Optimization of the number of transportation devices in a flexible manufacturing system using event graphs

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Optimisation du Nombre d'Unités de Transport dans un Atelier Flexible à l'Aide des Graphes d'Evénements

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Résumé:

Une étape importante dans la conception d'ateliers flexibles qui utilisent des ressources de transport à trajets variables est la définition du nombre de ces ressources de transport nécessaires pour atteindre une productivité donnée. Une méthode utilisée pour déterminer ce nombre consiste à considérer un ensemble de scénari réalistes, à déterminer le nombre de ressources de transport nécessaires pour chaque scénario, et à garder finalement le nombre minimal trouvé.

Dans cette publication, nous proposons une approche par séparation et évaluation pour définir le nombre minimal de ressources de transport nécessaires pour un scénario donné. Cette approche est basée sur une modélisation à l'aide des graphes d'événements.

Mots clefs:

Ateliers flexibles, Systèmes de transport, Réseaux de Petri, Graphes d'événements, Véhicules filo-guidés, Véhicules radio-guidés, Optimisation, Modélisation.

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OPTIMIZATION OF THE NUMBER OF TRANSPORTATION DEVICES IN A FLEXIBLE MANUFACTURING SYSTEM USING EVENT GRAPHS

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Abstract:

An important step when designing a Flexible Manufacturing System (FMS) using Automated Guided Vehicles (AGVs) is the definition of the number of AGVs to be used in order to reach a given productivity. A way to define this number is to consider several scenari, to define the minimal number of AGVs required by each scenario, and to keep the maximal of these numbers.

In this paper, we propose a Branch and Bound (B&B) approach to define the minimal number of AGVs require for a scenario. This approach is based on an Event Graph (EG) formulation.

Keywords:
Flexible Manufacturing Systems, Transportation Systems, Petri nets, Event Graphs, Automated Guided Vehicles, Optimization, Modelling.

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1. INTRODUCTION

One of the decisions to be made when designing Flexible Manufacturing Systems (FMSs) using Automated Guided Vehicles (AGVs) concerns the number of AGVs to be used in the FMS in order to meet the customer requirements. At this point of the design process, the manufacturing resources have already been selected. The goal is, of course, to minimize the number of AGVs since they are usually very expensive, and since both the complexity of the management system and the level of the Work-In-Process (WIP) increase with the number of vehicles. A usual way to solve this problem is to consider several scenari, each scenario being a set of customer requirement ratios and, for each scenario, to minimize the number of AGVs required to maximize the productivity of the system. The number of AGVs needed is the maximum among the previous minima. Note that the scenari selected are supposed to be representative of the future customer requirement ratios. Furthermore, the maximal productivity obtained for each scenario should be equal to or greater than the customer requirements, otherwise at least some of the manufacturing resources would be overloaded, which would require to reconsider the choice of these resources.

This paper addresses the problem of minimizing the number of AGVs while reaching a given productivity when the ratios of the customer requirements are known. In that case, the control of the system is expressed by a sequence of part types associated to each manufacturing resource. These sequences reflect the required ratios. Such a sequence, called input sequence, defines the order the product types are manufactured by the corresponding resource. It has been showed in [4] and [5] that such a system can be modelled as a strongly connected Event Graph (EG) and that the problem to be solved is a Mixed Linear Programming (MLP) problem in which the integer variables are binary variables.

Several efficient heuristic approaches have been proposed in the past. In this paper, we propose an approach which leads to the optimal solution. This paper is organized as follows.

The problem is presented in Section 2. Another formulation, used to develop the Branch and Bound approach, is introduced in Section 3. The B&B approach is presented in Section 4. The upper bound required by the B&B approach is obtained using the so-called "Adjustment Algorithm" presented in [5]. This algorithm is given in Section 5. Section 6 is devoted to the computation of the lower bound. The B&B algorithm is summarized in Section 7. The algorithm is applied to a FMS in Section 8, and Section 9 is a conclusion.

The reader is supposed to be aware of the basics of Petri nets and, in particular, of the properties of EG which can be found in the above references as well as in [1], [2], [3 , [4] and [6].

2. PRESENTATION OF THE PROBLEM

The productivity of a FMS can be expressed in terms of the cycle time of its strongly connected EG model: the smaller the cycle time, the greater the productivity. Reaching a given productivity of a FMS is thus equivalent to reach a given cycle time C for its strongly connected EG model. Furthermore, minimizing the number of AGVs is equivalent to minimize a linear combination of the place markings, the coefficients of the linear combination being the elements of a p-invariant. As a consequence, the function to be minimized is invariant if we replace a given marking M_0 of the strongly connected EG by any marking M derived from M_0 (we denote it by $M \in R(M_0)$) by firing a sequence of transitions.

According to the previous remarks, the problem at hand consists of finding an initial marking M_0 which:

$$\textit{Minimizes } {}^tU.M_0 \tag{1}$$

s.t.

$$\pi(M_0) \le C \tag{2}$$

and $M_0(p) \in \{0,1,2\}$ for any $p \in P$, set of places

where U is a p-invariant and $\pi(M_0)$ is the cycle time of the strongly connected EG when the transitions are fired as soon as they are enabled, starting from M_0. Constraints $M_0(p) \in \{0,1,2\}$ have been proven in Laftit et al. [5].

It was proven in Chretienne [2] that:

$$\pi(M_0) = \underset{\gamma \in \Gamma}{\textit{Min}} \left[\mu(\gamma) / M_0(\gamma)\right] \tag{3}$$

where Γ is the set of elementary circuits of the strongly connected EG, $\mu(\gamma)$ is the sum of the firing times of the transitions belonging to γ, and $M_0(\gamma)$ is the number of tokens in the places belonging to γ when M_0 applies.

