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***Bisimulation for higher-order process calculi***

Davide Sangiorgi

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# Bisimulation for higher-order process calculi

Davide Sangiorgi

Programme 2 — Calcul symbolique, programmation et génie logiciel  
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**Abstract:** A *higher-order process calculus* is a calculus for communicating systems which contains higher-order constructs like communication of terms. We analyse the notion of *bisimulation* in these calculi. We argue that both the standard definition of bisimulation (i.e., the one for CCS and related calculi), as well as *higher-order bisimulation* [AGR88, Bou89, Tho90] are in general unsatisfactory, because over-discriminating.

We propose and study a new form of bisimulation for such calculi, called *context bisimulation*, which yields a more satisfactory discriminating power. A drawback of context bisimulation is the heavy use of universal quantification in its definition. A major goal of the paper is to find characterisations which make bisimilarities easier to verify.

An important role in our theory is played by the *factorisation theorem*: When comparing the behaviour of two processes, it allows us to “isolate” subcomponents which might cause differences, so that the analysis can be concentrated on them.

**Key-words:** Bisimulation, higher-order process calculi

(Résumé : *tsvp*)

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# Bisimulation pour les calculs de processus d'ordre supérieur

**Résumé :** Un *calcul de processus d'ordre supérieur* est un calcul pour des systèmes de communication qui contient des opérateurs d'ordre supérieur comme communications des termes. Nous analysons la notion de *bisimulation* pour ces calculs. Nous retenons que la définition standard de bisimulation (c-à-d, celle pour CCS et les calculs s'y rapportant), ainsi que celle de *higher-order bisimulation* [AGR88, Bou89, Tho90] sont en general non satisfaisantes, car trop discriminantes.

Nous proposons et étudions une nouvelle forme de bisimulation pour de tels calculs, appelée *context bisimulation*, qui offre un pouvoir de discrimination plus satisfaisant. Un inconvénient à la context bisimulation est l'utilisation massive de quantification universelle dans sa définition. L'objet principal de cet article est de trouver des caractérisations qui facilitent la vérification des bisimulations.

Le *théorème de factorisation* joue un rôle important dans notre théorie. Lorsque l'on compare le comportement de deux processus, il permet d' "isoler" des sous-composants qui peuvent provoquer des différences; l'analyse peut donc se concentrer sur eux.

**Mots-clé :** Bisimulation, calculs de processus d'ordre supérieur

# 1 Introduction

Recently, various process calculi have been proposed which allow us to describe mobile systems, i.e. concurrent systems whose communication topology may change dynamically. We can categorise these calculi into *first-order calculi* like  $\pi$ -calculus [MPW92], in which only *names* (i.e., ports, or channels) can be communicated, and *higher-order calculi* like CHOCS [Tho90],  $\gamma$ -calculus [Bou89], Higher-Order  $\pi$ -calculus [San92], in which *agents* (i.e., terms of the language) can be communicated. Higher-order calculi are formally closer to the  $\lambda$ -calculus, whose basic computational step —  $\beta$ -reduction — involves term instantiation. This paper reports our study of the notion of *bisimulation* in higher-order calculi.

Bisimulation was originally introduced by Milner and Park [Mil80, Par81] for CCS-like languages, in which mobility is not explicitly present, and since then it has become a fundamental concept in the theory of concurrency. In mobility-free languages bisimulation is defined on top of a labeled transition system, which describes the operational behaviour of processes, by imposing the following circular requirement: Two processes are *bisimilar* if any action by one of them can be matched by an equal action from the other in such a way that the resulting derivatives are again bisimilar. Note that two matching actions must be syntactically *identical*. This condition is generally too strong in calculi with mobility. For instance, in name-passing calculi it does not respect alpha-conversion on names [MPW92]. But in higher-order calculi the damage goes well beyond alpha-conversion. We illustrate the kind of problems which arise using the simple process-passing calculus described by the following grammar (roughly, the language we shall use in the paper):

$$P ::= \bar{a}. \langle P_1 \rangle P_2 \mid a.(X)P \mid P_1 \mid P_2 \mid \nu a P \mid X \mid !P \mid \mathbf{0}$$

This calculus is similar to Thomsen's Plain CHOCS [Tho93], and is a second-order fragment of the Higher-Order  $\pi$ -calculus [San92]. Informally, process  $\bar{a}. \langle P_1 \rangle P_2$  can perform an output action at  $a$  emitting  $P_1$  and then continues as  $P_2$ . Process  $a.(X)P$  can receive a process at  $a$ , say  $Q$ , and then continues as  $P\{Q/X\}$ . Symbol  $X$  represents a process variable,  $\mid$  is parallel composition and  $\mathbf{0}$  is inaction. A replication  $!P$  stands, intuitively, for an infinite number of copies of  $P$  in parallel. Finally,  $\nu a P$  is the restriction operator, which declares  $a$  as a new name, different from all other names. Restriction is a static binder, as the ' $\lambda$ ' of the  $\lambda$ -calculus. We shall abbreviate output and input prefixes as  $\bar{a}.P$  and  $a.P$ , respectively, when the process received or emitted is not important.

In this calculus, the definition of bisimulation used for CCS or  $\pi$ -calculus breaks obvious algebraic laws, such as the commutativity of parallel composition. For instance, in general

we would distinguish processes  $\bar{a}. \langle P \mid Q \rangle \mathbf{0}$  and  $\bar{a}. \langle Q \mid P \rangle \mathbf{0}$ , since the actions they perform may have syntactically different object parts, namely  $P \mid Q$  and  $Q \mid P$ .

The approach taken by Thomsen [Tho90], following earlier ideas by Astesiano and Boudol [AGR88, Bou89], is to require *bisimilarity* rather than *identity* of the processes emitted in a higher-order output action. This form of bisimulation, called *higher-order bisimulation*, seems troublesome when restriction is a static binder, as in our setting. (By contrast, higher-order bisimulation appears to work well in calculi using dynamic binding as in [AGR88, AGR92, Bou89] or in the calculus CHOCS [Tho90]; the meaning of dynamic binding for the restriction construct is explained in the concluding section; in this paper we *only* deal with static binding.) For instance, take

$$P \stackrel{\text{def}}{=} \bar{a}. \langle \mathbf{0} \rangle \mathbf{0}, \quad Q \stackrel{\text{def}}{=} \nu m (\bar{a}. \langle m.\mathbf{0} \rangle \mathbf{0}). \quad (1)$$

Processes  $P$  and  $Q$  differ in the value carried by  $\bar{a}$ , which is  $\mathbf{0}$  in the former, and  $m.\mathbf{0}$  in the latter. Moreover, since  $\nu$  is a static binder, the output by  $Q$  causes the extrusion of name  $m$ , i.e., the scope of the restriction  $\nu m$  is enlarged to embrace the recipient of  $m.\mathbf{0}$ . To see an example, the interaction between  $Q$  and  $a.(X)R$  is (by alpha-conversion we can assume that  $m$  does not occur in  $R$ ):

$$(a.(X)R) \mid Q = (a.(X)R) \mid \nu m (\bar{a}. \langle m.\mathbf{0} \rangle \mathbf{0}) \xrightarrow{\tau} \nu m (R\{m.\mathbf{0}/X\} \mid \mathbf{0}).$$

In the derivative, since  $m$  is restricted and does not occur in  $R$ , process  $m.\mathbf{0}$  will never find a partner to communicate with. It is therefore a deadlocked process, and as such semantically the *same* as  $\mathbf{0}$ . Indeed, in any context  $P$  and  $Q$  give rise to the same interactions and, accordingly, should be considered equivalent. Unfortunately, they are *not* higher-order bisimilar. Higher-order bisimulation “forgets” restrictions which are extruded in an output, like the restriction on  $m$  in  $Q$ . For instance,  $P$  and  $Q$  are distinguished because the processes they transmit, namely  $\mathbf{0}$  and  $m.\mathbf{0}$ , are not equivalent. One could think of adjusting this example by imposing a different treatment of the extruded name  $m$ , thus comparing  $\mathbf{0}$  with  $\nu m (m.\mathbf{0})$  rather than  $m.\mathbf{0}$ . But this approach is certainly wrong and can be disastrous in other situations. For instance, if  $T$  is a deadlocked process with  $m$  free in it, like  $\nu n (n.m.\mathbf{0})$ , then this choice equates processes

$$\nu m (\bar{a}. \langle m.R \rangle \bar{m}.\mathbf{0}) \quad \text{and} \quad \nu m (\bar{a}. \langle T \rangle \bar{m}.\mathbf{0}) \quad (2)$$

which by contrast have completely different possibilities of interactions (the former can communicate at  $m$  with the recipient of  $m.R$  and thus activate a copy of  $R$ ). This approach would also yield the law

$$\nu m (\bar{a}. \langle P \rangle Q) = \bar{a}. \langle \nu m P \rangle Q$$

Yet in the first process all copies of  $P$  activated by its recipient share the name  $m$ , whereas in the second one, the name  $m$  is private to each copy. Higher-order bisimulation appears over-discriminating even if the restriction operator is omitted. Consider, for instance,

$$P_1 \stackrel{\text{def}}{=} \bar{a}. \langle \mathbf{0} \rangle !m.\mathbf{0} \quad P_2 \stackrel{\text{def}}{=} \bar{a}. \langle m.\mathbf{0} \rangle !m.\mathbf{0}. \quad (3)$$

They are not equated according to higher-order bisimulation. This is unsatisfactory because the replication  $!m.\mathbf{0}$  covers the difference between  $\mathbf{0}$  and  $m.\mathbf{0}$ , regardless of how many copies

of them are used. These inconvenients of higher-order bisimulation have also been noted by other people, including Roberto Amadio, Robin Milner and Eugenio Moggi.

The above counterintuitive equalities of higher-order bisimulation are due to the fact that, in an output, the object part and the continuation are examined separately. Above all, this prevents a satisfactory treatment of the channels private to the two. For the reader familiar with non-interleaving process algebras, this should recall the problems of *distributed bisimulation* [CH89] in presence of restriction. One might then try to follow the solution for the latter proposed by Boudol et al. in [BCHK94] and based on the introduction of *locations*. The idea is to keep the two components to analyse together but assign them different locations, which can be detected when an observable action is produced. Thus the observable actions of the two components can be distinguished and yet, there can be private names and communications between them. An inconvenient of this approach is that it requires an extension of the syntax of the language.

Instead, our choice has been to avoid the separation between object part and continuation of an output action by explicitly taking into account the *context* in which the emitted agent is supposed to go. The resulting bisimulation, called *context bisimulation*, can be given an elegant formulation using the syntactic constructs of *abstraction* and *concretion*, borrowed from [Mil91, Hen93]. Abstractions and concretions are expressions of the form  $(X)Q$  and  $\nu \tilde{z}\langle R \rangle Q$ , respectively. With these, input and output transitions can be written in the form

$$P \xrightarrow{a} (X)Q \quad \text{and} \quad P \xrightarrow{\bar{a}} \nu \tilde{z}\langle R \rangle Q,$$

the former meaning “ $P$  is willing to receive an agent at  $a$ , say  $R$ , and continue as  $P\{R/X\}$ ”, and the latter meaning “by extruding names  $\tilde{z}$ , process  $P$  can emit  $R$  at  $a$  and evolve to  $Q$ ”. A pseudo application between an abstraction  $(X)P$  and a concretion  $\nu \tilde{z}\langle R \rangle Q$  can be defined thus:

$$((X)P) \bullet (\nu \tilde{z}\langle R \rangle Q) \stackrel{\text{def}}{=} \nu \tilde{z}(P\{R/X\}|Q)$$

(with possible renaming of  $\tilde{z}$  to avoid capture of names in  $P$ ). Now, using  $C, D$  for concretions,  $F, G$  for abstractions,  $\approx_{\text{Ct}}$  for context bisimulation, and using the weak arrow  $P \xRightarrow{\bar{a}} D$  to abstract from silent steps, the bisimilarity clause of context bisimulation on the outputs of two processes  $P$  and  $Q$  is:

$$\begin{aligned} \text{whenever } P \xrightarrow{\bar{a}} C, \text{ there exists } D \text{ s.t. } Q \xRightarrow{\bar{a}} D \\ \text{and } F \bullet C \approx_{\text{Ct}} F \bullet D, \text{ for all abstractions } F. \end{aligned} \tag{4}$$

(here,  $F$  plays the role of a possible recipient of the process emitted by  $P$  and  $Q$ ). We impose the symmetric requirement on inputs:

$$\begin{aligned} \text{whenever } P \xrightarrow{a} F, \text{ there exists } G \text{ s.t. } Q \xRightarrow{a} G \\ \text{and } C \bullet F \approx_{\text{Ct}} C \bullet G, \text{ for all concretions } C. \end{aligned} \tag{5}$$

Context bisimulation equates the processes in (1) and (3), and distinguishes those in (2), as we wished to do. A drawback of context bisimulation is the universal quantifications over abstractions in (4) and over concretions in (5), which can make it hard, in practice, to use this equivalence. We shall therefore look for simpler characterisations, which do not require



universal quantifications. Our best candidate for this will be *normal bisimulation*: It shows that it is possible to reason fairly efficiently in a higher-order calculus, notwithstanding its sophisticated transitional semantics.

A crucial role in our study is played by *triggers* and by the *factorisation theorem*. A trigger is an elementary process whose only functionality is to activate a copy of another process. We shall use triggers to perform process transformations which make the treatment of higher-order constructs easier. The most important of such transformations is indeed the factorisation theorem, which states that, by means of triggers, a subprocess of a given process can be factorised out.

To prove that context bisimulation and normal bisimulation coincide, we shall go through an intermediate characterisation, namely *triggered bisimulation*. This is a bisimilarity relation with extremely simple clauses on input and output actions. However, it is only defined on the subclass of *triggered agents*, roughly, agents in which triggers only can be exchanged in communications. Triggers agents will represent for us a sort of “normal form” for agents. The factorisation theorem will be used to transform every agent into a triggered agent.

*Related work.* Very recently, a few studies of bisimulations similar to context bisimulation have been conducted. Amadio [Ama93] — who has proposed it independently from us — uses it to study the encoding of Plain CHOCS into  $\pi$ -calculus; Amadio and Dams [AD95] propose an extension of Hennessy and Milner’s modal logics which characterises this behavioural equivalence; Hansen and Kleist [HK94] have analysed a form of asynchronous higher-order calculus and showed that *late*, *early* and *open* (this terminology is borrowed from the  $\pi$ -calculus literature) variants of (strong) context bisimulation coincide.

Several other studies of higher-order process calculi appear in the literature; without claiming to be exhaustive, we can recall the works by Hennessy [Hen93], who considered the denotational approach, and by Astesiano et al. [AGR92], in the setting of generalised algebraic specifications (we shall comment on these two works in the concluding section); by Nielson [Nie89], who has mainly focused on types as a means of ensuring more reliable programs; by Nierstrasz [Nie], who has tried to combine  $\pi$ -calculus and  $\lambda$ -calculus with the purpose of defining a uniform framework for the semantics of concurrent object-oriented languages; by Kennaway and Sleep [KS85], Strom and Yemini [SY85], Holmstrom [Hol83], Leth [Let92], Giacalone, Mishra, and Prasad [GMP89], whose emphasis, however, is more on programming language and implementation issues.

*Structure of the paper.* In Section 2 we give the formal syntax and the transitional semantics of the higher-order calculus used as test-calculus in the paper. In Section 3 we put forward context bisimulation, probably the most intuitive adjustment of higher-order bisimulation for eliminating its drawbacks discussed above. In Section 4 we introduce triggers and we prove the factorisation theorem. This is used in Section 5, to define a mapping  $\mathcal{T}$  which transforms every agent into a triggered agent, that is an agent in which every higher-order communication is the communication of a trigger. In Sections 6 and 7, by exploiting  $\mathcal{T}$ , we are able to prove simpler characterisations of context bisimulation, namely triggered bisimulation and normal bisimulation; the former is even simpler than the latter, but is only defined on the subclass of triggered agents. In Section 8, we discuss the extension of the theory presented to a richer calculus, namely the Higher-Order  $\pi$ -calculus [San92]; this —

using the terminology in [San92] — is a  $\omega$ -order calculus, whereas the calculus of Section 2 or Plain CHOCS are second-order calculi.

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## 2 The language and its transition semantics

The calculus we use is similar to Thomsen’s Plain CHOCS [Tho93]; hence, processes can be passed around. The main differences from Plain CHOCS are: The use of abstractions and concretions to represent input and output prefixes; the presence of *first-order names*, i.e., names which carry nothing. These are opposed to *higher-order names*, i.e., names used to exchange processes. For the theory we shall develop, the presence of first-order names is not necessary, but makes the presentation of various results easier.

