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Abstract: C.E. Shannon in his Information Theory defined a rate of information transmission of a transmitter-receiver couple. We use this concept to define three models of image redundancy. First we apply information theory to a simple model considering an image as a set of isolated pixels. Then we introduce a Markov Random Field model to take into account the neighbourhood of a pixel. We show that we have to determine some parameters of the MRF in order to obtain sufficient statistics from common satellite images, and we propose a measure based on a generalized Ising model. Our third model considers the correspondence between grey level vectors of cliques. We introduce a distance in the grey level space to solve the problem of insufficient statistics. Finally, results for the proposed definitions are presented for some synthetic and a large variety of SPOT XS1, XS2 and XS3 image triples and are compared to the classical correlation coefficient measure.

Key-words: Image redundancy, entropy, mutual information, Markov Random Fields.

(Résumé : tsvp)

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Modélisations de la redondance d'images

Résumé : Dans sa théorie de l'information C.E. Shannon définit le taux de transmission d'information d'un couple émetteur-récepteur. Nous partons de ce concept pour définir trois modèles de la redondance d'images. D'abord nous appliquons la théorie de l'information à un modèle simple où une image est considérée comme un ensemble de pixels isolés. Ensuite nous introduisons un modèle de champ de Markov pour prendre en compte le voisinage d'un pixel. Nous montrons que nous devons déterminer quelques paramètres du modèle markovien pour obtenir des statistiques suffisantes à partir d'images satellitaires courantes, et nous proposons une mesure pour le modèle d'Ising généralisé. Le troisième modèle considère la correspondance entre vecteurs de niveaux de gris sur les cliques. Pour résoudre le problème de statistiques insuffisantes nous introduisons une distance dans l'espace des niveaux de gris. Enfin sont présentés les résultats de l'application des différentes définitions proposées et du coefficient de corrélation sur quelques images synthétiques et un grand nombre de triplets d'images SPOT XS1, XS2 et XS3.

Mots-clé : Redondance d'images, entropie, information mutuelle, champs de Markov.

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1 Introduction

When we are exposed to a multi-sensor fusion problem, a fundamental preliminary issue is to determine whether the information provided by the sensors are essentially the same or substantially different. In other words : are the sensors redundant or complementary ?

This question can be examined either by an a priori or an a posteriori approach. The former uses physical models of the sensors, the physical objects, external conditions and so on, whereas the latter studies the information provided by the sensors, in our case images. We have chosen this second approach and will be speaking of image redundancy rather than sensor redundancy.

Image redundancy should represent the quantity of common information in two images. That is, knowing one image, to what extent are we able to predict the other one. Redundancy and resemblance do not characterize the same properties. For example two random images may look quite the same, but it is difficult to predict one knowing the other, so the redundancy is weak. On the other hand an image and its negative are strongly redundant since it is easy to predict one knowing the other although they do not necessarily resemble. Redundancy is more related to a correlation rather than to a resemblance.

In [2], Giraudon and Houzelle imagine two images being connected by a virtual channel and consider one image as the transmitter and the other as the receiver. They can then apply Shannon's information theory [5] and use the notion of rate of information transmission (called system mutual information in more recent information theory literature) as a measure of image redundancy. Information theory has also been applied to image classification. Maître in [4] defines a *spatial entropy* which is the entropy of the probability distribution of class label configurations on the neighbours of a pixel, and a *global entropy* of the joint probability distribution of a pixel's grey level, its class label and the configuration of class labels on its neighbours. The radiometric entropy (i.e. the entropy of the probability distribution of grey levels), in addition to the global and spatial entropies, are defined globally and for each class. Maître uses these entropies to study the convergence properties of iterative contextual classification algorithms. Lohmann [3] does supervised texture classification based on co-occurrence features. He models the co-occurrence feature vector sampled in some image window as a multinomial distribution. This enables him to compute the mutual information of the co-occurrence feature vectors of the window and of each class, and attribute the class giving highest mutual information to the pixel in the center of the window.

We make the model of Giraudon and Houzelle more explicit. They use information theory to define the entropy of an image and a measure for the redundancy of two images. Their approach amounts to define the quantity of information (i.e. the basic definition of the information theory) as the one produced by the event of a pixel's grey level occurring, knowing the image's histogram. The stochastic model (such a model is necessary to apply information theory) consists in considering an isolated pixel and conferring a probability distribution to it given by the normalized histogram.

Our goal is to use the measure of the redundancy in an image fusion process for classification. In order to obtain a good classification we will require to consider not only isolated pixels but also their neighbourhood. Therefore we need to obtain a definition of redundancy which takes into account the neighbourhood of a pixel. We will develop two models, one based on the theory of Markov Random Fields (MRF), and another using one of the key concepts of this theory : the cliques.

In our first approach we define the information as the one produced by the event of a pixel's grey level occurring, knowing the grey levels of all other pixels. We extend Hamming's definition of the entropy of a Markov chain [1] to a Markov random field and propose a definition of the redundancy of two images in terms of conditional probabilities with respect to the neighbourhood of a pixel. To eliminate the statistical insufficiency we must apply the Hammersley-Clifford theorem and determine parameters of the Markov model. We propose a definition based on the generalized Ising model.

