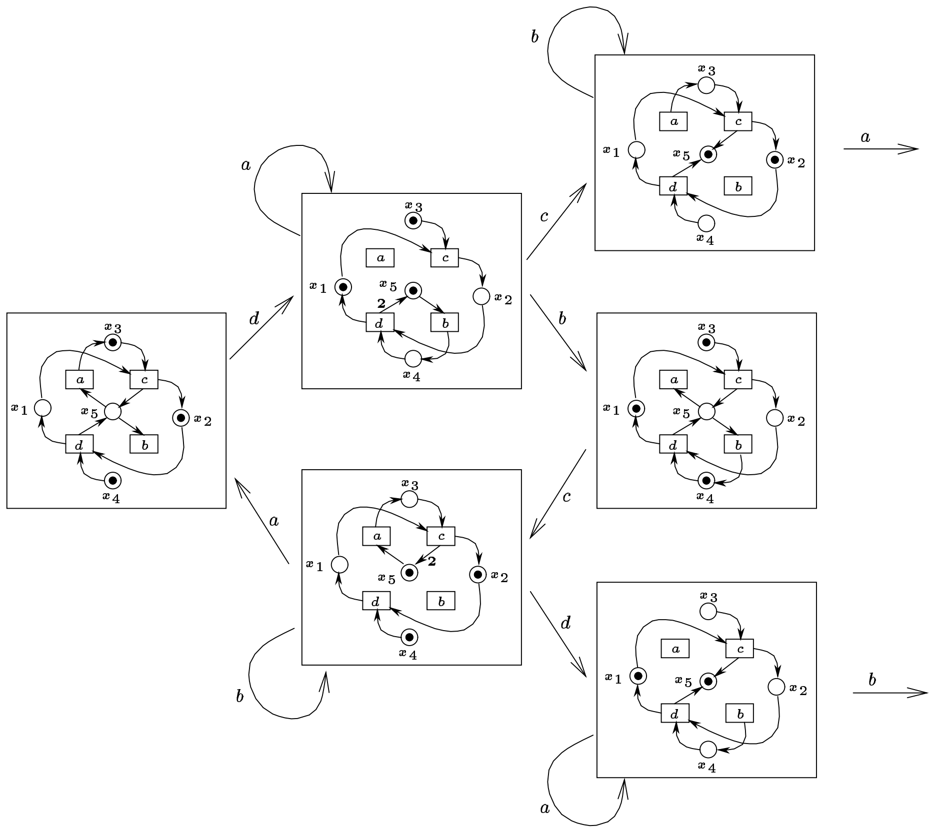


Figure 2: a selfmodifying net (here the stratified Petri net of Fig. 1) can be viewed as a Petri net whose structure evolves along computation. We adopt the usual pictorial representation for Petri net: an arrow from place x to event e (resp. from e to x) labelled n means $\Delta(x, e) = -n$ (resp. $\Delta(x, e) = n$); the absence of label means $n = 1$.

3 The Synthesis of Stratified Petri Nets

Definition 5 (Synthesis Problem for Stratified Petri Nets)
The synthesis problem for stratified Petri nets consists in the following. Given a finite and connected transition system $T \subseteq S \times E \times S$*, find a stratified Petri net* $N = (E, \mu) \in \mathbf{st\text{-}PNets}(1, n)$ *and an injective state encoding* $\rho : S \to \mathbb{N}^n$ *such that*

$$\begin{array}{rlcl} (i) & s \xrightarrow{e} s' & \Rightarrow & \rho(s)[e{>}\rho(s') \\ (ii) & \rho(s)[e{>}M & \Rightarrow & \exists s' \in S \;:\; s \xrightarrow{e} s' \;\wedge\; \rho(s') = M \end{array}$$

When the above two conditions are satisfied, ρ is termed a representation of transition system T by net N. These conditions state that T is isomorphic to its ρ-image in the marking graph N^* and that this image is a forward-closed subtransition system of N^*, we call *full forward embedding* a morphism of transition systems fulfilling these conditions and denote $\rho : T \tilde{\hookrightarrow} N^*$ this situation.

The fact that we have assume the presence of a fictituous parameter is by no means restrictive. Actually, if we had instead defined a representation of a transition system T by a net $N = (E, \mu) \in \mathbf{st\text{-}PNets}(0, n)$ to be a full forward embedding $\rho : T \tilde{\hookrightarrow} \mathbf{mg}(N)$ of T into the marking graph of N, then necessarily $\rho(s)(1)$ has the same value for all states and without loss of generality one can assume that this value is 1 because if it is not the case i.e. $\rho(s)(1) = k \neq 1$ then we can replace $N = (E, \mu)$ by $\tilde{N} = (E, \tilde{\mu})$ where