Thus, constraints (2) can be rewritten as:

$$M_0(\gamma) \ge \lceil \mu(\gamma) / C \rceil \quad \text{for any } \gamma \in \Gamma \tag{4}$$

where $\lceil a \rceil$ is the smallest integer greater than or equal to a.

Inequalities (4) can further be rewritten as:

$$D.M_0 \ge B \tag{5}$$

where:

$$d_{ij} = \begin{cases} 1 & \text{if place } p_j \text{ belongs to the i} - \text{th elementary circuit} \\ 0 & \text{otherwise} \end{cases}$$

for i = 1 to r and j = 1 to $|P|$ (r is the number of elementary circuits and $|P|$ the number of places),

$B = [b_1, \ldots, b_r]$, with $b_i = \lceil \mu(\gamma_i)/C \rceil$, γ_i being the i-th elementary circuit.

The problem at hand, referred to as $\mathcal{P}_0$, becomes:

find the initial marking M_0 which:

Minimizes ${}^tU.M_0$

s.t.

$D.M_0) \geq B$

and $M_0(p) \in \{0,1,2\}, \forall\ p \in P$

This problem is an integer linear programming problem whose variables are the initial numbers of tokens in each place. The drawback of this formulation is that the number of constraints (5) is equal to the number of elementary circuits, which is unpredictable and usually very large. It is the reason why another formulation has been proposed by Laftit et al. [5]. This formulation is partially based on the results obtained by Ramchandani [8] and Ramamoorthy [7].

This problem, introduced in Laftit et al. [5], is referred to as $\mathcal{P}_1$ in the remainder of the paper and is expressed as follows:

Minimize ${}^tU.M_0$

s.t.

$$S_{{}^op}(1) + \theta_{{}^op} \leq S_{p^o}(1) + C.M_0(p),\ \forall\ p \in P \tag{6}$$

$$S_t(1) \in (-\theta_t, C-\theta_t],\ \forall\ t \in T \tag{7}$$

$$M_0(p) \in \{0,1,2\},\ \forall\ p \in P \tag{8}$$

where T is the set of transitions, ${}^op$ (resp. p^o) is the input (resp. output) transition of place p, $S_t(1)$ is the instant at which transition t starts firing for the first time, and θ_t is the firing time of transition t.

The number of variables to be defined when solving $\mathcal{P}_1$ is $|P|+|T|$ ($|P|$ integer variables $M_0(p)$ and $|T|$ real variables $S_t(1)$). This should be compared to the $|P|$ variables $M_0(p)$ to be defined when solving $\mathcal{P}_0$. But the number of constraints (6) and (8) is equal to $2|P|+|T|$ when solving $\mathcal{P}_1$, while the number of constraints (5) is unpredictable when solving $\mathcal{P}_0$. Thus, $\mathcal{P}_1$ is *a priori* easier to solve than $\mathcal{P}_0$. The following result, due to Laftit et al. [5], links the solutions of $\mathcal{P}_0$ and $\mathcal{P}_1$.

Result 1:

The restriction of the optimal solution of $\mathcal{P}_1$ to $\{M_0(p)\}_{p\in P}$ is an optimal solution of $\mathcal{P}_0$.

Note also that, due to relation (6), the solution obtained is periodic.

In the approach present in this paper, we aim at solving $\mathcal{P}_1$ for the EG model of an FMS.

3. A NEW FORMULATION

The purpose of this section is to provide an alternative formulation of problem $\mathcal{P}_1$ which will be used to develop the Branch and Bound approach.

We first prove that the solution space of problem $\mathcal{P}_1$ can be reduced by using some properties of the optimal solutions.

Result 2:

Problem $\mathcal{P}_1$ is equivalent to problem $\mathcal{P}_2$ defined as follows:

Minimize ${}^tU.M_0$

s.t.

$$\theta_{op} \leq S_{p^o}(1) - S_{op}(1) + C.M_0(p) < C + \theta_{op},\ \forall\ p \in P \qquad (6')$$

$$S_t(1) \in (-\theta_t, C-\theta_t],\ \forall\ t \in T \qquad (7)$$

$$M_0(p) \in \{0,1,2\},\ \forall\ p \in P \qquad (8)$$

$$M_0(q) \leq 1 + M_0(p),\ \forall\ p,q \in P \text{ with } p^o = q^o \qquad (9)$$

Proof:

Let us first show that any solution $\{S_t(1), M_0\}$ satisfying constraints (6'), (7) and (8) also satisfies constraint (9). For this purpose, consider two places p and q such that $p^o = q^o = t$.

From constraint (6'),

$$C.M(q) < C + \theta_{oq} + S_{oq}(1) - S_t(1)$$

Since $\theta_{oq}(1) - S_{oq}(1) \leq C$,

$$C.M(q) < 2\ C - S_t(1) \qquad (10)$$

Again from constraint (6'),

$$- S_t(1) \leq - \theta_{op} - S_{op}(1) + C.M_0(p)$$

Since $\theta_{op} + S_{op}(1) > 0$,

$$- S_t(1) < C.M_0(p) \qquad (11)$$

Combining (10) and (11), we obtain:

$$M(q) < 2 + M_0(p)$$

Since M(q) and M(p) are integers,

$$M(q) \leq 1 + M_0(p)$$

In the remainder of the proof, we establish the equivalence between $\mathcal{P}_1$ and $\mathcal{P}_2$ by showing that there exists an optimal solution of $\mathcal{P}_1$ which satisfies constraint (6').