Thus, let  $\mathcal{F}$  be the infinite set of first-order names, and  $\mathcal{H}$  the infinite set of higher-order names. Then,  $\overline{\mathcal{F}} \stackrel{\text{def}}{=} \{\overline{m} : m \in \mathcal{F}\}$ ,  $\overline{\mathcal{H}} \stackrel{\text{def}}{=} \{\overline{a} : a \in \mathcal{H}\}$ ,  $\mathcal{N} \stackrel{\text{def}}{=} \mathcal{F} \cup \mathcal{H}$  and  $\overline{\mathcal{N}} \stackrel{\text{def}}{=} \overline{\mathcal{F}} \cup \overline{\mathcal{H}}$ . The special symbol  $\tau$ , which does not occur in  $\mathcal{N}$  or  $\overline{\mathcal{N}}$ , denotes a silent step. We let  $\mu$  range over  $\mathcal{N} \cup \overline{\mathcal{N}} \cup \{\tau\}$ , and  $\ell$  range over  $\mathcal{F} \cup \overline{\mathcal{F}} \cup \{\tau\}$  (the set of CCS-like actions). By convention, if  $\ell \neq \tau$  then  $\overline{\ell} = \ell$ . We use symbols  $x, y, z$  for names in  $\mathcal{N}$ ; symbols  $m, n$  for names in  $\mathcal{F}$ ; and symbols  $a, b, c$  for names in  $\mathcal{H}$ . We also assume an infinite set of *process variables*, ranged over by  $X, Y, Z$ .

**Definition 2.1** *The syntactic categories of our language and their grammar are:*

$$\begin{array}{ll} \text{Processes } P & := a.F \mid \overline{a}.C \mid \ell.P \mid P_1 \mid P_2 \mid \nu a.P \mid X \mid !P \mid \mathbf{0} \\ \text{Abstractions } F & := (X)P \\ \text{Concretions } C & := \nu x.C \mid \langle P_1 \rangle P_2 \\ \text{Agents } A & := P \mid F \mid C \end{array}$$

$P, Q, R$  and  $T$  will range over processes,  $F$  and  $G$  over abstractions,  $C$  and  $D$  over concretions,  $A$  and  $B$  over agents.

An abstraction  $(X)P$  binds all free occurrences of  $X$  in  $P$ ; similarly a restriction  $\nu x.P$  binds the free occurrences of  $x$  in  $P$ . These binders give rise in the expected way to the definitions of alpha conversion, free variables and free names of an agent  $A$ , respectively  $\text{fv}(A)$  and  $\text{fn}(A)$ .

An agent is *closed* if it has no free variable.  $\mathcal{A}g$  is the set of all agents;  $\mathcal{A}g^\circ$  is the set of all closed agents. Similarly,  $\mathcal{P}r$  and  $\mathcal{P}r^\circ$  are the sets of all processes and of all closed processes.  $P\{\tilde{Q}/\tilde{X}\}$  denotes the componentwise and simultaneous substitution of variables  $\tilde{X}$  with processes  $\tilde{Q}$  (where it is assumed that the members of  $\tilde{X}$  are distinct). We often abbreviate  $\nu x_1 \dots \nu x_n.A$  as  $\nu x_1, \dots, x_n.A$ . In a statement, we shall say that a name is *fresh* to mean that it is different from any other name occurring in agents of the statement. In a prefix  $\mu.A$  we call  $\mu$  the *subject*.

We shall only admit *standard concretions*, i.e., expressions  $\nu \tilde{x} \langle P_1 \rangle P_2$  where names in  $\tilde{x}$  are pairwise distinct and  $\tilde{x} \subseteq \text{fn}(P_1)$ . Indeed, the remaining concretions have little significance: In  $\nu \tilde{x} \langle Q \rangle P$ , by alpha conversion, names  $\tilde{x}$  can be assumed to be distinct; and if

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Alpha	$P$ and $Q$ alpha convertible	
	$Q \xrightarrow{\mu} A$	implies $P \xrightarrow{\mu} A$
Prefix	$\mu. A \xrightarrow{\mu} A$	
Parallelism	$P_1 \xrightarrow{\mu} A$	implies $P_1 \mid P_2 \xrightarrow{\mu} A \mid P_2$
First-order communication	$P_1 \xrightarrow{m} P'_1$	
	$P_2 \xrightarrow{\bar{m}} P'_2$	implies $P_1 \mid P_2 \xrightarrow{\tau} P'_1 \mid P'_2$
Higher-order communication	$P_1 \xrightarrow{a} F$	
	$P_2 \xrightarrow{\bar{a}} C$	implies $P_1 \mid P_2 \xrightarrow{\tau} F \bullet C$
Restriction	$P \xrightarrow{\mu} A, \mu \notin \{x, \bar{x}\}$	implies $\nu x P \xrightarrow{\mu} \nu x A$
Replication	$P \mid !P \xrightarrow{\mu} A$	implies $!P \xrightarrow{\mu} A$

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Table 1: The transition system

$x \notin \text{fn}(Q) \cup \{\tilde{x}\}$  then in  $\nu x, \tilde{x} \langle Q \rangle P$  name  $x$  can be pushed inwards, resulting in the standard concretion  $\nu \tilde{x} \langle Q \rangle (\nu x P)$ . In the following, we therefore assume that if  $x \notin \text{fn}(Q) \cup \{\tilde{x}\}$ , then  $\nu x, \tilde{x} \langle Q \rangle P$  denotes  $\nu \tilde{x} \langle Q \rangle (\nu x P)$ .

We wish to extend restriction to operate on abstractions, and (a form of) parallel composition to operate on abstractions and concretions:

if  $F = (X) Q$  then

if  $X \notin \text{fv}(P)$  then  $F \mid P$  denotes  $(X) (Q \mid P)$   
 (and similarly for  $P \mid F$ ),  
 and  $\nu x F$  denotes  $(X) \nu x F$ ;

if  $C = \nu \tilde{x} \langle Q \rangle R$  then

if  $\tilde{x} \cap \text{fn}(P) = \emptyset$  then  $C \mid P$  denotes  $\nu \tilde{x} \langle Q \rangle (R \mid P)$   
 (and similarly for  $P \mid C$ ).

We now present the operational semantics of the calculus. First, we define an operation of pseudo-application between an abstraction  $F = (X) P$  and a concretion  $C = \nu \tilde{x} \langle Q \rangle R$ . By alpha conversion, we can assume that  $\tilde{x} \cap \text{fn}(F) = \emptyset$  and then we set

$$C \bullet F \stackrel{\text{def}}{=} \nu \tilde{x} (R \mid P\{Q/X\})$$

and, symmetrically,

$$F \bullet C \stackrel{\text{def}}{=} \nu \tilde{x} (P\{Q/X\} \mid R).$$

Similarly, we define an operation of application between an abstraction  $F = (X) P$  and a process thus:

$$F \circ Q \stackrel{\text{def}}{=} P\{Q/X\}$$

The operational semantics of the calculus is reported in Table 1. We have omitted the symmetric of the parallelism and communication rules. There are these forms of judgements:

$$\begin{aligned} P &\xrightarrow{a} F && \text{(higher-order input transition at port } a) \\ P &\xrightarrow{\bar{a}} C && \text{(higher-order output transition at port } a) \\ P &\xrightarrow{\ell} Q && \text{(first-order transition)} \end{aligned}$$

where  $P, Q, F, C \in Ag$ . In turn, a first-order transition can be a first-order input (if  $\ell \in \mathcal{F}$ ), a first-order output (if  $\ell \in \overline{\mathcal{F}}$ ), or an interaction (if  $\ell = \tau$ ).

In the remainder of the paper we work up to alpha conversion; thus “=” denotes syntactic equality up to alpha conversion. We shall normally put enough brackets in the expressions so to avoid precedence ambiguities among the operators. However, to reduce the number of brackets, in a few places we shall assume the following syntactic rules: Substitutions and metanotations “•” and “o” have the highest syntactic precedence; the abstraction and concretion constructs the lowest; parallel composition has weaker precedence than the other process constructs. For instance,  $\langle P \rangle !m.R | Q$  stands for  $\langle P \rangle ((!m.R) | Q)$ , and  $F \bullet C | Q \{R/X\}$  stands for  $(F \bullet C) | (Q \{R/X\})$ .

If  $\mathcal{R}$  is a relation on processes, we write  $P \mathcal{R} Q$  to that  $(P, Q) \in \mathcal{R}$ . Moreover,  $\mathcal{R}_1 \mathcal{R}_2$  is the composition of the two relations  $\mathcal{R}_1$  and  $\mathcal{R}_2$ .

### 3 Context bisimulation

We shall study behavioural equivalences based on bisimulation for the language of the previous section. To overcome the counteintuitive equalities of higher-order bisimulation examined in Section 1, we propose the *context bisimilarity* relation below.

**Definition 3.1 (strong context bisimulation)** *A relation  $\mathcal{R} \subseteq Pr^o \times Pr^o$  is a strong context simulation if  $P \mathcal{R} Q$  implies*

1. whenever  $P \xrightarrow{\ell} P'$ , there exists  $Q'$  s.t.  $Q \xrightarrow{\ell} Q'$  and  $P' \mathcal{R} Q'$ ,
2. whenever  $P \xrightarrow{a} F$ , there exists  $G$  s.t.  $Q \xrightarrow{a} G$  and  $C \bullet F \mathcal{R} C \bullet G$ , for all concretions  $C$ .
3. whenever  $P \xrightarrow{\bar{a}} C$ , there exists  $D$  s.t.  $Q \xrightarrow{\bar{a}} D$  and  $F \bullet C \mathcal{R} F \bullet D$ , for all abstractions  $F$ .

A relation  $\mathcal{R}$  is a strong context bisimulation, briefly  $\sim_{Ct}$ -bisimulation, if  $\mathcal{R}$  and  $\mathcal{R}^{-1}$  are strong context simulations. We say that  $P, Q$  are strongly context bisimilar, briefly  $P \sim_{Ct} Q$ , if  $P \mathcal{R} Q$ , for some  $\sim_{Ct}$ -bisimulation  $\mathcal{R}$ .  $\square$

Relation  $\sim_{Ct}$  is generalised to abstractions, concretions and open agents as expected.

**Definition 3.2**

- For closed abstractions  $F_1$  and  $F_2$ , we set  $F_1 \sim_{Ct} F_2$  if  $C \bullet F_1 \sim_{Ct} C \bullet F_2$  for all closed concretions  $C$ .

- For closed concretions  $C_1$  and  $C_2$ , we set  $C_1 \sim_{Ct} C_2$  if  $F \bullet C_1 \sim_{Ct} F \bullet C_2$  for all closed abstractions  $F$ .
- For open agents  $A_1$  and  $A_2$  with  $fv(A_1, A_2) \subseteq \{\tilde{X}\}$ , set  $A_1 \sim_{Ct} A_2$  if  $A_1\{\tilde{P}/\tilde{X}\} \sim_{Ct} A_2\{\tilde{P}/\tilde{X}\}$  for all closed processes  $\tilde{P}$ .

**Proposition 3.3**  $\sim_{Ct}$  is an equivalence relation. □

**Example 3.4** Let

$$\begin{aligned} P_1 &\stackrel{def}{=} \bar{a}. \langle \mathbf{0} \rangle \mathbf{0} \\ P_2 &\stackrel{def}{=} \nu m (\bar{a}. \langle m.\mathbf{0} \rangle \mathbf{0}) \end{aligned}$$

We argued in Section 1 that  $P_1$  and  $P_2$  should be equated. We show that, indeed,  $P_1 \sim_{Ct} P_2$ . The set of all pairs of the form

$$(\nu m (R\{m.\mathbf{0}/X\}), R\{\mathbf{0}/X\}) \text{ with } m \notin fn(R)$$

contains the pair  $(P_1, P_2)$  and is a  $\sim_{Ct}$ -bisimulation. We sketch the argument.

Since  $m$  is restricted, process  $\nu m (R\{m.\mathbf{0}/X\})$  cannot perform a visible action at  $m$ ; moreover, since  $m$  occurs in  $R\{m.\mathbf{0}/X\}$  only in input position, no interaction along  $m$  can occur. Therefore, any transition for  $\nu m (R\{m.\mathbf{0}/X\})$  is of the form

$$\nu m (R\{m.\mathbf{0}/X\}) \xrightarrow{\mu} \nu m (A\{m.\mathbf{0}/X\})$$

and we also have

$$R\{\mathbf{0}/X\} \xrightarrow{\mu} A\{\mathbf{0}/X\}.$$

Suppose  $A$  is a concretion, say  $\nu \tilde{x} \langle Q_1 \rangle Q_2$  with  $X$  free in  $Q_1$  and  $Q_2$ . Then, for all abstractions  $F \stackrel{def}{=} (Y) Q_3$  we have:

$$\begin{aligned} F \bullet \nu m (A\{m.\mathbf{0}/X\}) &= \nu m (Q_3\{Q_1/Y\}\{m.\mathbf{0}/X\} \mid Q_2\{m.\mathbf{0}/X\}) \\ &= \nu m ((Q_3\{Q_1/Y\} \mid Q_2)\{m.\mathbf{0}/X\}) \stackrel{def}{=} Q_4, \\ F \bullet A\{\mathbf{0}/X\} &= (Q_3\{Q_1/Y\} \mid Q_2)\{\mathbf{0}/X\} \stackrel{def}{=} Q_5 \end{aligned}$$

and  $(Q_4, Q_5)$  are in  $\mathcal{R}$ .

Some simple laws for  $\sim_{Ct}$ :

**Lemma 3.5**

1.  $P_1 \mid P_2 \sim_{Ct} P_2 \mid P_1$ ;
2.  $P_1 \mid (P_2 \mid P_3) \sim_{Ct} (P_1 \mid P_2) \mid P_3$ ;
3.  $P \mid \mathbf{0} \sim_{Ct} P$ ;
4.  $\nu x \nu y A \sim_{Ct} \nu y \nu x A$ ;
5. if  $x \notin fn(P_2)$ , then  $(\nu x P_1) \mid P_2 \sim_{Ct} \nu x (P_1 \mid P_2)$ ;
6.  $!P \sim_{Ct} P \mid !P$ . □

We shall use the up-to technique below for establishing bisimilarity results.

**Definition 3.6** A relation  $\mathcal{R} \subseteq \mathcal{P}r^o \times \mathcal{P}r^o$  is a strong context simulation up-to  $\sim_{Ct}$  if  $P \mathcal{R} Q$  implies

1. whenever  $P \xrightarrow{\ell} P'$ , there exists  $Q'$  s.t.  $Q \xrightarrow{\ell} Q'$  and  $P' \sim_{Ct} \mathcal{R} \sim_{Ct} Q'$ ,
2. whenever  $P \xrightarrow{a} F$ , there exists  $G$  s.t.  $Q \xrightarrow{a} G$  and  $C \bullet F \sim_{Ct} \mathcal{R} \sim_{Ct} C \bullet G$ , for all concretions  $C$ .
3. whenever  $P \xrightarrow{\bar{a}} C$ , there exists  $D$  s.t.  $Q \xrightarrow{\bar{a}} D$  and  $F \bullet C \sim_{Ct} \mathcal{R} \sim_{Ct} F \bullet D$ , for all abstractions  $F$ .

**Proposition 3.7** If  $\mathcal{R}$  is a strong context simulation up-to  $\sim_{Ct}$  then  $\mathcal{R} \subseteq \sim_{Ct}$ .

PROOF: Use a diagram-chasing argument to show that  $\sim_{Ct} \mathcal{R} \sim_{Ct}$  is a strong context bisimulation.  $\square$

### 3.1 Congruence properties of context bisimulation

Context bisimulation is preserved by all operators of the language. As far as proofs are concerned, the most difficult one is congruence for object constructor (i.e.,  $P \sim_{Ct} Q$  implies  $a.\langle P \rangle R \sim_{Ct} a.\langle Q \rangle R$ ); this case is specific of the higher-order setting to which our language belongs. To derive this, we need to prove a congruence result w.r.t. substitutions:

**Proposition 3.8** Let  $P_1, P_2, A \in Ag$ ; then  $P_1 \sim_{Ct} P_2$  implies  $A\{P_1/X\} \sim_{Ct} A\{P_2/X\}$ .

PROOF: See Appendix A.  $\square$

**Theorem 3.9 (congruence of  $\sim_{Ct}$ )** Let  $P_i, A_i \in Ag$ .

1.  $A_1 \sim_{Ct} A_2$  implies:  $\nu x A_1 \sim_{Ct} \nu x A_2$ ,  
 $\mu. A_1 \sim_{Ct} \mu. A_2$ .
2.  $P_1 \sim_{Ct} P_2$  implies:  $P_1 \mid Q \sim_{Ct} P_2 \mid Q$ ,  
 $!P_1 \sim_{Ct} !P_2$ ,  
 $\langle P_1 \rangle Q \sim_{Ct} \langle P_2 \rangle Q$ ,  
 $\langle Q \rangle P_1 \sim_{Ct} \langle Q \rangle P_2$ ,  
 $(X) P_1 \sim_{Ct} (X) P_2$ .

PROOF: Each clause can either be derived from Proposition 3.8, or is straightforward on its own.  $\square$

**Corollary 3.10**

1.  $F_1 \sim_{Ct} F_2$  and  $C_1 \sim_{Ct} C_2$  imply  $F_1 \bullet C_1 \sim_{Ct} F_2 \bullet C_2$ ;
2.  $F_1 \sim_{Ct} F_2$  and  $P_1 \sim_{Ct} P_2$  imply  $F_1 \circ P_1 \sim_{Ct} F_2 \circ P_2$ .  $\square$

### 3.2 Weak context bisimulation

To introduce weak context bisimulation, we first define weak transitions, where  $\tau$  steps are absorbed. Thus  $\Longrightarrow$  is the reflexive and transitive closure of  $\xrightarrow{\tau}$ , and  $P \xRightarrow{\mu} A$  holds if there is  $P'$  s.t.  $P \Longrightarrow P'$  and  $P' \xrightarrow{\mu} A$ . Finally,  $P \xRightarrow{\widehat{\mu}} A$  is  $P \Longrightarrow A$  if  $\mu = \tau$  and  $P \xRightarrow{\mu} A$  otherwise.

