Surface Reconstruction: GNCs and MFAs

Mads Nielsen

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

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Mads Nielsen

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Abstract: Noise-corrupted signals and images can be reconstructed by minimization of a Hamiltonian. Often the Hamiltonian is a non-convex functional. The solution of minimum energy can then be approximated by the Graduated Non-Convexity (GNC) algorithm developed for the weak membrane by Blake and Zisserman. The GNC approximates the non-convex solution space by a convex solution space, and varies the solution space slowly towards the non-convex solution space. Earlier work used Mean Field Annealing (MFA) to relax the Hamiltonian in the general case. It is often claimed that MFA leads to a GNC algorithm. It is shown that this is not necessarily the case, and especially not the case for earlier MF approximations of the weak membrane. In the case of the weak membrane, MFA might lead to predictable and inexpedient results. Two automatic and proved GNC-generating methods are presented. One is using a Gaussian filtering of the smoothness term and is called Smoothness Focusing (SF). The other is using a Gaussian filtering of the *a priori* distribution of the derivative in a Maximum A Posteriori estimation scheme, and is called Probability Focusing (PF). The algorithms are experimentally compared to the Blake-Zisserman GNC and shown competitive.

Key-words: Surface Reconstruction, Energy minimization, Relaxation, Bayesian Etimation, Statistical Physics

(Résumé : tsvp)

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Unité de recherche INRIA Sophia-Antipolis
2004 route des Lucioles, BP 93, 06902 SOPHIA-ANTIPOLIS Cedex (France)
Téléphone : (33) 93 65 77 77 – Télécopie : (33) 93 65 77 65

Reconstruction de Surfaces: GNCs et MFAs

Résumé : Les images ou les signaux bruités peuvent être reconstruits par la minimisation d'un hamiltonien. Souvent, l'hamiltonien est une fonctionnelle non convexe. La solution d'énergie minimale peut donc être approchée par l'algorithme GNC (Graduated Non Convexity) de Blake et Zisserman. Le GNC fait une approximation de la fonction énergie non convexe par une autre fonction convexe qu'il fait varier doucement vers la fonction initiale. Les précédentes approches ont utilisé le Mean Field Annealing (MFA) pour relaxer le hamiltonien. Il est souvent affirmé que l'algorithme MFA est équivalent à l'algorithme GNC. Nous montrons que ceci n'est pas nécessairement le cas, en particulier pour les premières approximations de la membrane faible. Dans le cas de la membrane faible, l'algorithme MFA peut conduire à des résultats prévisibles et indésirables. Nous présentons deux méthodes automatiques qui génèrent effectivement un algorithme GNC. La première utilise un filtrage gaussien du terme de lissage. La deuxième utilise un filtrage gaussien de la distribution *a priori* de la dérivée dans un modèle d'estimation du maximum a posteriori. Ces algorithmes sont comparés au GNC de Blake et Zisserman. Les résultats sont au moins aussi bons.

Mots-clé : Reconstruction des surfaces, regularisation, minimisation de l'énergie, physique statistique

1 Introduction

Regularization is a method of reformulating ill-posed inverse problems as well-posed problems as done by Tikhonov and Arsenin [1]. This reformulation implies the addition of a stabilizing term, followed by a global minimization of a energy function, yielding a unique solution. Applied to the problem of reconstruction of a noise-corrupted surface, the energy function or Hamiltonian (the terms are used interchangeably) can be expressed as follows:

$$E(\tilde{s}) = \int_{\Omega} (\tilde{s} - c)^2 + f(\tilde{s}) dA$$

where $\tilde{s}$ is the reconstruction, Ω is the domain of the reconstruction, c is the measurements, dA is an area element of Ω, and f is the stabilizing functional. Tikhonov uses a stabilizing functional which is a sum of the squares of derivatives of $\tilde{s}$, and obtains a convex energy function, making the minimization is simple.

Geman and Geman [2] introduced a discontinuity field in the reconstruction and proposed thereby the model of the weak membrane, which is formulated in the continuous case by Mumford and Shah [3]. The discontinuity field is incorporated directly in the stabilizing functional by Blake and Zisserman [4], using what Rangarajan and Cheleppa [5] call the adiabatic approximation. The resulting stabilizing term will no longer lead to a convex solution space, therefore we will not use the term regularization, but surface reconstruction, because several solutions of minmum energy might theoretically exist.

In general the stabilizing term reflects the *a priori* knowledge of the surface, and other stabilizing functionals might be appropriate for other reconstruction problems. The stabilizing functional can be interpreted in terms of information theory, as an entropy measure [6], or in terms of Bayesian Estimation using Maximum A Posteriori (MAP) estimation as an M-estimator [7]. Given a noise model $P(c|s)$, where s is the ideal surface, which is to be reconstructed and an *a priori* distribution of surfaces $P(s)$, we can calculate the *a posteriori* probability of a given surface using Bayes formula:

$$P(s|c) = \frac{P(c|s)P(s)}{P(c)}$$

where $P(c)$ is a normalizing constant or in terms of statistical physics the partition function. Typically, the *a priori* distribution is given in terms of the derivative of the surface, which is why we use the notation $P(\nabla s)$. Using a noise model of additive, Gaussian, uncorrelated noise, and no spatial correlation of $P(\nabla s)$ we can find the minus-log-probability function of a surface as

$$E(\tilde{s}) = \sum(\tilde{s} - c)^2 + f(\nabla \tilde{s}) \tag{1}$$

where $f(\nabla \tilde{s}) = -\log P(\nabla \tilde{s})$, and ∇ is a difference operator in the discrete approximation. In terms of statistical physics E is called the Gibbs energy. Because the minus-log function is monotonically decreasing, a minimization of the Gibbs energy correspond to a maximization of the *a posteriori* probability. In the following we call the function f in Equation 1 the smoothness function. The properties of the reconstruction and the convexity of the solution space depends strongly on the smoothness function. Thikonov [1] used a parabolic smoothness function, Blake and Zisserman [4] used a thresholded parabolic smoothness function, and we [8] used a Lorentzian estimator. In the following, we will not in general refer to any specific smoothness function.

Geman and Geman [2] used Simulated Annealing (SA) to find the minimum of Equation 1, while Blake and Zisserman [4] introduced the deterministic and approximative Graduated Non-Convexity (GNC) algorithm in the case of the weak membrane and showed that it is up to 50 times faster than SA [9]. Geiger and Girosi [10] and Bilbro et. al. [11] used the Mean Field Annealing (MFA) formalism to create a deterministic version of the SA, and claimed it creates GNC-like algorithms.

This paper concerns how the concept of GNC can be generalized, which criteria an algorithm has to fulfill to be a GNC, and how previous work such as MFA is explained in terms of GNC. In Section 2 the Blake and Zisserman GNC is sketched and the criteria of a GNC algorithm are emphasized. In Section 3 the MFA is sketched, and it is shown, that it is not leading to a GNC algorithm in the case of the weak membrane, but to an algorithm which in some cases yields predictable and inexpedient results. In Section 4 two alternatives to the MFA are given in the general case. They are proven to yield GNC algorithms and are fully automatic.