Consider an optimal solution $\{S_t(1), M_0\}$ of $\mathcal{P}_1$. Assume that constraint (6') does not hold for place p_1, i.e.:

$$S_{p_1^o}(1) - S_{op_1}(1) + C.M_0(p_1) \geq C + \theta_{op_1} \qquad (12)$$

We consider a new marking M_1 defined as follows:

$M_1(p) = M_0(p)\ \forall\ p \notin p_1$ and $M_1(p_1) = M_0(p_1) - 1$.

Clearly,

$$S_{p^o}(1) - S_{op}(1) + C.M_1(p) \ge \theta_{op} \text{ for all } p \in P \qquad (13)$$

Furthermore, relation (12) implies that:

$$\begin{aligned} C.M_0(p_1) &\ge C + \theta_{op_1} + S_{op_1}(1) - S_{p_1{}^o}(1) \\ &> C + \theta_{op_1{}^o} + (-\theta_{op_1}) - (C - \theta_{p_1{}^o}) \qquad \text{(constraint (7))} \\ &\ge 0 \end{aligned}$$

Since $M_0(p_1)$ is an integer,

$$M_0(p_1) \ge 1$$

which implies that

$$0 \le M_0(p_1) < M_0(p_1) \le 2$$

These inequalities together with relation (13) show that $\{S_t(1), M_1\}$ is a solution to $\mathcal{P}_1$. Since $\{S_t(1), M_0\}$ is an optimal solution to $\mathcal{P}_1$ and since ${}^tU.M_1 \le {}^tU.M_0$, $\{S_t(1), M_1\}$ is also an optimal solution to $\mathcal{P}_1$.

If constraint (6') still does not hold, the above reasoning is repeated until an optimal solution satisfying (6') is obtained. This solution is clearly an optimal solution to $\mathcal{P}_2$.

Q.E.D.

Let us notice that constraint (9) is considered in $\mathcal{P}_2$ event though it is redundant. As can be expected, this constraint can be used to obtain tight bounds when the integrity is relaxed.

Finally, as in [5], we extend the EG model of an FMS such that at least one optimal initial marking assigns at most one token to each place. For this purpose, for each place p of the model, one additional place p' and one additional transition t' whose firing time is equal to zero are introduced, as shown in Figure 1.

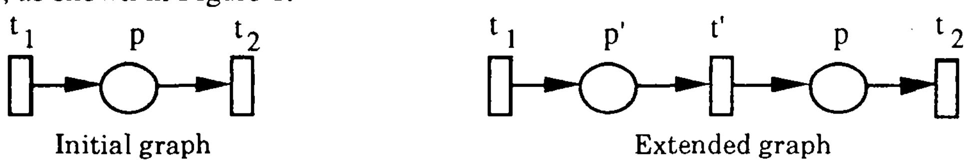

Fig. 1: Extension of the initial graph

In the remainder of this paper, we denote by P_0 and T_0 the set of initial places and the set of initial transitions and by P_s and T_s the set of places p' and the set of transitions t' added to the model. Clearly, in the extended model, there exists an optimal solution such that $M_0(p) \le 1$ and $M_0(p') \le M_0(p)$. Thus problem $\mathcal{P}_2$ becomes problem $\mathcal{P}_3$ defined as follows:

$$\text{Min } {}^tU.M_0$$

s.t.

$$\theta_{op'} \le S_{p^o}(1) - S_{op'}(1) + C.[M_0(p) + M_0(p')] < C + \theta_{op'}, \forall\, p \in P_0 \qquad (14)$$

$$S_t(1) \in (-\theta_t, C-\theta_t], \forall\, t \in T_0 \qquad (15)$$

$$M_0(p), M_0(p') \in \{0,1\}, \forall\, p \in P_0, \forall\, p' \in P_s \qquad (16)$$

$$M_0(q') \le M_0(p), \forall\, p,q \in P_0 \text{ with } p^o = q^o \qquad (17)$$

The B&B approach presented latter is based on this formulation. In the following, we denote by P and T the set of places and the set of transitions of the extended model, i.e. $P = P_0 \cup P_s$ and $T = T_0 \cup T_s$.

4. A Branch and Bound Approach for Solving $\mathcal{P}_3$

a. Initializing the Branch and Bound process

The Branch and Bound (B&B) approach starts by defining the root of the B&B tree. For this purpose, we choose a transition t_0 and we set $S_{t_0}(1) = 0$: since the solution we are looking for is periodic, we know that each transition fires at least once on any period of length C, and thus we can choose the beginning of such a period at any point in time, and in particular at the instant a given transition starts firing.

In order to increase the efficiency of the algorithm, we choose the transition t_0 such that $\sum_{p \in {}^o t_0} u_p$ is maximal, where u_p is the coefficient of the p-invariant U corresponding to p. If several transitions lead to the maximal value, we choose the one which has the maximal number of input places. If several transitions are still candidates, we choose one of them at random.

Since t_0 starts at time 0, and since a place contains at most one token initially, we set $M_0(p) = 1$ for $p \in {}^o t_0$.

b. Computation of the descendants of a node

Let us consider, at a given stage of the B&B process, a node w which has no descendants. Such a node is called a leaf of the B&B tree. We denote by $P^f(w)$ the set of places whose marking has been defined before reaching node w. The next step consists of selecting the place $p^* \in P \backslash P^f(w)$ to be marked next.