**Definition 3.11 (weak context bisimulation)** *A relation  $\mathcal{R} \subseteq \mathcal{P}r^o \times \mathcal{P}r^o$  is a weak context simulation if  $P \mathcal{R} Q$  implies*

1. *whenever  $P \xrightarrow{\ell} P'$ , there exists  $Q'$  s.t.  $Q \xRightarrow{\widehat{\ell}} Q'$  and  $P' \mathcal{R} Q'$ ;*
2. *whenever  $P \xrightarrow{a} F$ , there exists  $G$  s.t.  $Q \xRightarrow{a} G$  and  $C \bullet F \mathcal{R} C \bullet G$ , for all concretions  $C$ ;*
3. *whenever  $P \xrightarrow{\bar{a}} C$ , there exists  $D$  s.t.  $Q \xRightarrow{\bar{a}} D$  and  $F \bullet C \mathcal{R} F \bullet D$ , for all abstractions  $F$ .*

*A relation  $\mathcal{R}$  is a weak context bisimulation, briefly  $\approx_{C_t}$ -bisimulation, if  $\mathcal{R}$  and  $\mathcal{R}^{-1}$  are weak context simulations. We say that  $P, Q$  are weakly context bisimilar, briefly  $P \approx_{C_t} Q$ , if  $P \mathcal{R} Q$ , for some  $\approx_{C_t}$ -bisimulation  $\mathcal{R}$ .*

**Remark 3.12 (delay and late bisimulations)** In the formulation of weak context bisimulation above, agents are immediately tested after a visible action (recall that  $\xRightarrow{\mu}$  stands for  $\Longrightarrow \xrightarrow{\mu}$ ). This treatment of weak transitions is characteristic of *delay bisimulation*, studied in CCS-like languages in [Wei89]. Moreover, in the input and output clauses of Definition 3.11 the existential quantifier precedes the universal one. This is characteristic of *late bisimulation* [MPW92], as opposed to *early bisimulation*, in which the order of the quantifiers is exchanged. We shall show in Section 7.1 that, for weak context bisimulation, the late and early versions coincide.

We think that the “late delay schema” fits well the machinery of abstractions and concretions adopted. First, it well describes the complementarity between abstractions and concretions. Secondly, it seems natural to require that abstractions and concretions do not evolve on their own, but only after meeting a complementary agent. Another compelling motivation for formulating a late bisimulation as a delay bisimulation is that otherwise the resulting relation might not be an equivalence relation; this, for instance, happens in the  $\pi$ -calculus [San93].  $\square$

Weak context bisimulation is extended to concretions, abstractions and open agents in the same way as the strong equivalence (Definition 3.2); thus, for concretions  $C_1$  and  $C_2$ , we have  $C_1 \approx_{C_t} C_2$  if  $C_1 \bullet F \approx_{C_t} C_2 \bullet F$  for all closed abstractions  $F$ . The congruence results for  $\sim_{C_t}$  in Proposition 3.3 and Theorem 3.9 can be extended to  $\approx_{C_t}$ , with a completely analogous proof (see also Remark A.5 in Appendix A).

**Theorem 3.13**  *$\approx_{C_t}$  is an equivalence relation and is preserved by all operators of the language.*  $\square$

We shall use the following up to technique to establish weak-context bisimilarity results.

**Definition 3.14** A relation  $\mathcal{R} \subseteq \mathcal{P}r^o \times \mathcal{P}r^o$  is a weak context simulation up-to  $\approx_{Ct}$  if  $P \mathcal{R} Q$  implies

1. whenever  $P \xrightarrow{\ell} P'$ , there exists  $Q'$  s.t.  $Q \xrightarrow{\widehat{\ell}} Q'$  and  $P' \approx_{Ct} \mathcal{R} \approx_{Ct} Q'$ ,
2. whenever  $P \xrightarrow{a} F$ , there exists  $G$  s.t.  $Q \xrightarrow{a} G$  and  $C \bullet F \approx_{Ct} \mathcal{R} \approx_{Ct} C \bullet G$ , for all concretions  $C$ .
3. whenever  $P \xrightarrow{\bar{a}} C$ , there exists  $D$  s.t.  $Q \xrightarrow{\bar{a}} D$  and  $F \bullet C \approx_{Ct} \mathcal{R} \approx_{Ct} F \bullet D$ , for all abstractions  $F$ .

**Proposition 3.15** If  $\mathcal{R}$  is a weak context simulation up-to  $\approx_{Ct}$  then  $\mathcal{R} \subseteq \approx_{Ct}$ .

PROOF: By showing that  $\approx_{Ct} \mathcal{R} \approx_{Ct}$  is a  $\approx_{Ct}$ -bisimulation.  $\square$

Weak context bisimulation is the relation we are mostly interested in, since it abstracts away from silent steps of processes. We shall look for characterisations of this behavioural equivalence which do not use the heavy universal quantifications in its input clause (quantification on concretions) and in its output clause (quantification on abstractions). We shall use strong context bisimulation as an auxiliary relation.

## 4 The factorisation theorem

The main result of this section is the *factorisation theorem*. It allows us to factorise out certain subagents of a given agent. Thus, a complex process can be decomposed into the parallel composition of simpler processes.

The assertion of the factorisation theorem uses a special kind of agents called *triggers* and the metanotation  $A\{z := B\}$ . We introduce this metanotation, and prove some algebraic properties about it, in Section 4.1; we present triggers in Section 4.2.

### 4.1 Distributivity properties of private replications

We write  $A\{z := B\}$  as an abbreviation for  $\nu z(A \mid !z.B)$ , under the assumption that  $z$  occurs free in  $A$  and  $B$  only in output subject position.

Intuitively, in  $A\{z := B\}$ , agent  $B$  represents a “local environment” for  $A$  and  $z$  is a “pointer” that allows  $A$  to access this local environment; alternatively, we can think of  $B$  as a resource with owner  $A$  and  $z$  as a trigger with which a copy of the resource may be activated.

Remember that  $z$  is restricted, and hence *not free*, in  $A\{z := B\}$ , in the same way as  $x$  is not free in the  $\lambda$ -expression  $M\{y/x\}$ . Indeed, we chose curly brackets for the above abbreviation because  $\{z := B\}$  behaves just like a substitution in  $A\{z := B\}$ . For instance, if  $B$  is an abstraction and  $z$  a higher-order name, then  $A\{z := B\}$  behaves as the agent obtained from  $A$  by substituting ‘ $B \circ R \mid Q$ ’ for any subexpressions ‘ $\bar{z}. \langle R \rangle Q$ ’. This because, given the



side condition on the use of  $z$  in  $A$  and  $B$ , the only possible effect of the prefix  $\bar{z}. \langle R \rangle Q$  is to trigger a copy of  $B$  with argument  $R$ . For example, a little thinking should convince the reader that if  $a$  is not free in  $R, R', P, F$ , then the visible behaviour of  $(\bar{a}. \langle R \rangle \bar{a}. \langle R' \rangle P) \{a := F\}$  is the same as that of  $F \circ R \mid F \circ R' \mid P$ .

The results in this and in the next section show that, indeed, the metanotation  $\{z := B\}$  has algebraic properties similar to those of substitutions. We shall sometimes call  $\{z := B\}$  an *implicit substitution*. We give  $\{z := B\}$  the same precedence as substitutions; thus  $Q \mid P \{z := B\}$  stands for  $Q \mid (P \{z := B\})$ .

Theorem 4.2 shows that  $\{z := B\}$  distributes over all operators of the language. To prove this, we first need to show that  $\{z := B\}$  distributes over process substitutions (in the same way as in Section 3.1, to prove the congruence of  $\sim_{Ct}$ , we first needed the result on substitutions).

**Proposition 4.1** *Let  $A, P, B \in Ag$ . Suppose that  $z \notin fn(B)$  and that  $z$  occurs free in  $A$  and  $P$  only in output subject position. Then*

$$A\{P/X\} \{z := B\} \sim_{Ct} A \{z := B\} \{P \{z := B\}/X\}.$$

PROOF: See Appendix B. □

**Theorem 4.2 (distributivity of  $\{z := B\}$ )** *Suppose that  $z \notin fn(B)$  and that  $z$  occurs free in  $A, P, Q$  and  $C$  only in output subject position. Then the following results on open agents hold:*

1.  $(\nu x A) \{z := B\} \sim_{Ct} \nu x (A \{z := B\})$ , if  $x \notin fn(B) \cup \{z\}$ .
2.  $(P \mid Q) \{z := B\} \sim_{Ct} P \{z := B\} \mid Q \{z := B\}$ .
3.  $(!P) \{z := B\} \sim_{Ct} !(P \{z := B\})$ .
4.  $(\mu. A) \{z := B\} \sim_{Ct} \mu. (A \{z := B\})$ , if  $\mu \neq z$ .
5.  $(\bar{z}. C) \{z := B\} \sim_{Ct} (\tau. C \bullet B) \{z := B\}$  (here  $z$  is a higher-order name and  $B$  an abstraction).
6.  $(\bar{z}. P) \{z := B\} \sim_{Ct} (\tau. (P \mid B)) \{z := B\}$  (here  $z$  is a first-order name and  $B$  a process).
7.  $\mathbf{0} \{z := B\} \sim_{Ct} \mathbf{0}$ .
8.  $(\langle Q \rangle P) \{z := B\} \sim_{Ct} \langle Q \{z := B\} \rangle (P \{z := B\})$ .

PROOF: Each case either can be derived using Proposition 4.1, or is straightforward on its own. □

**Remark 4.3** In the two results above, the requirement  $z \notin fn(B)$  could be weakened to “ $z$  free in  $B$  only in output subject position”, at the price of some more work in the proofs. This extra power is not necessary for the our purposes. □

## 4.2 Triggers

A trigger is a process of the form  $\overline{m}.0$ ; we write  $\text{Tr}_m$  to denote a trigger whose free name is  $m$ .

The assertion of the factorisation theorem reads as follows. Take an agent  $A$ , and suppose we can extract an expression  $Q$  from certain components of  $A$ , in form of an explicit substitution; i.e., for some agent  $A'$  and variable  $Y$ , it holds that  $A = A'\{Q/Y\}$ : Using a trigger  $\text{Tr}_m$ , for some fresh  $m$ , the explicit substitution can be transformed into an implicit one, obtaining  $(A'\{\text{Tr}_m/Y\})\{m := Q\}$ . By contrast with  $A\{Q/Y\}$ , in  $(A'\{\text{Tr}_m/Y\})\{m := Q\}$  each copy of  $Q$  is activated when it is needed using the trigger  $m$ .

**Example 4.4** *If  $P \stackrel{\text{def}}{=} Q \mid \overline{a}.\langle Q \rangle R$ , then  $P = (X \mid \overline{a}.\langle X \rangle R)\{Q/X\}$  and, applying the factorisation theorem,*

$$\begin{aligned} P &= (X \mid \overline{a}.\langle X \rangle R)\{\text{Tr}_m/X\}\{m := Q\} \\ &= (\text{Tr}_m \mid \overline{a}.\langle \text{Tr}_m \rangle R)\{m := Q\} \end{aligned}$$

**Lemma 4.5** *For each  $P \in \mathcal{Pr}$ , it holds that  $\tau.P \approx_{\text{Ct}} P$ .* □

We derive the factorisation theorem from Lemma 4.6, which shows us that the effect of using a trigger is precisely to add a  $\tau$ -action on the head of the replaced expression.

**Lemma 4.6** *For every  $A, R \in \text{Ag}$  with  $m \notin \text{fn}(A, R)$ , it holds that*

$$A\{\tau.R/X\} \sim_{\text{Ct}} A\{\text{Tr}_m/X\}\{m := R\}.$$

PROOF: By induction on the structure of  $A$ . The basic case is when  $A = Y$  and is immediate using Theorem 4.2. For the inductive cases, as an example we show the case of parallel composition. We have:

$$\begin{aligned} (P_1 \mid P_2)\{\tau.R/X\} &= \\ P_1\{\tau.R/X\} \mid P_2\{\tau.R/X\} &\sim_{\text{Ct}} \quad (\text{induction twice}) \\ P_1\{\text{Tr}_m/X\}\{m := R\} \mid P_2\{\text{Tr}_m/X\}\{m := R\} &\sim_{\text{Ct}} \quad (\text{Theorem 4.2(2)}) \\ (P_1\{\text{Tr}_m/X\} \mid P_2\{\text{Tr}_m/X\})\{m := R\} &= \\ (P_1 \mid P_2)\{\text{Tr}_m/X\}\{m := R\} & \quad \square \end{aligned}$$

**Theorem 4.7 (factorisation theorem)** *For every  $A, Q \in \text{Ag}$  with  $m \notin \text{fn}(A, Q)$ , it holds that*

$$A\{Q/X\} \approx_{\text{Ct}} A\{\text{Tr}_m/X\}\{m := Q\}.$$

PROOF: From Lemma 4.6,  $A\{\text{Tr}_m/X\}\{m := Q\} \sim_{\text{Ct}} A\{\tau.Q/X\}$ . Since, by Lemma 4.5,  $\tau.Q \approx_{\text{Ct}} Q$  and  $\approx_{\text{Ct}}$  is a congruence relation, we can infer  $A\{\tau.Q/X\} \approx_{\text{Ct}} A\{Q/X\}$ . □

In the remainder of the paper, for weak context bisimulation or other weak equivalences, the adjective “weak” might be omitted.

## 5 Triggered agents

In this section we introduce the class of *triggered agent*. They represent a sort of “normal form” for the agents of the calculus. Most important, there is a very simple characterisation of context bisimulation on triggered agents, called *triggered bisimulation*. We shall exploit the factorisation theorem to transform every agent into a triggered agent. The transformation allows us to use the simpler theory of triggered agents to reason about the set of all agents. The distinguishing feature of triggered agents is that every communication among them is the exchange of a trigger.

**Definition 5.1 (triggered agents)** *The grammar for triggered agents is obtained from that of ordinary agents in Definition 2.1 by replacing the productions for concretion with the production*

$$\text{Concretions } C \quad := \quad \nu \tilde{x} (\langle \text{Tr}_m \rangle P_1) \{m := P_2\} \text{ with } m \notin \text{fn}(P_1, P_2) \cup \{\tilde{x}\}.$$

In other words, we place the additional requirement that all concretions be in the above “triggered” form. Recall that  $\nu \tilde{x} (\langle \text{Tr}_m \rangle P_1) \{m := P_2\}$  stands for  $\nu m \langle \text{Tr}_m \rangle \nu \tilde{x} (P_1 \mid !m. P_2)$ . We write  $\mathcal{T}Ag$  and  $\mathcal{T}Ag^\circ$  for the classes of triggered agents and of closed triggered agents, respectively.  $\mathcal{TP}r$  and  $\mathcal{TP}r^\circ$  are the subclasses of triggered processes and of closed triggered processes.

We give a mapping  $\mathcal{T}$  which transforms every agent  $A$  into the triggered agent  $\mathcal{T}[A]$ . The mapping is defined inductively on the structure of  $A$ ; it acts as a homomorphism on all constructs of the language except concretions, for which we have:

$$\mathcal{T}[\langle Q \rangle P] = (\langle \text{Tr}_m \rangle \mathcal{T}[P]) \{m := \mathcal{T}[Q]\} \quad \text{where } m \text{ is a fresh name}$$

For instance, we have:

$$\begin{aligned} \mathcal{T}[\langle a.(X) X \rangle \mid \bar{a}. \nu x \langle Q \rangle P] &= \left( a.(X) X \right) \mid \bar{a}. \nu x \left( (\langle \text{Tr}_m \rangle \mathcal{T}[P]) \{m := \mathcal{T}[Q]\} \right) \\ &= \left( a.(X) X \right) \mid \bar{a}. \nu m \langle \text{Tr}_m \rangle \nu x \left( \mathcal{T}[P] \mid !m. \mathcal{T}[Q] \right) \end{aligned}$$

**Theorem 5.2 (correctness of  $\mathcal{T}$ )** *For each  $A \in Ag$ :*

1.  $\mathcal{T}[A]$  is a triggered agent;
2.  $\mathcal{T}[A] \approx_{Ct} A$ .

PROOF: Assertion (1) is straightforward. Assertion (2) can be proved by induction on the structure of  $A$ . The only case in which  $\mathcal{T}$  does not act as a homomorphism is when  $A$  is a concretion of the form  $\langle R \rangle P$ . If  $m \notin \text{fn}(A)$ , then we have:

$$\begin{aligned} \langle R \rangle P &\approx_{Ct} \quad (\text{Theorem 4.7}) \\ (\langle \text{Tr}_m \rangle P) \{m := R\} &\approx_{Ct} \quad (\text{induction twice}) \\ (\langle \text{Tr}_m \rangle \mathcal{T}[P]) \{m := \mathcal{T}[R]\} &= \mathcal{T}[A] \quad \square \end{aligned}$$

It is useful to see the operational correspondence between  $P$  and  $\mathcal{T}[P]$ . Transformation  $\mathcal{T}$  may expand the number of silent steps in a process. But the behaviour is otherwise the

same. The expansion is due to the fact that if in  $P$  a process  $Q$  is transmitted and used  $n$  times then, in  $\mathcal{T}[P]$ ,  $n$  interactions are required to activate the copies of  $Q$ , as the following example shows.

