# Control of Systems with Incomplete Information and Finite Memory Controllers

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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

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# Control of systems with incomplete information and finite memory controllers

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**Abstract:** We study the control of conservative discrete time dynamical systems when there are incomplete information and finite memory controllers. We state results for the asymptotic stability in the case of finite dimension and in the case of infinite dimension, we give an example where it is impossible to get asymptotic stability.

**Key-words:** Asymptotic stability, conservative systems, control without regret, incomplete information, finite memory controllers, discrete time systems.

*(Résumé : tsvp)*

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# Contrôle de systèmes avec information incomplète et contrôleurs de mémoire finie

**Résumé :** On considère ici le contrôle d'un système dynamique sans amortissement, à temps discrets et avec des observations incomplètes, où les contrôleurs ont mémoire finie. On établit quelques résultats sur la stabilité asymptotique dans le cas de dimension finie. Dans le cas de dimension infinie, on présente un exemple où il n'est pas possible d'obtenir la stabilité asymptotique.

**Mots-clé :** Stabilité asymptotique, systèmes sans amortissement, contrôle sans regrets, observations incomplètes, contrôleurs avec mémoire finie, systèmes à temps discrets.mots-clé

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## 1 Introduction

Here we consider conservative systems (i.e. without natural damping), when there are only available partial information and finite memory controllers. The objective of this paper is to analyze the control of these systems, both in the case of finite or infinite dimension. We deal only with the case of discrete time systems.

Our final aim is to highlight the severe limitations associated with the control of non damped systems (for large dimension or infinte dimension systems). These limitations have been recognized in the literature (see e.g. [2]-[4]) and we want here to analyze its presence in the case of conservative discrete time systems (let us mention that some related results are presented in [3]).

## 2 Finite dimensional case

We will study here the special case where the system state is described by a point of $\mathbb{C}^N$. The evolution is given by the equation

$$\left| \begin{array}{l} y^{n+1} = Q\, y^n + v^n \qquad n = 1, 2, \dots, \\ \\ y^0 = x, \end{array} \right. \tag{1}$$

where $Q$ is a unitary $N \times N$ matrix, with the following form

$$Q = \begin{pmatrix} 0 & I_{(N-1)\times(N-1)} \\ 1 & 0 \end{pmatrix}. \tag{2}$$

For each discrete time we know the observation $\omega^n \in \mathbb{C}$,

$$\omega^n = \langle \theta, y^n \rangle, \quad \theta \in \mathbb{C}^N, \quad \|\theta\| = 1, \tag{3}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathbb{C}^N$, i.e. $\langle v, w \rangle = \sum_{j=1}^{N} \overline{v_j} \cdot w_j$. We will also use the notation $v^* = \bar{v}^t$, and so $\langle v, w \rangle = v^* w$.

The control available at each time is any vector $v \in \mathbb{C}^N$ (and so, the system is obviously controllable), but the controller has finite memory, i.e. the controller is unable to remember the past observations, and in consequence, we can only apply a feedback control which must be computed using the present observation.

**Remark 1** *To simplify the analysis we have studied here the case of one step memory but a similar analysis can be also carried out for a m-step memory.*

**Remark 2** *We have analyzed here the case $X = \mathbb{C}^N$ to simplify the presentation of the properties shown in section 2.2, but the case $X = \Re^N$ can be studied analogously with similar results.*

## 2.1 Control synthesis

In order to design a feedback control that stabilizes the system, and eventually, steers it asymptotically to zero, we use the *"Control without regret"* (see [5]) methodology.

Let

$$J(v, y) = \|Q\, y + v\|^2 \tag{4}$$

be the cost function, where $\|\cdot\|$ is the euclidean norm in $\mathbb{C}^N$.

If $\omega$ is an observation, the set of possible states that produce this observation is

$$G = \{y : \langle \theta, y \rangle = \omega\}.$$

Obviously, by virtue of (3), these elements have the following form

$$y = \omega\theta + u \tag{5}$$

for some $u \in \theta^{\perp}$, i.e. $\langle \theta, u \rangle = 0$.