$$\tilde{\mu}(e) = \left[\begin{array}{c|c} 1 & 0_{1,n-1} \\ \hline k \times u_1 & \\ \cdot & A \\ \cdot & \\ k \times u_{n-1} & \end{array}\right] \quad \text{if} \quad \mu(e) = \left[\begin{array}{c|c} 1 & 0_{1,n-1} \\ \hline u_1 & \\ \cdot & A \\ \cdot & \\ u_{n-1} & \end{array}\right]$$

then $\tilde{\rho} : T \tilde{\hookrightarrow} \mathbf{mg}(N)$ where $\tilde{\rho}(s) = (1, v_1, \ldots, v_{n-1})$ when $\rho(s) = (k, v_1, \ldots, v_{n-1})$ is an alternative representation that satisfies this property ($\tilde{\rho}(s)(1) = 1$). Finally by dropping the first row in $\tilde{\mu}(e)$ and the first component in $\tilde{\rho}(s)$ we obtain a representation in the sense of the previous definition.

3.1 Regions

Suppose that $N = N_1 \circ N_2 \ldots \circ N_K$ where each $N_i = (E, \mu_i)$ is an atomic net; i.e. a net with only one place x_i. Notice that $\mu_i(e)$ is of the form $\mu_i(e) = \left[\begin{array}{cc} \Delta_i(e) & 1 \end{array}\right]$ where $\Delta_i : E \to \mathbb{Z}^i$ and that $\left[\begin{array}{cc} \mu_i(e) & 0_{1,K-i} \end{array}\right]$ is the i^{th} row of $\mu(e)$. Let $\rho : T \stackrel{\sim}{\hookrightarrow} N^*$ be a representation of T, then we have the sequence of approximations of the state encoding function $\rho_k : S \to \mathbb{N}^k$ for $1 \leq k \leq n$ where $\rho_k(s)(i) = \rho(s)(i)$ gives the content of place x_i for $1 \leq i \leq k$ in the marking representing state s, and by convenience we add the *null approximation* $\rho_0 : S \to \mathbb{N}^0$ that takes each state to the empty vector. We call *extension* of place x_i the pair of mappings $r_i = (\sigma_i, \Delta_i)$ where $\sigma_i : S \to \mathbb{N}$ is given by $\sigma_i(s) = \rho(s)(i)$ i.e. σ_i records the values of each state in place x_i. If $\rho : S \to \mathbb{N}^k$ is some state encoding, we let $\overline{\rho} : S \to \mathbb{N}^{1+k}$ denote the 1-*shift* of ρ defined by $\overline{\rho}(s)(1) = 1$ and $\overline{\rho}(s)(1+i) = \rho(s)(i)$, then $\Delta_i(e) \cdot \overline{\rho}_{i-1}(s)$ records the modification of place x_i due to the firing of event e in state s, and r_i is a $\overline{\rho}_{i-1}$-region of T according to the following definition.

Definition 6 *Let $T \subseteq S \times E \times S$ a transition system and $\rho : S \to \mathbb{N}^m$ a state encoding function, a ρ-region of T is a pair of mappings $r = (\sigma, \Delta)$ where $\sigma : S \to \mathbb{N}$ and $\Delta : E \to \mathbb{Z}^m$ such that $s \xrightarrow{e} s' \;\Rightarrow\; \sigma(s') = \sigma(s) + \Delta(e) \cdot \rho(s)$*

If $\rho : S \to \mathbb{N}$ is the trivial state encoding, then $\overline{\rho}$-regions coincide with the regions for pure Petri nets (the so-called *generalized regions*).

3.2 Representation Theorem

Definition 7 *A stratification by regions of a transition system $T \subseteq S \times E \times S$ is a sequence $R = (r_1, r_2, \ldots, r_n)$ of pairs of mappings $r_i = (\sigma_i, \Delta_i)$ where $\sigma_i : S \to \mathbb{N}$ and $\Delta_i : E \to \mathbb{Z}^i$, such that if we let $\rho_i : S \to \mathbb{N}^i$ be given by $\rho_i(s)(j) = \sigma_j(s)$ for $1 \leq j \leq i$ then r_i is a $\overline{\rho}_{i-1}$-region.*

Thus if $N = (E, \mu) \in$ **st-PNets**$(1, n)$, the extensions of r_i of the places x_i (for $1 \leq i \leq n$) form a stratification by regions of its marking graph N^*. Conversely

Observation 8 *Let $R = (r_1, r_2, \ldots, r_n)$ be a stratification by regions of a transition system $T \subseteq S \times E \times S$, the regions $r_j = (\sigma_i, \Delta_i)$ are the extensions of* *places of the net $N_R = (E, \mu_R)$ where*

$$\mu_R(e)(i,j) = \begin{cases} \Delta_i(e)(j) & \textit{if} \quad i > j \\ 1 & \textit{if} \quad i = j \\ 0 & \textit{if} \quad i < j \end{cases}$$

for $1 \leq i, j \leq n$. Moreover the mapping $\rho : S \to I\!N^n$ given by $\rho(s)(i) = \sigma_i(s)$ is a morphism of transition system from T to N_R^*. ρ is a (canonical) representation of T by N_R if and only if the following two conditions are met:

1. $s \neq s' \quad \Rightarrow \quad \exists i : 1 \leq i \leq n-1 \quad \sigma_i(s) \neq \sigma_i(s')$,
2. $s \overset{e}{\not\to} \quad \Rightarrow \quad \exists i : 1 \leq i \leq n-1 \quad \sigma_i(s) + \mu_i(e) \cdot \overline{\rho}_i(s) < 0.$

We say, in the first case, that place x_i distinguishes states s and s' and, in the second case, that place x_i inhibits event e in state s. The sequence $R = (r_1, \ldots, r_n)$ is termed a solution of the synthesis problem for T.

3.3 Polynomial time Solution

If T admits some solution $R = (r_1, \ldots, r_n)$, then any stratification by regions $R' = (r'_1, \ldots, r'_{n'})$ of T may be extended into a solution R'' for T. The most trivial way consists in letting $R'' = (r'_1, \ldots, r'_{n'}, r''_{n'+1}, \ldots, r''_{n'+n})$ where $r''_{n'+j}$ is obtained by "shifting" r_j in the sense that $\sigma''_{n'+j}(s) = \sigma_j(s)$ and

$$\Delta''_{n'+j}(e)(k) = \begin{cases} \Delta_j(e)(0) & \text{if} \quad k = 0 \\ 0 & \text{if} \quad 1 \leq k \leq n-1 \\ \Delta_j(e)(k-n+1) & \text{if} \quad k \geq n \end{cases}$$

Hence in order to establish the following

Theorem 9 *The synthesis problem for stratified Petri nets has a polynomial time solution.*

we just have to show that the following two problems have polynomial time solutions.

State Separation Problem (SSP)

Instance. A finite transition system $T \subseteq S \times E \times S$, a state encoding function $\rho : S \to I\!N^m$, and a pair $(s_1, s_2) \in S \times S$ of distinct states.
Question. Is there a ρ-region (σ, Δ) which distinguishes states s_1 and s_2, i.e. such that $\sigma(s_1) \neq \sigma(s_2)$?

Event-State Separation Problem (ESSP)
Instance. A finite transition system $T \subseteq S \times E \times S$, a state encoding function $\rho : S \to I\!N^m$, and a pair $(s, e) \in S \times E$ where event e is not enabled in state s.
Question. Is there a ρ-region (σ, Δ) which inhibits event e in state s, i.e. such that $\sigma(s) + \Delta(e) \cdot \rho(s) < 0$?

The set of ρ-regions is partially ordered by letting $r_1 \leq r_2$ when $r_1 = (\sigma_1, \Delta)$ and $r_2 = (\sigma_2, \Delta)$ where $\sigma_1(s) \leq \sigma_2(s)$ for all states s, then any instance of a separation problem that can be solved by r_2 can also be solved by r_1, and thus it is sufficient to consider minimal regions only. Now minimal regions are in bijective correspondence with *abstract regions* which are those mappings $\Delta : E \to Z\!\!Z^m$ for which some $\sigma : S \to I\!N^n$ exists making the pair $r = (\sigma, \Delta)$ a ρ-region. Actually since $s \xrightarrow{e} s \Rightarrow \sigma(s') = \sigma(s) + \Delta(e) \cdot \rho(s)$ and since transition system T is assumed to be connected, the mapping σ is determined by Δ up to an additive constant, and the minimal region associated with r is $r_{min} = (\sigma_{min}, \Delta)$ where $\sigma_{min}(s) = \sigma(s) - k$ where $k = \mathbf{min}\{\sigma(s) |\ s \in S\}$.

In the sequel, we show that abstract ρ-regions form a module, a base of which can be computed in polynomial time and we describe polynomial time solutions for the above two separation problems.