The choice of p* is made according to an order defined on the set of places. In this order, the places belonging to P_0 are ordered first (P_0 is the set of initial places). In most of the optimal solutions obtained when considering FMSs, the markings of the places of the initial models (i.e. models not yet extended) are less than or equal to one, which means that, in the extended models, $M(p') = 0$ for $p' \in P_s$. Thus, it is likely to obtain the optimal solution after just considering the places belonging to P_0, which drastically reduces the branches to be explored. This explains the reason why the places of P_0 are ordered first. The places of P_0 are further ordered in the decreasing order of the corresponding element of the p-invariant U. The explanation is straight forward: following this order, the partial values of the objective function grow faster which, in turn, favour sooner decisions not to explore branches. This reduces the number of nodes to take into account, and thus the computation burden.

The order of the places belonging to P_s (which follows the places of P_0) is random.

Thus, p* is the place belonging to $P \backslash P^f(w)$ which is ordered first according to the order defined above.

Starting from node w, we create two descendants w_0 and w_1 as shown in Figure 2.

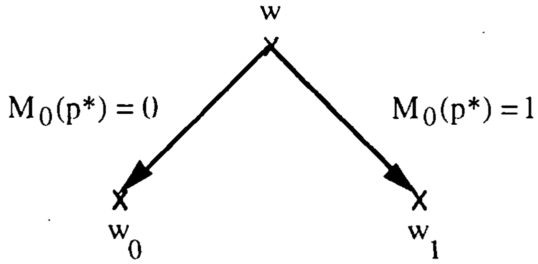

Fig. 2: Descendants of a node w

For the new node w_0, if p^* is an initial place, i.e. $p^* \in P_0$, according to constraint (17), $M_0(q') = 0$ for all places q' and such that $q^o = (p^*)^o$. In this case, $P^f(w_0) = P^f(w) \cup \{p^*\} \cup \{q' / q^o = (p^*)^o\}$. For the new node w_1, $P^f(w_1) = P^f(w) \cup \{p^*\}$.

For w_0 and w_1, we compute a lower and an upper bound of the optimal solution, taking into account the places whose marking has been previously fixed. The lower bound is obtained by relaxing the integrity constraint of the markings $M_0(p)$ for $p \notin P^f(w_i)$ with $i = 0$ or 1. If the relaxed problem does not have solution, then the node at hand does not contain solution to problem $\mathbf{P}_3$ and it is a leaf of the global B&B tree. Otherwise, if the lower bound is greater that the smallest upper bound previously obtained (including the upper bound computed for the current node), then the node at hand is a leaf of the global B&B tree (i.e. no further descendant will be considered starting from this node). Otherwise, two new descendants of this node will be considered, as defined above.

The computation of the upper and the lower bounds is presented in the following sections.

5. Computation of the Upper Bound

The computation of the upper bound is based on the Adjustment Algorithm (AA) due to Laftit et al. [5]. Let us briefly recall this algorithm.

a. The Adjustment Algorithm (AA)

The AA is a heuristic algorithm which applies to problem $\mathbf{P}_0$ (i.e. to the initial EG model). It starts with a feasible solution, that is a solution which verifies constraints (5). It then proceeds iteratively. At each operation, one token is removed from the system as long as the marking remains feasible. The token which is removed is the one which leads to the maximal decrease of the objective function while minimizing the least the degree of freedom of the system.

The degree of freedom of a place $p \in P$ is defined by:

$$df(M_0, p, C) = \underset{\gamma \in \Gamma_p}{\mathcal{Min}}(M_0(\gamma) - \mu(\gamma) / C)$$

where Γ_p is the set of elementary circuits which contain p, and $\lfloor df(M_0, p, C) \rfloor$ is the maximal number of tokens which can be removed from place p while keeping the cycle time of the system less than C.

The variation of the degree of freedom of the system when removing a token from place p is given by:

$$Var(p) = \sum_{q \in P} u_q [df(M_0, q, C) - df(M_1, q, C)]$$

where u_q is the element of the p-invariant U corresponding to place q, and M_1 is the marking derived from M_0 by removing one token from q.

Finally, place p from which one token is removed is a place such that:

$$\frac{Var(p)}{u_p} = \underset{q \in E(M_0)}{Min} \frac{Var(q)}{u_q} \qquad (18)$$

where $E(M_0) = \{p \setminus p \in P \text{ and } df(M_0, p, C) \geq 1\}$. We notice that $E(M_0)$ is the set of places from which one token can be removed while keeping the solution feasible.

Criterion (18) provides a good trade off between a small reduction of the degree of freedom of the system and a great reduction of the value of the objective function.

If the place p selected is such that $M_0(p) = 0$, we have to find M_1 derived from M_0 by firing some transitions until $M_1(p) \geq 1$. At least one of such marking exists. The algorithm continues by replacing M_0 by M_1.

The algorithm stops when $E(M_0) = \emptyset$, i.e. when removing any of the remainder tokens would result in a non feasible marking. The AA can be summarized as follows:

1. Find a feasible marking.
2. If $E(M_0) = \emptyset$, M_0 is a solution to the problem. Stop.
3. Define $p \in E(M_0)$ such that:
   $$\frac{Var(p)}{u_p} = \underset{q \in E(M_0)}{Min} \frac{Var(q)}{u_q}$$
4. If $M_0(p^*) \geq 1$, set $M_0(p^*) = M_0(p^*) - 1$ and $M_0(p) = M_0(p)$ for $p \in P$ and $p \neq p^*$.
   Otherwise, find M_1 derived from M_0 by firing some transitions and such that $M_1(p^*) \geq 1$, set $M_0(p^*) = M_1(p^*) - 1$ and $M_0(p) = M_1(p)$ for $p \in P$ and $p \neq p^*$.
5. Go to 2.

Usually, the initial feasible solution M_0 is chosen as $M_0(p) = 1$ for any $p \in P$. This marking was proven to be feasible as soon as $C \geq \underset{t \in T}{Max}\, \theta_t$.

