Flip-Flop Nets

Vincent Schmitt

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

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Vincent Schmitt *

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Abstract: The so-called synthesis problem for nets which consists in deciding whether a given automaton is isomorphic to the case graph of a net and then constructing the net has been solved for various type of nets ranging from elementary nets to Petri nets. Though P/T nets admits polynomial time synthesis algorithms, the synthesis problem for elementary nets is known to be NP-complete. Applying the principle of generalized regions inherited from the P/T nets representation to the boolean setting gives rise to flip-flop nets. These nets are a slight generalization of elementary nets and admits a polynomial time synthesis.

Key-words: Net Synthesis, Condition/Event Nets, Regions, Algorithms

(Résumé : tsvp)

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*Email: Vincent.Schmitt@irisa.fr

Unité de recherche INRIA Rennes
IRISA, Campus universitaire de Beaulieu, 35042 RENNES Cedex (France)
Téléphone : (33) 99 84 71 00 – Télécopie : (33) 99 84 71 71

Les réseaux à bascules

Résumé : Le problème de synthèse de réseaux qui consiste à décider pour un automate donné s'il correspond au graphe de marquage d'un réseau et dans l'affirmative de construire ce réseau est résolu pour diverses classes de réseau allant des réseaux élémentaires aux réseaux de Petri. Si la synthèse des réseaux de Petri admet une solution polynomiale, celle des réseaux élémentaires a été prouvée NP-complète. Les réseaux à bascules sont introduits en appliquant le principe des régions généralisées hérités des réseaux de Petri au cas booléen. Ces réseaux sont aussi une généralisation des réseaux élémentaires et admettent une synthèse polynomiale.

Mots-clé : Synthèse de réseaux, réseaux conditions/événements, régions, algorithmes

1 Introduction

Ehrenfeucht and Rozenberg introduced in [ER90] the concept of regions in simple graphs labeled on arcs. Their aim was to give a characterization of graphs that occur as case graphs of elementary nets. In their study, regions of a particular graph are sets of vertices which are either entered or exited, or left invariant by all arcs with identical label. A region may be seen as a property satisfied exactly by the vertices it contains. Conversely, a vertex may be represented unambiguously by the set of regions it belongs to, if there exists for every pair of distinct vertices a region separating the pair - i.e. containing exactly one of the two vertices. Now all arcs with the same label carry the same changes of properties and a label may be represented as the mutual difference between the respective sets of properties which are satisfied by the source and target vertices of an arbitrary arc with that label. This representation is unambiguous due to simpleness, and it supplies enough data for reconstructing the graph if for each label and each vertex originating no arc with that label, there exists a region containing all the origins of the arcs with that label but not containing the considered vertex - such a region is said to separate this vertex from the given label. The so-called regional axioms specify these two different requirements for separation between vertices and between vertices and labels of arcs. Given a simple graph labeled on arcs, may be derived from that graph an elementary net with one place per region and one transition per label, and with flow relations set according to the mutual differences between source and target vertices identified with sets of places, or markings. The case graph of the net assembled in this way is isomorphic to the given graph if and only if the given graph is an elementary transition system, which means essentially that it satisfies the two regional axioms.

It is worth noting that in case when the given graph is already the case graph of a net, the above construction produces a saturated version of that net, with extra places but unchanged behaviour. In [DR92], Desel and Reisig observed that one may optimize the construction by computing just enough regions in the given graph to ensure the validity of the regional axioms. Following this observation Bernardinello proved that the set of minimal regions (w.r.t. set inclusion) is adequate for that purpose [Ber93]. Nevertheless, the problem of deciding whether the separation axioms are valid in a given graph is NP-complete [BBD95b].

The correspondence between elementary transition systems and elementary net systems is an instance of the concrete dualities between transition systems and nets established in [BD95]. Those correspondences are defined for arbitrary types of nets by varying the concept of regions and adapting accordingly the statement of the two separation axioms. For instance, in the case of P/T nets, regions in a graph G may be defined as morphisms from G to the Cayley graph of $\mathbb{Z}$, representing $\mathbb{N}$-valued properties of vertices in G instead of boolean properties. Remarkably, this leads to polynomial algorithms for the synthesis of bounded P/T nets with marking graph isomorphic to a given graph [BBD95a].