First we look for a control which *doesn't make it worse than doing nothing*, that is,

$$J(v,y) \leq J(0,y) \quad \forall y \in G.$$

By (1) and (4), it must be verified that

$$\|Q\,y + v\|^2 - \|Q\,y\|^2 \leq 0\,.$$

From this inequaltity we get

$$2\,\mathrm{Re}\,\langle Qy, v\rangle + \|v\|^2 \leq 0\,,$$

and so, from (5) and trasposing $Q$, we get

$$2\,\mathrm{Re}\,\langle \omega\theta, Q'v\rangle + 2\,\mathrm{Re}\,\langle u, Q'v\rangle + \|v\|^2 \leq 0 \qquad \forall u \,/\, \langle \theta, u\rangle = 0. \tag{6}$$

In order that (6) holds, it must be $\langle u, Q'v\rangle = 0$ for any $u$ in the subspace $\theta^\perp$, condition that is verified iff $Q'v = \alpha\theta$ for some $\alpha \in \mathbb{C}$. Hence the set $V$ of non harmful controls is

$$V = \Big\{ v : v = \alpha Q\theta,\ \alpha \in \mathbb{C},\, 2\,\mathrm{Re}\,\langle \omega\theta, Q'v\rangle + \|v\|^2 \leq 0 \Big\}\,. \tag{7}$$

Now we find the maximum prejudice produced by these controls due to the lack of information about the real state of the system. That is, we evaluate

$$\hat{J}(v) = \max_{y \in G}\ J(v,y) - J(0,y)$$

or, taking into account (5) and (7), the equivalent expression

$$\hat{J}(v) = \max_{u \perp \theta} \Big( 2\,\mathrm{Re}\,\langle \omega\theta, Q'v\rangle + \|v\|^2 \Big)\,.$$

As this last expression is independent of $u$, we have

$$\hat{J}(v) = 2\,\mathrm{Re}\,\langle \omega\theta, \alpha\theta\rangle + |\alpha|^2\,\|\theta\|^2 = 2\,\mathrm{Re}(\bar{\omega}\alpha) + |\alpha|^2\,.$$

Finally we choose in $V$ the control $\hat{v}$ which minimizes the maximum prejudice, that is

$$\hat{J}(\hat{v}) = \min_{\alpha Q\theta \in V} \Big( 2\,\mathrm{Re}\,(\bar{\omega}\alpha) + |\alpha|^2 \Big)\,.$$

This minimum has the value $\hat{J}(\hat{v}) = -\,|\omega|^2$ and is attained when $\alpha = -\omega$, hence the minimum regret control is

$$\hat{v} = -Q\theta\omega = -Q\theta\,\langle \theta, y\rangle\,. \tag{8}$$

So, for the control at the $n$-th time step, we have

$$v^n = -Q\,\theta\,\langle \theta, y^n \rangle$$

and the evolution equation of the closed loop system is

$$y^{n+1} = Q(I - \theta\theta^*)y^n. \tag{9}$$

Defining the operator

$$T_c = Q(I - \theta\theta^*), \tag{10}$$

from (3) it is easy to check that $\|T_c\| \leq 1$.

Of course, since we are interested in the asymptotic stability of the system, we want the following relation to hold

$$\lim_{\nu \to \infty} \|T_c^{\nu}\, x\| = 0 \qquad \forall x \in \mathbb{C}^N. \tag{11}$$

## 2.2 Asymptotic stability

Clearly, if $\|T_c\| < 1$ then (11) is verified, but generally it is $\|T_c\| = 1$.

We will show here that (11) holds iff $\left\|T_c^N\right\| < 1$.

- Obviously $\left\|T_c^N\right\| < 1 \Rightarrow$ (11).
- In the other sense, if $\left\|T_c^N\right\| = 1$ there exists $y$ such that $\left\|T_c^N y\right\| = \|y\| = 1$. From here it easy to check that $\|T_c^p y\| = 1\ \forall p = 0, \ldots, N$.

  From (10) we get that $T_c^p y = Q^p y$. Then from (2) it is $T_c^N y = y$, and this relation forbids (11) to hold.

We will present below a necessary and sufficient condition for the validity of the relation $\left\|T_c^N\right\| < 1$.