**Example 5.3** Let  $P \stackrel{\text{def}}{=} (\bar{a}. \langle Q \rangle R) \mid a.(X)(X \mid X)$ . Then

$$P \xrightarrow{\tau} R \mid Q \mid Q \stackrel{\text{def}}{=} P'$$

In  $\mathcal{T}[P]$  this is simulated using two additional interactions:

$$\begin{aligned} \mathcal{T}[P] &= \bar{a}. (\langle \text{Tr}_m \rangle \mathcal{T}[R]) \{m := \mathcal{T}[Q]\} \mid a.(X)(X \mid X) \\ &\xrightarrow{\tau} \sim_{\text{Ct}} \left( \mathcal{T}[R] \mid \text{Tr}_m \mid \text{Tr}_m \right) \{m := \mathcal{T}[Q]\} \\ &= \left( \mathcal{T}[R] \mid \bar{m}.\mathbf{0} \mid \bar{m}.\mathbf{0} \right) \{m := \mathcal{T}[Q]\} \\ &\xrightarrow{\tau} \xrightarrow{\tau} \sim_{\text{Ct}} \left( \mathcal{T}[R] \mid \mathcal{T}[Q] \mid \mathcal{T}[Q] \right) \{m := \mathcal{T}[Q]\} \\ &\sim_{\text{Ct}} \mathcal{T}[R] \mid \mathcal{T}[Q] \mid \mathcal{T}[Q] = \mathcal{T}[P'] \end{aligned}$$

**Lemma 5.4** For all  $F$  and  $C$ , it holds that  $\mathcal{T}[F \bullet C] \approx_{\text{Ct}} \mathcal{T}[F] \bullet \mathcal{T}[C]$ .

PROOF: Let  $F \stackrel{\text{def}}{=} (X)P$  and  $C \stackrel{\text{def}}{=} \nu \tilde{x} \langle Q \rangle R$ . We have:

$$\begin{aligned} \mathcal{T}[F] \bullet \mathcal{T}[C] &= ((X) \mathcal{T}[P]) \bullet \nu \tilde{x} (\langle \text{Tr}_m \rangle \mathcal{T}[R]) \{m := \mathcal{T}[Q]\} \\ &\sim_{\text{Ct}} \nu \tilde{x} ((\mathcal{T}[P] \{ \text{Tr}_m / X \} \mid \mathcal{T}[R]) \{m := \mathcal{T}[Q]\}) \\ &\approx_{\text{Ct}} \nu \tilde{x} (\mathcal{T}[P] \{ \mathcal{T}[Q] / X \} \mid \mathcal{T}[R]) \\ &= \mathcal{T}[\nu \tilde{x} (P \{ Q / X \} \mid R)] \\ &= \mathcal{T}[F \bullet C] \end{aligned}$$

where the use of  $\approx_{\text{Ct}}$  is due to Theorem 4.7. □

**Lemma 5.5 (operational correspondence for  $\mathcal{T}$  on strong transitions)**

1. (a) If  $P \xrightarrow{\mu} A$  and  $\mu \neq \tau$ , then  $\mathcal{T}[P] \xrightarrow{\mu} \sim_{\text{Ct}} \mathcal{T}[A]$ ;  
 (b) if  $P \xrightarrow{\tau} P'$ , then  $\mathcal{T}[P] \xrightarrow{\tau} \approx_{\text{Ct}} \mathcal{T}[P']$ .
2. The converse of (1), i.e. :  
 (a) If  $\mathcal{T}[P] \xrightarrow{\mu} A'$  and  $\mu \neq \tau$ , then there is  $A$  s.t.  $P \xrightarrow{\mu} A$  and  $\mathcal{T}[A] \sim_{\text{Ct}} A'$ ;  
 (b) if  $\mathcal{T}[P] \xrightarrow{\tau} P''$ , then there is  $P'$  s.t.  $P \xrightarrow{\tau} P'$  and  $P'' \approx_{\text{Ct}} \mathcal{T}[P']$ .

PROOF: By transition induction. We only consider the rule for parallel composition for assertion 1(a) in the case that  $\mu$  is a higher-order output, and the higher-order communication rule for assertion 1(b).

Suppose  $P_1 \mid P_2 \xrightarrow{\bar{a}} C \mid P_2$ , for some  $C$  s.t.  $P_1 \xrightarrow{\bar{a}} C$ . By induction,

$$\mathcal{T}[P_1] \xrightarrow{\bar{a}} \sim_{\text{Ct}} \mathcal{T}[C].$$

Hence

$$\mathcal{T}[P_1 \mid P_2] = \mathcal{T}[P_1] \mid \mathcal{T}[P_2] \xrightarrow{\bar{a}} \sim_{\text{Ct}} \mathcal{T}[C] \mid \mathcal{T}[P_2].$$

Let  $C = \nu \tilde{x} \langle Q \rangle R$ . Then  $\mathcal{T}[C] = \nu m \langle \text{Tr}_m \rangle \nu \tilde{x} (R \mid !m.Q)$  and

$$\mathcal{T}[C] \mid \mathcal{T}[P_2] = \nu m \langle \text{Tr}_m \rangle \nu \tilde{x} (R \mid !m.Q) \mid \mathcal{T}[P_2]$$

and, assuming  $\tilde{x} \cap \text{fn}(\mathcal{T}[P_2]) = \emptyset$ , by Lemma 3.5,

$$\begin{aligned} &\sim_{\text{Ct}} \nu m \langle \text{Tr}_m \rangle \nu \tilde{x} (R \mid \mathcal{T}[P_2] \mid !m.Q) \\ &= \mathcal{T}[\nu \tilde{x} \langle Q \rangle (R \mid P_2)] = \mathcal{T}[C \mid P_2] \end{aligned}$$

Now, the communication rule. Thus, suppose  $P_1 \mid P_2 \xrightarrow{\tau} F \bullet C$ , for some  $F$  and  $C$  s.t.  $P_1 \xrightarrow{a} F$  and  $P_2 \xrightarrow{\bar{a}} C$ . By induction,  $\mathcal{T}[P_1] \xrightarrow{a} \sim_{\text{Ct}} \mathcal{T}[F]$  and  $\mathcal{T}[P_2] \xrightarrow{\bar{a}} \sim_{\text{Ct}} \mathcal{T}[C]$ . Hence  $\mathcal{T}[P_1 \mid P_2] = \mathcal{T}[P_1] \mid \mathcal{T}[P_2] \xrightarrow{\tau} \sim_{\text{Ct}} \mathcal{T}[F] \bullet \mathcal{T}[C]$  and, by Lemma 5.4,  $\mathcal{T}[F] \bullet \mathcal{T}[C] \approx_{\text{Ct}} \mathcal{T}[F \bullet C]$ .  $\square$

## 6 Triggered bisimulation

We show that the class  $\mathcal{TP}r^\circ$  of triggered processes is amenable to an analysis in which only triggers are exchanged with an external observer. We start by showing that the class  $\mathcal{TP}r^\circ$  is closed with respect to the production of such actions (Lemma 6.2) and then we define a bisimulation in which only this kind of actions is taken into account.

**Lemma 6.1** *If  $A \in \mathcal{T}Ag$ , then for every  $\text{Tr}_m$  we have  $A\{\text{Tr}_m/X\} \in \mathcal{T}Ag$ .*  $\square$

**Lemma 6.2** *Suppose  $P \in \mathcal{TP}r^\circ$ . It holds that:*

1. *If  $P \xrightarrow{\ell} P'$ , then  $P' \in \mathcal{TP}r^\circ$ ;*
2. *if  $P \xrightarrow{a} F$ , then for all  $m$ ,  $F \circ \text{Tr}_m \in \mathcal{TP}r^\circ$ ;*
3. *if  $P \xrightarrow{\bar{a}} C$ , then there is  $P' \in \mathcal{TP}r^\circ$  s.t.  $C = \nu m \langle \text{Tr}_m \rangle P'$  and, moreover, for some  $\tilde{x}, P''$  and  $R$  with  $m \notin \text{fn}(P'', R) \cup \{\tilde{x}\}$ , we have  $P' \sim_{\text{Ct}} \nu \tilde{x} (P'' \mid !m.R)$ .*

PROOF: A simple transition induction; for assertion (2), use Lemma 6.1.  $\square$

**Definition 6.3 (triggered bisimulation)** *A relation  $\mathcal{R} \subseteq \mathcal{TP}r^\circ \times \mathcal{TP}r^\circ$  is a triggered simulation if  $P \mathcal{R} Q$  implies, for  $m \notin \text{fn}(P, Q)$ :*

1. *whenever  $P \xrightarrow{\ell} P'$ , there exists  $Q'$  s.t.  $Q \xrightarrow{\widehat{\ell}} Q'$  and  $P' \mathcal{R} Q'$ ,*
2. *whenever  $P \xrightarrow{a} F$ , there exists  $G$  s.t.  $Q \xrightarrow{a} G$  and  $F \circ \text{Tr}_m \mathcal{R} G \circ \text{Tr}_m$ ,*
3. *whenever  $P \xrightarrow{\bar{a}} \nu m \langle \text{Tr}_m \rangle P'$ , there exists  $Q'$  s.t.  $Q \xrightarrow{\bar{a}} \nu m \langle \text{Tr}_m \rangle Q'$  and  $P' \mathcal{R} Q'$ .*

$\mathcal{R}$  is a triggered bisimulation, briefly  $\approx_{\text{Tr}}$ -bisimulation, if  $\mathcal{R}$  and  $\mathcal{R}^{-1}$  are triggered simulations. We say that  $P$  and  $Q$  are triggered bisimilar, briefly  $P \approx_{\text{Tr}} Q$ , if  $P \mathcal{R} Q$ , for some  $\approx_{\text{Tr}}$ -bisimulation  $\mathcal{R}$ .

Definition 6.3 is consistent for, by Lemma 6.2,  $P', Q', F \circ \text{Tr}_m$  and  $G \circ \text{Tr}_m$  are triggered processes. Note that for clauses (2) and (3) it is enough to take *some*  $m \notin \text{fn}(P, Q)$ . Any other choice of  $m$  — with  $m \notin \text{fn}(P, Q)$  — represents the same kind of test on the processes since, intuitively, the difference between the two choices is expressed by an injective mapping on names. This observation is to evidence the simplicity of the clauses of Definition 6.3 compared to those in the definition of context bisimulation: In clause (3) of Definition 6.3, no universal quantification and no check whatsoever on the agents emitted in the output is necessary; similarly, in clause (2) a single (fresh) trigger is used, whereas in context bisimulation every agent which can possibly be received in the input is taken into account.

Relation  $\approx_{\text{Tr}}$  is extended to  $\mathcal{T}Ag$  accordingly; in particular, in the case of open agents free variables are instantiated with fresh triggers only.

**Definition 6.4**

- For closed triggered abstractions  $F_1$  and  $F_2$ , we set  $F_1 \approx_{\text{Tr}} F_2$  if  $F_1 \circ \text{Tr}_m \approx_{\text{Tr}} F_2 \circ \text{Tr}_m$ , for some fresh name  $m$ .
- For closed triggered concretions  $\nu m \langle \text{Tr}_m \rangle P$  and  $\nu m \langle \text{Tr}_m \rangle Q$ , we set  $\nu m \langle \text{Tr}_m \rangle P \approx_{\text{Tr}} \nu m \langle \text{Tr}_m \rangle Q$  if  $P \approx_{\text{Tr}} Q$ .
- For open triggered agents  $A_1$  and  $A_2$  with  $\text{fv}(A_1, A_2) = \{X_1, \dots, X_n\}$ , if  $\{m_1, \dots, m_n\}$  is a set of distinct fresh names, then we set  $A_1 \approx_{\text{Tr}} A_2$  if  $A_1 \{ \text{Tr}_{m_1} / X_1, \dots, \text{Tr}_{m_n} / X_n \} \approx_{\text{Tr}} A_2 \{ \text{Tr}_{m_1} / X_1, \dots, \text{Tr}_{m_n} / X_n \}$ .

**Definition 6.5** A relation  $\mathcal{R} \subseteq \mathcal{TP}r^\circ \times \mathcal{TP}r^\circ$  is a triggered bisimulation up-to  $\approx_{\text{Tr}}$  if  $P \mathcal{R} Q$  implies, for  $m \notin \text{fn}(P, Q)$ :

1. whenever  $P \xrightarrow{\hat{\ell}} P'$ , there exists  $Q'$  s.t.  $Q \xrightarrow{\hat{\ell}} Q'$  and  $P' \approx_{\text{Tr}} \mathcal{R} \approx_{\text{Tr}} Q'$ ,
2. whenever  $P \xrightarrow{a} F$ , there exists  $G$  s.t.  $Q \xrightarrow{a} G$  and  $F \circ \text{Tr}_m \approx_{\text{Tr}} \mathcal{R} \approx_{\text{Tr}} G \circ \text{Tr}_m$ ,
3. whenever  $P \xrightarrow{\bar{a}} \nu m \langle \text{Tr}_m \rangle P'$ , there exists  $Q'$  s.t.  $Q \xrightarrow{\bar{a}} \nu m \langle \text{Tr}_m \rangle Q'$  and  $P' \approx_{\text{Tr}} \mathcal{R} \approx_{\text{Tr}} Q'$ .

**Proposition 6.6** If  $\mathcal{R}$  is a triggered bisimulation up-to  $\approx_{\text{Tr}}$ , then  $\mathcal{R} \subseteq \approx_{\text{Tr}}$ . □

## 6.1 Weak triggered and context bisimulations coincide

Next we prove that, on triggered processes,  $\approx_{\text{Tr}}$  and  $\approx_{\text{Ct}}$  coincide. First, we need some properties of  $\approx_{\text{Tr}}$ .

**Lemma 6.7**  $P \approx_{\text{Tr}} Q$  implies:

1.  $\nu x P \approx_{\text{Tr}} \nu x Q$ ;
2.  $P \mid R \approx_{\text{Tr}} Q \mid R$ .

PROOF: We only examine (2). For this, we prove that

$$\mathcal{R} = \{(\nu \tilde{y}(P_1 | R), \nu \tilde{y}(P_2 | R)) : P_1, P_2, R \in \mathcal{TP}r^\circ \text{ and } P_1 \approx_{\text{Tr}} P_2\}$$

is a  $\approx_{\text{Tr}}$ -bisimulation. Suppose  $(\nu \tilde{y}(P_1 | R), \nu \tilde{y}(P_2 | R)) \in \mathcal{R}$  and  $\nu \tilde{y}(P_1 | R) \xrightarrow{\mu} Q_1$ . We proceed by case analysis on the rule used to infer this action. If the higher-order communication rule has been used with  $P_1$  performing the output, then for some  $m$ , we have

$$P_1 \xrightarrow{\bar{a}} \nu m \langle \text{Tr}_m \rangle P'_1, \quad R \xrightarrow{a} F, \quad \mu = \tau, \quad \text{and} \quad Q_1 = \nu \tilde{y}, m(P'_1 | F \circ \text{Tr}_m).$$

Since  $P_1 \approx_{\text{Tr}} P_2$ , assuming  $m$  not free in  $P_2$  we have

$$P_2 \xRightarrow{\bar{a}} \nu m \langle \text{Tr}_m \rangle P'_2 \quad \text{with} \quad P'_1 \approx_{\text{Tr}} P'_2, \quad \text{and} \quad \nu \tilde{y}(P_2 | R) \xRightarrow{\tau} \nu \tilde{y}, m(P'_2 | F \circ \text{Tr}_m).$$

This is enough because by Lemma 6.2, we have  $P'_1, P'_2, F \circ \text{Tr}_m \in \mathcal{TP}r^\circ$ . All other cases are similar.  $\square$

**Proposition 6.8** *Let  $P, Q \in \mathcal{TP}r^\circ$ ; then  $P \approx_{\text{Ct}} Q$  implies  $P \approx_{\text{Tr}} Q$ .*

PROOF: We prove that

$$\mathcal{R} = \{(P_1, P_2) : P_1, P_2 \in \mathcal{TP}r^\circ \text{ and } P_1 \approx_{\text{Ct}} P_2\}$$

is a  $\approx_{\text{Tr}}$ -bisimulation. Let  $(P_1, P_2) \in \mathcal{R}$  and suppose that  $P_1 \xrightarrow{\mu} A$ . The only non-trivial case is when  $\mu$  is a higher-order output, so we only consider this case. Thus suppose  $\mu = \bar{a}$ ; by Lemma 6.2, then  $A = \nu m \langle \text{Tr}_m \rangle P'_1$ . By definition of  $\approx_{\text{Ct}}$  and Lemma 6.2, there exists  $P'_2$  s.t.  $P_2 \xRightarrow{\bar{a}} \nu m \langle \text{Tr}_m \rangle P'_2$  and for every  $G$  (by alpha conversion we can assume that  $m \notin \text{fn}(G)$ ):

$$\nu m (G \circ \text{Tr}_m | P'_1) \approx_{\text{Ct}} \nu m (G \circ \text{Tr}_m | P'_2). \quad (6)$$

In order to close the bisimulation we have to show that  $P'_1 \approx_{\text{Ct}} P'_2$ . Lemma 6.2 tells us that there exist  $\tilde{y}_1, \tilde{y}_2, R_1, R_2, P''_1, P''_2$  s.t.