3.3.1 The Module of ρ-Regions

Let $T \subseteq S \times E \times S$ be a deterministic and connected finite transition system. Let $\partial^0, \partial^1 : T \to S$ and $\ell : T \to E$ denote respectively the *source*, *target*, and *labelling* functions given by $\partial^0(t) = s$, $\partial^1(t) = s'$, and $\ell(t) = e$ for $t = s \xrightarrow{e} s' \in T$. Let $\partial_{\Delta,\rho}(t) = \Delta(\ell(t)) \cdot \rho(\partial^0(t))$, this expression measures the variation of σ along transition t: $\sigma(\partial^1(t)) = \sigma(\partial^0(t)) + \partial_{\Delta,\rho}(t)$. Therefore the algebraic sum of the values of $\partial_{\Delta,\rho}$ along any closed path of the transition system should vanished. In order to make that matter precise, we borrow some terminology from algebraic topology (see e.g. [20]). We define the *0-chains* $v \in C_0(T)$ and the *1-chains* $c \in C_1(T)$ of the transition system T as the vectors $v \in (S \to \mathbb{Z})$ and $c \in (T \to \mathbb{Z})$ respectively. $C_0(T)$ and $C_1(T)$ are free $\mathbb{Z}$-modules [2] (generated by S and T respectively) and we shall present their elements as formal sums: $v = \sum v_i \cdot s_i$ and $c = \sum c_j \cdot t_j$ where $v_i = v(s_i)$ and $c_j = c(t_j)$. The *boundary* operator $\partial : C_1(T) \to C_0(T)$ is given by $\partial(c) = \sum c_j \cdot (\partial^1(t_j) - \partial^0(t_j))$ for $c = \sum c_j \cdot t_j$. A *cycle* is a 1-chain with no boundary, and $Z(T) = \{c \in C_1(T) | \partial(c) = 0\}$ denotes the $\mathbb{Z}$-module of cycles of the transition system T. We recall that the *scalar product* of two vectors $\alpha = \sum \alpha_i \cdot x_i$ and $\beta = \sum \beta_i \cdot x_i$ of the free $\mathbb{Z}$-module $\mathbb{Z}{<}X{>}$ is the integer $\alpha \cdot \beta = \sum \alpha_i \cdot \beta_i$.

Observation 10 *$r = (\sigma, \Delta)$ is a ρ-region of transition system T if and only if: $\forall c \in C_1(T) \quad \sigma \cdot \partial(c) = \partial_{\Delta,\rho}(c)$ where $\partial_{\Delta,\rho} : \mathbb{Z} < T > \to \mathbb{Z}$ is the linear map given by $\partial_{\Delta,\rho}(t) = \Delta(\ell(t)) \cdot \rho(\partial^0(t))$.*

By linearity, the above condition is equivalent to: $\forall t \in T \; \sigma \cdot \partial(t) = \partial_{\Delta,\rho}(t)$ where transition t is identified with the chain reduced to that transition. Now this equation tells us $\sigma(\partial^1(t))$ - $\sigma(\partial^0(t)) = \partial_{\Delta,\rho}(t)$, which is exactly the condition that characterizes ρ-regions. ∎

Observation 11 *$\Delta : E \to \mathbb{Z}$ is an abstract ρ-region of T if and only if the value of $\partial_{\Delta,\rho}$ vanishes along every cycle: $\forall c \in Z(T) \quad \partial_{\Delta,\rho}(c) = 0$*

The condition is necessary in view of the earlier observation. Conversely, suppose this condition holds and let $s_0 \in S$ be an arbitrary state; we set $\Delta(s_0, s)$ $= \partial_{\Delta,\rho}(c)$ where c is an arbitrary chain from s_0 to s, i.e. a chain c such that $\partial c = s - s_0$ (this is well defined because $\partial_{\Delta,\rho}$ vanishes along every cycle). Let $\mathbf{min}(s_0, \Delta) = \mathbf{min}\{\Delta(s_0, s) | \; s \in S\}$, and $\sigma_{\Delta,s_0}(s) = \Delta(s_0, s)$ - $\mathbf{min}(s_0, \Delta)$, then $(\sigma_{\Delta,s_0}; \Delta)$ is a ρ-region. Moreover the ρ-regions associated with the abstract region Δ are all regions of the form $(\sigma_{k,\Delta,s_0}; \Delta)$ where $\sigma_{k,\Delta,s_0}(s) = \sigma_{\Delta,s_0}(s) + k$ for some $k \in \mathbb{N}$. Thus $(\sigma_{\Delta,s_0}; \Delta)$ is the minimal region associated with the abstract region Δ. In particular σ_{Δ,s_0} does not depend upon the choice of s_0 and will be from now on denoted σ_Δ; this fact follows also by noticing:

$$\sigma_\Delta(s) = \Delta(s_0, s) - \mathbf{min}\{\Delta(s_0, s') | \; s' \in S\} = \mathbf{max}\{\Delta(s', s) | s' \in S\}$$

[2]we shall denote $\mathbb{Z}{<}X{>}$ the free $\mathbb{Z}$-module generated by set X

where $\Delta(s,s') = \partial_{\Delta,\rho}(c)$ for any c such that $\partial c = s - s'$. ∎

Abstract ρ-regions of transition system T form a finite $\mathbb{Z}$-sub-module of $\mathbb{Z}<A>$ defined by the equations $\partial_{\Delta,\rho}(c_i) = 0$ with unknowns $x_{e,i} = \Delta(e)(i)$ where $\{c_1,\ldots,c_{\nu(T)}\}$ is a basis of cycles of T, more precisely