The AA has been proven to be very efficient in the sense that it provides solutions very close to the optimum and, in several cases, equal to the optimum.

b. Using the AA to compute the upper bound

As stated in the previous section, an upper bound is computed for each node of the B&B tree. Thus, we have to take into account the fact that some of the places of the extended model are already marked.

Note that the B&B approach is based on the extended model. We also notice that at node w of the B&B tree, the marking $M_0(p)$ has been determined for any place $p \in P^f(w)$. We apply the AA algorithm to the extended model to obtain an upper bound by starting from the initial marking M_w defined as follows:

$$M_w(p) = \begin{cases} M_0(p) & \text{if } p \in P^f(w) \\ 1 & \text{otherwise} \end{cases}$$

M_w is a feasible marking if problem $\mathcal{P}_3$ has a solution when taking into account markings $M_0(p)\ \forall\ p \in P^f(w)$ and when relaxing the integrity constraint of $M_0(p)\ \forall\ p \notin P^f(w)$. In other words, M_w is feasible if the lower bound presented in the next section is found.

6. Computation of the Lower Bound

A lower bound is computed for each node w of the B&B tree. To reach this goal, we first solve a relaxed LP problem $\mathcal{P}^*(w)$ derived from $\mathcal{P}_3$. We then use the properties established hereafter to obtain additional constraints from the solution of $\mathcal{P}^*(w)$. We denote by $\mathcal{P}^{**}(w)$ problem $\mathcal{P}^*(w)$ completed by these new constraints. The solution of $\mathcal{P}^{**}(w)$ provides the desired lower bound.

a. Problem $\mathcal{P}^*(w)$

Problem $\mathcal{P}^*(w)$ is formulated as follows:

$$\mathcal{M}inimize \sum_{p \in P} u_p x_p \tag{19}$$

s.t.

$$\theta_{op} \le S_{po}(1) - S_{op'}(1) + C\,(x_p + x_{p'}) < C + \theta_{op'},\ \forall\ p \in P_0 \tag{20}$$

$$S_t(1) \in (-\theta_t, C-\theta_t],\ \forall\ t \in T_0 \tag{21}$$

$$x_p = M_0(p),\ \forall\ p \in P^f(w) \tag{22}$$

$$0 \le x_p \le 1,\ \forall\ p \in P \backslash P^f(w) \tag{23}$$

Clearly, $\mathcal{P}^*(w)$ is not different from problem $\mathcal{P}_3$ except that the markings of places $p \in P^f(w)$ are given and that the markings for the other places are supposed to be real numbers.

b. Introducing new constraints to problem $\mathcal{P}^*(w)$

b_1. If variables x_p represent the number of tokens in place p of the initial model (i.e. the total number of tokens in place p and the place p' corresponding to p in the extended model), then a necessary and sufficient condition to obtain a cycle time of the system less than C is:

$$\sum_{p\in\gamma} x_p \geq \left\lceil \frac{\sum_{t\in\gamma}\theta_t}{C} \right\rceil \quad \forall\gamma\in\Gamma \tag{24}$$

where Γ is the set of elementary circuits of the initial EG model.

Thus, adding constraints (24) to problem $\mathcal{P}^*(w)$ would improve the lower bounds. Unfortunately, the number of elementary circuits is too big to introduce constraints (24) for all of them. We thus use the solution of $\mathcal{P}^*(w)$ to select the constraints which are the most interesting to improve (i.e. increase) the lower bound. Using relations (20), it is easy to prove that:

$$\sum_{p\in\gamma} x_p \geq \frac{\sum_{t\in\gamma}\theta_t}{C}, \quad \forall\gamma\in\Gamma$$

where $\{x_p\}_{p\in P}$ is the solution of $\mathcal{P}^*(w)$, but (24) may be violated for some of the $\gamma\in\Gamma$. The idea is thus to add to problem $\mathcal{P}^*(w)$ the constraints (24) which are violated by the solution of $\mathcal{P}^*(w)$.

For each place $p \in P_0$, we consider the set Γ_p of circuits containing p and determining the one, denoted by γ_p, which minimizes $\left(\sum_{p\in\gamma} x_p - \frac{1}{C}\sum_{t\in\gamma}\theta_t\right)$. This elementary circuit can be determined polynomially.

Constraints (24) related to the circuit γ_p are added to problem $\mathcal{P}^*(w)$.

b_2. Constraints (24) concern the elementary circuits. The constraints introduced hereafter concern the paths of the event graph. We need some preliminary results to establish these new constraints.

Result 3:

We consider a path $\sigma = (t_1, p_1, t_2, p_2, \ldots, t_{n-1}, p_{n-1}, t_n)$ of the event graph. In a periodic operation node, a necessary condition for the system to have a cyclic time less than or equal to C is that σ contains at least $\left\lfloor \sum_{i=2}^{n}\theta_{t_i} / C \right\rfloor$ tokens.