The second model that takes into account the neighbourhood of a pixel, is a quite natural extension of the Isolated-pixels model. Instead of the correspondence pixel to pixel, it considers the one between vectors of neighbouring pixels (i.e. *cliques* in markovian terminology). We obtain a measure which is applicable to large images, whereas smaller images do not provide enough statistical data. In order to further increase the statistics we introduce a distance in the grey level space.

We consider images where the sites $s \in S = \{s_1, ..., s_N\}$ have grey levels $x_s \in \Lambda = \{g_1, ..., g_M\}$.

We will use the notation $x = [x_{s_1}, ..., x_{s_N}]$.

2 The Isolated Pixels model

Every x_{s_i} is considered to be the i^{th} realization of the stochastic variable X_s whose sample space is Λ and whose probability distribution is the normalized grey level occurrence frequency, f_x, (normalized histogram).

$$\forall g \in \Lambda \quad P(X_s = g) = f_x(g) = \frac{1}{N} \sum_{s \in S} \delta(x_s - g)$$

$$\text{where} \quad \delta(0) = 1 \ \text{ and } \ \delta(x) = 0 \ \text{ for } \ x \neq 0$$

Considering X_s as an information source, we can apply the information theory and define the quantity of **information** produced by a realization x_s of X_s :

$$I(x_s) = \log \frac{1}{P(X_s = x_s)} = -\log f_x(x_s) \tag{1}$$

The **entropy** of X_s is defined as the average of the information for all possible values x_s weighted according to the probability of x_s, in other words, it is the mathematical expectation of the information :

$$H(X_s) = \sum_{g \in \Lambda} P(X_s = g) \log \frac{1}{P(X_s = g)} = -\sum_{g \in \Lambda} f_x(g) \log f_x(g) \tag{2}$$

This is the entropy of X_s when we consider only grey level occurrence probabilities. It is uniquely determined by the image histogram.

If we use the base 2 logarithm, this quantity represents the optimal average number of bits needed to code the grey level of a pixel knowing the image histogram (see [1]).

Similarly for a second image Y : $H(Y_s) = -\sum_{g \in \Lambda} f_y(g) \log f_y(g)$

Now, let the realization x_{s_i} of the source X_s (transmitter) be the input to an information channel and the realization y_{s_i} of the source Y_s (receiver) be the output of this channel (or vice versa). We can then define the **joint entropy** of the transmitter and the receiver as the mathematical expectation of the information produced by a realization of the variable (X_s, Y_s) :

$$\begin{aligned} H(X_s, Y_s) &= \sum_{g_1 \in \Lambda} \sum_{g_2 \in \Lambda} P(X_s = g_1, Y_s = g_2) \log \frac{1}{P(X_s = g_1, Y_s = g_2)} \\ &= -\sum_{g_1 \in \Lambda} \sum_{g_2 \in \Lambda} f_{xy}(g_1, g_2) \log f_{xy}(g_1, g_2) \end{aligned} \tag{3}$$

$$\text{where} \quad f_{xy}(g_1, g_2) = \frac{1}{N} \sum_{s \in S} \delta(x_s - g_1)\delta(y_s - g_2)$$

Equation (3) expresses the minimum number of bits needed to code the grey level of a site in one image and the grey level of the corresponding site in the other image, knowing their joint histogram.

We also define the **conditional entropies** :

$$\begin{aligned} H(X_s \mid Y_s) &= \sum_{g_1 \in \Lambda} \sum_{g_2 \in \Lambda} P(X_s = g_1, Y_s = g_2) \log \frac{1}{P(X_s = g_1 \mid y_s = g_2)} \\ &= -\sum_{g_1 \in \Lambda} \sum_{g_2 \in \Lambda} f_{xy}(g_1, g_2) \log \frac{f_{xy}(g_1, g_2)}{f_y(g_2)} \qquad (4) \\ \text{and} \quad H(Y_s \mid X_s) &= -\sum_{g_1 \in \Lambda} \sum_{g_2 \in \Lambda} f_{xy}(g_1, g_2) \log \frac{f_{xy}(g_1, g_2)}{f_x(g_1)} \end{aligned}$$

It is easy to show that these conditional entropies can be expressed as the difference between the joint and individual entropies :

$$H(X_s \mid Y_s) = H(X_s, Y_s) - H(Y_s)$$

$$H(Y_s \mid X_s) = H(X_s, Y_s) - H(X_s)$$

$H(X_s \mid Y_s)$ is sometimes called equivocation (Shannon).

It measures the average ambiguity of the input when the output is observed. In other words it is the minimum quantity of information required to be added to the output to recover the input. It is zero if we can define a function φ from Λ to Λ so that $\forall s \in S \quad x_s = \varphi(y_s)$.

The **system mutual information** or **rate of information transmission** is defined as the difference between the entropy of the transmitter and the equivocation :

$$\begin{aligned} R(X_s, Y_s) &= H(X_s) - H(X_s \mid Y_s) \\ &= \sum_{g_1 \in \Lambda} \sum_{g_2 \in \Lambda} P(X_s = g_1, Y_s = g_2) \log \frac{P(X_s = g_1, Y_s = g_2)}{P(X_s = g_1)P(Y_s = g_2)} \\ &= \sum_{g_1 \in \Lambda} \sum_{g_2 \in \Lambda} f_{xy}(g_1, g_2) \log \frac{f_{xy}(g_1, g_2)}{f_x(g_1) f_y(g_2)} \qquad (5) \end{aligned}$$