The purpose of this paper is to fill in the gap between elementary nets and P/T-nets as regards the complexity of the synthesis problem. Namely, we search for a class of condition/event nets which contains the class of elementary nets but nevertheless gives way to polynomial synthesis algorithms. These nets, called in the sequel flip-flop nets, are the boolean counterpart to P/T nets : for these nets, regions in a graph G are essentialy morphisms from G to the Cayley graph of the boolean group $\mathbf{2}$. This leads, as we shall see, to polynomial synthesis algorithms because $\mathbf{2}$ is also a ring.

Since we focus on decision algorithms, we confine our attention to finite graphs and finite nets in the remaining of the paper, organized as follows. Section 2 introduces flip-flop nets and the terminology. Section 3 studies an adequate concept of regions for flip-flop nets and shows that checking validity of the separation axioms w.r.t these regions (in a finite graph) reduces to solving finite linear systems over the boolean ring. Altogether this provides a polynomial algorithm for the synthesis of flip-flop nets. A short conclusion completes the paper.

2 Flip-flop nets

In this section we first introduce flip-flop nets, then we recall the basis of P/T nets and we make a parallel between the two families of nets, showing that flip-flop nets are the boolean counterpart of P/T nets.

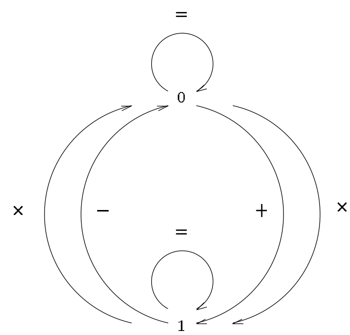

Figure 1: the transition system $\mathcal{T}_\mathbf{2}$

Definition 1 (Flip-flop nets and their markings) *A flip-flop net is a triple $N = (P, E, F)$, where P and E are disjoint sets of places and events respectively, and F is a map from $E \times P$ to the set $E' = \{=, +, -, \times\}$, called the flow map. A marking of N is a map from P to $\mathbf{2} = \{0, 1\}$. Let $\mathcal{M}(N)$ denote the set of markings of N.*

The behaviour of flip-flop nets is determined by the so-called firing rule.

Definition 2 (Firing rule for flip-flop nets) *Let $N = (P, E, F)$ and $M \in \mathcal{M}(N)$. An event $e \in E$ may be fired at M, resulting in a transition $M[e > M'$, if and only if exists for all $p \in P$ a corresponding transition $M(p) \stackrel{F(e,p)}{\rightarrow} M'(p)$ in the transition system $\mathcal{T}_\mathbf{2}$ displayed in Fig. 1.*

A few remarks are in order. Observe that an event e cannot be disabled at any marking M by a place p such that $F(e, p) =$"$\times$", since p may be switched by e either from 0 to 1 or from 1 to 0 according to the case $M(p) = 0$ or $M(p) = 1$. This explains the name of flip-flop nets. Observe also that if we ignore this particularity and thus restrict F to range over $\{+, -, =\}$, we recover exactly the elementary nets of [ER90].

Definition 3 (Marking graph) *The marking graph of a flip-flop net $N = (P, E, F)$ is the transition system $MG(N) = (\mathcal{M}(N), E, T)$ with the set of transitions $T = \{M \stackrel{e}{\rightarrow} M' \mid M[e > M'\}$.*

It is worth noting that every place p of a net $N=(P,E,F)$ induces a corresponding morphism (η_p,σ_p) from $MG(N)$ to $\mathcal{T}_{\mathbf{2}}$, given by $\eta_p(e)=F(p,e)$ for e in E and $\tau_p(M)=M(p)$ for M in $\mathcal{M}(N)$ (recall that a morphism (η,σ) between transition systems (Q,E,T) and (Q',E',T') is a pair of maps $\eta : E \longrightarrow E'$, $\sigma : Q \longrightarrow Q'$ such that $q \stackrel{e}{\to} q' \ \in T \Rightarrow \sigma(q) \stackrel{\eta(e)}{\to} \sigma(q') \ \in T'$).

Let us now proceed by comparing flip-flop nets and P/T nets. We recall first the usual definition of P/T nets.