Proof:

The operation node being periodic, relation (6) holds, and thus:

$$S_{t_i}(1) + \theta_{t_i} \leq S_{t_{i+1}}(1) + C.M_0(p_i), \text{ for } i = 1,2,\ldots,n-1 \tag{25}$$

By adding relations (25), we obtain:

$$S_{t_1}(1)+\sum_{i=1}^{n-1}\theta_{t_i} \le S_{t_n}(1)+C\sum_{i=1}^{n-1}M_0(p_i)$$

But, according to relations (8):

$S_{t_n}(1) \le C - \theta_{t_n}$.

and thus:

$$S_{t_1}(1)+\sum_{i=1}^{n-1}\theta_{t_i} \le C-\theta_{t_n}+C\sum_{i=1}^{n-1}M_0(p_i)$$

or:

$$S_{t_1}(1)+\sum_{i=1}^{n}\theta_{t_i} \le C+C\sum_{i=1}^{n-1}M_0(p_i) \tag{26}$$

But, according to relations (8), we also have:

$S_{t_1}(1) > -\theta_{t_1}$

thus:

$$-\theta_{t_1}+\sum_{i=1}^{n}\theta_{t_i} < C+C\sum_{i=1}^{n-1}M_0(p_i)$$

or:

$$\sum_{i=2}^{n}\theta_{t_i} < C+C\sum_{i=1}^{n-1}M_0(p_i)$$

This leads to:

$$\sum_{i=1}^{n-1}M_0(p_i) > \left(\sum_{i=2}^{n}\theta_{t_i}/C\right)-1$$

The $M_0(p_i)$'s being integers:

$$\sum_{i=1}^{n-1}M_0(p_i) \ge \left\lfloor \sum_{i=2}^{n}\theta_{t_i}/C \right\rfloor$$

Q.E.D.

Corollary

If, in a path $\sigma = (t_1, p_1, t_2, p_2, \ldots, t_{n-1}, p_{n-1}, t_n)$ of the event graph, t_1 is the transition t_0, i.e. the transition belonging to the root of the B&B tree, then the number of tokens in σ should be at least equal to $\left\lfloor \sum_{i=2}^{n}\theta_{t_i}/C-\varepsilon \right\rfloor$ when the operation node is periodic in order to reach a cycle time less than or equal to C. ε is a positive real which can be as small as possible.

Proof:

If $t_1 = t_0$, then $S_{t_1}(1) = 0$, and inequality (26) becomes:

$$\sum_{i=1}^{n}\theta_{t_i} \le C+C\sum_{i=1}^{n-1}M_0(p_i)$$

or:

$$\sum_{i=1}^{n-1} M_0(p_i) \geq \left(\sum_{i=1}^{n} \theta_{t_i} / C\right) - 1$$

and:

$$\sum_{i=1}^{n-1} M_0(p_i) \geq \left\lfloor \sum_{i=1}^{n} \theta_{t_i} / C - \varepsilon \right\rfloor$$

Q.E.D.

The constrains related to the path are derived from Result 3, the corollary, as well as from inequalities (24).

We denote by w the current node. Let us consider a place p whose marking has not been decided yet, i.e. $p \notin P^f(w)$. Let us also consider the marking M_f with $M_f(p) = M_0(p)$, $\forall\, p \in P^f(w)$, and $M_f(p) = 0$, $\forall\, p \notin P^f(w)$. For each transition $t \in T$ and $t \neq {}^o p$, we compute a path $\sigma_0(t, {}^o p)$ joining t to ${}^o p$ which

$$\textbf{Maximizes} \sum_{q \in \sigma_0(t, {}^o p) \cap p} \left(\theta_{q^o} - C.M_f(q)\right)$$

Notice that $\sigma_0(t, {}^o p)$ is the path among those joining t to ${}^o p$ in which a maximal number of tokens is needed to reach the cycle time C and that it can be determined polynomially.

Consider now path $\sigma(t,p)$ obtained from $\sigma(t, {}^o p)$ by adding p and p^o.

Let $\mu(t,p)$ be the number of tokens assigned to path $\sigma(t,p)$ according to the optimal solution $\{x_q\}_{q \in P}$ of problem $\mathcal{P}^*(w)$, i.e. $\mu(t,p) = \sum_{q \in \sigma(t,p)} x_q$. Let:

$$\Delta(t,p) = \begin{cases} \left\lceil \sum_{i=1}^{n} \theta_{t_i} / C \right\rceil - \mu(t,p) & \text{if } \sigma(t,p) \text{ is an elementary circuit} \\ \left\lfloor \sum_{i=2}^{n} \theta_{t_i} / C \right\rfloor - \mu(t,p) & \text{if } \sigma(t,p) \text{ is not a circuit and } t \neq t_0 \\ \left\lfloor \sum_{i=1}^{n} \theta_{t_i} / C - \varepsilon \right\rfloor - \mu(t,p) & \text{if } \sigma(t,p) \text{ is not a circuit and } t = t_0 \end{cases}$$

$\Delta(t,p)$ is the minimal number of tokens which should be introduced in $\sigma(t,p)$ to meet the necessary condition for the system to reach a cycle time less than or equal to C.

We then define t^* such that:

$$\Delta(t^*, p) = \mathop{\textbf{Max}}_{t \in T} \Delta(t,p)$$

$\sigma(t^*,p)$ is the path which ends up at p^o and to which the larger number of tokens should be added to reach a cycle time less than or equal to C.

Finally, for each $p \notin P^f(w)$, we introduce the following constraint:

$$\sum_{q \in \sigma(t^*,p) \cap p} x_q \geq \mu(t^*,p) + \Delta(t^*,p) \quad \text{for } p \notin P^f(w) \tag{26}$$

$\mathcal{P}^{**}(w)$ is obtained by adding to $\mathcal{P}^*(w)$ constraints (24) and (26) as explained in this section. The solution of $\mathcal{P}^{**}(w)$ is the lower bound corresponding to node w.

If either $\mathcal{P}^*(w)$ or $\mathcal{P}^{**}(w)$ has no solution, the node w cannot lead to an optimal solution of the problem at hand, and no further computation is made starting from w.

7. THE B&B ALGORITHM

The B&B algorithm is summarized hereafter.