$$P'_1 \sim_{\text{Ct}} \nu \tilde{y}_1 (P''_1 | !m. R_1), \quad P'_2 \sim_{\text{Ct}} \nu \tilde{y}_2 (P''_2 | !m. R_2) \quad (7)$$

where  $m \notin \text{fn}(R_1, R_2, P''_1, P''_2) \cup \{\tilde{y}_1\} \cup \{\tilde{y}_2\}$ . Suppose  $m'$  is a fresh name. Now, changing  $m$  for  $m'$  does not affect the equivalence among processes, i.e. we have

$$\begin{aligned} \nu \tilde{y}_1 (P''_1 | !m. R_1) &\approx_{\text{Ct}} \nu \tilde{y}_2 (P''_2 | !m. R_2) \quad \text{iff} \\ \nu \tilde{y}_1 (P''_1 | !m'. R_1) &\approx_{\text{Ct}} \nu \tilde{y}_2 (P''_2 | !m'. R_2) \end{aligned}$$

Therefore we get  $P'_1 \approx_{\text{Ct}} P'_2$  if we can prove the latter equivalence. The advantage with this is that we shall be able to exploit (6). Since  $m$  is not free in  $\nu \tilde{y}_1 (P''_1 | !m'. R_1)$  and  $\nu \tilde{y}_2 (P''_2 | !m'. R_2)$ , using the factorisation theorem (4.7) we have

$$\begin{aligned} \nu \tilde{y}_1 (P''_1 | !m'. R_1) &\approx_{\text{Ct}} \\ \nu \tilde{y}_1 \left( (P''_1 | !m'. \text{Tr}_m) \{m := R_1\} \right) &\sim_{\text{Ct}} \\ \nu m \left( !m'. \text{Tr}_m | \nu \tilde{y}_1 (P''_1 | !m. R_1) \right) &\sim_{\text{Ct}} \quad (\text{by (7)}) \\ \nu m \left( !m'. \text{Tr}_m | P'_1 \right) &\stackrel{\text{def}}{=} Q_1 \end{aligned}$$

and similarly,

$$\nu \tilde{y}_2 (P_2'' \mid !m'. R_2) \approx_{\text{Ct}} \nu m (!m'. \text{Tr}_m \mid P_2') \stackrel{\text{def}}{=} Q_2$$

Now  $Q_1 \approx_{\text{Ct}} Q_2$  can be derived from (6), for  $G \stackrel{\text{def}}{=} (X) (!m'. X)$ .  $\square$

**Proposition 6.9** *Let  $P, Q \in \mathcal{TP}r^o$ ; then  $P \approx_{\text{Tr}} Q$  implies  $P \approx_{\text{Ct}} Q$ .*

PROOF: We show that

$$\mathcal{R} = \{(P, Q) : P \approx_{\text{Tr}} Q\}$$

is a  $\approx_{\text{Ct}}$ -bisimulation up-to  $\approx_{\text{Ct}}$ . Let  $P \mathcal{R} Q$ . The most interesting case is to see how higher-order input actions of  $P$  are matched by  $Q$ , and we consider this case only. Thus, suppose  $P_1 \xrightarrow{a} F_1$ . Since  $P_1 \approx_{\text{Tr}} P_2$ , there is  $F_2$  s.t.  $P_2 \xrightarrow{a} F_2$  and, if  $m$  is fresh,

$$F_1 \circ \text{Tr}_m \approx_{\text{Tr}} F_2 \circ \text{Tr}_m. \quad (8)$$

To close the bisimulation, we have to show that for all  $C$ ,

$$F_1 \bullet C \approx_{\text{Ct}} \mathcal{R} \approx_{\text{Ct}} F_2 \bullet C \quad (9)$$

Let  $C = \nu \tilde{y} \langle Q \rangle R$ . We have

$$\begin{aligned} F_1 \bullet C &= \nu \tilde{y} (F_1 \circ Q \mid R) \approx_{\text{Ct}} \text{(by the factorisation theorem)} \\ \nu \tilde{y} ((F_1 \circ \text{Tr}_m) \{m := Q\} \mid R) &\approx_{\text{Ct}} \text{(Theorem 5.2(2))} \\ \nu \tilde{y} ((F_1 \circ \text{Tr}_m) \{m := T[[Q]]\} \mid T[[R]]) &\stackrel{\text{def}}{=} P_1' \end{aligned}$$

and, similarly,  $F_2 \bullet C \approx_{\text{Ct}} \nu \tilde{y} ((F_2 \circ \text{Tr}_m) \{m := T[[Q]]\} \mid T[[R]]) \stackrel{\text{def}}{=} P_2'$ . We have  $P_1', P_2' \in \mathcal{TP}r^o$ . From (8), since  $\approx_{\text{Tr}}$  is preserved by parallel composition and restriction, we infer  $P_1' \approx_{\text{Tr}} P_2'$ , thus concluding the proof of (9).  $\square$

**Corollary 6.10** *Relations  $\approx_{\text{Tr}}$  and  $\approx_{\text{Ct}}$  coincide on closed triggered processes.*  $\square$

Using the factorisation theorem, Corollary 6.10 can be generalised to open agents.

## 7 Normal bisimulation

The mapping to triggered agents is a useful tool for reasoning with higher-order processes. For instance, in [San92] we used the mapping as an intermediate step to define a compilation from the Higher-Order  $\pi$ -calculus to the  $\pi$ -calculus and to prove its full abstraction. In this section, we exploit the mapping to derive a characterisation of context bisimulation, called *normal bisimulation*, which does not have universal quantifications in the clauses of its definition. Normal bisimulation is not as simple as triggered bisimulation, but the former is defined on the whole class of agents of the calculus, whereas the latter is only defined on triggered agents.

The name “normal bisimulation” is to indicate that it is obtained by “normalising” the clauses of context bisimulation. Let us present the definition first; then we shall comment on it. In the following,  $\text{Ab}_m$  denotes the abstraction  $(X) !m. X$ .



**Definition 7.1 (normal bisimulation)** A relation  $\mathcal{R} \subseteq \mathcal{P}r^o \times \mathcal{P}r^o$  is a normal simulation if  $P \mathcal{R} Q$  implies, for  $m \notin \text{fn}(P, Q)$ :

1. whenever  $P \xrightarrow{\ell} P'$ , there exists  $Q'$  s.t.  $Q \xrightarrow{\widehat{\ell}} Q'$  and  $P' \mathcal{R} Q'$ ;
2. whenever  $P \xrightarrow{a} F$ , there exists  $G$  s.t.  $Q \xrightarrow{a} G$  and  $F \circ \text{Tr}_m \mathcal{R} G \circ \text{Tr}_m$ ;
3. whenever  $P \xrightarrow{\bar{a}} C$ , there exists  $D$  s.t.  $Q \xrightarrow{\bar{a}} D$  and  $C \bullet \text{Ab}_m \mathcal{R} D \bullet \text{Ab}_m$ .

A relation  $\mathcal{R}$  is a normal bisimulation, briefly  $\approx_{Nr}$ -bisimulation, if  $\mathcal{R}$  and  $\mathcal{R}^{-1}$  are normal simulations. We say that  $P$  and  $Q$  are normal bisimilar, briefly  $P \approx_{Nr} Q$ , if  $P \mathcal{R} Q$ , for some  $\approx_{Nr}$ -bisimulation  $\mathcal{R}$ .

As for triggered bisimulation, we should stress that in clauses (2) and (3) of Definition 7.1 it is enough to pick *some* fresh name  $m$ , since the specific choice of the fresh name does not affect the equivalence of the resulting processes. The extension of  $\approx_{Nr}$  to abstractions and open agents is defined as for triggered bisimulation, therefore only employing fresh triggers.

The idea of normal bisimulation comes from the results on the discriminating power given by triggers and shown in Sections 4 and 6. To test the equivalence between  $F \stackrel{\text{def}}{=} a.(X)P$  and  $G \stackrel{\text{def}}{=} a.(X)Q$  we do not have to try *every* applications  $F \bullet C$  and  $G \bullet C$ , but it is enough to verify that  $P$  and  $Q$  are equivalent when  $X$  is instantiated with a *single* (fresh) trigger  $\text{Tr}_m$ . In fact, by the congruence properties of  $\approx_{Ct}$ ,

$$F \circ \text{Tr}_m = P\{\text{Tr}_m/X\} \approx_{Ct} Q\{\text{Tr}_m/X\} = G \circ \text{Tr}_m$$

implies that for any  $\tilde{y}, R$  and  $T$

$$\nu \tilde{y} \left( (P\{\text{Tr}_m/X\}) \{m := R\} | T \right) \approx_{Ct} \nu \tilde{y} \left( (Q\{\text{Tr}_m/X\}) \{m := R\} | T \right); \quad (10)$$

then, since by the factorisation theorem

$$(P\{\text{Tr}_m/X\}) \{m := R\} \approx_{Ct} P\{R/X\} \quad \text{and} \quad (Q\{\text{Tr}_m/X\}) \{m := R\} \approx_{Ct} Q\{R/X\},$$

from (10), if  $C \stackrel{\text{def}}{=} \nu \tilde{y} \langle R \rangle T$ , we get

$$F \bullet C = \nu \tilde{y} (P\{R/X\} | T) \approx_{Ct} \nu \tilde{y} (Q\{R/X\} | T) = G \bullet C.$$

Similar reasoning can be used to justify the clause of normal bisimulation on output actions, w.r.t. that of context bisimulation. It may be useful however to see why in the requirement of this clause, namely

$$C \bullet \text{Ab}_m \approx_{Nr} C \bullet \text{Ab}_m$$

the abstraction  $\text{Ab}_m$  cannot be made simpler without losing discriminating power. We recall that if  $C \stackrel{\text{def}}{=} \nu \tilde{x} \langle P_1 \rangle P_2$  and  $D \stackrel{\text{def}}{=} \nu \tilde{y} \langle Q_1 \rangle Q_2$ , then  $C \bullet \text{Ab}_m \approx_{Nr} D \bullet \text{Ab}_m$  means that

$$\nu \tilde{x} (P_2 | !m.P_1) \approx_{Nr} \nu \tilde{y} (Q_2 | !m.Q_1) \quad (11)$$

We show that both the guard on  $m$  and the replication are necessary. First of all, we need the guard  $m$  on  $P_1$  and  $Q_1$ : For, otherwise, if  $R$  is a process like  $!n.0$ , which can perform

an unbounded number of identical visible actions, then the processes  $\bar{a}.(R)\mathbf{0}$  and  $\bar{a}.\langle\mathbf{0}\rangle R$  would be made equivalent. But they are not context bisimilar; for instance they can be distinguished when interacting with a process like  $a.(X)\mathbf{0}$  which discharges what it receives at  $a$ .

Now, the use of replication. Suppose we eliminate it from (11). Then let  $R \stackrel{\text{def}}{=} n.n.\bar{p}.\mathbf{0}|\bar{n}.\mathbf{0}$  and consider  $P \stackrel{\text{def}}{=} \nu n (\bar{a}.\langle R\rangle\mathbf{0})$  and  $Q \stackrel{\text{def}}{=} \bar{a}.\langle\mathbf{0}\rangle\mathbf{0}$ . Again,  $P$  and  $Q$  would become equivalent since  $\nu n (\mathbf{0} | m.R)$  and  $\mathbf{0} | m.\mathbf{0}$  are indistinguishable. However  $P$  and  $Q$  can be differentiated when interacting with a process like  $a.(X)(X | X)$  which makes two copies of the input run in parallel, since then the action  $c$  of  $R$  can be observed. In fact, only in the *linear* calculus, where in each expression a variable may occur free at most once, the replication in (11) can be avoided.

We now prove formally that  $\approx_{\text{Ct}}$  and  $\approx_{\text{Nr}}$  coincide. The inclusion  $\approx_{\text{Ct}} \subseteq \approx_{\text{Nr}}$  is obvious, since the requirements in the definition of  $\approx_{\text{Nr}}$  are a subset of those in the definition of  $\approx_{\text{Ct}}$ . To prove the opposite inclusion, we use Lemma 7.2 below, which relates weak transitions of  $P$  and  $T[P]$ . Its proof is obtained using the operational correspondence on strong transitions in Lemma 5.5.

**Lemma 7.2**

1. if  $P \xrightarrow{\mu} A$ , then  $T[P] \xrightarrow{\mu} \approx_{\text{Tr}} T[A]$ ;
2. The converse: If  $T[P] \xrightarrow{\mu} A'$ , then there is  $A$  s.t.  $P \xrightarrow{\mu} A$  and  $A' \approx_{\text{Tr}} T[A]$ .  $\square$

**Lemma 7.3** For all concretions  $C$ , if  $m$  is fresh, then  $T[C] = \nu m \langle \text{Tr}_m \rangle T[C \bullet \text{Ab}_m]$ .  $\square$

**Theorem 7.4** Relations  $\approx_{\text{Nr}}$  and  $\approx_{\text{Ct}}$  coincide on  $\text{Pr}^\circ \times \text{Pr}^\circ$ .

PROOF: We only have to prove the inclusion  $\approx_{\text{Nr}} \subseteq \approx_{\text{Ct}}$ . For this, we show that

$$\mathcal{R} = \{(T[P], T[Q]) : P \approx_{\text{Nr}} Q\}$$

is a  $\approx_{\text{Tr}}$ -bisimulation up-to  $\approx_{\text{Tr}}$  (Definition 6.5). This is enough, because on triggered processes  $\approx_{\text{Tr}}$  and  $\approx_{\text{Ct}}$  coincide (Corollary 6.10), and  $\mathcal{T}$  respects  $\approx_{\text{Ct}}$  (Theorem 5.2(2)): Hence from  $T[P] \approx_{\text{Tr}} T[Q]$ , we can infer  $P \approx_{\text{Ct}} Q$ . Let  $(T[P], T[Q]) \in \mathcal{R}$ ; the only non-trivial case is to show how higher-order output actions of  $T[P]$  are matched by  $T[Q]$ .

Suppose  $T[P] \xrightarrow{\bar{a}} C'$ . By Lemma 7.2(2),  $C$  exists s.t.

$$P \xrightarrow{\bar{a}} C \text{ and } C' \approx_{\text{Tr}} T[C].$$

Since  $P \approx_{\text{Nr}} Q$ , there exist  $D$  s.t.  $Q \xrightarrow{\bar{a}} D$  and

$$D \bullet \text{Ab}_m \approx_{\text{Nr}} C \bullet \text{Ab}_m. \tag{12}$$

Further, by Lemma 7.2(1),

$$T[Q] \xrightarrow{\bar{a}} D' \approx_{\text{Tr}} T[D].$$

By Lemma 7.3,  $T[C] = \nu m \langle \text{Tr}_m \rangle T[C \bullet \text{Ab}_m]$  and  $T[D] = \nu m \langle \text{Tr}_m \rangle T[D \bullet \text{Ab}_m]$ . Summarising, we have:

$$\begin{aligned} T[P] &\xrightarrow{\bar{a}} \approx_{\text{Tr}} \nu m \langle \text{Tr}_m \rangle T[C \bullet \text{Ab}_m] \\ T[Q] &\xrightarrow{\bar{a}} \approx_{\text{Tr}} \nu m \langle \text{Tr}_m \rangle T[D \bullet \text{Ab}_m] \end{aligned}$$

and, since by (12),  $C \bullet \text{Ab}_m \approx_{\text{Nr}} D \bullet \text{Ab}_m$ , we have  $T[C \bullet \text{Ab}_m] \mathcal{R} T[D \bullet \text{Ab}_m]$ . This is enough, because  $\mathcal{R}$  is a  $\approx_{\text{Tr}}$ -bisimulation up-to  $\approx_{\text{Tr}}$ .  $\square$

Theorem 7.4 can be extended to open agents, thus obtaining:

**Corollary 7.5** *Relations  $\approx_{\text{Nr}}$  and  $\approx_{\text{Ct}}$  coincide on  $\mathcal{A}g \times \mathcal{A}g$ .*  $\square$

**Remark 7.6** Now that we have proved that  $\approx_{\text{Ct}}$  and  $\approx_{\text{Nr}}$  coincide, one might wonder why we did not introduce  $\approx_{\text{Nr}}$  directly. The reason is that we would have needed  $\approx_{\text{Ct}}$  to prove the congruence of  $\approx_{\text{Nr}}$  over parallel composition. Moreover, it is easier to convince ourselves of the naturalness of context bisimulation; we can then accept normal bisimulation as a simpler characterisation of the former.  $\square$

## 7.1 Late and early equivalences

As pointed out in Remark 3.12, the formulation of weak context bisimulation in Definition 3.11 is in the *late* style. In the early style, the order of quantifiers in the input and output clauses is reversed. Thus,  $\mathcal{R}$  is an *weak early context bisimulation* if  $P \mathcal{R} Q$  implies:

1. whenever  $P \xrightarrow{\ell} P'$ , there exists  $Q'$  s.t.  $Q \xrightarrow{\widehat{\ell}} Q'$  and  $P' \mathcal{R} Q'$ ;
2. whenever  $P \xrightarrow{a} F$ , then for all concretions  $C$  there exists  $G$  s.t.  $Q \xrightarrow{a} G$  and  $C \bullet F \mathcal{R} C \bullet G$ ;
3. whenever  $P \xrightarrow{\bar{a}} C$ , then for all abstractions  $F$  there exists  $D$  s.t.  $Q \xrightarrow{\bar{a}} D$  and  $F \bullet C \mathcal{R} F \bullet D$ .