Definition 4 (P/T nets) *A P/T net is a triple $N=(P,T,W)$, where P and T are disjoint sets, of places and transitions respectively, and $W : P\times T \longrightarrow \mathbb{Z}$ is the weight function. A marking of N is a map from P to $\mathbb{N}$. Let $\mathcal{M}(N)$ denote the set of markings of N. A transition $t\in T$ is enabled at marking M if and only if $\forall p \in P, M(p)+W(p,t)\geq 0$. A transition t enabled at M can fire; in doing so, it produces a new marking M', determined uniquely from M by the relation $M[t>M'$ if and only if $\forall p\in P, M'(p)=M(p)+W(p,t)$. The marking graph of N is the transition system $MG(N)=(\mathcal{M}(N),E,T)$, where $T=\{M \stackrel{t}{\to} M' \ \mid M[t>M'\}$.*

The classical definition for the firing rule may be restated equivalently in the following form :

Definition 5 (firing rule for P/T nets) *Let $N=(P,T,W)$ and $M\in\mathcal{M}(N)$. An event $t\in T$ may be fired at M, resulting in a transition $M[t>M'$, if and only if exists for all $p\in P$ a corresponding transition $M(p) \stackrel{W(p,t)}{\to} M'(p)$ in the transition system $\mathcal{T}_{\mathbb{Z}}$ displayed in Fig.2.*

Note that $\mathcal{T}_{\mathbb{Z}}$ is the induced subgraph of the Cayley graph of $\mathbb{Z}$ on the set of positive nodes. Observe also that every place p in $N=(P,T,W)$ induces a corresponding morphism (η_p,σ_p) from $MG(N)$ to $\mathcal{T}_{\mathbb{Z}}$, given by $\eta_p(t)=W(p,t)$ for $t\in T$ and $\sigma_p(M)=M(p)$ for $p\in P$. This establishes the parallel between P/T-nets and flip-flop nets. Nevertheless, $\mathcal{T}_{\mathbf{2}}$ is not an induced subgraph of the Cayley graph of $\mathbf{2}=(\{0,1\},+,0)$, displayed in Fig.3. The reasons for this mismatch are the following : if the Cayley graph of $\mathbf{2}$ appeared in place of $\mathcal{T}_{\mathbf{2}}$ in the definition of flip-flop nets, flow maps would be forced to range in $\{=,\times\}$,

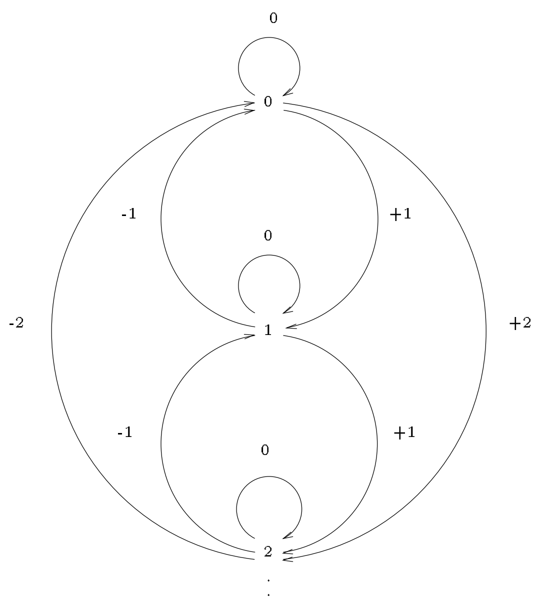

Figure 2: the transition system $\mathcal{T}_{\mathbb{Z}}$

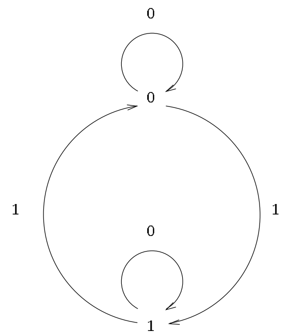

Figure 3: the Cayley Graph of $\mathbf{2} = (\{0,1\}, +, 0)$

leaving no way to disable events at particular markings. The trick we have used for that purpose is to split the action 1 (or $\times$) into two partially defined actions $+$ and $-$ operating respectively at 0 and 1. This technique is in fact implicitly used in the definition of elementary nets, where the action 1 of the group disappears.

