

## A Fiber-Based ACL Model for Geometrical and Mechanical simulations

Sandra Martelli, Ammar Joukhadar, Stefano Zaffagnini, Maurilio Marcacci, S. Lavallee, G. Champleboux

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S. Martelli, A. Joukhadar, S. Zaffagnini  
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———— THÈME 3 ————



*Rapport  
de recherche*





## A FIBER-BASED ACL MODEL FOR GEOMETRICAL AND MECHANICAL SIMULATIONS

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M. Marcacci, S. Lavallee‡, G. Champleboux.

Thème 3 — Interaction homme-machine,  
images, données, connaissances  
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**Abstract:** ACL (Anterior Cruciate Ligament) has a fundamental role in knee biomechanics and ACL insufficiency is probably the most common knee pathology. Although widely investigated, ACL mathematical models are still controversial in simulations of knee kinematics. We have developed a new ACL model, made of 9 curvilinear independent fibers, corresponding to the individual surface fibers, which are described as a succession of 20 punctual physical particles linked by linear viscoelastic relations, and they can twist, bend and stretch under applied forces. The model has been validated on 4 fresh pig knees and has given very consistent results to describe ACL anatomy, function and mechanics. Simulations of passive kinematics give coherent results on fibers elongations, forces and ligament deformations, confirming the expected correlation between fibers' strain and stress. The advantage and the feature of this method with respect to previous models is the possibility to take into account more accurately ACL anatomy, mechanical properties and the ligament behaviour during range of motion. All input geometrical data, can be acquired in fixed position, respect sizes and shapes of the ligament, envisaging a possible application on human ACL to develop surgical planning and simulation.

**Key-words:** Physics, dynamics, ligament

*(Résumé : tsvp)*

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## **Simulation du comportement mécanique et géométrique de l'ACL basée sur une représentation par fibres**

**Résumé :** Nous présentons un nouveau modèle de l'ACL (Anterior Cruciate Ligament) dans lequel on représente le ACL par 9 fibres indépendants. Chaque fibre est constitué de 20 masses ponctuelles liées entre elles par des connecteurs de force visco-élastiques. Cette représentation permet de simuler le comportement de l'ACL en traction et en flexion. Le modèle a été testé sur 4 genoux frais de couchons, et il a donné des résultat consistants avec l'anatomie de l'ACL, son fonctionnement et son mécanique. La simulation du mouvement passive de l'ACL a donné des résultats cohérents concernant l'élongation des fibres, les forces, et les déformation de ligament.

**Mots-clé :** Physique, dynamique, ligament

## Contents

1	INTRODUCTION	6
2	Outline of the <i>RobotΦ</i> system	6
3	MATERIAL AND METHODS	8
4	RESULTS	15
5	DISCUSSION	27
6	CONCLUSIONS	29

## List of Figures

1	<i>LS, TS</i> and <i>JS</i> connectors. . . . .	9
2	Passive Range of Motion executed by the senior surgeon and recorded by FARO arm . . . . .	9
3	Digitization of ACL fibers in complete flexion . . . . .	10
4	Fixed and mobile reference system for data and mathematical model . . . . .	11
5	Spline interpolation of acquired points on fiber n.4 of pig n.4 . . . . .	13
6	3D model of pig n.4 ACL (femoral insertion shown at top, posterior fibers on the left, anterior fibers on the right) . . . . .	14
7	Distances between insertions of anterior fibers of pig n.4 during PROM . . . . .	15
8	Distances between insertions of posterior fibers of pig n.4 during PROM . . . . .	16
9	Distances between insertions of medial fiber and lateral fiber of pig n.4 during PROM . . . . .	17
10	Length of anterior fibers of pig n.4 during PROM computed by the model . . . . .	18
11	Length of posterior fibers of pig n.4 during PROM computed by the model . . . . .	19
12	Length of medial fiber and lateral fiber of pig n.4 during PROM computed by the model . . . . .	20
13	Forces exerted by anterior fibers of pig n.4 during PROM computed by the model . . . . .	21
14	Forces exerted by posterior fibers of pig n.4 during PROM computed by the model . . . . .	22
15	Forces exerted by medial fiber and lateral fiber of pig n.4 during PROM computed by the model . . . . .	23
16	Resulting force exerted by ACL on insertions during PROM computed by the model . . . . .	24
17	ACL cross-section areas of pig n.4 during PROM computed by the model (1 the most proximal, 3 the most distal) . . . . .	25
18	ACL volume of pig n.4 during PROM computed by the model . . . . .	26
19	ACL deformations during PROM, computed by the model for pig. 4 . . . . .	27

# 1 INTRODUCTION

The knowledge of ACL (Anterior Cruciate Ligament) anatomical features and mechanical behaviour is fundamental for optimization of ACL reconstruction, restoration of knee stability and prevention of ACL injuries. ACL biomechanics therefore has been extensively investigated and models of ACL have been proposed to verify consequences of functional hypothesis and to predict its behaviour under specific conditions. Nevertheless information about ACL are far away from a complete knowledge and further investigations are needed to integrate microscopic with macroscopic descriptions, and geometrical with mechanical features. In fact descriptions of the ligament as linear or non linear elastic lines between femoral and tibial insertions [9, 3] give controversial results [5], although popular and useful for first order approximations. More complex models, such as the viscoelastic one by Woo et al. [12] or FEM [2], have not been applied to simulate knee kinematics or are not stable enough and reliable for wide ranges of motion. We think that this lack of agreement on quantitative results and interpretation of ACL functional behaviour is due to a poor integration between ACL anatomy and mechanical properties. Present models therefore are suitable for global knee modeling or adapted to very strict conditions. The goal of this paper was to develop a model of ACL based on more accurate anatomical data and mechanical properties. The model has been used to describe ACL deformations, function and behaviour during passive knee motion, because kinematics is the basic physical condition to compare functional results.

In the following sections we will; first give an over view of the dynamic modeling system *RobotΦ* system used to modelize the ligament; describe the material and méthode used to get data on ligament; discuss the obtained results and conclude.

## 2 Outline of the *RobotΦ* system

The *RobotΦ* system [8, 7] basically allows the construction of complex physical scenes involving various interacting objects and robots (i.e. articulated quasi-rigid objects), and the “control” of these scenes using appropriate force-based operators. In practice, the user can: (1) construct the physical models of a set of objects (rigid, deformable or articulated objects), (2) control the motions of these objects by applying selected external forces, (3) simulate the dynamic behavior of the involved objects, and (4) define and record the motion parameters which are relevant for programming the Robotic application considered. The main features of our approach are the following:

- *RobotΦ* is a generalization of the spring/mass system. An object is represented by a network of interacting primitive connected by appropriate damper/spring connectors. A primitive is an object whose motion and deformations, between two time step and under the effect of a given force, can be given by an existing dynamic model (dynamics of point, dynamics of rigid object, Lagrangian formulation, etc..). At the moment, the available models are the point mechanics and the rigid body mechanics.