We denote the largest weak early context bisimulation by  $\approx_{\text{Ct}}^E$ . Hansen and Kleist [HK94] have proved that late and early strong context bisimulations coincide on an asynchronous variant of Plain CHOCS. The theory developed in this paper yields a straightforward proof of the result for the weak equivalences.

**Corollary 7.7** *Relations  $\approx_{\text{Ct}}$  and  $\approx_{\text{Ct}}^E$  coincide.*

PROOF: Clearly,  $\approx_{\text{Ct}} \subseteq \approx_{\text{Ct}}^E$ : Every late bisimulation is an early bisimulation. On the other hand, an early bisimulation is also a normal bisimulation: Hence, by Corollary 7.5,  $\approx_{\text{Ct}}^E \subseteq \approx_{\text{Ct}}$ .  $\square$

By contrast, late and early bisimulations are usually different in first-order calculi like the  $\pi$ -calculus [MPW92, PS93]. Note also that in normal and triggered bisimulation the late vs. early issue disappears, for the responsible quantifiers are absent.

## 8 Extensions

In this section, we discuss the extension of the theory presented to a richer calculus, namely the Higher-Order  $\pi$ -calculus, briefly  $\text{HO}\pi$  [San92]. We shall focus on the higher-order fragment of  $\text{HO}\pi$ , thus ignoring first-order features like communication of names and abstraction over names. (This fragment of)  $\text{HO}\pi$  can be thought of as an  $\omega$ -order extension of the process-passing language in Section 2. Also, we shall stick to a monadic calculus (only one agent can be transmitted at a time); polyadicity can be easily accommodated.

In the CHOCS-like language of Section 2, only processes can be passed around. In  $\text{HO}\pi$ , besides processes, abstractions can be passed too. Moreover, abstractions can be of arbitrary high order. We explain what the order of an abstraction is. All abstractions seen so far are of the form  $(X)P$ , where  $X$  is a process variable which may appear in  $P$ . If  $*$  is taken to be the type of all processes, then from a function-theoretic point of view,  $(X)P$  has type  $* \rightarrow *$ , for  $(X)P$  takes a process and returns a process. We shall say that  $(X)P$  is a second-order abstraction (first-order abstractions being abstractions on names, as found in the full  $\text{HO}\pi$ ). The syntax of  $\text{HO}\pi$  allows agent-variables, rather than just process-variables, and an application construct  $A_1 \circ A_2$  (sometimes written  $A_1 \langle A_2 \rangle$  in the literature); thus, one can write meaningful abstractions of order greater than two. An example is

$$G \stackrel{\text{def}}{=} (Y) (Q \mid Y \circ Q)$$

$G$  takes an agent of type  $* \rightarrow *$  and yields back a process. Therefore  $G$  has type  $(* \rightarrow *) \rightarrow *$ . Abstraction  $G$  has order three, the order being determined by the level of arrow-nesting in the type. If  $F$  is  $(X) (P \mid X)$ , then  $G \circ F$  yields  $Q \mid F \circ Q = Q \mid P \mid Q$ .

In the same way, we can construct fourth-order abstractions, fifth-order abstractions and so forth. In this sense  $\text{HO}\pi$  is a  $\omega$ -order calculus: There is no bound on the order of agents which can be written and communicated. Abstractions can also be passed around, like in

$$P \stackrel{\text{def}}{=} (\bar{a}. \langle G \rangle \bar{b}. \langle F \rangle \mathbf{0}) \mid a. (Z) b. (Y) (Z \circ Y)$$

where  $G$  and  $F$  are the abstractions above defined, and then we have (garbage-collecting  $\mathbf{0}$  processes)

$$P \xrightarrow{\tau} \xrightarrow{\tau} G \circ F = Q \mid P \mid Q.$$

There is also a type discipline on names, so to avoid disagreement on what is carried or expected at a given name. For instance, in process  $P$  above name  $a$  would have type  $(* \rightarrow *) \rightarrow *$  and name  $b$  type  $* \rightarrow *$ . Types are extended to concretions in the expected way: If  $G$  has type  $T$ , then  $\langle G \rangle P$  has type  $T \rightarrow *$ . The pseudo-application  $\bullet$  and the application  $\circ$  are only allowed between agents of compatible types. Below, we write  $A : T$  if  $A$  has type  $T$ .

The definition of context bisimulation can be easily extended to  $\text{HO}\pi$  — one just have to take into account the type informations. More interesting is to see how to extend the definitions of trigger bisimulation and normal bisimulation. We shall briefly consider the latter. We first have to understand what is the appropriate notion of trigger in  $\text{HO}\pi$ . A trigger of type  $T = S \rightarrow *$  at  $m$  is

$$\text{Tr}_m^T \stackrel{\text{def}}{=} (X) \bar{m}. \langle X \rangle \mathbf{0}$$

where  $X$  is a variable of type  $S$ . Thus a trigger takes an argument  $X$  and export it in the action at  $m$ . The appearance and the use of the argument  $X$  is the new ingredient w.r.t. the triggers of Section 4.2. Dually, for  $T = (S \rightarrow *) \rightarrow *$ , the abstraction  $\text{Ab}_m$  used in the output clause of normal bisimulation becomes

$$\text{Ab}_m^T \stackrel{\text{def}}{=} (Y) !m.(X)(Y \circ X)$$

where  $Y$  has type  $S \rightarrow *$  and  $X$  has type  $S$ . Note that  $\text{Tr}_m^T : T$  and  $\text{Ab}_m^T : T$ .

Thus, the requirements of normal bisimulation over  $\text{HO}\pi$  processes are the following:  $P \approx_{\text{Nr}} Q$  implies:

- Whenever  $P \xrightarrow{\ell} P'$ , there exists  $Q'$  s.t.  $Q \xRightarrow{\widehat{\ell}} Q'$  and  $P' \approx_{\text{Nr}} Q'$ ;
- for all types  $T = S \rightarrow *$ , whenever  $P \xrightarrow{a} F$  with  $F : T$ , there exists  $G : T$  s.t.  $Q \xRightarrow{a} G$  and  $F \circ \text{Tr}_m^S \approx_{\text{Nr}} G \circ \text{Tr}_m^S$ , for some fresh  $m$ ;
- for all  $T$ , whenever  $P \xrightarrow{\bar{a}} C$  with  $C : T$ , there exists  $D : T$  s.t.  $Q \xRightarrow{\bar{a}} D$  and  $C \bullet \text{Ab}_m^T \approx_{\text{Nr}} D \bullet \text{Ab}_m^T$ , for some fresh  $m$ .

In this way, normal bisimulation and context bisimulation coincide over  $\text{HO}\pi$  processes (for details, see [San92]).

## 9 Conclusions and future work

In this paper we have considered a few bisimilarity equivalences for higher-order process calculi, in particular context bisimulation and normal bisimulation. Normal bisimulation specifies some minimum requirements, or tests, on higher-order actions and hence provides us with a useful mathematical tool to verify agent equivalences; the characterisation in terms of context bisimulation gives us a measure of the power of such tests and reinforces the naturalness of the equivalence. We have isolated a subclass of agents, called *triggered agents*, which represent some sort of “normal form” for agents. We have shown that on triggered agents context and normal bisimulations coincide with a third one, namely triggered bisimulation, which is the simplest of the bisimilarities examined.

In [San92] we also compare context bisimulation and normal bisimulation with barbed congruence, introduced in [MS92] as a tool to *uniformly* define bisimulation-based equivalences in different calculi. It is shown in [San92] that under certain conditions on the syntax of the calculus, barbed congruence coincides with the “early non-delay” version of context bisimulation. In this paper, we have chosen the “late delay” schema because it seems more appropriate with an operational semantics based on the abstraction and concretion constructs.

At a first glance, it appears surprising that the bisimilarity clauses of normal bisimulation and triggered bisimulation contain no form of universal quantifications on matching actions. For instance, in normal bisimulation to see whether an input action  $P \xrightarrow{a} F$  is matched by the input action  $Q \xrightarrow{a} G$ , it suffices to examine the derivative abstractions  $F$  and  $G$  on a *single* argument (a trigger): This guarantees that  $F$  and  $G$  are equivalent on *all*

possible arguments. This simplicity contrasts with what happens in first-order calculi like the  $\pi$ -calculus, where the bisimilarity clauses require the examination of different arguments (as in late and early bisimulations [MPW92, PS93]) or, at least, impose multiple name instantiations (as in open bisimulation [San93]). But the extra complexity of first-order calculi can be understood and justified with their greater expressiveness: See for instance [San95], where it is shown that the expressiveness of a Plain CHOCS-like calculus is the same as that of a sublanguage of the  $\pi$ -calculus obeying some strong syntactic constraints.

It would be interesting to know at which extent the results presented here depend upon the choice of operators in our higher-order calculus. An important condition seems to be that context bisimulation be a congruence relation — the factorisation theorem would fail otherwise. Thus, for instance, adding summation in an unconstrained way would be dangerous; guarded summation, however, would be acceptable [San92].

We hope that the results presented, e.g., the factorisation theorem and normal bisimulation, can contribute to the development of a manageable and solid theory for higher-order process calculi. In [San92], we used these results to prove the full abstraction of a compilation from  $\text{HO}\pi$  to  $\pi$ -calculus. As argued in [San92, Hen93], the possibility of encoding higher-order calculi at first order does not mean that the former are superfluous: Higher-order constructs arise in many applications, for instance operating systems, and it is advantageous to be able to use them and to reason with them *explicitly*, i.e. not via an encoding.

The factorisation theorem and normal bisimulation might also be useful on first-order or mobility-free calculi, to develop a theory of equality of contexts or of open terms. Progress in this direction has been made by Peter Sewell [Sew95], using techniques related to ours.

We have compared the weak delay versions of context bisimulation, normal bisimulation and triggered bisimulation. We believe that similar results can be obtained for the *branching* [GW89] versions of these bisimilarities. We do not know at present if they could be established also for the *strong* versions of the equivalences, where  $\tau$ -actions have the same weight as visible actions. Indeed, some of our central technical results, like the factorisation theorem and the full abstraction of the mapping to triggered agents, are only true in the weak case.

We have carried out an operational study of higher-order process calculi. The denotational approach has been investigated by Matthew Hennessy [Hen93], who has given a model for (a slight variant of) Thomsen's CHOCS [Tho90]. The model is constructed from a domain equation and is proved to be fully abstract w.r.t. a notion of may testing. The language CHOCS differs from Plain CHOCS, and hence from the calculus used in this paper, in an important aspect: in CHOCS the restriction operator is a *dynamic* binder, whereas in the latter calculi it is a *static* binder. To see the difference, suppose  $b$  is a name which occurs free in  $P$  and  $Q$ . Having static binding, an interaction between  $a.(X)X$  and  $\nu b(\bar{a}. \langle P \rangle Q)$  is

$$(a.(X)X) \mid \nu b(\bar{a}. \langle P \rangle Q) \xrightarrow{\tau} \nu b(P \mid Q)$$

and a scope extrusion of  $\nu b$  accompanies the movement of process  $P$ . By contrast, having dynamic binding we would get:

$$(a.(X)X) \mid \nu b(\bar{a}. \langle P \rangle Q) \xrightarrow{\tau} P \mid \nu b Q.$$

Note that the reduction has destroyed the privacy of the  $b$ -link between  $P$  and  $Q$ : The free occurrences of  $b$  in  $P$  have evaded the restriction which embraced them. The main advantage of dynamic binding is an easier semantics (operationally and denotationally); but static binding facilitates the analysis of a program from its text. Our and Hennessy's works are tailored to the specific discipline chosen for restriction. In both cases, it appears that the theory developed would not support a different discipline.

It would be interesting to understand whether our formulations of bisimulation for higher-order calculi can be expressed within the framework of generalised algebraic specifications of Astesiano et al. [AGR92]. They introduce algebraic structures called *observational structures* — roughly first-order signatures equipped with a notion of observation — where processes can be treated as data. Most important, observational structures can be assigned a modal logic to describe observational properties of terms of the structures. The comparison with [AGR92] could shed lights on how to define modal logics for the bisimulations which we have analysed.

## References

- [AD95] R. Amadio and M. Dam. Reasoning about higher-order processes. In Proceedings of TAPSOFT '95, volume 715 of *Lecture Notes in Computer Science*. Springer Verlag, 1995. To appear.
- [AGR88] E. Astesiano, A. Giovini, and G. Reggio. Generalized bisimulation in relational specifications. In *STACS '88*, volume 294 of *Lecture Notes in Computer Science*, pages 207–226. Springer Verlag, 1988.
- [AGR92] E. Astesiano, A. Giovini, and G. Reggio. Observational structures and their logic. *Theoretical Computer Science*, 96:249–283, 1992.
- [Ama93] R. Amadio. On the reduction of CHOCS bisimulation to  $\pi$ -calculus bisimulation. In E. Best, editor, *Proceedings of CONCUR '93*, volume 715. Springer Verlag, 1993.
- [BCHK94] G. Boudol, I. Castellani, M. Hennessy, and A. Kiehn. A theory of processes with localities. *Formal Aspects of Computing*, 6:165–200, 1994.
- [Bou89] G. Boudol. Towards a lambda calculus for concurrent and communicating systems. In *TAPSOFT '89*, volume 351 of *Lecture Notes in Computer Science*, pages 149–161, 1989.
- [CH89] I. Castellani and M. Hennessy. Distributed bisimulation. *JACM*, 10(4):887–911, 1989.
- [GMP89] A. Giacalone, P. Mishra, and S. Prasad. FACILE, a symmetric integration of concurrent and functional programming. In J. Diaz and F. Orejas, editors, *TAPSOFT '89*, volume 352 of *Lecture Notes in Computer Science*, pages 189–209. Springer Verlag, 1989.

- [GW89] R.J. van Glabbeek and W. P. Weijland. Branching time and abstraction in bisimulation semantics. In G.X. Ritter, editor, *Information Processing '89*, pages 613–618. Elsevier Science, 1989.
- [Hen93] M. Hennessy. A fully abstract denotational model for higher-order processes. In *8th LICS Conf.* IEEE Computer Society Press, 1993.
- [HK94] M. Hansen and J. Kleist. Process calculi with asynchronous communication. Master thesis, Aalborg University (supervisor Hans Hüttel.), August 1994.
- [Hol83] S. Holmstrom. PFL: A functional language for parallel programming. In *Declarative Programming Workshop*, Programming Methodology Group, Chalmers University of Technology, University of Goteborg, Sweden, 1983.
- [KS85] J.R. Kennaway and M.R. Sleep. Syntax and informal semantics of DyNe, a parallel language. In *Proceedings of the Workshop on the Analysis of Concurrent Systems 1983*, volume 207 of *Lecture Notes in Computer Science*, pages 222–230. Springer Verlag, 1985.
- [Let92] L. Leth. *Functional Programs as Reconfigurable Networks of Communicating Processes*. PhD thesis, Imperial College, London University, 1992.
- [Mil80] R. Milner. *A Calculus of Communicating Systems*, volume 92 of *Lecture Notes in Computer Science*. Springer Verlag, 1980.
- [Mil91] R. Milner. The polyadic  $\pi$ -calculus: a tutorial. Technical Report ECS-LFCS-91-180, LFCS, Dept. of Comp. Sci., Edinburgh Univ., October 1991. Also in *Logic and Algebra of Specification*, ed. F.L. Bauer, W. Brauer and H. Schwichtenberg, Springer Verlag, 1993.
- [MPW92] R. Milner, J. Parrow, and D. Walker. A calculus of mobile processes, (Parts I and II). *Information and Computation*, 100:1–77, 1992.
- [MS92] R. Milner and D. Sangiorgi. Barbed bisimulation. In W. Kuich, editor, *19th ICALP*, volume 623 of *Lecture Notes in Computer Science*, pages 685–695. Springer Verlag, 1992.
- [Nie] O. Nierstrasz. Towards an object calculus. In M. Tokoro, O. Nierstrasz, P. Wegner, and A. Yonezawa, editors, *ECOOP '91 Workshop on Object Based Concurrent Programming*, Geneva, Switzerland, 1991, volume 612 of *Lecture Notes in Computer Science*. Springer Verlag.
- [Nie89] F. Nielson. The typed  $\lambda$ -calculus with first-class processes. In *Proc. PARLE '89*, volume 366 of *Lecture Notes in Computer Science*. Springer Verlag, 1989.
- [Par81] D.M. Park. Concurrency on automata and infinite sequences. In P. Deussen, editor, *Conf. on Theoretical Computer Science*, volume 104 of *Lecture Notes in Computer Science*. Springer Verlag, 1981.



- [PS93] J. Parrow and D. Sangiorgi. Algebraic theories for name-passing calculi. Technical Report ECS–LFCS–93–262, LFCS, Dept. of Comp. Sci., Edinburgh Univ., 1993. To appear in *Information and Computation*.
- [San92] D. Sangiorgi. *Expressing Mobility in Process Algebras: First-Order and Higher-Order Paradigms*. PhD thesis CST–99–93, Department of Computer Science, University of Edinburgh, 1992.
- [San93] D. Sangiorgi. A theory of bisimulation for the  $\pi$ -calculus. Technical Report ECS–LFCS–93–270, LFCS, Dept. of Comp. Sci., Edinburgh Univ., 1993. Extended Abstract in *Proc. CONCUR '93*, volume 715 of Lecture Notes in Computer Science, Springer Verlag.
- [San95] D. Sangiorgi. Internal mobility and agent-passing calculi. To appear in Proc. ICALP 95, Lecture Notes in Computer Science, Springer Verlag.
- [Sew95] P. Sewell. Forthcoming PhD thesis, Dep. Computer Science, University of Edinburgh, 1995.
- [SY85] R.E. Strom and S. Yemini. The NIL distributed systems programming language: a status report. In *Seminar on Concurrency 1984*, volume 197 of *Lecture Notes in Computer Science*, pages 512–523. Springer Verlag, 1985.
- [Tho90] B. Thomsen. *Calculi for Higher Order Communicating Systems*. PhD thesis, Department of Computing, Imperial College, 1990.
- [Tho93] B. Thomsen. Plain CHOCS. *Acta Informatica*, 30:1–59, 1993.
- [Wei89] W.P. Weijland. *Synchrony and Asynchrony in Process Algebra*. PhD thesis, University of Amsterdam, 1989.