Previously, we prove the following technical result:

**Theorem 1** $\left\|T_c^N\right\| < 1 \Leftrightarrow \left\{\theta, Q'\theta, \ldots, (Q')^{N-1}\theta\right\}$ *are linearly independent.*

**Proof.** $\Leftarrow$)Let us suppose $\left\|T_c^N\right\| = 1$, then it must exists a vector $y$ with norm 1 such that $\left\|T_c^N y\right\| = 1$. But this means that

$$\left\|T_c^N y\right\| = 1, \quad \left\|T_c^{N-1} y\right\| = 1, \ldots, \quad \|T_c y\| = 1$$

and by considering that $I - \theta\theta'$ is a *Projection Operator* we can affirm that

$$\langle \theta, y \rangle = 0, \langle \theta, Qy \rangle = 0, \ldots, \left\langle \theta, Q^{N-1} y \right\rangle = 0,$$

or trasposing $Q$

$$\langle \theta, y \rangle = 0, \langle Q'\theta, y \rangle = 0, \ldots, \left\langle (Q')^{N-1}\theta, y \right\rangle = 0.$$

If the vectors

$$\left\{\theta, Q'\theta, \ldots, (Q')^{N-1}\theta\right\} \tag{12}$$

were linearly independent, then we would have that $y = 0$, which contradicts the assumption $\|y\| = 1$. So, it is necessarily $\left\|T_c^N\right\| < 1$.

$\Rightarrow$)Let us suppose that $\left\{\theta, Q'\theta, \ldots, (Q')^{N-1}\theta\right\}$ are linearly dependent, then $\exists\, y \neq 0$ such that

$$\langle \theta, y \rangle = 0, \langle Q'\theta, y \rangle = 0, \ldots, \left\langle (Q')^{N-1}\theta, y \right\rangle = 0,$$

or, in an equivalent form

$$\langle \theta, y \rangle = 0, \langle \theta, Qy \rangle = 0, \ldots, \left\langle \theta, Q^{N-1} y \right\rangle = 0.$$

This relation implies that

$$T_c^p y = Q^p y \qquad \forall p = 0, 1, \ldots$$

and in consequence

$$\left\|T_c^N\right\| = 1.$$

This contradiction implies that $\left\{\theta, Q'\theta, \ldots, (Q')^{N-1}\theta\right\}$ are linearly independent.

■

We will give now a condition which assures the linear independence of the vectors in (12).

Let $\xi \in \mathbb{C}^N$, we define the vector $\eta \in \mathbb{C}^N$ such that for $i = 1, \ldots, N$,

$$\eta_i = \left\langle \theta, Q^{i-1}\xi \right\rangle . \tag{13}$$

Taking into account (2), we have

$$\eta_i = \sum_{j=1}^{N} \overline{\theta_j} \left(Q^{i-1}\xi\right)_j = \sum_{j=1}^{N} \overline{\theta_j}\, \xi_{j+i-1}$$

(where the sum $j + i - 1$ is understood as the mod$\,(n)$ addition). Then $\eta$ is the circular convolution of $\bar{\theta}$ with $\xi$ (see [6] pp. 33-33), i.e.

$$\eta = \bar{\theta} * \xi \,.$$

Now, we use the Discrete Fourier Transform (DFT). Let us remember that the definition of DFT in this case is

$$\hat{\theta}_i = \sum_{j=0}^{N-1} W^{\,j\,i} \theta_{j+1}, \quad i = 0, \ldots, N-1,$$

where $W = e^{\frac{2\,\pi}{N}\sqrt{-1}}$. By the properties of DFT, we have

$$\hat{\eta} = \overline{\hat{\theta}} \cdot \hat{\xi},$$

where the product is understood as the product component by component, i.e.

$$\hat{\eta}_i = \overline{\hat{\theta}}_i \cdot \hat{\xi}_i \;, \quad \text{for } \; i = 0, \ldots, N-1.$$

Now we can prove the following necessary and sufficient condition to get asymptotic stability.