Next section shows that this detail has in fact a strong impact on the complexity of the synthesis problem : since this problem is NP-complete for elementary nets, whereas it lies in P for flip-flop nets or P/T nets.

3 Regions in finite transition systems

The goal of this section is to solve the synthesis problem stated as follows :

Problem 6 *Given a pointed transition system $\mathcal{T}$, decide whether there exists a marked flip-flop net with case graph isomorphic to $\mathcal{T}$, and if so construct that net.*

The following definitions makes this statement precise.

Definition 7 *A pointed transition system $\mathcal{T} = (Q, E, T, *_{\mathcal{T}})$ is a transition system (Q, E, T) with a distinguished state $*_{\mathcal{T}}$ called the start state. The set* *of reachable states of $\mathcal{T}$ is the inductive closure of $\{*_{\mathcal{T}}\}$ with respect to the forward transition relation T. $\mathcal{T}$ is said to be accessible if all states $q \in Q$ are reachable. A morphism of pointed transition system is a morphism of transition systems preserving start states.*

Definition 8 *A marked flip-flop net is a 4-uple $\mathcal{N} = (P, E, F, M_0)$ where (P, E, F) is a flip-flop net and M_0 is one of its markings. The case graph of $\mathcal{N}$, $CG(\mathcal{N})$, is the pointed transition system consisting in the induced restriction of the marking graph $MG(N)$ on the subset of markings reachable from M_0, with M_0 as start state.*

We will present a polynomial algorithm solving problem 6. The solution we propose is based on an adaptation to flip-flop nets of the general idea of regions in transition systems, which we now explain.

As noted earlier, each place p in a flip-flop net N gives rise to a corresponding morphism of transition systems $(\eta_p, \sigma_p) : MG(N) \longrightarrow \mathcal{T}_2$. In order to decide whether exists a marked flip-flop net $\mathcal{N}$ with case graph isomorphic to a given pointed transition system $\mathcal{T}$, the idea is to consider all morphisms $(\eta, \sigma) : \mathcal{T} \longrightarrow \mathcal{T}_2$ as possible places in $\mathcal{N}$ with flow map $F(e, (\eta, \sigma)) = \sigma(e)$ and initial marking $M_0((\eta, \sigma)) = \sigma(*_{\mathcal{T}})$. Now for any $(\eta, \sigma) : \mathcal{T} \longrightarrow \mathcal{T}_2$, if we set $R = \sigma^{-1}(1)$, then one of the following relations (i) or (ii) must hold for each event a :

$i)$ $\forall q, q', (q \xrightarrow{a} q' \ \Rightarrow (q \in R \Leftrightarrow q' \in R))$

$ii)$ $\forall q, q', (q \xrightarrow{a} q' \ \Rightarrow (q \in R \Leftrightarrow q' \notin R))$

Conversely every set of states R such that (i) or (ii) holds for each event a coincides with the inverse image $\sigma^{-1}(1)$ by some morphism $(\eta, \sigma) : \mathcal{T} \longrightarrow \mathcal{T}_2$ (where $\eta(a) =$"$=$" if and only if the relation (i) is satisfied for a). This motivates the following definition of regions.

Definition 9 (Regions in a transition system) *A region R in a transition system (Q, E, T) is a set of states $R \subseteq Q$ satisfying : $\forall a \in E$,*
$(\forall q, q' \in Q, (q \xrightarrow{a} q' \ \Rightarrow (q \in p \Leftrightarrow q' \in p))) \vee (\forall q, q' \in Q, (q \xrightarrow{a} q' \ \Rightarrow (q \in p \Leftrightarrow q' \notin p)))$

Regions provide a simple criterion for recognizing those graphs which are isomorphic to case graphs of elementary nets. This criterion may be stated as follows.