1. Select t_0 as shown in Subsection (3.a).
2. Order the set of places (see Subsection 3.b).
3. Create the root by setting $M_0(p) = 1$ for $p \in {}^o t_0$, and denote this root by w.
4. Compute the lower bound lb(w) at node w by:
   - solving $\mathcal{P}^*(w)$,
   - creating the additional constraints,
   - solving $\mathcal{P}^{**}(w)$,

   as shown in Section 6.
5. If either $\mathcal{P}^*(w)$ or $\mathcal{P}^{**}(w)$ does not have solution, remove w from the set $\mathcal{N}$ of pending nodes and go to step 8.
6. Compute the upper bound at node w (see Section 5) and denote by ub(w) the smallest upper bound obtained at this point.
7. If $lb(w) > ub(w)$, remove w from the set $\mathcal{N}$ of pending nodes.

   Otherwise:

   (i) if w has two descendants, compute them and add them to the set $\mathcal{N}$ of pending nodes, otherwise keep the solution corresponding to w if it is the first solution obtained, or if this solution is better than any of the solutions obtained previously,

   (ii) remove w from the set $\mathcal{N}$ of pending nodes.
8. If $\mathcal{N} \neq \emptyset$, choose one $w \in \mathcal{N}$ and go to step 4, otherwise print the best solution obtained and stop the computation.

8. APPLICATION TO THE DESIGN OF AN FMS

In this section, we apply the above algorithm to the design of an FMS.

a. Introduction of the problem

The FMS at hand is composed of four machines M_1, M_2, M_3 and M_4. It can produce three types of products denoted by T_1, T_2 and T_3. The manufacturing processes of these product types are given in Figure 3. The numbers in parentheses are the manufacturing times.

We neglect the transportation times for simplicity, but taking into account the transportation times would not modify the approach presented hereafter.

We assume that the production ratios considered in the current scenario are 1/4, 1/4 and 1/2 for T_1, T_2 and T_3, respectively.

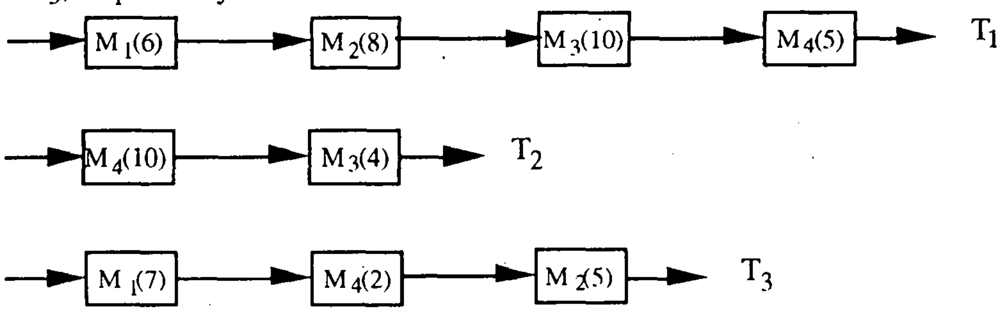

Fig. 3: Manufacturing processes of the product types

The goal is to reach a given productivity using a number of AGVs which is as small as possible.

The input sequences of the machines, which verify the production ratios, are:

for M_1 : T_1, T_3, T_3

for M_2 : T_1, T_3, T_3

for M_3 : T_1, T_2

for M_4 : T_1, T_2, T_3, T_3

The time necessary to manufacture a complete sequence which fits with the required ratios, say T_1, T_2, T_3, T_3, is:

for M_1 : $6 + 2 \times 7 = 20$

for M_2 : $8 + 2 \times 5 = 18$

for M_3 : $10 + 4 = 14$

for M_4 : $5 + 10 + 2 \times 2 = 19$

We know (see [4]) that it is always possible to fully utilize the bottle-neck machine, which is M_1 in this case. In other words, it is always possible to manufacture a sequence T_1, T_2, T_3, T_3 in steady state in a cycle time of length $C = 20$. The goal is to reach this goal with a number of AGVs which is as small as possible.

b. Modelling

The way to model a manufacturing system whose operation mode is periodic was extensively explained in [4] and [5]. The model corresponding to the example considered in this section in given in Figure 4.

This model is an event graph which exposes three types of elementary circuits which are:

(i) **The process circuits**, which model the manufacturing processes. For instance $<t_1, p_2, t_2, p_3, t_3, p_4, t_4, p_1, t_1>$ is a process circuit. These circuits model the physical part of the system, and the tokens which belong to these circuits represent AGVs;