Observation 12 *Let $\{c_1,\ldots,c_{\nu(T)}\}$ be a basis of cycles of the transition system $T \subseteq S \times E \times S$. The mapping $\Delta : E \to \mathbb{Z}^m$ is an abstract ρ-region of T if and only if viewed as a vector $[x_{e,i} = \Delta(e)(i) |\ e \in E \ \wedge\ 1 \le i \le m]$ of $\mathbb{Z}<E \times m>$ it is a solution of the system of linear equations $\{\lambda_{j,e,i} \cdot x_{e,i} |\ 1 \le j \le \nu(T)\}$ with integral coefficients $\lambda_{j,e,i} = \sum_{t \in T}\{c_j(t) \cdot \rho(\partial^0(t))(i) |\ \ell(t) = e\}$.*

The algorithm of von zur Gathen and Sieveking (see [24]) computes in polynomial time a basis of vectors $\{\Delta_1,\ldots,\Delta_t\}$ for the module of abstract regions viewed as a sub-module of $\mathbb{Z}<E \times m>$, given a basis of cycles for T. In view of the following proposition (see [14] and [20]) computing a basis of cycles for a finite and connected transition system amounts to constructing a spanning tree and can also be performed in polynomial time.

Proposition 13 *Let $T \subseteq S \times E \times S$ be a finite and connected transition system. Let $U \subseteq T$ be a spanning tree, then each transition $t \in T \setminus U$ determines a cycle c^t with arcs in $U \cup \{t\}$. The cycles c^t, for t ranging over $T \setminus U$, form a basis of cycles of transition system T, with dimension $\nu(T) = |T| - |U|$. The number $\nu(TS)$, called the* Betti number *of T, is an invariant of the transition system (i.e. it does not depend upon the chosen spanning tree).*

3.3.2 State Separation Regions

We recall that the minimal region $(\sigma_\Delta; \Delta)$ associated with the abstract region Δ is given by $\sigma_\Delta = \mathbf{max}\{\Delta(s',s) | s' \in S\}$ where c is any chain from s' to s i.e. such that $\partial c = s - s'$.

Observation 14 *The minimal region associated with abstract region $\Delta : E \to \mathbb{Z}^m$ distinguishes between states s and s' if and only if $\Delta(s',s) \neq 0$.*

Actually suppose $\Delta(s',s) > 0$ and let s'' such that $\sigma_\Delta(s') = \Delta(s'',s')$, then $\sigma_\Delta(s') < \Delta(s'',s') + \Delta(s',s) = \Delta(s'',s) \le \sigma_\Delta(s)$. ∎

Since $\Delta(s',s) = \partial_{\Delta,\rho}(c) = \sum_{t\in T} c(t)\cdot[\Delta(\ell(t))\cdot\rho(\partial^0(t))]$ for c such that $\partial c = s - s'$ is an expression linear in Δ we deduce

Corollary 15 *Two states s and s' of T are distinguished by some ρ-region if and only if $\Delta(s',s)\neq 0$ for some some abstract ρ-region $\Delta\in\{\Delta_1,\ldots\Delta_t\}$ in the basis of the module of abstract ρ-regions for T.*

Thus it is sufficient to compute the array with entries $\Delta_i(s',s)$ for $1\le i\le t$ and $s,s'\in S$ which takes polynomial time. Of course from a pratical point of view, we choose a particular state s_0 and compute the array $\Delta_i(s_0,s)$ indexed by $1\le i\le t$ and $s\in S$ since $\Delta_i(s',s)=\Delta_i(s_0,s)-\Delta_i(s_0,s')$.

3.3.3 Inhibitor Regions

Observation 16 *The minimal region associated with abstract region $\Delta : E\to \mathbb{Z}^m$ inhibits event e in state s if and only if:*

$$\forall s'\in S \quad \Delta(s',s)+\Delta(e)\cdot\rho(s)<0 \tag{6}$$

Let $\Delta=\sum_{1\le i\le t}\lambda_i\cdot\Delta_i$, then the system of inequations (6) is satisfied if and only if the vector $(\lambda_1,\cdots,\lambda_t)$ is an integral solution of the system of inequations

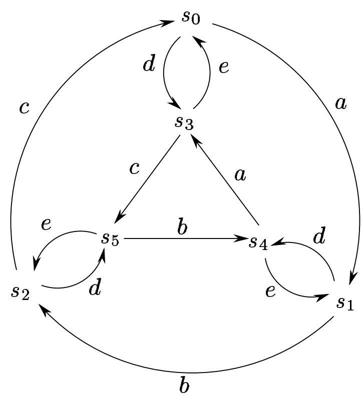
$$\{\sum_{i=1}^{t}\alpha_i^{s'}\cdot\lambda_i<0 |\; s'\in S\}$$

where $\alpha_i^{s'}=\Delta_i(s',s)+\Delta_i(e)\cdot\rho(s)$. Now a system of linear inequations

$$MX\le(-1)^n \tag{7}$$

where M is an integral matrix and $(-1)^n =< -1,\ldots,-1 > (\in\mathbb{Z}^n)$ has an integral solution iff it has a rational solution. The method of Khachiyan (see [24] p.170) may be used to decide upon the feasability of (7) and to compute a rational solution, if it exists, in polynomial time. Thus, every instance of the second separation axiom is solved up to a multiplicative factor, or shown unfeasible, in polynomial time.