- The geometrical form of an object is represented by a set of convex polyhedra. One polyhedron can be associated to a part of a primitive (the case of a concave rigid object), to a primitive (the case of convex rigid object) or to more than one primitive (a deformable object represented by a set of point masses). These polyhedra help to get the same geometrical model for all primitives, to simplify the semi-automatic construction of the physical representation from the geometrical one and to simplify the calculation of the interaction between two primitives from the same object or two primitives from two different objects.
- Primitives have to be connected by means of a set of connectors in order to make a solid object. This is why, each primitive has to have the same physical representation. Each primitive is discretized into a set of punctual masses (particles), which respect locally and globally the inertial properties of the primitive. Connectors can relate two particles from two primitives but not two particles from the same primitive, ie, that when a deformable object, which is represented by a set of particles, each particle is considered as a primitive.
- Three types of connectors are considered in our model:
  - *linear connector LS* which connects a pair of particles. This connector specifies the distance to maintain between these two particles. The force which is associated to this linear constraint can be expressed by  $F = -\lambda\Delta d - \mu\dot{d}$ , where  $\lambda$  and  $\mu$  are constants,  $d$  is the distance between the two particles and  $\dot{d}$  is the relative speed of the two particles.
  - *torsion connector TS* which connects three particles. This connector specifies the angle to maintain between these three particles. The force which is associated to this angular constraint can be expressed by  $F = -\lambda\Delta\theta - \mu\dot{\theta}$ , where  $\theta$  is the angle formed by the three particles and  $\dot{\theta}$  is the angular speed. The force given by this connector depends only on the angle between particles and not to the distance between them. If we represent an angular constraints by a set of three linear connectors instead of an angular one, we get a wrong behavior, because we relate the flexion force with the traction one, and consequently the length of an object may depend to its curvature.
  - *The joint connector JS* which connect a set of four particles. This connector specifies the angle to maintain between three of these particles (like the case of the torsion connector), and the rotation axis which is defined using the fourth particle. This type of connector is useful for modeling revolute joints.

To simulate the plastic behavior of an object, we allow that the rest position of these connectors be variable. It changes in function to the internal tension of the connector.

- Three types of physical interactions are considered for simulating the dynamic behavior of objects: viscous-elastic collision, static & kinetic friction, and viscosity.

- The complexity of the algorithm which computes at each time step the position/ deformation/ interaction /contact of the object is  $O(n)$ , where  $n$  is the number of particles of the object model.
- The constructing of a physical model of an object can be done semi-automatically basing on its geometrical model. This construction respect the inertial properties of the real objet[7].
- The motion equation is solved by an adaptive approache based on the conservation of the mecanical energy. This approache allows us to estimate the incurred error, to avoid numeriasl divergence, and to reduce the execution time[6].

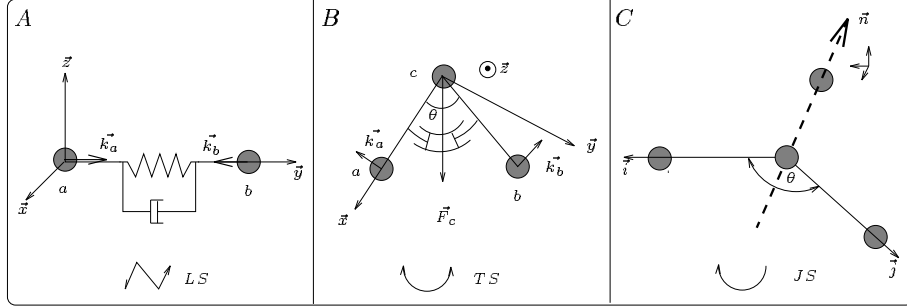


Figure 1: *LS*, *TS* and *JS* connectors.

### 3 MATERIAL AND METHODS

Four fresh pig knees have been used, cutting specimens approximately 15 cm above and 20 below the joint line. Muscles were removed while patella and capsula were kept intact. Femur was fixed with a rigid clamping device while tibia was left mobile and attached to a 6dof passive arm, used to record positions orientations and points when necessary. First of all a manual passive motion was recorded from complete flexion to full extension (set to 0).

Then tibia was rigidly fixed in the recorded complete flexion position, ACL was exposed, resecting patella, capsula, menisci and all other ligaments. In these conditions ACL surface was digitized using the passive arm previously detached from tibia. ACL is digitized from proximal to distal, following surface fibers. Surface fibers were digitized around the whole ligament starting from the medial fiber and taking care to distribute acquisitions around the structure (Fig.3).

Couples of femoral insertion points and tibial points of ACL surface fibers were also explicitly recorded and insertion boundaries were followed more carefully. Points were expressed in a fixed reference system solidal to femur but without any special anatomical feature, slightly different from case to case (Fig.4).

Anyway as points on the tibia are fixed with respect to the position sensor during passive trajectory, their coordinates can be computed at each recorded position according to the following formula:



Figure 2: Passive Range of Motion executed by the senior surgeon and recorded by FARO arm



Figure 3: Digitization of ACL fibers in complete flexion

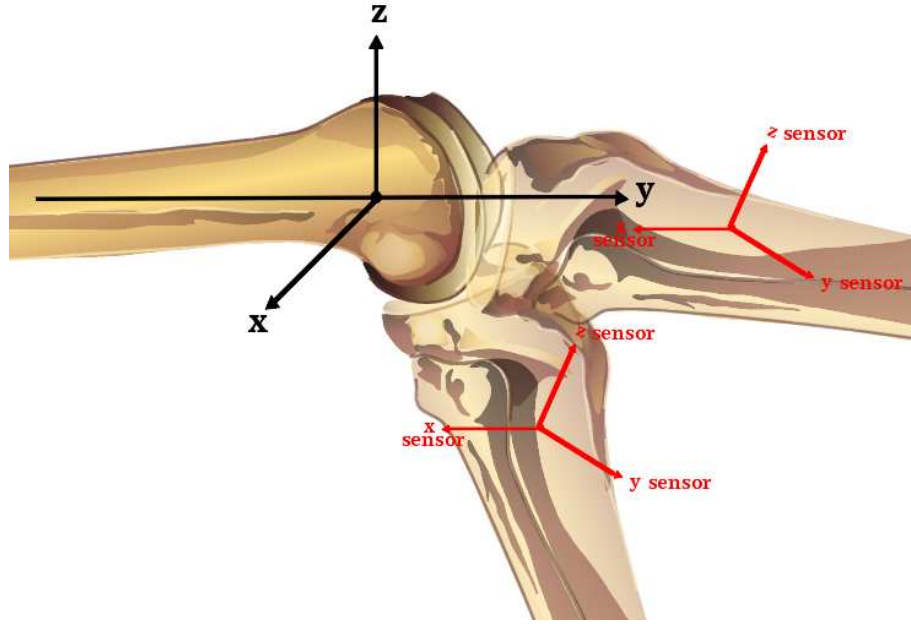


Figure 4: Fixed and mobile reference system for data and mathematical model

$T_0 = 4 \times 4$ : matrix expressing sensor's position and orientation at complete flexion.