Theorem 10 *A pointed transition system is isomorphic to the case graph of some flip-flop net if and only if it is accessible and separated, which means that the following conditions are satisfied :*
1. *For every pair of distinct states, there exists a region which contains exactly one of them,*
2. *For every action a not enabled at some state q, there exists a region which does not contains q but contains all the sources and none of the targets of the transitions labeled by a.*
Moreover, when these conditions are satisfied, the transition system is deterministic, which means that $q \xrightarrow{a} q' \ \wedge q \xrightarrow{a} q'' \ \Rightarrow q' = q''$.

proof:($\Rightarrow$) Let $\mathcal{N}$ be a marked flip-flop net. For any place p, the sets $p^{\ni} = \{M \in CG(\mathcal{N}) \mid M(p) = 1\}$ and its complement $C(p^{\ni})$ are regions in $CG(\mathcal{N})$. Hence $CG(\mathcal{N})$ satisfies the first condition of separation. Now assume event e is not enabled at marking M, by definition of the firing rule, there exists a place p such that either $M(p) = 0$ and $F(p,e) =$"$-$" or $M(p) = 1$ and $F(p,e) =$"$+$". In the first case, $M'[e > M'' \Rightarrow (M' \in p^{\ni} \wedge M'' \notin p^{\ni})$ for every accessible markings M' and M'', and $M \notin p^{\ni}$. In the second case, $M'[e > M'' \Rightarrow (M' \in C(p^{\ni}) \wedge M'' \notin C(p^{\ni}))$ and $M \notin C(p^{\ni})$. Thus, $CG(\mathcal{N})$ satisfies the second condition of separation. Moreover $CG(\mathcal{N})$ is deterministic and accessible by construction.

($\Leftarrow$) Given a pointed transition system $\mathcal{T} = (S, A, T, *_{\mathcal{T}})$, let $\mathcal{N}(\mathcal{T}) = (P, A, F, M_0)$ be the marked net with set of places P defined as the set of regions x in $\mathcal{T}$, initial marking M_0 given by $x \in M_0 \Leftrightarrow *_{\mathcal{T}} \in x$, and flow map F as follows :

- $F(x,a) =$"$=$" if and only if $s \xrightarrow{a} s' \ \Rightarrow (s \in x \Leftrightarrow s' \in x)$,
- $F(x,a) =$"$-$" if and only if $s \xrightarrow{a} s' \ \Rightarrow (s \in x \wedge s' \notin x)$,
- $F(x,a) =$"$+$" if and only if $s \xrightarrow{a} s' \ \Rightarrow (s \notin x \wedge s \in x)$, and

- $F(x,a) =$ "$\times$" if and only if $(\exists s \in x, s \xrightarrow{a}\ ) \wedge (\exists s \notin x, s \xrightarrow{a}\ ) \wedge (s \xrightarrow{a} s' \Rightarrow (s \in x \Leftrightarrow s' \notin x))$.

Assuming $\mathcal{T}$ is accessible and separated, let us show that the marked net $\mathcal{N}(\mathcal{T})$ has a case graph isomorphic to $\mathcal{T}$. For any state s of $\mathcal{T}$, let $s^{\in}$ denote (the characteristic function of) the set of regions which s belongs to. Thus in particular $M_0 = (*_{\mathcal{T}})^{\in}$. Because $\mathcal{T}$ satisfies the first condition of separation, the map $()^{\in} : S \longrightarrow \mathcal{P}(P)$ is one-to-one. If $s \xrightarrow{a} s'$ then for every region x, $s^{\in}(x) \stackrel{F(x,a)}{\rightarrow} s'^{\in}(x) \in \mathcal{T}_{\mathbf{2}}$ by the definition of the flow map, hence $s^{\in}[a > s'^{\in} \in MG(\mathcal{N})$, and since $M_0 = (*_{\mathcal{T}})^{\in}$ and $\mathcal{T}$ is accessible, $s^{\in}$ is a reachable marking for every $s \in S$. This proves that $(id_A, ()^{\epsilon}) : \mathcal{T} \longrightarrow CG(\mathcal{N}(\mathcal{T}))$ is a morphism of pointed transition system. Now, if $s^{\in}[a > M$ then $F(x,a) =$ "$-$" and $s^{\in}(x) = 1$ for any region x containing all the sources and none of the targets of the transitions labeled by a. Because $\mathcal{T}$ satisfies the second condition of separation, necessarily there must exist in that case a transition t with origin s and label a. Moreover, the target of this transition is uniquely determined, because $s \xrightarrow{a} s'$ entails $(s)^{\in}[a > (s')^{\in}$, i.e. $(s')^{\in} = M$, and $()^{\in} : S \longrightarrow \mathcal{P}(P)$ is one-to-one. This proves that $\mathcal{T}$ is deterministic, and also that the range of $()^{\in}$ is equal to the set of reachable markings of $\mathcal{N}$. Thus $(id_A, ()^{\in})$ is an isomorphism of pointed transition system, as was to prove. ∎