3.3.4 An Example



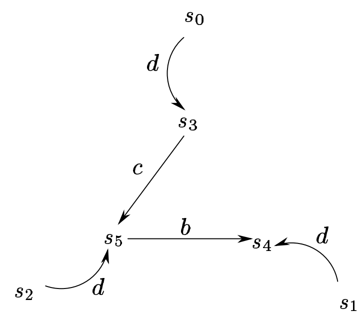


Figure 3: transition system T (on the left) and one of its spanning tree (on the right

Let us consider the transition system of Fig. (3) with the indicated spanning tree. Each transition t that does not belong to the spanning tree induces a cycle c_t of T:

t	c_t
$s_0 \xrightarrow{a} s_1$	$(s_0 \xrightarrow{a} s_1) + (s_1 \xrightarrow{d} s_4) - (s_5 \xrightarrow{b} s_4) - (s_3 \xrightarrow{c} s_3) - (s_0 \xrightarrow{d} s_3)$
$s_3 \xrightarrow{e} s_0$	$(s_3 \xrightarrow{e} s_0) + (s_0 \xrightarrow{d} s_3)$
$s_2 \xrightarrow{c} s_0$	$(s_3 \xrightarrow{e} s_0) + (s_0 \xrightarrow{d} s_3) + (s_3 \xrightarrow{c} s_5) - (s_2 \xrightarrow{d} s_5)$
$s_4 \xrightarrow{a} s_3$	$(s_4 \xrightarrow{a} s_3) + (s_3 \xrightarrow{c} s_5) + (s_5 \xrightarrow{b} s_4)$
$s_2 \xrightarrow{b} s_1$	$(s_2 \xrightarrow{b} s_1) + (s_1 \xrightarrow{d} s_4) - (s_5 \xrightarrow{b} s_4) - (s_2 \xrightarrow{d} s_5)$
$s_5 \xrightarrow{e} s_2$	$(s_5 \xrightarrow{e} s_2) + (s_2 \xrightarrow{d} s_5)$
$s_4 \xrightarrow{e} s_1$	$(s_4 \xrightarrow{e} s_1) + (s_1 \xrightarrow{d} s_4)$

Let $\overline{\rho}_0 : S \to I\!N$ be the partial state encoding which takes every state to 1. Abstract $\overline{\rho}_0$-regions of T are those mappings $\Delta : E \to Z\!\!Z$ which satisfy the equations

$$\partial_{\Delta,\overline{\rho}_1}(c_t) = \sum_{t' \in T} c_t(t') \cdot [\Delta(\ell(t')) \cdot \overline{\rho}_0(\partial^0(t'))] = 0$$

induced by the cycles c_t. For instance $c_{s_0 \xrightarrow{a} s_1}$ gives the equation

$$\Delta(a) + \Delta(d) - \Delta(b) - \Delta(c) - \Delta(d) = 0$$

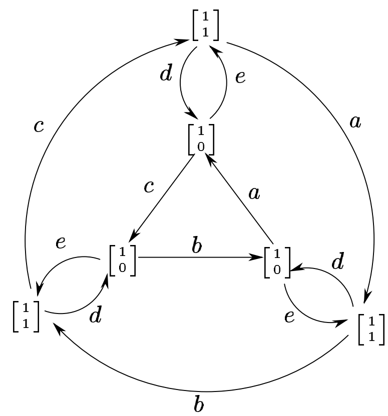


Figure 4: transition system T together with the partial state encoding $\overline{\rho}_1 : S \to I\!\!N^2$ where $\overline{\rho}_1(s)(1) = 1$ and $\overline{\rho}_1(s)(2) = \sigma_{\Delta_1}(s)$

Altogether these equations reduce to the system

$$x_a = x_b = x_c = 0 \quad \text{and} \quad x_d + x_e = 0$$

where $x_a = \Delta(a)$, $x_b = \Delta(b)$, $x_c = \Delta(c)$, $x_d = \Delta(d)$, and $x_e = \Delta(e)$. The module of abstract $\overline{\rho}_0$-regions has dimension 1 with base element

$$\Delta_1 = \begin{array}{c|c|} a & 0 \\ b & 0 \\ c & 0 \\ d & -1 \\ e & 1 \end{array}$$

the partial state encoding $\overline{\rho}_1 : S \to I\!\!N^2$ given by $\overline{\rho}_1(s)(1) = 1$ and $\overline{\rho}_1(s)(2) = \sigma_{\Delta_1}(s)$ is depicted in Fig. (4). Abstract $\overline{\rho}_1$-regions of T are those mappings