**Theorem 2** *Let $\theta \in \mathbb{C}^N$ and $\hat{\theta}_i$ its Fourier components. Then the vectors*

$$\left\{\theta, Q'\theta, \ldots, (Q')^{N-1}\theta\right\}$$

*are linearly independent if and only if $\hat{\theta}_i \neq 0$, for $i = 0, \ldots, N-1$.*

**Proof.** $\Rightarrow$) Let us suppose that $\hat{\theta}_{i^*} = 0$, then defining

$$\xi = (\mathrm{DFT})^{-1}(0, \underbrace{\ldots, 0, 1, 0, \ldots}_{\text{at the } i^* \text{ position}}, 0)$$

we have that

$$\hat{\eta} = \overline{\hat{\theta}} \cdot \hat{\xi} = 0 \Rightarrow \eta = 0$$

relation that implies the linear dependence of the vectors $\left\{\theta, Q'\theta, \ldots, (Q')^{N-1}\theta\right\}$.

$\Leftarrow$) Let us suppose that the vectors $\left\{\theta, Q'\theta, \ldots, (Q')^{N-1}\theta\right\}$ are linearly dependent, then there exists $\xi \in \mathbb{C}^N$, $\xi \neq 0$, such that

$$\langle \theta, \xi \rangle = 0, \;\; \langle Q'\theta, \xi \rangle = 0, \;\; \ldots, \;\; \left\langle (Q')^{N-1}\theta, \xi \right\rangle = 0.$$

Therefore, using the definition (13), we have

$$\eta_i = 0 \;\; \text{for all} \;\; i = 1, \ldots, N.$$

In consequence, $\eta = 0$, and so, $\hat{\eta} = 0$. Hence $\overline{\hat{\theta}}_i \cdot \hat{\xi}_i = 0$, for $i = 0, \ldots, N-1$, but using the hypothesis $\hat{\theta}_i \neq 0$, for $i = 0, \ldots, N-1$, we have $\hat{\xi}_i = 0$, for $i = 0, \ldots, N-1$ and so $\hat{\xi} = 0$. This contradiction implies that the vectors $\left\{\theta, Q'\theta, \ldots, (Q')^{N-1}\theta\right\}$ are linearly independent.

■

From Theorem 1 and 2, we get the result we were looking for:

**Corollary 3** *The closed loop system (9) is asymptotically stable iff*

$$\hat{\theta}_i \neq 0 \quad \forall i = 0, \ldots, N-1.$$

**Remark 3** *Although we have proved that the operator $T_c$ is asymptotically stable, generally the rate of convergence can be very slow when $N$, the space dimension, increases. What is worse, even for a small $N$, the rate of convergence can be dismally slow if some $\hat{\theta}_i$ is close enough to 0.*

## 2.3 Examples

In the following pictures we will show some characteristics of the evolution of the closed loop system when the above presented design methodology is applied in two cases where $N = 10$.

In the first case we have chosen random values for $x$ and $\theta$. In this case all the Fourier coefficients of $\theta$ are far away from 0, see figure [1].