Observe from the above proof that if $\mathcal{T}$ is accessible and separated, then any subset of regions in $\mathcal{T}$ containing enough elements for witnessing satisfaction of the two conditions of separation, let $P' \subseteq P$, induces a corresponding subnet $\mathcal{N}'(\mathcal{T}) = (P', E, F', M_0')$ with case graph isomorphic to $\mathcal{T}$ (where F' and M_0' are the induced restriction of the respective maps F and M_0). This observation is crucial for extracting from the above criterion an efficient synthesis algorithm. The construction of our polynomial algorithm relies on an algebraic characterization of regions in $\mathcal{T}$ by systems of equations in the boolean ring. Let us now describe this algebraic setting which will subsequently be used to compute a minimal set of regions P' in $\mathcal{T}$ inducing a net $\mathcal{N}'(\mathcal{T})$ with case graph isomorphic to $\mathcal{T}$.

We introduce first some necessary terminology on graphs. Let $\mathcal{T} = (S, A, T)$ be a transition system. A finite sequence of transitions of $\mathcal{T}$, $(q_i^1 \xrightarrow{a_i} q_i^2\ )_{1 \leq i \leq n}$ is a path in $\mathcal{T}$ if $q_i^2 = q_{i+1}^1$ for all $i < n$, and then q_1^1 and q_n^2 are the two extremities of that path. Let T_{sym} be the extended set of transitions $q \xrightarrow{a} q'$ where either $q \xrightarrow{a} q' \in T$ or $q' \xrightarrow{a} q \in T$. A chain in $\mathcal{T}$ is a path in the transition system consisting of (S, A, T_{sym}), and a chain with identical extremities is called a cycle. For any chain $c = (q_i^1 \xrightarrow{a_i} q_i^2)_{1 \leq i \leq n}$, let $l(c) = a_1 ... a_n \in A^\star$, and for any word $w \in A^\star$, let $\pi(w)$ denote the Parikh image of w.

The algebraic characterization of regions in an accessible transition system may now be stated.

Theorem 11 *For any accessible transition system $\mathcal{T} = (S, A, T, *_\mathcal{T})$, pairs of complementary regions $(x, C(x))$ in $\mathcal{T}$ are in bijective correspondence with maps ρ from A to* **2** *such that $\tilde{\rho}$, the unique monoid morphism from $A^\star$ to $\mathbb{Z}/2\mathbb{Z}$ extending ρ, satisfies for all cycles c : $\tilde{\rho}(l(c)) = 0$.*

proof: Given a region x in $\mathcal{T}$, let $\eta_x : A \longrightarrow \{=, +, -, \times\}$ be the the map $\eta_x(e) = F(x, e)$ where F is the flow map defined for the net $\mathcal{N}(\mathcal{T})$ in the proof of theorem 10. Let $\rho_x = chg \cdot \eta_x$ where $chg : \{=, +, -, \times\} \longrightarrow \mathbf{2}$ is defined by $chg(=) = 0$ and $chg(-) = chg(+) = chg(\times) = 1$, and let $\tilde{\rho}_x$ be the unique monoid morphism from $A^\star$ to $\mathbb{Z}/2\mathbb{Z}$ extending ρ_x. Clearly $\tilde{\rho}_x = \tilde{\rho}_{C(x)}$ because $chg \cdot \eta_x = chg \cdot \eta_{C(x)}$. From the commutativity of $\mathbb{Z}/2\mathbb{Z}$, two words with the same Parikh image must have the same image under $\tilde{\rho}_x$. Moreover, for any chain c of $\mathcal{T}$, $\tilde{\rho}_x(l(c))$ counts modulo 2 edges in c with exactly one extremity in x, thus $\tilde{\rho}_x(l(c)) = 0$ if c is a cycle. Conversely, let $\tilde{\rho}$ be a morphism from $A^\star$ to $\mathbb{Z}/2\mathbb{Z}$, such that $\tilde{\rho}(l(c)) = 0$ for every cycle c in $\mathcal{T}$. Note that for a fixed node s, $\tilde{\rho}(l(p))$ takes a constant value for arbitrary paths p from $*_\mathcal{T}$ to s. Then define x as the set of nodes s such that $\tilde{\rho}(l(p)) = 0$ for some path p from $*_\mathcal{T}$ to s. Hence $*_\mathcal{T} \in x$ and $C(x)$ is the set of nodes reached by initial paths p such that $\tilde{\rho}(l(p)) = 1$. We are going to check that x and $C(x)$ is a pair of complementary regions. Let $s \xrightarrow{a} s' \in T$, and let p an arbitrary path $*_\mathcal{T}$ to s. Depending on whether $\rho(a) = 0$ or $\rho(a) = 1$, one of the following two situations is met. If $\rho(a) = 0$ then $s \in x$ if and only if $\tilde{\rho}(l(p)) = 0$ if and only if $\tilde{\rho}(l(p) \cdot a) = 0$ if and only if $s' \in x$. If $\rho(a) = 1$ then $s \in x$ if and only if $\tilde{\rho}(l(p)) = 0$ if and only if $\tilde{\rho}(l(p) \cdot a) = 1$ if and only if $s' \notin x$. Thus, x and $C(x)$ are regions, as required. ∎