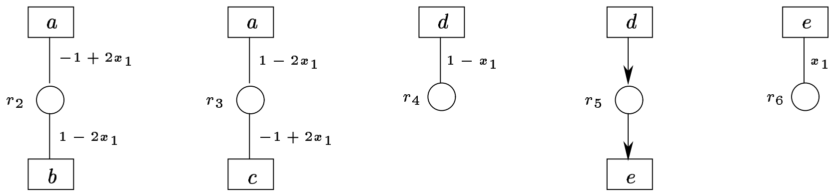


Figure 5: places r_i associated with the base of abstract $\overline{\rho}_1$-regions of T

$\Delta : E \to \mathbb{Z}^2$ which satisfy the following equations

t	$\partial_{\Delta_1,\overline{\rho}_1}(c_t) = 0$
$s_0 \xrightarrow{a} s_1$	$x_{a,1} + x_{a,2} + x_{d,1} + x_{d,2} - x_{b,1} - x_{c,1} - x_{d,1} - x_{d,2} = 0$
$s_3 \xrightarrow{e} s_0$	$x_{e,1} - x_{d,1} - x_{d,2} = 0$
$s_2 \xrightarrow{c} s_0$	$x_{c,1} + x_{c,2} + x_{d,1} + x_{d,2} + x_{c,1} - x_{d,1} - x_{d,2} = 0$
$s_4 \xrightarrow{a} s_3$	$x_{a,1} + x_{c,1} + x_{b,1} = 0$
$s_2 \xrightarrow{b} s_1$	$-x_{b,1} - x_{b,2} + x_{d,1} + x_{d,2} - x_{b,1} - x_{d,1} - x_{d,2} = 0$
$s_5 \xrightarrow{e} s_2$	$x_{e,1} - x_{d,1} - x_{d,2} = 0$
$s_4 \xrightarrow{e} s_1$	$x_{e,1} - x_{d,1} - x_{d,2} = 0$

where $x_{e,i} = \Delta(e)(i)$ for $e \in \{a, b, c, d, e\}$ and $i \in \{0, 1\}$. This system simplifies to

$$\begin{aligned} x_{a,2} &= -2x_{a,1} \\ x_{b,2} &= -2x_{b,1} \\ x_{c,2} &= -2x_{c,1} \\ x_{a,1} + x_{b,1} + x_{c,1} &= 0 \\ x_{d,1} + x_{d,2} + x_{e,1} &= 0 \end{aligned}$$

The module of abstract $\overline{\rho}_1$-regions of T has dimension 5 with base

$$\Delta_2 = \begin{array}{c|cc|} & 1 & 2 \\ \hline a & -1 & 2 \\ b & 1 & -2 \\ c & 0 & 0 \\ d & 0 & 0 \\ e & 0 & 0 \end{array} \quad \Delta_3 = \begin{array}{c|cc|} & 1 & 2 \\ \hline a & 1 & -2 \\ b & 0 & 0 \\ c & -1 & 2 \\ d & 0 & 0 \\ e & 0 & 0 \end{array} \quad \Delta_4 = \begin{array}{c|cc|} & 1 & 2 \\ \hline a & 0 & 0 \\ b & 0 & 0 \\ c & 0 & 0 \\ d & 1 & -1 \\ e & 0 & 0 \end{array} \quad \Delta_5 = \begin{array}{c|cc|} & 1 & 2 \\ \hline a & 0 & 0 \\ b & 0 & 0 \\ c & 0 & 0 \\ d & 1 & 0 \\ e & -1 & 0 \end{array} \quad \Delta_6 = \begin{array}{c|cc|} & 1 & 2 \\ \hline a & 0 & 0 \\ b & 0 & 0 \\ c & 0 & 0 \\ d & 0 & 0 \\ e & 0 & 1 \end{array}$$

The minimal regions r_i associated with abstract regions Δ_i for $2 \leq i \leq 6$ are depicted in Fig. (5). One may verify that the sequence $(x_1, x_2, x_3, x_4, x_5)$ where the x_i's are the minimal regions associated respectively with Δ_1, Δ_5, Δ_3, Δ_2

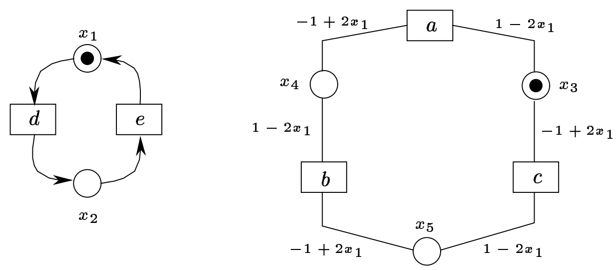


Figure 6: stratified Petri net associated with the solution $(x_1, x_2, x_3, x_4, x_5)$ where the x_i's are the minimal regions associated respectively with Δ_1, Δ_5, Δ_3, Δ_2 and $-\Delta_2 - \Delta_3$