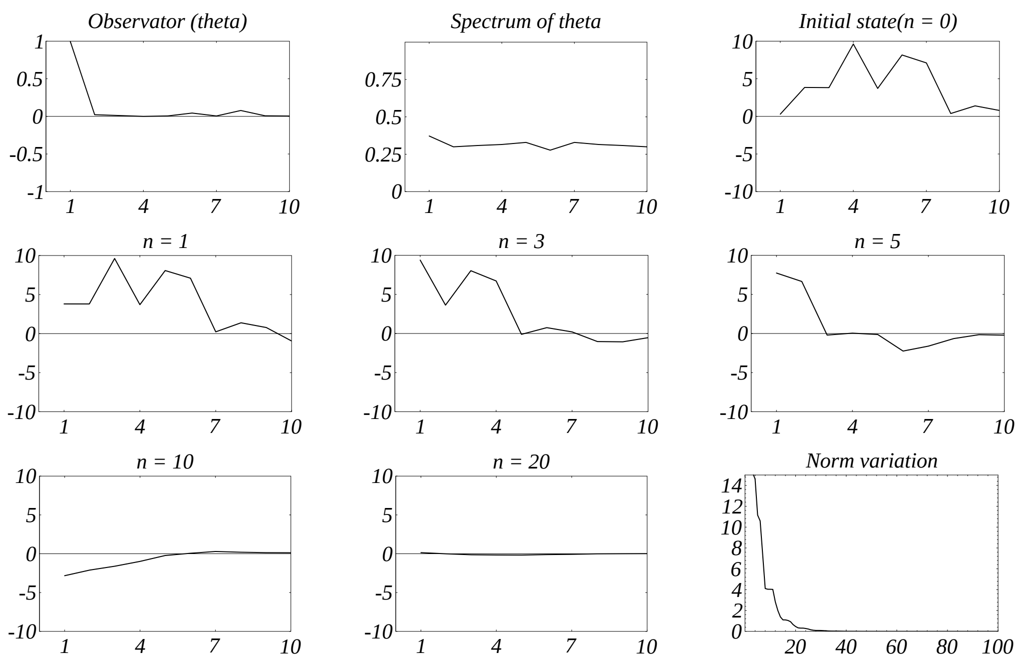


Figure 1: Evolution when all the coefficients are far away from 0.

In the second case we have chosen $\theta$ with one Fourier coefficient near to 0, see figure [2].

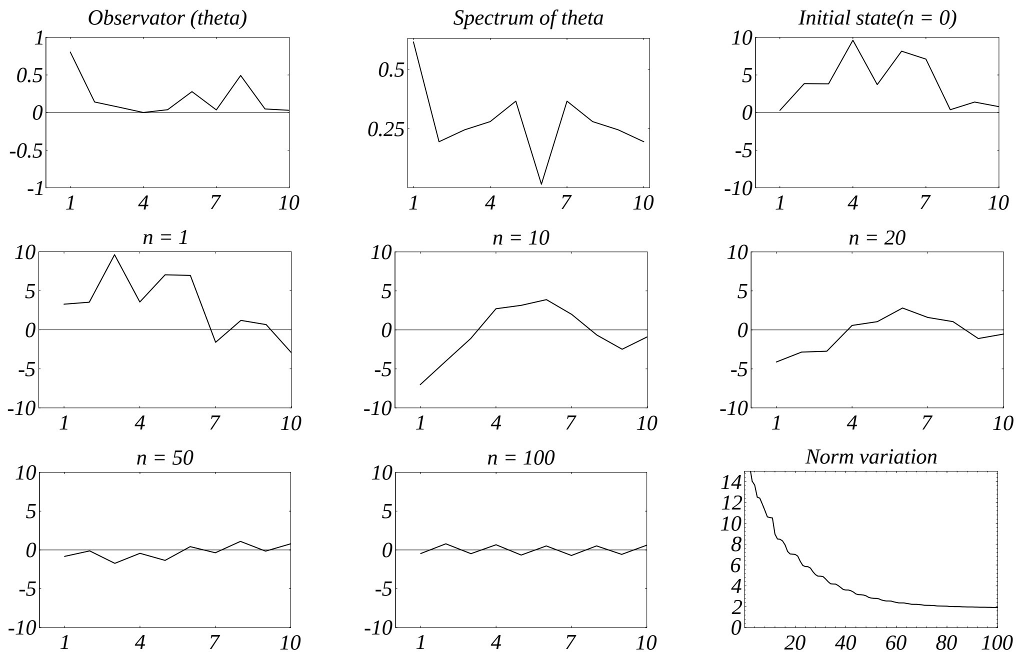


Figure 2: Evolution when there is a Fourier coefficient close to 0.

# 3 Infinite dimensional case

## 3.1 Description of a conservative system and its control

Let us consider now an infinite dimensional system where the states are elements of the set $X = L^2\left((0,1)\,;\mathbb{C}\right)$ and the discrete time evolution is given by

$$\left|\begin{array}{l} y^{n+1} = Q\,y^n + v^n \qquad n = 0,1,2,\ldots, \\ \\ y^0 = x, \end{array}\right. \tag{14}$$

where $v^n$ is an arbitrary element of $X$ and $Q$ is an operator verifying that $\|Qx\| = \|x\|$ for any $x \in X$.