On the ground of the algebraic characterization of regions supplied by the above theorem, we concentrate in the rest of the section on the problem of constructing an efficient algorithm for the synthesis of finite flip-flop nets from finite accessible transition systems.

For convenience we introduce some notations. From now on, $\mathcal{T} = (S, A, T, *_{\mathcal{T}})$ will denote a finite accessible transition system with alphabet $A = \{a_1, ..., a_n\}$. We let $[i]$ denote the class modulo 2 of $i \in I\!N$, and extend this notation to n-vectors by setting $[< i_1, ..., i_n >] =< [i_1], ..., [i_n] > (\in \mathbf{2}^n)$ for any $< i_1, ..., i_n >\in I\!N^n$. We represent a map $\rho : A \longrightarrow \mathbf{2}$ by the corresponding n-vector $\vec{\rho} =< \rho(a_1), ..., \rho(a_n) >$.

Now recall that the set of cycles in a finite transition system $\mathcal{T}$ is a vector space of finite dimension, generated from an independent basis $\mathcal{C}(\mathcal{T})$. Therefore, the set of maps ρ from A to $\mathbf{2}$ inducing regions in $\mathcal{T}$ is the set of solutions of a finite system of linear equations in $Z\!\!Z/2Z\!\!Z$, with one equation $\vec{\rho} \cdot [\pi(\vec{l(c)})] = 0$ for each cycle c in the basis $\mathcal{C}(\mathcal{T})$. The following proposition borrowed from [GM85], shows that the number of equations in that system is in fact polynomial in the size of $\mathcal{T}$.

Proposition 12 (Gondran and Minoux) *Let $G = (S, T)$ be a finite graph with p connected components. Let $U \subseteq T$ be maximal such that the graph (S, U) is a forest (i.e. does not contain any cycle). For any $t \in T\backslash U$, let c^t be the unique cycle whose set of arcs is included in $U \cup \{t\}$. Then the set of cycles c^t, where t ranges over $T\backslash U$, forms a basis of cycles of G, with dimension $|T| - |S| + p$.*

In case when G is an accessible transition system, the maximal forest of the proposition may naturally be chosen among the spanning trees rooted at the start state. Constructing a spanning tree takes polynomial time, hence computing a basis of cycles for G takes polynomial time. In the sequel, let $Span(\mathcal{T})$ and $\mathcal{C}(\mathcal{T})$ be a fixed spanning tree and a fixed basis of cycles for $\mathcal{T}$. In order to decide whether an arbitrary pointed transition system is isomorphic to the case graph of some flip-flop net, one has to check satisfaction of the two conditions of separation stated in theorem 10. In the particular case of a finite and accessible transition system $\mathcal{T}$, this amounts to solve a finite series of systems of linear equations in $Z\!\!Z/2Z\!\!Z$. Namely, $\mathcal{T}$ is isomorphic to the case graph of a flip-flop net if and only if every instance of the following two separation problems 13 and 14 is solvable.