$T_i = 4 \times 4$ : matrix expressing sensor's position and orientation during passive extension  $i = 1 \dots n$

$P =$  points on tibia in absolute coordinates.

$P_i = P$  at the  $i^{th}$  position of the passive trajectory

Then,

$$P_i = T_i T_0^{-1}(P)$$

This let us compute distances between fibers' insertion points along passive trajectory with the sensor's accuracy plus numerical one (0.3 mm + 0.2 mm).

To build an accurate anatomical model we use all points collected on ACL surface fibers. Points on each fiber were interpolated by a bicubic spline and 20 equidistant knots were recorded on this curve (Fig.5)

The computed  $20 \times 9$  points were used as initial knots for the physical model of the ACL. This is made of independent surface fibers made of knots linked by visco-elastic connectors which produce a force  $F$ :

$\vec{F} = -\lambda \Delta x - \mu \dot{x}$  where,  $\lambda$  is the elasticity factor which define the stiffness of the ligament and  $\mu$  the damping factor which represente the energy dissipation.

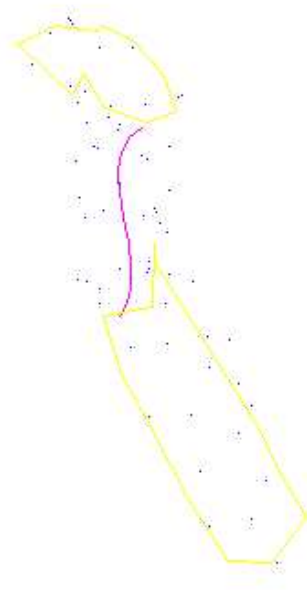


Figure 5: Spline interpolation of acquired points on fiber n.4 of pig n.4

The elastic constant  $\lambda$  was computed from global mechanical data reported in literature ([1] and [11]) considering the mesh approximately as a cylindric one with 9 parallel fibers, made of 20 serial elements so that  $k = 75 * 20/9$  when  $X$  is expressed in mm and  $F$  in Newton. Viscosity  $\mu$  is not estimated in litterature and has been determined empirically to achieve a geometrical reasonable behaviour. In all tests (=5000). In addition to these basic relations fibers were linked by an angular dumper, describing the small resistance to flexion of the ligament and whose functional equation was :

$\vec{F} = -\lambda\Delta\theta - \mu\dot{\theta}$  where  $\theta$  is the angle formed by three points. As ACL has a volumetric consistency but its horizontal mechanical properties have not been estimated, we assume its interfiber relations are neglectable and we just link them by a certain viscosity, simulating the ligament density ( = 2.0). The ligament's mass of 35 gr was uniformly distributed among knots. Femur was kept fixed giving it the whole leg's mass and tibial insertion is rigid as it is attached to the bone. The resulting ACL model was that of a deformable object with complex non linear elastic properties whose tibial insertion can be driven along the recorded trajectory to observe ACL deformation, fibers' elongations and forces on attachment sites (Fig.6).

Passive motion was achieved as a succession of equilibrium positions, using a control force giving tibial insertion a very small speed ( $< 0.1mm/s$ ), almost null at the recorded position ( $0.04mm/s$ ). The motion was given by the dynamic simulation system *RobotPhi* described and reported by Joukhadar in [8, 7]. This system is based on the spring/damper model and use an adaptive time step based[6]

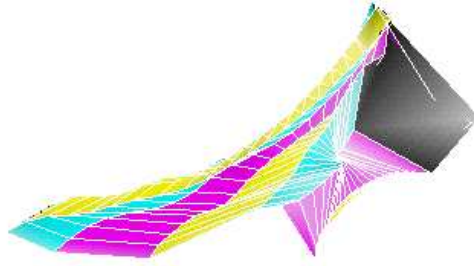


Figure 6: 3D model of pig n.4 ACL (femoral insertion shown at top, posterior fibers on the left, anterior fibers on the right)

in order to solve the equation of the motion/deformation. This adaptive approach allows us to estimate the incurred error, to avoid numerical divergence and to reduce the execution time. Manual passive motion and acquisition errors have been estimated, by repeatability tests and accuracy computation, as  $\pm 1$  mm. The solution of the physical model is thus computed with a smaller maximum position error.

## 4 RESULTS

The ACL model was used to compute the geometrical and the physical behaviour of the whole ligament during PROM. In fact even when displacement of ACL insertions is known, the global and inner deformation of the ligament and its fibers is still an open debate. To compare input data with previous results in literature the distance between femoral and tibial insertions was tracked for all fibers. Although individual differences exist, anterior fibers are taut in flexion and slack in extension, posterior fibers are slack in flexion and tight in extension, one medial and one lateral fibers are almost isometric.