The observation $\omega^n \in \mathbb{C}$ is given by $\omega^n = \langle \theta, y^n \rangle$ where $\theta \in X$, $\|\theta\| = 1$ and $\langle \cdot, \cdot \rangle$ denote the inner product in $X$.

For the elements of the set $X$, we will use the following representation (in this section we will use the notation $i = \sqrt{-1}$)

$$x(s) = \sum_{n=0}^{\infty} c_n e^{i2\pi ns},$$

with $c_n \in \mathbb{C}$. With this representation, we will use the operator $Q$ defined by

$$(Qx)(s) = \sum_{n=0}^{\infty} c_n e^{i2\pi(n+1)(s+h)} = \sum_{n=1}^{\infty} c_{n-1} e^{i2\pi n(s+h)},$$

where $h$ is an irrational number.

In this case, as in the previous one, we have:

- the optimal *"sans regret"* control is:

$$v^n = -Q\theta \langle \theta, y^n \rangle \tag{15}$$

- the closed loop evolution operator is $T_c = Q(I - \theta\theta^*)$,
- $\|T_c\| \leq 1$.

Now we give an example where, in spite of the fact that all the Fourier coefficients of $\theta$ are nonzero, the operator $T_c$ is not asymptotically stable.

In order to do that, we will show the existence of an initial state $x(s)$ such that $\|y^n\| = \|T_c^n x\| > \delta > 0$, for all $n$ natural.

### 3.2 Counter-example description

Let

- $0 < \gamma < 1$,
- the observer $\theta(s) = \sqrt{1-\gamma^2} \sum_{n=0}^{\infty} \gamma^n e^{i2\pi ns}$

the initial state $x(s) = e^{i2\pi ps}$ where $p$ is an integer greater than 1.
It is easy to check that $\|x\| = \|\theta\| = 1$.

From (15) and (14) we have

$$y^n = Qy^{n-1} - Q\theta \left\langle \theta, y^{n-1} \right\rangle ,$$

where $\langle \theta, x \rangle = \int_0^1 \bar{\theta}(s)x(s)\, ds = \sum_{n=0}^{\infty} \bar{\theta}_n x_n$ , and $\theta_n$ , $x_n$ are the respective Fourier coefficients.

If we define $e_n = e^{i2\pi ns}$, then $\{e_n : n \in \mathbb{N}\}$ is a base of $X$.

Let $y_j^n$ be the $j$-th component of $y^n$ in this base, then $y_j^n = \langle y^n, e_j \rangle$ .

We define now the functions $z^n$:

$$z^n = y^n - y_{n+p}^n e_{n+p} ,$$

where $p$ is the integer in the definition of the initial state $x(s)$

The evolution equation for the new state $z$ is the following one

$$\begin{aligned}
z^n &= Qy^{n-1} - \left\langle \theta, y^{n-1} \right\rangle Q\theta - y_{n+p}^n e_{n+p} \\
&= Q(z^{n-1} + y_{n+p-1}^{n-1} e_{n+p-1}) - \left\langle \theta, z^{n-1} + y_{n+p-1}^{n-1} e_{n+p-1} \right\rangle Q\theta - y_{n+p}^n e_{n+p} \\
&= T_c z^{n-1} + y_{n+p-1}^{n-1} e^{i2\pi(n+p)h} e_{n+p} - y_{n+p-1}^{n-1} \gamma^{n+p-1} Q\theta - y_{n+p}^n e_{n+p} \\
&= T_c z^{n-1} + \left( y_{n+p-1}^{n-1} e^{i2\pi(n+p)h} - y_{n+p}^n \right) e_{n+p} - y_{n+p-1}^{n-1} \gamma^{n+p-1} Q\theta, \qquad (16)
\end{aligned}$$

where we have used the property $Qe_n = e^{i2\pi nh} e_{n+1}$.