Problem 13 *For each pair of non-identical initial paths p, p' in $Span(\mathcal{T})$, including possibly the empty path ϵ, solve the system $(S^1_{p,p'})$, with unknown $\vec{\rho}$, assembled from the equations*

$$\vec{\rho} \cdot [\pi(\vec{l(c)})] = 0,$$

for any $c \in \mathcal{C}(\mathcal{T})$, plus the equation,

$$\vec{\rho} \cdot ([\pi(\vec{l(p)})] + [\pi(\vec{l(p')})]) = 1.$$

Problem 14 *For each action $a_i \in A$ and for each path p from $*_\mathcal{T}$ to s in $Span(\mathcal{T})$ such that $s \xrightarrow{a_i} \notin \mathcal{T}$, solve the system $S^2_{a_i,p}$ with unknown $\vec{\rho}$ assembled from the equations*

$$\vec{\rho} \cdot [\pi(\vec{l(c)})] = 0,$$

for $c \in \mathcal{C}(\mathcal{T})$ and for every chain c in $Span(\mathcal{T})$ between two nodes enabling a_i in $\mathcal{T}$, plus the equation

$$\rho(a_i) = 1,$$

and the equation

$$\vec{\rho} \cdot ([\pi(\vec{l(p)})] + [\pi(\vec{l(p')})]) = 1,$$

for one (arbitrary) path p' from $*_\mathcal{T}$ to s' in $Span(\mathcal{T})$ such that $s' \xrightarrow{a_i}$ in $\mathcal{T}$.

All the above systems have size polynomial in the size of $\mathcal{T}$, each of them may be solved by Gaussian elimination which takes polynomial time, and their total number is bounded by $|S|^2 + |S| \times |A|$. Hence deciding whether $\mathcal{T}$ is isomorphic to the case graph of some flip-flop net takes polynomial time. Furthermore, when all systems $S^1_{p,p'}$ and $S^2_{a_1,p}$ have solutions $\rho^1_{p,p'}$ and $\rho^2_{a_1,p}$ inducing regions $x^1_{p,p'}$ and $x^2_{a_1,p}$, one may construct the desired flip-flop net as the induced restriction of $\mathcal{N}(\mathcal{T})$ on the subset of places $x^1_{p,p'}$ and $x^2_{a_i,p}$. Summing up, one obtains the following.

Theorem 15 *Let $\mathcal{T}$ be a finite accessible labeled graph, then one may decide whether $\mathcal{T}$ is isomorphic to the case graph of some finite flip-flop net, and if so construct that net, in polynomial time.*

4 Conclusion

Applying the principle of generalized regions inherited from the P/T nets representation to the boolean setting gives rise to flip-flop nets. We think that this model of machine is worth care, for various reasons. First, it occurs in a very natural way as the boolean counterpart of the traditionnal P/T net. Flip-flop nets are a very simple and slight generalization of elementary nets, and they admit moreover polynomial-time synthesis which makes their use practically possible. This latter point is certainly the main advantage of these nets over elementary nets.

We should note that the synthesis for flip-flop nets is much simpler than the one given for P/T nets in [BBD95a]. This is due to the following fact. The firing rule of an event at a certain marking for flip-flop nets can be expressed with a finite number of equalities due to the fact that $\mathbb{Z}/2\mathbb{Z}$ is finite, whereas for P/T nets, this rule is expressed with an inequality which cannot be reduced to a finite number of equalities.

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Unité de recherche INRIA Lorraine, Technopôle de Nancy-Brabois, Campus scientifique,
615 rue du Jardin Botanique, BP 101, 54600 VILLERS LÈS NANCY
Unité de recherche INRIA Rennes, Irisa, Campus universitaire de Beaulieu, 35042 RENNES Cedex
Unité de recherche INRIA Rhône-Alpes, 46 avenue Félix Viallet, 38031 GRENOBLE Cedex 1
Unité de recherche INRIA Rocquencourt, Domaine de Voluceau, Rocquencourt, BP 105, 78153 LE CHESNAY Cedex
Unité de recherche INRIA Sophia-Antipolis, 2004 route des Lucioles, BP 93, 06902 SOPHIA-ANTIPOLIS Cedex

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