Fibers during PROM are curvilinear and their shape changes from a straight line to twisted and bending curves, these distances don't always correspond either to fibers' length or to their resisted elongations. Maps of real fibers' (curvilinear) lengths are much more regular, and appear almost constant in model simulations:

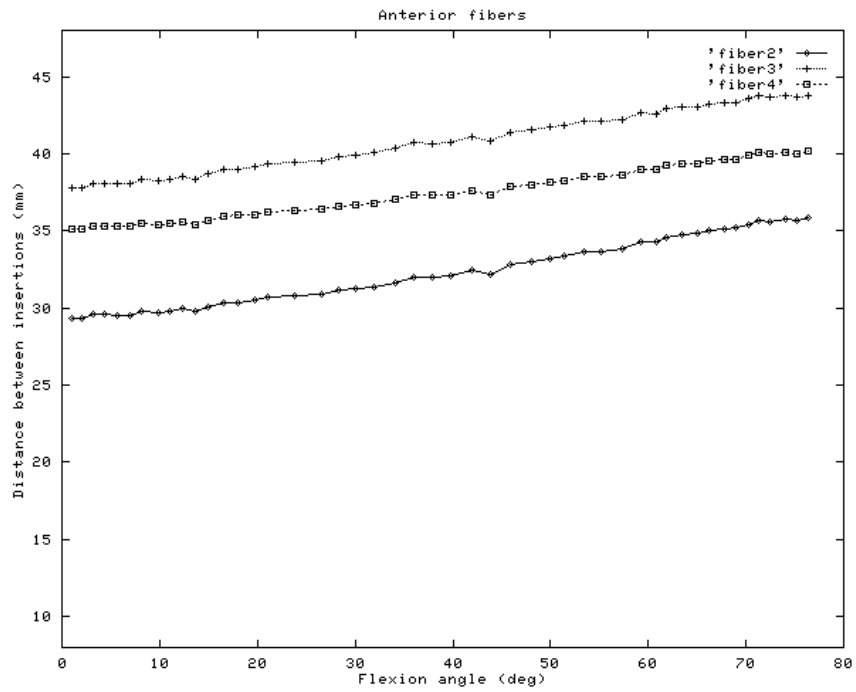


Figure 7: Distances between insertions of anterior fibers of pig n.4 during PROM

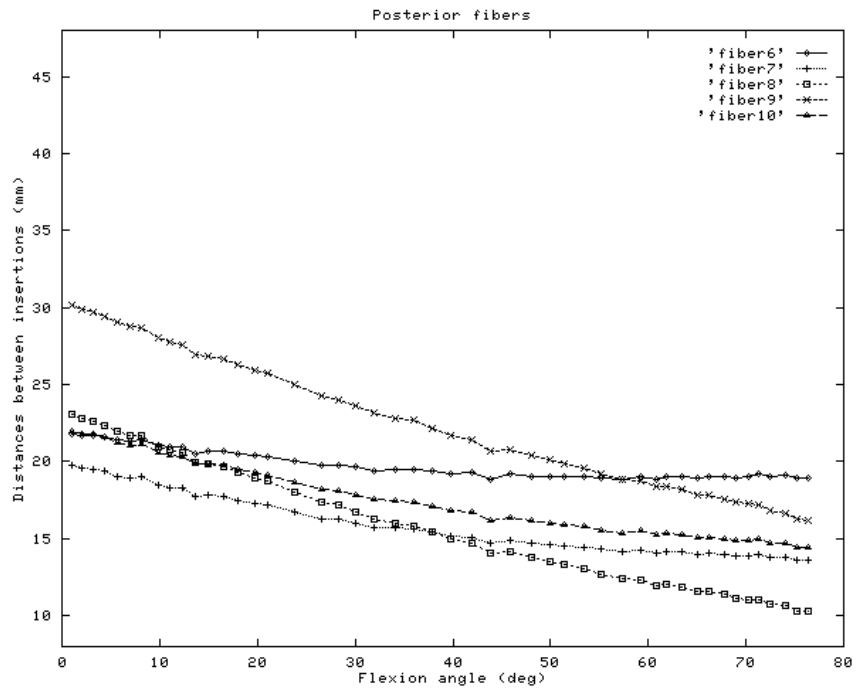


Figure 8: Distances between insertions of posterior fibers of pig n.4 during PROM

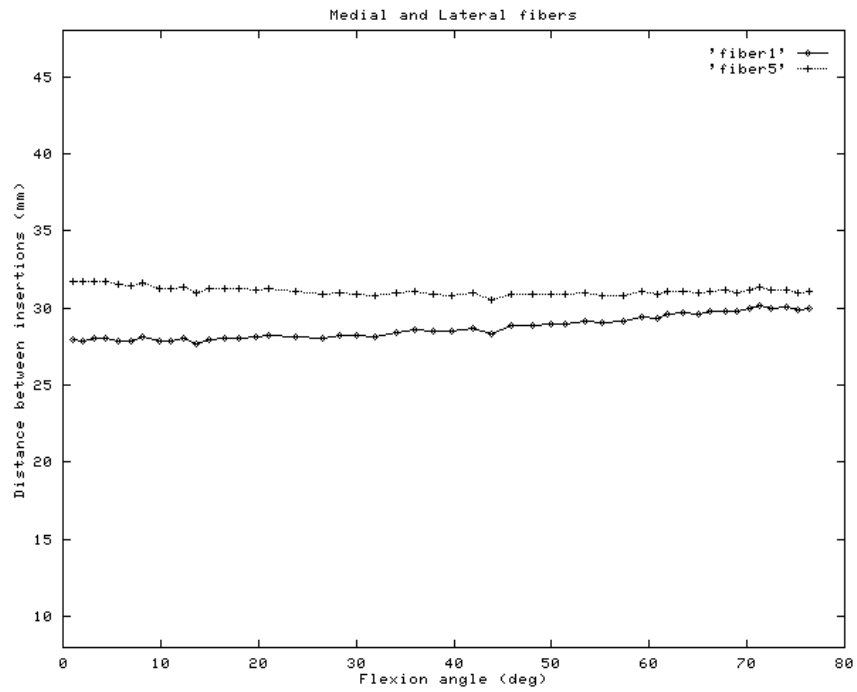


Figure 9: Distances between insertions of medial fiber and lateral fiber of pig n.4 during PROM

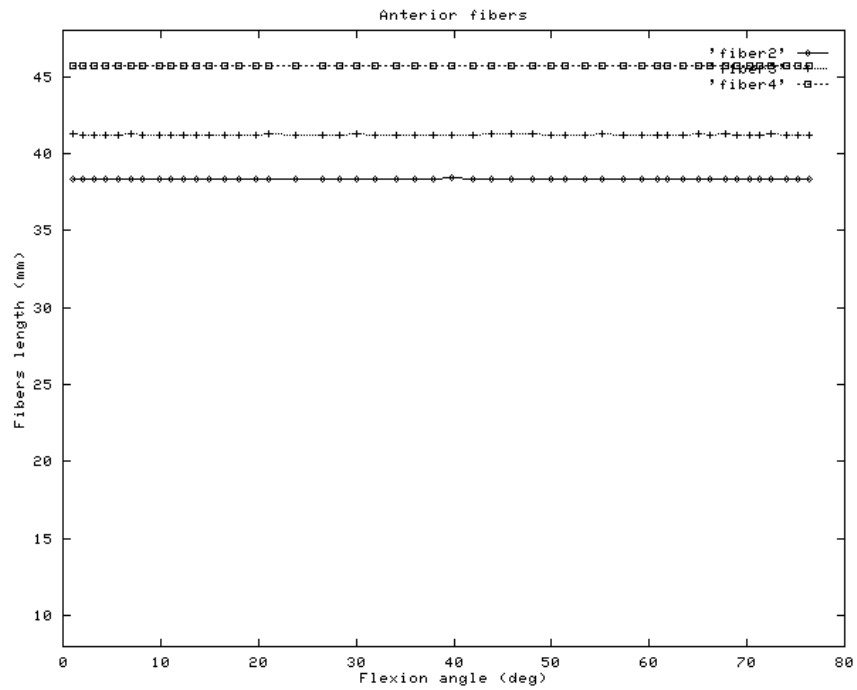


Figure 10: Length of anterior fibers of pig n.4 during PROM computed by the model