In order to see expression (16) with more detail, let us write

$$y^n = Qy^{n-1} - Q\theta \left\langle \theta, y^{n-1} \right\rangle ,$$

then the $j$-th component of $y^n$ can be calculated as

$$\langle y^n, e_j \rangle = \left\langle Qy^{n-1}, e_j \right\rangle - \langle Q\theta, e_j \rangle \left\langle \theta, y^{n-1} \right\rangle .$$

Using the operator $Q^*$, from the property $Q^* e_j = e_{j-1} e^{-i2\pi nh}$, we have

$$y_j^n = y_{j-1}^{n-1} e^{i2\pi jh} - \gamma^{j-1} e^{i2\pi jh} \left\langle \theta, y^{n-1} \right\rangle . \tag{17}$$

From (17) and letting $j = n+p$ we have

$$\begin{aligned} z^n &= T_c z^{n-1} + y_{n+p-1}^{n-1} e^{i2\pi(n+p)h} e_{n+p} - y_{n+p-1}^{n-1} \gamma^{n+p-1} Q\theta \\ &\quad - \left( y_{n+p-1}^{n-1} e^{i2\pi(n+p)h} - \gamma^{n+p-1} e^{i2\pi(n+p)h} \left\langle \theta, y^{n-1} \right\rangle \right) e_{n+p} \\ &= T_c z^{n-1} + \gamma^{n+p-1} e^{i2\pi(n+p)h} \left\langle \theta, y^{n-1} \right\rangle e_{n+p} - y_{n+p-1}^{n-1} \gamma^{n+p-1} Q\theta . \end{aligned}$$

By computing the norm of both members and using the triangular inequality we obtain

$$\|z^n\| \leq \left\| T_c z^{n-1} \right\| + \gamma^{n+p-1} + \gamma^{n+p-1} \left| y_{n+p-1}^{n-1} \right| .$$

Then

$$\|z^n\| \leq \left\| T_c z^{n-1} \right\| + 2\,\gamma^{n+p-1} .$$

By considering that $\|T_c\| \leq 1$ we have

$$\|z^n\| \leq \left\| z^{n-1} \right\| + 2\,\gamma^{n+p-1}$$

and from this recurrent relation it follows that

$$\|z^n\| \leq \sum_{j=1}^{n} 2\gamma^{p+j-1} = 2\gamma^p \frac{1-\gamma^n}{1-\gamma} .$$

Returning to $y^n$, we have

$$\|y^n\| \geq \left| \|z^n\| - \left\| y_{n+p}^n e_{n+p} \right\| \right| \geq \left| y_{n+p}^n \right| - \|z^n\| . \tag{18}$$

From the equation (17), it results that

$$\begin{aligned} y_{n+p}^n &= \left( y_{n+p-2}^{n-2} e^{i2\pi(n+p-1)h} - \gamma^{n+p-2} e^{i2\pi(n+p-1)h} \left\langle \theta, y^{n-2} \right\rangle \right) e^{i2\pi(n+p)h} \\ &\quad - \gamma^{n+p-1} e^{i2\pi(n+p)h} \left\langle \theta, y^{n-1} \right\rangle \\ &= y_p^0 e^{i2\pi(p+1)h} - \gamma^p e^{i2\pi(p+1)h} \left\langle \theta, y^0 \right\rangle - \cdots - \gamma^{n+p-1} e^{i2\pi(n+p)h} \left\langle \theta, y^{n-1} \right\rangle . \end{aligned}$$

Therefore

$$\left| y^n_{n+p} \right| \geq 1 - \gamma^p \sum_{k=0}^{n-1} \gamma^k \,. \tag{19}$$

Finally by substituting(19) in (18) we have

$$\|y^n\| \geq 1 - 3\gamma^p \frac{1-\gamma^n}{1-\gamma}$$

and so, choosing $\gamma$ small enough, we can affirm that $\|y^n\|$ is greater than a positive constant for all $n$.

# Conclusions

We have seen in the previous sections that

- The issues of:
  - partial information on the state of the system that must be stabilized
  - finite memory of the available controllers (and any realistic or implementable controller necesarily has a finite memory)

  produce heavy limitations on the control of systems with large of infinite dimension when there is no natural damping.

- The methodology of *"controls without regret"* enables us to design (under suitable conditions) controllers that stabilize the system and that never increase the energy of the system.

- There are conditions that assure the stabilization of the system (in some sense, these conditions mean that the system is controllable and observable).

- These conditions work in the case of finite dimension, but are not sufficient in the case of infinite dimension, as is shown by an exemple presented.

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