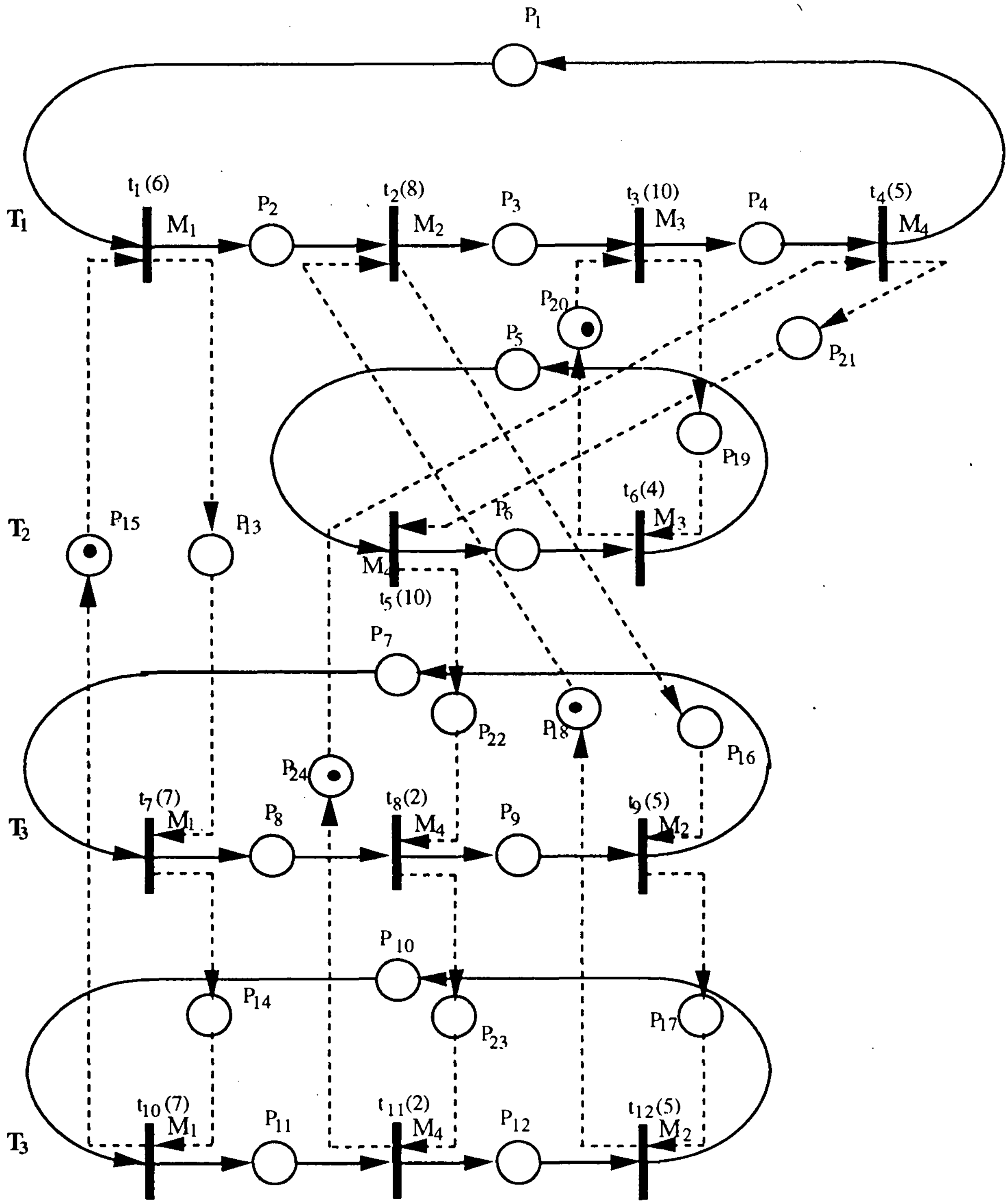

Fig. 4: The event graph model of the FMS

(ii) **The command circuits**, which model the control of the system. One command circuit is associated to each machine. For instance $<p_{15}, t_1, p_{13}, t_7, p_{14}, t_{10}, p_{15}>$ is a command circuit. There is exactly one token in each command circuit, which prevents two transitions corresponding to the same machine to be fired simultaneously. This token represents information;

(iii) **The mixed circuits**, partially composed of parts of the command circuits, and partially of parts of the process circuits. For instance $<t_{11}, p_{24}, t_4, p_1, t_1, p_{13}, t_7, p_{14}, t_{10}, p_{11}, t_{11}>$ is a mixed circuit. The number of mixed circuits is usually very high even for quite small models.

c. Problem formulation and solution

The p-invariant used to minimize the number of AGVs is the vector ${}^tU = [u_1, u_2, \ldots, u_{24}]$ such that:

$$u_i = \begin{cases} 1 & \text{if } i = 1,2,\ldots,12 \\ \alpha & \text{if } i = 13,14,\ldots,24 \end{cases}$$

We choose $\alpha = 10^4$ in order to limit to 1 the number of tokens in the command circuits, as required by the model.

Furthermore C= 20.

Any transition can be chosen as transition t_0 since each $t \in T$ has one input place belonging to a command circuit and one output place belonging to a process circuit. We choose $t_0 = t_1$. Thus, $M_0(p_1) = M_0(p_{15}) = 1$.

The places are ordered as follows:

- places belonging to command circuits are ordered first: $p_{13}, p_{14}, \ldots, p_{24}$;
- then places belonging to process circuits follow: $p_2, p_3, \ldots, p_{12}$.

The places used to build the extended model (which is not represented) follow place p_{12}.

In this example, the algorithm created 16 levels (i.e. examined 16 nodes, including the root). The final result is:

$$M_0 = (0,1,1,1,1,0,1,0,0,1,0,0,1,0,0,0,0,1,0,1,0,0,0,1)$$

We thus need three tokens in the process circuit corresponding to T_1, one token in the process circuit corresponding to T_2 and one in each of the process circuits corresponding to T_3.

Thus, assuming that each AGV carries one product, we need 6 AGVs if we want to manufacture a set (T_1, T_2, T_3, T_3) in steady state during each period of 20 units of time. The algorithm also provides the initial location of the AGVs, which means that, in transient state, we may have to let some AGVs unloaded during some periods, until the steady state is reached.

9. CONCLUSION

This paper provides a Branch-and-Bound approach to define the minimal number of AGVs to be used in order to reach a given productivity. Extended numerical experiences conducted by Sauer [9] show that event graph models with one hundred places and one hundred transitions can be solved, which corresponds to the size of common Flexible Manufacturing Systems.

Further research should concern the improvement of the lower bounds by finding new properties of the optimal solution. We should also find a way to mark several places at each iteration of the Branch-and-Bound algorithm.

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