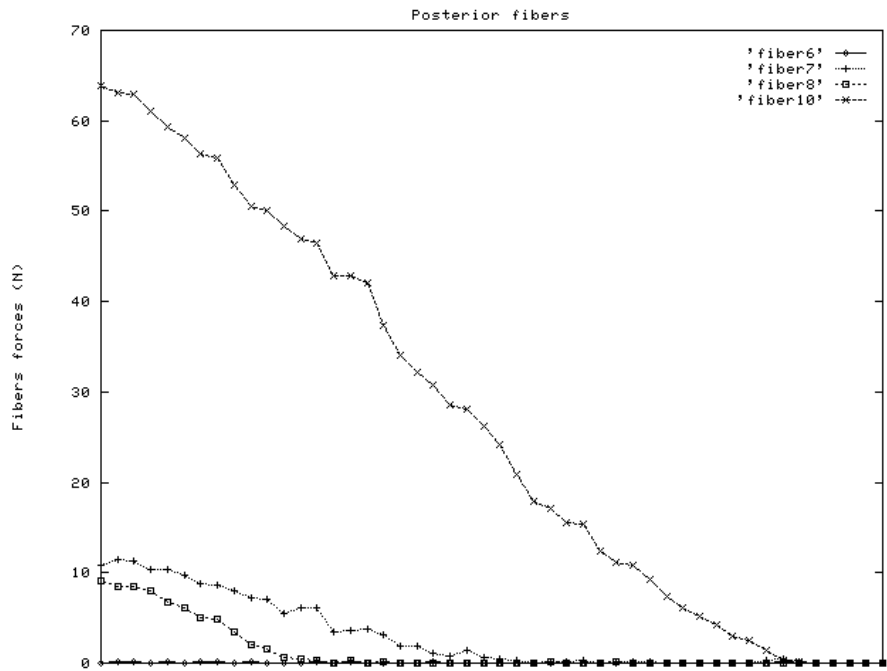


Figure 11: Length of posterior fibers of pig n.4 during PROM computed by the model

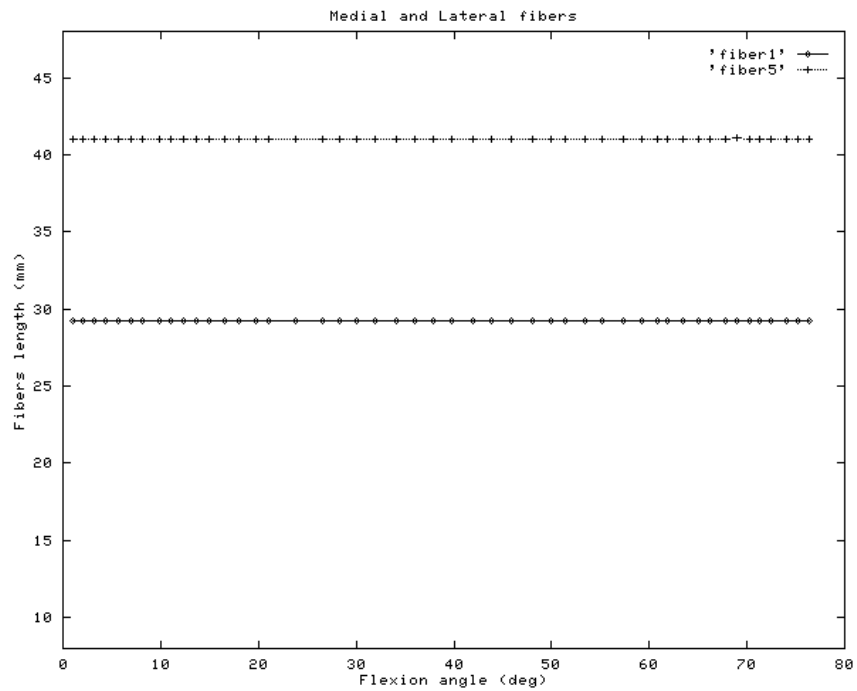


Figure 12: Length of medial fiber and lateral fiber of pig n.4 during PROM computed by the model

Forces exerted by single fibers on attachment sites are the result of fibers' tension, when a fiber is taut over its initial length. Maps of forces during manual PROM are shown in Figs.13 14 15:

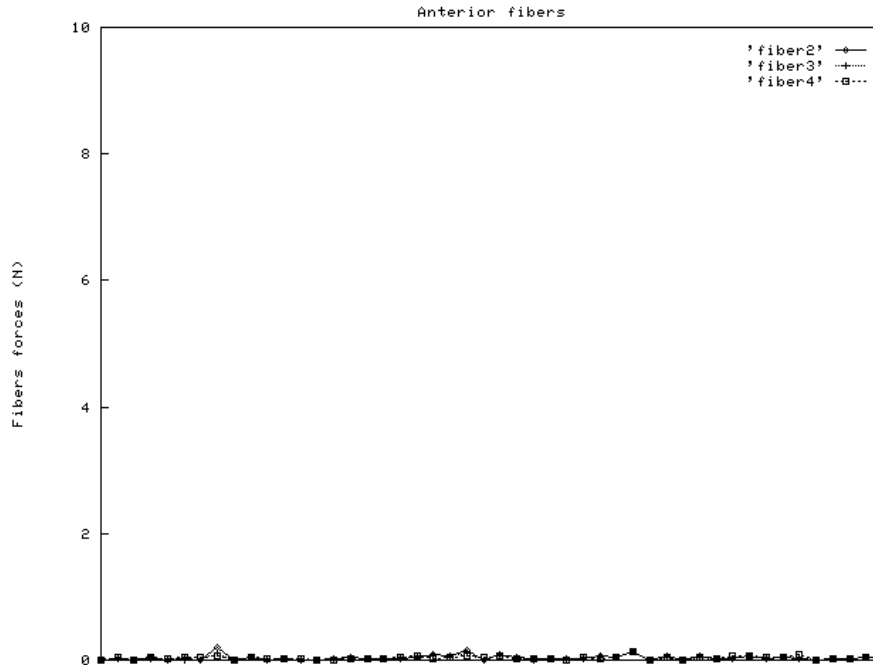


Figure 13: Forces exerted by anterior fibers of pig n.4 during PROM computed by the model

Globally the resulting force on ACL insertions (thought as applied to barycenter) increases during extension, as a result of progressive recruitment of posterior fibers, as shown in fig.14 for pig n.1 and pig n.4.