## A Proof of Proposition 3.8

In this appendix we prove Proposition 3.8, that is that  $\sim_{Ct}$  is a congruence on substitutions. To prove the general result, we first have to prove some instances of it (Lemma A.2).

**Lemma A.1** *If  $!P \xrightarrow{\mu} A$ , then  $A \sim_{Ct} A_1 \mid !P$ , for some  $A_1$  s.t.  $P \mid P \xrightarrow{\mu} A_1$ .*

PROOF: By transition induction. □

**Lemma A.2** *For all  $P_1, P_2, R \in \mathcal{Pr}$ ,  $P_1 \sim_{Ct} P_2$  implies*

1.  $\nu x P_1 \sim_{Ct} \nu x P_2$ ;
2.  $P_1 \mid R \sim_{Ct} P_2 \mid R$ ;
3.  $!P_1 \sim_{Ct} !P_2$ .

PROOF: We only examine (2) and (3). It is enough to prove the result on closed expressions. For (2), we show that

$$\mathcal{R} = \{(\nu \tilde{x}(P_1 | R), \nu \tilde{x}(P_2 | R)) : P_1 \sim_{\text{Ct}} P_2\} \cup \sim_{\text{Ct}}$$

is a bisimulation up-to  $\sim_{\text{Ct}}$ . Take  $(\nu \tilde{x}(P_1 | R), \nu \tilde{x}(P_2 | R)) \in \mathcal{R}$  with  $P_1 \sim_{\text{Ct}} P_2$  and suppose that  $\nu \tilde{x}(P_1 | R) \xrightarrow{\mu} A$ . There are five cases to consider:

**Case 1**  $P_1 \xrightarrow{\mu} A_1$  and  $A = \nu \tilde{x}(A_1 | R)$ .

Use the definition of  $\sim_{\text{Ct}}$  and simple algebraic manipulations with the laws of Lemma 3.5.

**Case 2**  $R \xrightarrow{\mu} A_1$  and  $A = \nu \tilde{x}(P_1 | A_1)$ .

Similar to case 1.

**Case 3**  $P_1 \xrightarrow{\bar{a}} C_1$ ,  $R \xrightarrow{a} F$ ,  $\mu = \tau$  and  $A = \nu \tilde{x}(C_1 \bullet F)$ .

By definition of  $\sim_{\text{Ct}}$ ,  $P_1 \sim_{\text{Ct}} Q_1$  and  $P_1 \xrightarrow{\bar{a}} C_1$  imply that  $P_2 \xrightarrow{\bar{a}} C_2$  and  $G \bullet C_1 \sim_{\text{Ct}} G \bullet C_2$ , for all  $G$ . Using this and assertion (1) of this lemma, we infer  $\nu \tilde{x}(P_2 | R) \xrightarrow{\tau} \nu \tilde{x}(C_2 \bullet F) \sim_{\text{Ct}} \nu \tilde{x}(C_1 \bullet F)$ , which closes up the bisimulation.

**Case 4**  $P_1 \xrightarrow{a} F_1$ ,  $R \xrightarrow{a} C$ ,  $\mu = \tau$  and  $A = \nu \tilde{x}(F_1 \bullet C)$ .

Similar to case 3.

**Case 5** Interaction between  $P_1$  and  $R$  along a first-order name.

This case is simpler than cases (3) and (4).

Now, assertion (3) of the lemma. We show that

$$\mathcal{R} \stackrel{\text{def}}{=} \{(!P | R, !Q | R) : P \sim_{\text{Ct}} Q\}$$

is a  $\sim_{\text{Ct}}$ -bisimulation up-to  $\sim_{\text{Ct}}$ . We first notice that since  $\sim_{\text{Ct}}$  is transitive and preserved by parallel composition (assertion (2) of this lemma),  $P \sim_{\text{Ct}} Q$  implies  $P | P \sim_{\text{Ct}} Q | Q$ .

We show that  $!Q | R$  can match any action from  $!P | R$ , say  $!P | R \xrightarrow{\mu} T_1$ . We only examine the case in which  $\mu$  is a silent action and originates from  $!P$ ; the remaining cases are similar. Thus, assume  $T_1 = P_1 | R$ , for some  $P_1$  s.t.  $!P \xrightarrow{\tau} P_1$ . By Lemma A.1, if  $!P \xrightarrow{\tau} P_1$  then  $P_1 \sim_{\text{Ct}} P'_1 | !P$ , for some  $P'_1$  s.t.  $P | P \xrightarrow{\tau} P'_1$ . Since  $Q | Q \sim_{\text{Ct}} P | P$ , there is  $Q'_1$  s.t.  $Q | Q \xrightarrow{\tau} Q'_1 \sim_{\text{Ct}} P'_1$ ; therefore,  $!Q \xrightarrow{\tau} \sim_{\text{Ct}} !Q | Q'_1$ . From these facts, and using the congruence of  $\sim_{\text{Ct}}$  for parallel composition, we have:

$$\begin{array}{l} !P | R \xrightarrow{\tau} \sim_{\text{Ct}} !P | P'_1 | R \\ !Q | R \xrightarrow{\tau} \sim_{\text{Ct}} !Q | P'_1 | R. \end{array}$$

This is enough, since  $(!P | P'_1 | R, !Q | P'_1 | R) \in \mathcal{R}$ .  $\square$

We say that a variable  $X$  is *guarded* in an agent  $A$  if  $X$  only occurs in subexpressions of  $A$  of the form  $\mu.B$ .

**Lemma A.3** *Suppose that  $fv(P) = \{X\}$  and that  $X$  is guarded in  $P$ . Then, for all  $Q$ ,*

1. *If  $P \xrightarrow{\mu} A$ , then  $P\{Q/X\} \xrightarrow{\mu} A\{Q/X\}$ ;*

2. If  $P\{Q/X\} \xrightarrow{\mu} A'$ , then there is  $A$  s.t.  $P \xrightarrow{\mu} A$  and  $A' = A\{Q/X\}$ .

PROOF: By transition induction.  $\square$

**Lemma A.4** Suppose  $X_1 \in fv(P_1)$  and that  $X_2 \notin fv(P_1)$ . Then there exists  $P_2$  such that

1.  $X_1$  is guarded in  $P_2$ ;
2.  $P_2\{X_1/X_2\} = P_1$ ;
3. if  $Q_1 \sim_{ct} Q_2$ , then  $P_1\{Q_1/X_1\} \sim_{ct} P_2\{Q_1/X_1, Q_2/X_2\}$ .

PROOF: Induction on the structure of  $P_1$ . The basis of the induction occurs when  $P_1 = X$  or  $P_1 = \mu.A$ . If  $P_1 = X$ , then define  $P_2 \stackrel{\text{def}}{=} P_1$  if  $X \neq X_1$  and  $P_2 \stackrel{\text{def}}{=} X_2$  if  $X = X_1$ ; if  $P_1 = \mu.A$ , then take  $P_2 \stackrel{\text{def}}{=} P_1$ .

When  $P_1$  is a process of the form  $R_1 \mid R_2$ ,  $\nu x R$ , or  $!R$ , use the inductive hypothesis on the  $R$  or  $R_i$ 's and (to prove assertion (3)) Lemma A.2.  $\square$

We are now ready to prove Proposition 3.8. We first recall its assertion.

**Proposition 3.8** Let  $P_1, P_2, A \in Ag$ ; then  $P_1 \sim_{ct} P_2$  implies  $A\{P_1/X\} \sim_{ct} A\{P_2/X\}$ .

PROOF: It suffices to prove the result for closed processes. Consider the set  $\mathcal{R}$  of all pairs of the form:

$$\left( P\{Q_2/X_1\}, P\{Q_1/X_1\} \right), \text{ with } Q_2 \sim_{ct} Q_1 \text{ and } X_1 \text{ guarded in } P.$$

We prove the following facts:

- (a)  $\mathcal{R}$  is a  $\sim_{ct}$ -bisimulation up-to  $\sim_{ct}$ ;
- (b) for all  $R$  with  $fv(R) \subseteq \{X\}$ , if  $Q_1 \sim_{ct} Q_2$ , then

$$R\{Q_2/X\} \mathcal{R} \sim_{ct} R\{Q_1/X\}.$$

Then the assertion of the proposition follows from (a) and (b) by transitivity of  $\sim_{ct}$ .

We first prove (b). We can apply Lemma A.4 to  $R$ ; let  $R'$  be the process returned. We have

$$\begin{aligned} R\{Q_2/X_1\} &= \text{(by Lemma A.4(2))} & (13) \\ R'\{Q_2/X_1, Q_2/X_2\} \mathcal{R} & \text{(} X_1 \text{ is guarded in } R' \text{ by Lemma A.4(1))} \\ R'\{Q_1/X_1, Q_2/X_2\} & \sim_{ct} \text{(by Lemma A.4(3))} \\ R\{Q_1/X_1\} & \end{aligned}$$

which proves fact (b).

We now prove fact (a). Let  $P\{Q_2/X_1\} \mathcal{R} P\{Q_1/X_1\}$ . We show that the actions of  $P\{Q_2/X_1\}$  can be matched by  $P\{Q_1/X_1\}$ . Since  $X_1$  is guarded in  $P$ , by Lemma A.3 we can infer all transitions for  $P\{Q_2/X_1\}$  and  $P\{Q_1/X_1\}$  from those of  $P$ . There are three cases:

**Case (a)**  $P \xrightarrow{\ell} P'$ .

Then we have the transitions

$$P\{Q_2/X_1\} \xrightarrow{\ell} P'\{Q_2/X_1\}, \text{ and } P\{Q_1/X_1\} \xrightarrow{\ell} P'\{Q_1/X_1\}.$$

By fact (b), it holds that  $P'\{Q_2/X_1\} \mathcal{R} \sim_{\text{Ct}} P'\{Q_1/X_1\}$ , which closes up the bisimulation.

**Case (b)**  $P \xrightarrow{\bar{a}} C$ .

Then we have the transitions

$$P\{Q_2/X_1\} \xrightarrow{\bar{a}} C\{Q_2/X_1\}, \text{ and } P\{Q_1/X_1\} \xrightarrow{\bar{a}} C\{Q_1/X_1\}$$

For every closed abstraction  $G$ , we have  $G \bullet (C\{Q_i/X_1\}) = (G \bullet C)\{Q_i/X_1\}$ ,  $i = 1, 2$ . Let  $R = G \bullet C$ . Then  $R\{Q_2/X_1\} \mathcal{R} \sim_{\text{Ct}} R\{Q_1/X_1\}$  follows by fact (b).

**Case (c)**  $P \xrightarrow{a} F$ .

Similar to case (b). □

**Remark A.5** The weak version of Proposition 3.8 (i.e.,  $R \approx_{\text{Ct}} Q$  implies  $A\{R/X\} \approx_{\text{Ct}} A\{Q/X\}$ ) can be proved along the same lines. We should mention, however, that in place of the “bisimulation up-to  $\sim_{\text{Ct}}$ ” technique (used in the proofs of Lemma A.2 and Proposition 3.8), one should use the following up-to technique for  $\approx_{\text{Ct}}$ :

Given a symmetric relation  $\mathcal{R} \subseteq \mathcal{P}r^o \times \mathcal{P}r^o$ , suppose that  $(P, Q) \in \mathcal{R}$  imply:

1. whenever  $P \xrightarrow{\ell} P'$ , there is  $Q'$  s.t.  $Q \xRightarrow{\widehat{\ell}} Q'$  and  $P' \sim_{\text{Ct}} \mathcal{R} \approx_{\text{Ct}} Q'$ ;
2. whenever  $P \xrightarrow{a} F$ , there exists  $G$  s.t.  $Q \xrightarrow{a} G$  and  $C \bullet F \sim_{\text{Ct}} \mathcal{R} \approx_{\text{Ct}} C \bullet G$ , for all concretions  $C$ .
3. whenever  $P \xrightarrow{\bar{a}} C$ , there exists  $D$  s.t.  $Q \xrightarrow{\bar{a}} D$  and  $F \bullet C \sim_{\text{Ct}} \mathcal{R} \approx_{\text{Ct}} F \bullet D$ , for all abstractions  $F$ .

Then  $\mathcal{R} \subseteq \approx_{\text{Ct}}$ .

The soundness of this technique can be established using a standard argument for up-to techniques. □

## B Proof of Proposition 4.1

In this appendix we prove Proposition 4.1, that is that  $\{z := B\}$  distributes over substitution, on the hypothesis that  $z$  is used only in output subject position. The presentation of this appendix is similar to that of Appendix A, where we inferred congruence for  $\sim_{\text{Ct}}$  over substitution. In both cases, we have first to prove some instance (Lemma A.2 and B.3, respectively) of the general result (Propositions 3.8 and 4.1, respectively). Moreover, the proof of Proposition 4.1 closely follows the proof of Proposition 3.8.

We write “ $x \text{ nsp } A$ ” to mean that name  $x$  occurs free in agent  $A$  only in output subject position. Our first target is to show that  $\{z := B\}$  distributes over parallel composition. We cannot quite follow the proof technique used by Milner to show the analogous result for the  $\pi$ -calculus [Mil91, Section 5.4], due to the higher-order setting in which we are working. For our proof, Lemmas B.1 and B.2 are needed. Lemma B.1 relates the actions of the processes  $Q$  and  $Q'$ , where  $Q'$  is obtained from  $Q$  through a substitution of one of its names. In the lemma, assertion (2) — the vice versa of (1) — is only possible because of the condition  $x, z \text{ nsp } Q$  which prevents new possible interactions from being generated as effect of the substitution  $Q\{z/x\}$ .

**Lemma B.1** *Suppose  $x, z \text{ nsp } Q$ . We have:*

1. *If  $Q \xrightarrow{\mu} A$ , then  $Q\{z/x\} \xrightarrow{\mu\{z/x\}} A\{z/x\}$ ;*
2. *if  $Q\{z/x\} \xrightarrow{\mu'} A'$ , then there exists  $A$  s.t.  
 $Q \xrightarrow{\mu} A$  with  $\mu' = \mu\{z/x\}$  and  $A' = A\{z/x\}$ .*

Moreover both in (1) and in (2), it holds that  $x, z \text{ nsp } A$ .

PROOF: A simple transition induction, both for (1) and for (2). □

**Lemma B.2** *Suppose that  $x, z_1 \text{ nsp } Q$ , that  $x, z_1 \notin \text{fn}(B)$  and that  $z_2 \notin \text{fn}(Q, B)$ . Then*

$$Q\{z_1/x\} \{z_1 := B\} \sim_{\text{Ct}} Q\{z_2/x\} \{z_1 := B\} \{z_2 := B\}.$$

PROOF: Below, for a generic process  $P$ , we write  $P\{\tilde{z} := \tilde{B}\}$  for  $P\{z_1 := B\}\{z_2 := B\}$ . Since  $z_2$  does not occur free in  $Q\{z_1/x\}\{z_1 := B\}$ , we have

$$Q\{z_1/x\} \{z_1 := B\} \sim_{\text{Ct}} Q\{z_1/x\} \{\tilde{z} := \tilde{B}\};$$

therefore it suffices to prove that  $Q\{z_1/x\} \{\tilde{z} := \tilde{B}\} \sim_{\text{Ct}} Q\{z_2/x\} \{\tilde{z} := \tilde{B}\}$ . For this, we show that the set  $\mathcal{R}$  of all pairs of the form

$$(Q\{z_1/x\} \{\tilde{z} := \tilde{B}\}, Q\{z_2/x\} \{\tilde{z} := \tilde{B}\})$$

with  $x, z_1 \text{ nsp } Q$ ,  $x, z_1 \notin \text{fn}(B)$  and  $z_2 \notin \text{fn}(Q, B)$ , is a  $\sim_{\text{Ct}}$ -bisimulation up-to  $\sim_{\text{Ct}}$ . Suppose  $P_1 \mathcal{R} P_2$ , for

$$P_1 \stackrel{\text{def}}{=} Q\{z_1/x\} \{\tilde{z} := \tilde{B}\}, \quad P_2 \stackrel{\text{def}}{=} Q\{z_2/x\} \{\tilde{z} := \tilde{B}\}.$$

We have to consider the actions of  $Q\{z_1/x\}$  and  $Q\{z_2/x\}$  which can cause an action of either  $P_1$  or  $P_2$ . In the following we make use of Lemma B.1 which tells us how to infer the actions for  $Q\{z_1/x\}$  and  $Q\{z_2/x\}$  from those of  $Q$ .