Table 1: $\sigma_\Delta = \mathbf{max}_{s'} \Delta(s', s) = \mathbf{max}\{\Delta(s_0, s) - \Delta(s_0, s') | \ s' \in S\}$

$\sigma_\Delta(s)$	s_0	s_1	s_2	s_3	s_4	s_5
Δ_1	1	1	1	0	0	0
Δ_5	0	0	0	1	1	1
Δ_2	0	1	0	0	1	0
Δ_3	1	0	0	1	0	0
$-\Delta_2 - \Delta_3$	0	0	1	0	0	1

and $-\Delta_2 - \Delta_3$ is a solution of the synthesis problem for T, the resulting net is represented in Fig. (6) with the marking associated with state s_0. These regions do provide an injective state encoding as shown in Table (1). The reader may wish to verify that they also provide a solution for every instance of the event-state separation problem. For example an abstract $\overline{\rho}_1$-region $\Delta = \sum_{i=2}^{6} \lambda_i \cdot \Delta_i$ inhibites event b in state s_3 if and only if

$$\forall s \in S \quad \Delta(s, s_3) + \Delta(b) \cdot \overline{\rho}_1(s_3) = \Delta(s, s_3) + \lambda_2 < 0$$

which gives the following system of inequations

$$\begin{array}{lcl} s = s_0 & : & -\lambda_1 + \lambda_5 + \lambda_2 < 0 \\ s = s_1 & : & -\lambda_1 + \lambda_3 + \lambda_5 < 0 \\ s = s_2 & : & -\lambda_1 + \lambda_3 + \lambda_5 + \lambda_2 < 0 \\ s = s_3 & : & \lambda_2 < 0 \\ s = s_4 & : & \lambda_3 < 0 \\ s = s_5 & : & \lambda_3 + \lambda_2 < 0 \end{array}$$

a solution of which is $\lambda_1 = \lambda_4 = \lambda_5 = 0$ and $\lambda_2 = \lambda_3 = -1$, thus the inhibiting region $\Delta = -\Delta_2 - \Delta_3$.

4 Conclusion

Stratified nets evolve by self-modification and therefore can be used for modelling dynamical processes whose structure evolves along computations. This feature is central for describing cooperative works in which mutually interacting processes cooperate in a dynamical way. It is, in that respect, a much simpler and a more tractable (via linear algebra tools) model than the calculus of mobile processes [21]. However there remains to investigate whether this model is flexible enough for modelling the dynamic systems of agents encountered in CSCW.

Due to the compromise between generality (selfmodification) and discipline (absence of circular dependencies), stratified Petri nets have an enhanced expressing power (with respect to ordinary Petri nets), but at the same time they may still be verified using the classical techniques of linear algebra, plus the techniques of abstract interpretation (by projecting up to a certain layer). In further researches we will investigate these techniques of verification of properties of stratified Petri nets.

As mentioned in the introduction stratified nets as well as self-modifying nets may be defined for arbitrary type of nets (e.g. the class of stratified flip-flop nets has also a polynomial time synthesis). Therefore this concept of stratified nets has a great flexibility; however if we want to stay in the realms of linear algebra it seems reasonable to restrict ourselves to nets that can be interpreted as linear automata (i.e. automata with linear representations) with coefficients in some semi-ring. By doing so, most of the results on the structural properties of Petri nets which are based on techniques of linear algebra should be preserved. Gondran and Minoux [14, 15] argued that semi-rings are the appropriate algebraic structure for applying linear algebraic techniques for optimization and enumeration problems in graphs. Linear systems over idempotent semi-rings (dioids) have also been studied [1, 12]. It might be interesting to investigate which algebraic structure is required for the type of nets so that one can recast the algorithm of [2] and extend it to the class of stratified nets. The class of partially ordered commutative rings [11] is probably a good candidate. Another benefit of considering nets over a semi-ring is their close relationship to the linear sequential machines [16] and to the family of automata introduced by Schützenberger in [26] which have a rich mathematical theory [9]. In particular Schützenberger proved a Kleene like theorem relating finite automata of this family with rational series. Rational series have in turn been applied to the control theory of dynamical systems where they appear as the generating series of finite dimensional bilinear (differential) systems [17]. Stratified Petri nets might be used for the description of discrete (or discretized) dynamical systems.

To sum up, we advocate stratified nets as a flexible mechanism for defining extensions of Petri nets with an enhanced expressivity and which are still amenable to automatic verification by linear algebra techniques. In this paper we have introduced stratified Petri nets and proved that their synthesis problem has a polynomial solution. However this paper does not provide evidences for the above claim which should be sustained by further research.

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