Therefore ACL action is the result of complex geometrical deformations of its 3D geometrical structure and appearance. The model tracks cross-section areas and 3d volume during PROM. Although a section approximately perpendicular to the ligament's axe has a different area according to its distance from insertions, it seems to stay almost equivalent during PROM, so that only isometric changes in boundaries occur.

Nevertheless fibers elongations and recruitment produce an effect in ACL "inside", and the 3D volume surrounded by corresponding knots changes significantly:

## 5 DISCUSSION

Maps of insertions' distances are in agreement with data on human, reported by several authors [4, 10, 5]. In fact, although a complete agreement in literature is not yet reached [5], most recent data confirm that anterior fibers are taut in flexion while posterior fibers in extension, Figs.7 8. Also the existance of isometric fibers like in Fig.8, has been previously claimed and measured by O'Connor [13], even if the exact position of this neutral fibers is not clear enough for comparison with

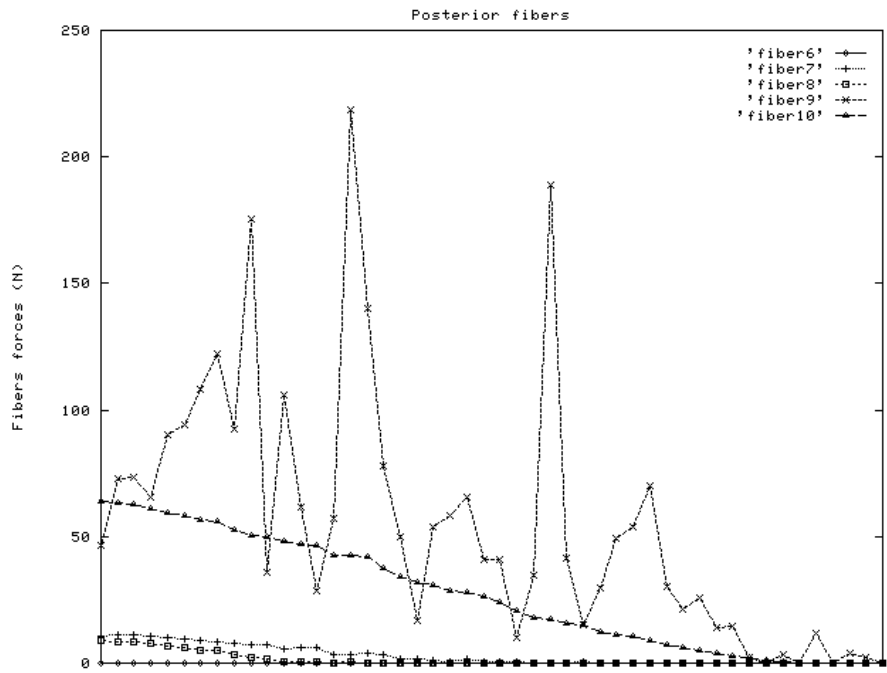


Figure 14: Forces exerted by posterior fibers of pig n.4 during PROM computed by the model

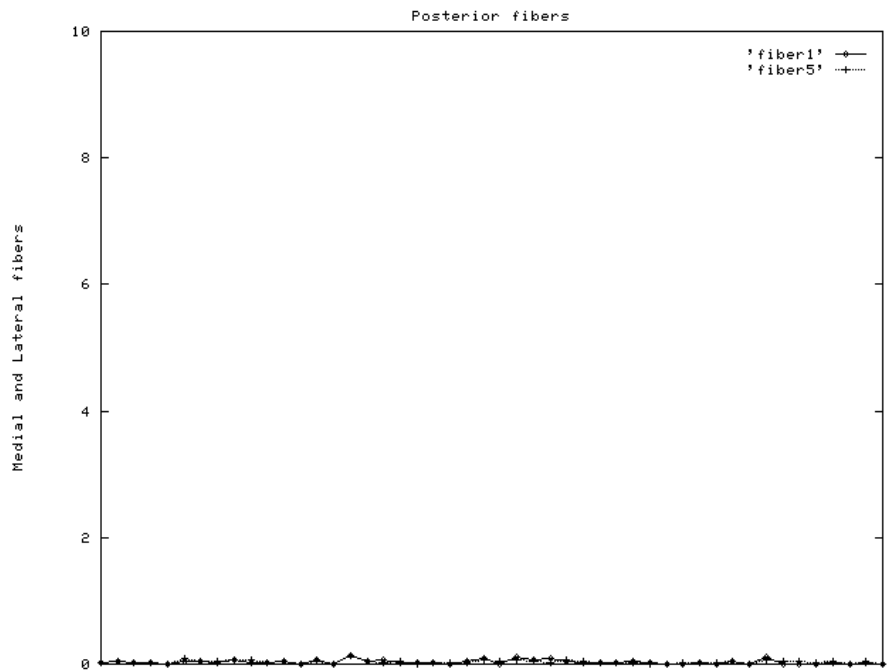


Figure 15: Forces exerted by medial fiber and lateral fiber of pig n.4 during PROM computed by the model