**Case (a)**  $Q \xrightarrow{\tau} Q'$ .

We get the actions

$$P_1 \xrightarrow{\tau} Q'\{z_1/x\} \{\tilde{z} := \tilde{B}\} \quad \text{and} \quad P_2 \xrightarrow{\tau} Q'\{z_2/x\} \{\tilde{z} := \tilde{B}\}.$$

Now we are back in  $\mathcal{R}$  because Lemma B.1 insures that  $Q'$  meets the conditions of  $\mathcal{R}$ .

**Case (b)**  $Q \xrightarrow{a} G$ .

By the conditions on  $x$  and  $z_1$  it must be  $a \notin \{x, z_1\}$ . We thus get the actions:

$$P_1 \xrightarrow{a} G\{z_1/x\} \{\tilde{z} := \tilde{B}\} \stackrel{\text{def}}{=} G_1 \quad \text{and} \quad P_2 \xrightarrow{a} G\{z_2/x\} \{\tilde{z} := \tilde{B}\} \stackrel{\text{def}}{=} G_2.$$

We have to prove that  $C \bullet G_1 \sim_{\text{Ct}} \mathcal{R} \sim_{\text{Ct}} C \bullet G_2$ , for every concretion  $C$ . Assuming, by alpha conversion, that  $x, z_1, z_2$  do not occur free in  $C$ , we have

$$C \bullet G_i \sim_{\text{Ct}} (C \bullet G)\{z_i/x\} \{\tilde{z} := \tilde{B}\}, \text{ for } i = 1, 2$$

which shows that  $C \bullet G_1 \sim_{\text{Ct}} \mathcal{R} \sim_{\text{Ct}} C \bullet G_2$  holds.

**Case (c)**  $Q \xrightarrow{\bar{a}} C$ , with  $a \notin \{x, z_1\}$ .

Similar to case (b).

**Case (d)**  $Q \xrightarrow{\bar{a}} C$ , with  $a \in \{x, z_1\}$ .

We assume that  $a = x$ , as the case  $a = z_1$  is similar. The transition by  $Q$  determines an interaction between  $Q\{z_1/x\}$  and  $!z_1.B$  in  $P_1$ , and an interaction between  $Q\{z_2/x\}$  and  $!z_2.B$  in  $P_2$ :

$$P_1 \xrightarrow{\tau} \sim_{\text{Ct}} (B \bullet C)\{z_1/x\} \{\tilde{z} := \tilde{B}\} \quad \text{and} \quad P_2 \xrightarrow{\tau} \sim_{\text{Ct}} (B \bullet C)\{z_2/x\} \{\tilde{z} := \tilde{B}\}.$$

and we are back in  $\mathcal{R}$ .

**Case (e)**  $Q \xrightarrow{\ell} Q'$ , with  $\ell \neq \tau$ .

Similar to the previous cases. □

**Lemma B.3** *Suppose that  $z \notin \text{fn}(B)$  and that  $z \text{ nsp } P, Q$ . Then*

1.  $(\nu x P) \{z := B\} \sim_{\text{Ct}} \nu x (P \{z := B\}), x \neq z$ .
2.  $(P \mid Q) \{z := B\} \sim_{\text{Ct}} P \{z := B\} \mid Q \{z := B\}$ .
3.  $(!P) \{z := B\} \sim_{\text{Ct}} ! (P \{z := B\})$ .

**PROOF:** The hardest cases are (2) and (3), and we only consider these. It is enough to show the result for closed agents. We start with (2).

Let  $x \notin \text{fn}(P, Q)$  and  $Q' \stackrel{\text{def}}{=} Q\{x/z\}$ . Then  $Q = Q'\{z/x\}$ ; moreover, if  $z' \notin \text{fn}(P, Q, B)$ , then

$$\begin{aligned} (P \mid Q) \{z := B\} &= \\ (P \mid Q')\{z/x\} \{z := B\} &\sim_{\text{Ct}} \quad (\text{by Lemma B.2}) \\ (P \mid Q')\{z'/x\} \{z := B\} \{z' := B\} &= \\ (P \mid Q'\{z'/x\}) \{z := B\} \{z' := B\} &\sim_{\text{Ct}} \quad (\text{since } z \notin \text{fn}(Q'\{z'/x\}, B) \text{ and } z' \notin \text{fn}(P, B)) \\ (P \{z := B\}) \mid (Q'\{z'/x\} \{z' := B\}) &= \quad (\text{using alpha conversion}) \\ P \{z := B\} \mid Q \{z := B\} & \end{aligned}$$

Now assertion (4) of the lemma. We show that the set  $\mathcal{R}$  of all pairs of the form

$$\left( Q \mid (!P) \{z := B\}, Q \mid !(P \{z := B\}) \right) \quad \text{with } z \notin \text{fn}(B) \text{ and } z \text{ nsp } P$$

is a  $\sim_{\text{Ct}}$ -bisimulation up-to  $\sim_{\text{Ct}}$ . An action of

$$Q \mid (!P) \{z := B\} \quad \text{or} \quad Q \mid !(P \{z := B\})$$

comes from an action of  $Q$  alone, an action of  $P$  alone, an interaction between  $Q$  and  $P$ , or an interaction between  $P$  and  $\{z := B\}$ . We analyse the cases of higher-order output action by  $P$  and of communication between  $P$  and  $\{z := B\}$ , supposing  $z$  is a higher-order name. Suppose  $P \xrightarrow{\bar{a}} C$ , with  $a \neq z$ . Since  $z \text{ nsp } P$ , also  $z \text{ nsp } C$ . Assuming  $z$  does indeed occur free in  $C$ , we have:

$$Q \mid (!P) \{z := B\} \xrightarrow{\bar{a}} \sim_{\text{Ct}} (C \mid Q \mid !P) \{z := B\} \stackrel{\text{def}}{=} C_1$$

and

$$Q \mid !(P \{z := B\}) \xrightarrow{\bar{a}} \sim_{\text{Ct}} (C \mid Q \mid !(P \{z := B\})) \{z := B\} \stackrel{\text{def}}{=} C_2$$

We show that, for all  $G$ , it holds that  $G \bullet C_1 \sim_{\text{Ct}} \mathcal{R} \sim_{\text{Ct}} G \bullet C_2$ . By alpha conversion, we can assume that  $z \notin \text{fn}(G)$  and, therefore, using assertion (2) of this lemma, we have

$$\begin{aligned} G \bullet C_1 &\sim_{\text{Ct}} (G \bullet C \mid Q \mid !P) \{z := B\} \\ &\sim_{\text{Ct}} (G \bullet C \mid Q) \{z := B\} \mid (!P) \{z := B\} \stackrel{\text{def}}{=} R_1 \end{aligned}$$

and

$$\begin{aligned} G \bullet C_2 &\sim_{\text{Ct}} (G \bullet C \mid Q \mid !(P \{z := B\})) \{z := B\} \\ &\sim_{\text{Ct}} (G \bullet C \mid Q) \{z := B\} \mid !(P \{z := B\}) \stackrel{\text{def}}{=} R_2 \end{aligned}$$

which is enough, since  $R_1 \mathcal{R} R_2$ .

Now we look at the case in which  $z$  is a higher-order name and there is an interaction between  $P$  and  $\{z := B\}$ . Suppose  $P \xrightarrow{\bar{z}} C$  is the action of  $P$ . We have

$$\begin{aligned} Q \mid (!P) \{z := B\} &\xrightarrow{\tau} \sim_{\text{Ct}} \\ Q \mid (C \bullet B \mid !P) \{z := B\} &\stackrel{\text{def}}{=} R_1 \end{aligned}$$

and

$$\begin{aligned} Q \mid !(P \{z := B\}) &\xrightarrow{\tau} \sim_{\text{Ct}} \\ Q \mid (C \bullet B) \{z := B\} \mid !(P \{z := B\}) &\stackrel{\text{def}}{=} R_2. \end{aligned}$$

It holds that  $z \text{ nsp } (C \bullet B \mid !P)$ . Therefore, using assertion (2) of this lemma,

$$R_1 \sim_{\text{Ct}} Q \mid (C \bullet B) \{z := B\} \mid (!P) \{z := B\} \stackrel{\text{def}}{=} R'_1$$

which, since  $R'_1 \mathcal{R} R_2$ , closes up the bisimulation.  $\square$

Recall that a variable  $X$  is *guarded* in an agent  $A$  if  $X$  only occurs in subexpressions of  $A$  of the form  $\mu. B$ .

**Lemma B.4** *Suppose that  $X_1 \in \text{fv}(A_1)$  and that  $X_2 \notin \text{fv}(A_1)$ . Then there exists  $A_2$  such that*

1.  $X_1$  is guarded in  $A_2$ ;
2.  $A_2\{X_1/X_2\} = A_1$ ;
3. for all  $Q$ , we have  $A_1\{Q/X_1\}\{z := B\} \sim_{\text{ct}} A_2\{Q/X_1, Q\{z := B\}/X_2\}\{z := B\}$ .

PROOF: By induction on the structure of  $A_1$ . The basis of the induction occurs when  $A_1 = X$  or  $A_1 = \mu.A$ . If  $A_1 = X$ , then define  $A_2 \stackrel{\text{def}}{=} A_1$  if  $X \neq X_1$ , and  $A_2 \stackrel{\text{def}}{=} X_2$  if  $X = X_1$ ; if  $A_1 = \mu.A$ , then take  $A_2 \stackrel{\text{def}}{=} A_1$ .

When  $A_1$  is a process of the form  $P_1 \mid P_2$ ,  $\nu x P$ , or  $!P$ , use the inductive hypothesis and Lemma B.3. As an example, we consider the case of parallel composition and prove assertion (3). Suppose  $A_1 = P_1 \mid P_2$ . By induction there exists  $P'_i$ ,  $i = 1, 2$  such that  $P'_i$  and  $P_i$  satisfy the assertion of the lemma. Define  $A_2 \stackrel{\text{def}}{=} P'_1 \mid P'_2$ . Abbreviating the substitution  $\{Q/X_1, Q\{z := B\}/X_2\}$  as  $\{\widetilde{Q}_F/\widetilde{x}\}$ , we have:

$$\begin{aligned}
(P_1 \mid P_2)\{Q/X_1\}\{z := B\} &= \text{(distributing the substitution)} \\
(P_1\{Q/X_1\} \mid P_2\{Q/X_1\})\{z := B\} &\sim_{\text{ct}} \text{(Lemma B.3(2))} \\
P_1\{Q/X_1\}\{z := B\} \mid P_2\{Q/X_1\}\{z := B\} &\sim_{\text{ct}} \text{(induction)} \\
P'_1\{\widetilde{Q}_F/\widetilde{x}\}\{z := B\} \mid P'_2\{\widetilde{Q}_F/\widetilde{x}\}\{z := B\} &\sim_{\text{ct}} \text{(reversing the steps)} \\
(P'_1 \mid P'_2)\{\widetilde{Q}_F/\widetilde{x}\}\{z := B\} & \quad \square
\end{aligned}$$

We are now ready to prove Proposition 4.1. We first recall its assertion:

**Proposition 4.1** Let  $A, P, B \in \mathcal{A}g$ . Suppose that  $z \notin \text{fn}(B)$  and that  $z$  occurs free in  $A$  and  $P$  only in output subject position. Then

$$A\{P/X\}\{z := B\} \sim_{\text{ct}} A\{z := B\}\{P\{z := B\}/X\}.$$

PROOF: It suffices to prove the assertion for closed processes. Consider the set  $\mathcal{R}$  of all pairs of the form:

$$\left( P\{Q/X_1\}\{z := B\}, P\{z := B\}\{Q\{z := B\}/X_1\} \right), \text{ with } X_1 \text{ guarded in } P.$$

We prove the following facts:

- (a)  $\mathcal{R}$  is a  $\sim_{\text{ct}}$ -bisimulation up-to  $\sim_{\text{ct}}$ ;
- (b) for all  $R$  with  $\text{fv}(R) \subseteq \{X\}$ ,

$$R\{Q/X_1\}\{z := B\} \sim_{\text{ct}} \mathcal{R} \sim_{\text{ct}} R\{z := B\}\{Q\{z := B\}/X_1\}.$$

The assertion of the theorem follows from (a) and (b) by transitivity of  $\sim_{\text{ct}}$ .

We first prove (b). We can apply Lemma B.4 to  $R$ ; let  $R'$  be the process returned. We have



$$\begin{aligned}
R\{Q/X_1\} \{z := B\} &\sim_{\text{Ct}} \text{ (by Lemma B.4(3))} \\
R'\{Q/X_1, Q\{z := B\}/X_2\} \{z := B\} &= \\
(R'\{Q\{z := B\}/X_2\})\{Q/X_1\} \{z := B\} &\mathcal{R} \\
&\text{(by Lemma B.4(1), } X_1 \text{ is guarded in } R') \\
(R'\{Q\{z := B\}/X_2\}) \{z := B\} \{Q\{z := B\}/X_1\} &= \text{ (} z \text{ is not free in } Q\{z := B\}) \\
R'\{z := B\} \{Q\{z := B\}/X_1, Q\{z := B\}/X_2\} &= \text{ (using Lemma B.4(2))} \\
R\{z := B\} \{Q\{z := B\}/X_1\} &
\end{aligned}$$

i.e., summarising:

$$R\{Q/X_1\} \{z := B\} \sim_{\text{Ct}} \mathcal{R} R\{z := B\} \{Q\{z := B\}/X_1\},$$

which proves fact (b).

We now prove fact (a). Let  $Q_1 \mathcal{R} Q_2$ , for

$$Q_1 \stackrel{\text{def}}{=} P\{Q/X_1\} \{z := B\} \quad Q_2 \stackrel{\text{def}}{=} P\{z := B\} \{Q\{z := B\}/X_1\}$$

with  $X_1$  guarded in  $P$ . Because  $X_1$  is guarded in  $P$ , we can infer all transitions for  $Q_1$  and  $Q_2$  from those of  $P$ . There are four cases to consider:

**Case 1**  $P \xrightarrow{\tau} P'$ .

Then we have the transitions

$$Q_1 \xrightarrow{\tau} P'\{Q/X_1\} \{z := B\} \stackrel{\text{def}}{=} Q'_1, \quad Q_2 \xrightarrow{\tau} P'\{z := B\} \{Q\{z := B\}/X_1\} \stackrel{\text{def}}{=} Q'_2.$$

By fact (b), we have  $Q'_1 \sim_{\text{Ct}} \mathcal{R} \sim_{\text{Ct}} Q'_2$ , which is enough because  $\mathcal{R}$  a bisimulation up-to  $\sim_{\text{Ct}}$ .

**Case 2**  $P \xrightarrow{\bar{a}} C$ .

First suppose  $a \neq z$ . Then we have the transitions

$$Q_1 \xrightarrow{\bar{a}} C\{Q/X_1\} \{z := B\} \quad Q_2 \xrightarrow{\bar{a}} C\{z := B\} \{Q\{z := B\}/X_1\}.$$

For every closed abstraction  $G$ , we have, using simple algebraic manipulations,

$$\begin{aligned}
G \bullet (C\{Q/X_1\} \{z := B\}) &\sim_{\text{Ct}} (G \bullet C)\{Q/X_1\} \{z := B\} \stackrel{\text{def}}{=} Q'_1 \\
G \bullet (C\{z := B\} \{Q\{z := B\}/X_1\}) &\sim_{\text{Ct}} (G \bullet C)\{z := B\} \{Q\{z := B\}/X_1\} \stackrel{\text{def}}{=} Q'_2
\end{aligned}$$

Now  $Q'_1 \sim_{\text{Ct}} \mathcal{R} \sim_{\text{Ct}} Q'_2$  holds by fact (b), for  $R = G \bullet C$ .

Suppose now that  $a = z$ . In this case the transition  $P \xrightarrow{\bar{z}} C$  determines an interaction with  $!z.B$  as follows in  $Q_1$  and  $Q_2$ :

$$\begin{aligned}
Q_1 &\xrightarrow{\tau} \sim_{\text{Ct}} (B \bullet C)\{Q/X_1\} \{z := B\} \stackrel{\text{def}}{=} Q'_1 \\
Q_2 &\xrightarrow{\tau} \sim_{\text{Ct}} (B \bullet C)\{z := B\} \{Q\{z := B\}/X_1\} \stackrel{\text{def}}{=} Q'_2
\end{aligned}$$

and  $Q'_1 \sim_{\text{Ct}} \mathcal{R} \sim_{\text{Ct}} Q'_2$  holds by fact (b).

**Case 3**  $P \xrightarrow{a} G$ .

Since  $z$  nsp  $P$ , it must be  $a \neq z$ ; then proceed as in case (2) for  $a \neq z$ .

**Case 4**  $P \xrightarrow{\ell} P'$  and  $\ell \neq \tau$ .

Similar to previous cases. □



Unité de recherche INRIA Lorraine, Technopôle de Nancy-Brabois, Campus scientifique,  
615 rue du Jardin Botanique, BP 101, 54600 VILLERS LÈS NANCY  
Unité de recherche INRIA Rennes, Irista, Campus universitaire de Beaulieu, 35042 RENNES Cedex  
Unité de recherche INRIA Rhône-Alpes, 46 avenue Félix Viallet, 38031 GRENoble Cedex 1  
Unité de recherche INRIA Rocquencourt, Domaine de Voluceau, Rocquencourt, BP 105, 78153 LE CHESNAY Cedex  
Unité de recherche INRIA Sophia-Antipolis, 2004 route des Lucioles, BP 93, 06902 SOPHIA-ANTIPOLIS Cedex

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