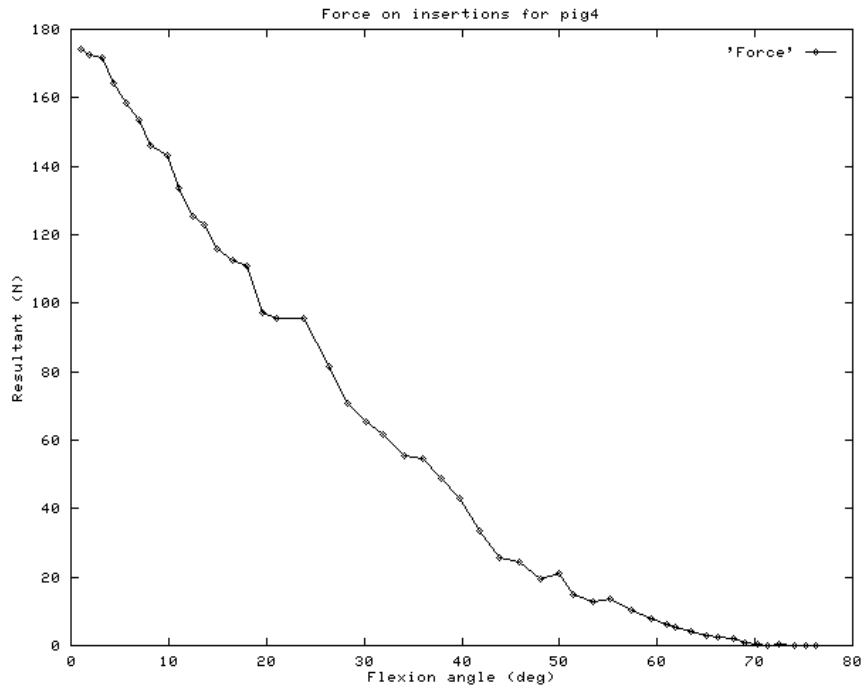


Figure 16: Resulting force exerted by ACL on insertions during PROM computed by the model

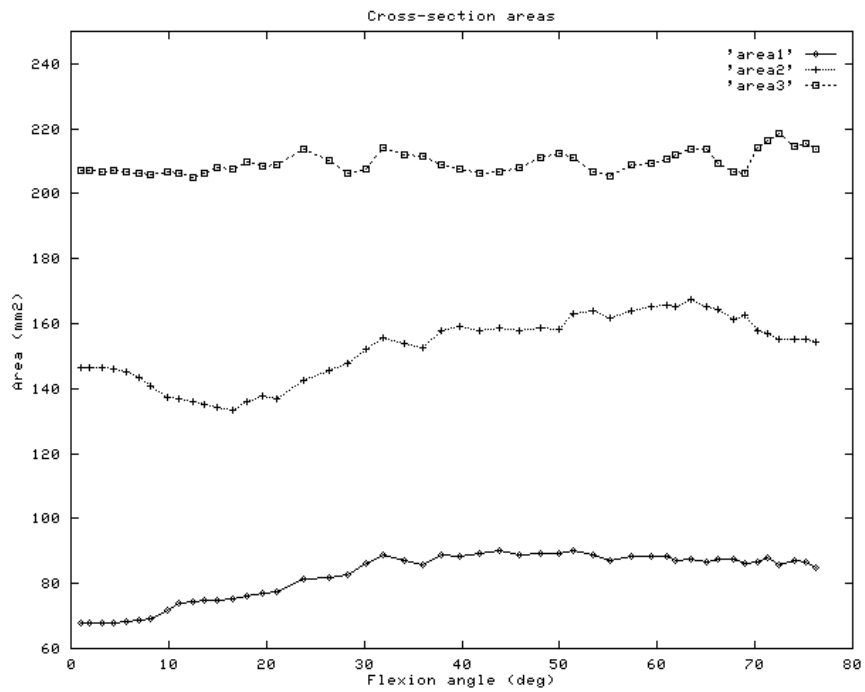


Figure 17: ACL cross-section areas of pig n.4 during PROM computed by the model (1 the most proximal, 3 the most distal)

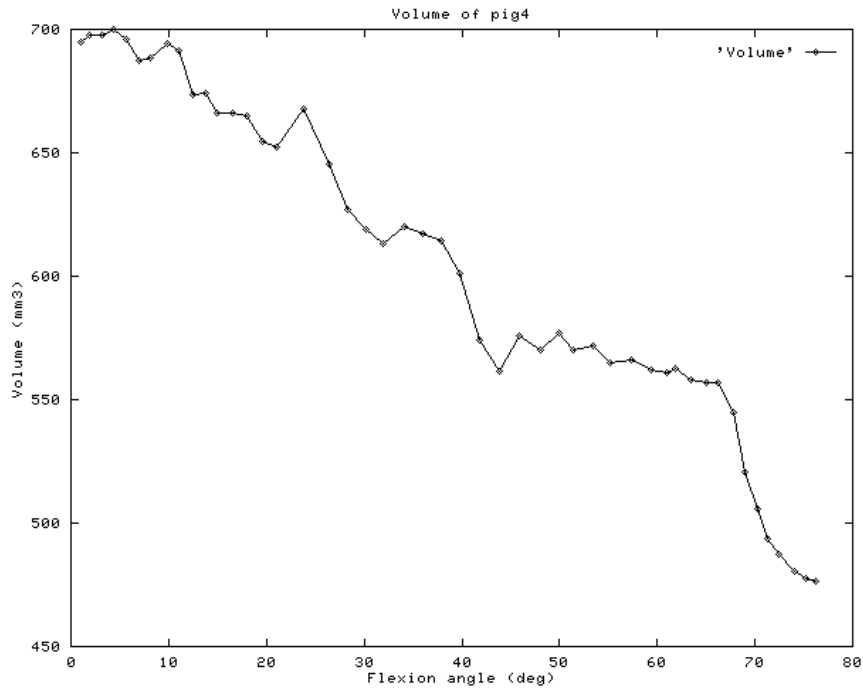


Figure 18: ACL volume of pig n.4 during PROM computed by the model

our data (on pigs). It can be noticed that maps of insertions distances show huge variations, both for anterior and posterior fibers (25). These inconsistencies are solved by our model, which considers a more accurate geometry of ACL fibers. Using the measured elastic constants, we obtain reasonable fibers elongations in the range [0]. Considering curvilinear fibers allows a more reliable estimation of fibers' current and initial length. Thus fiber's length in the model is generally bigger than the distance between insertions, because fibers bend and crimp when slack. They are equal when the curvilinear fiber of the model becomes tight and straight. From that point on, fiber's length coincides with the distance between insertions, until the fiber slackens again and keeps its constant initial length. As the initial position is complete flexion, anterior fibers appear exactly isometric during PROM, never stretching over flexion length (Fig.10). On the contrary, posterior fibers stay isometric until they extend and start to elongate as elastic lines (Fig.11) approaching extension.

As expected, tight fibers are active, that is they exert a force bigger than 0, so force plotting show non null values when they are straight and when an elongation happens. Although the model includes a certain resistance to compression, this is a sort of inertial factor, not affecting the global forces, which stays 0 when fibers are bent without elongation. Fibers seem to become active "near" the moment in which they stretch completely, that is when their length is equal to the insertions' distance. Anterior fibers appear almost useless during PROM, as they are charged very small forces. It is possible that they are the main constraint under stress conditions and not passive motion, but more probably this behaviour is due also to an overestimation of their initial length. Further tests on stress and dynamic activities are needed to verify how much the estimation of a good "rest" position

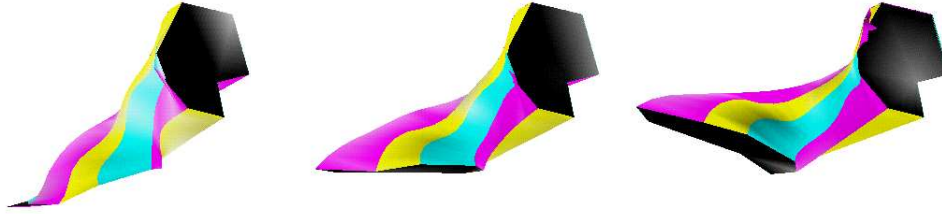


Figure 19: ACL deformations during PROM, computed by the model for fig. 4

is critical for the result. The choice of another initial position among those of passive trajectory or a small correction to rest length when visual inspection detects a too tight fiber, would avoid these cases of never active fibers. As an overview on fibers' forces and deformations, we can assess that fibers' behaviour is partially in agreement with Fuss hypothesis that fibers are inextensible lines [5]. However our model predicts a smooth and continuous transition from passive to active states, and gives non null resulting forces exerted by ACL during most PROM, instead of sudden and occasional states of fibers' activity only at maximum strain. This produces reasonable force plottings (Figs.13 14 15 16). Actually they cannot be compared with literature because of lack of references on pigs, but qualitative active fibers maps are in agreement with Mommersteeg [8], and confirm that 8-10 fibers are enough for a consistent and correct description of force transmission.

Geometrical data on shape and volume changes during PROM have been considered for analytic measurements only recently, for a more correct estimation of ACL interaction with other knee structures. Therefore a clear comparison with the literature is difficult also for cross sections and volume plotting especially on animals. Values seem realistic when compared with Woo's measurements, and vary from case to case, as expected in the clinical practice. The model predicts the isometry of cross-section areas (Fig.17) and the increase of volume (Fig.18). The first behaviour looks reasonable, but the latter can be only justified by the use of weak inner interactions of our surface fibers, so further experimental data would be interesting for comparison.

It is interesting to notice that the geometry of ACL, fibers and cross-sections vary on different individuals, but the mechanical parameters used by the model do not. This confirms studies on collagenous fibers and makes the model easily applicable to less invasive anatomical data acquisition, such as CT and MRI images. The model just needs the acquisition of initial fibers' curvature and shape in an anatomical position in PROM. We thus suggest the possibility of a future use on humans, for diagnosis and surgical simulation.

## 6 CONCLUSIONS

We have presented an anatomical model of ACL based on the description of surface fibers by physical particles linked by viscoelastic relations. The advantage of this method is that it is able to consider curvilinear fibers which are closer to anatomical fibers and describe more consistently fibers' behaviour also when they are slack or slightly tight. Fibers elongations and force maps given by the model are in agreement with literature and consistent with mechanical properties. Posterior fibers bend and twist in flexion, when they are not active, while they are progressively recruited during extension, becoming straight lines and then undergoing small strain (10%) as a result of the fibers' elongations, forces exerted by ACL on the tibia and femur increase in extension, because of the recruitment of posterior fibers. During PROM forces vary from 0 to 180N, and the 3D shape of ACL is submitted to huge changes, keeping cross-sections almost equivalent and increasing the global volume. This model is a first attempt to integrate microscopic hypothesis on ligament's biomechanics and macroscopic models. It is more accurate than previous models from the anatomical point of view, and provides a new effective correlation between fibers length and tension. Further work is probably necessary to improve the model accuracy and future tests will concern the physical relations set on the fibers : an initial non linear stress-strain curve can be substituted for the linear one currently used; stronger inner fibers could be added to improve the geometrical correspondance of the model; further tests under known stresses, such as drawer test, are needed to test the model flexibility and general applicability. The most challenging perspective for this ACL model is its application to surgical planning. Its mechanical accuracy makes it suitable for a future use in optimization of ACL replacement, if MRI or CT images let us identify surface fibers.

## References

- [1] Smith BA, Livesay GA, and Woo SLY. Biology and biomechanics of the anterior cruciate ligament. *Clinics in Sports Medicine*, 12(4):637–670, 1993.
- [2] MZ Benjaballah and a Shirazi-Adl. Biomechanics of the human knee joint in compression: reconstruction, mesh generation, and finite element analysis. *The knee*, 2(2):69–79, 1995.
- [3] L Blankevoort and R. huiskes. Ligament-bone interaction in a 3-dimensional model of the knee. *Biomechanics. Eng.*, 113:263–269, 1991.
- [4] L. Blankevoort, R Huiskes, and A de Lange. Recruitment of the knee joint ligaments. *Trans of ASME*, 113, 1991.
- [5] FK Fuss. Anatomy of the cruciate ligaments and their function in extension and flexion of the human knee joint. *Anat.*, (184):165–176, 1989.
- [6] A. Joukhadar and C. Laugier. Adaptive time step for fast converging dynamic simulation system. In *IEEE/International Conference on Intelligent Robots and Systems "IROS"*, Osaka, Japon, November 1996.

- [7] A. Joukhadar and C. Laugier. Dynamic simulation: Model, basic algorithms, and optimization. In *Workshop on the Algorithmic Foundations of Robotics*, Toulouse, France, July 1996.
- [8] Ammar Joukhadar. *robot $\phi$* : Dynamic modeling system for robotics applications. research report RR-2543, INRIA, May 1995.
- [9] DA Loch, Z Luo, J Lewis, and N Stewart. A theoretical model of the knee and acl: theory and experimental verification. *Biomechanics*, 25(1):81–90, 1992.
- [10] JK Koolos et al. TJ Mommersteeg. The fibre bundle anatomy of human cruciate ligaments. *Anat.*, (187):461–471, 1995.
- [11] J Wismans, F Velpaus, F Janssen, J Huson, and P Struben. A three-dimensional mathematical model of the knee joint. *Biomechanics*, 13:677–685, 1980.
- [12] SL Woo, GA Johnson, and BA Smith. Mathematical modeling of ligaments and tendons. *Trans. of ASME*, pages 468–473, 1993.
- [13] AB Zavatsky and JJ O’Connor. A model of human ligaments in the sagittal plane. part i: response to passive flexion. *Instn Mec Engrs*, 206:125 – 134, 1992.





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