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Leonidas Georgiadis, Wojciec Szpankowski, L. Tassiulas

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

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Leonidas GEORGIADIS
Wojciech SZPANKOWSKI - Leandros TASSIULAS

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ADAPTIVE POLICIES FOR SPATIAL REUSE RING NETWORKS

Leonidas Georgiadis
High Perf. Comp. and Comm.
IBM T. J. Watson Research Ctr.
P.O. Box 704
Yorktown Heights, NY 10598
U.S.A.

Wojciech Szpankowski*
Dept. Computer Science
Purdue University
Computer Science Build.
W. Lafayette, IN 47907
U.S.A.

Leandros Tassiulas
Dept. Electrical Engn.
Polytechnic University
6 Metrotech Center
Brooklyn, NY 11201
U.S.A.

Abstract

A slotted ring that allows simultaneous trasmissions of messages by different users is considered. Such a ring network is commonly called ring with *spatial resue*. It can achieve significantly higher throughtput than standard token rings but it also can lead to unfairness problems. Policies that operate in cycles and quarantee that a certain number (quota) of packets will be transmitted by every node in every cycle have been considered before to alleviate the unfairness. We consider here the problem of designing a policy that will result in a stable system whenever the arrival rates are within the stability region of a ring with spatial reuse (the stability region is defined as the set of node arrival rates for which there is a policy that makes the ring stable). We provide such a policy. No knowledge of arrival rates or message destination probabilities are required. The policy is an adaptive version of the quota policies and can be implemented with the same distributed mechanism. We shall use Lyapunov test function techniques together with the regenerative approach to derive our main results.

STRATEGIES ADAPTATIVES POUR LA REUTILISATION DU MEDIUM SUR UN RESEAU EN ANNEAU

Résumé

On considère un anneau slotté qui permet la transmission simultanée de paquets par différents utilisateurs. Un tel réseau est appelé généralement anneau avec réutilisation spatiale. Ce réseau peut fournir un débit beaucoup plus grand que l'anneau à jeton mais il ne fournit pas un service équitable. Nous considérons des stratégies qui sont basées sur des services cycliques et qui garantissent à chaque cycle un certain quota pour chacun des nœuds. Nous décrivons une politique qui conduit à un système stable quelques soient les taux arrivées dans le domaine de stabilité de l'anneau à jeton avec réutilisation spatial (ce domaine est défini comme l'ensemble des taux d'arrivée pour lesquels il existe une stratégie stable). Aucune connaissance des taux d'arrivées ou de la distribution des destinations n'est requise. Cette stratégie est une version adaptée des stratégies de type quota et peut être implémentée avec le même mécanisme distribué. Nous utiliserons la fonction de Lyapounov ainsi que l'approche régénérative pour démontrer nos résultats principaux.

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1 Introduction

We consider a ring with spatial reuse, i.e., a ring in which multiple simultaneous transmissions are allowed as long as they take place over different links (cf. [4, 5, 6]. Time is divided in slots and each slot is equal to the smallest transmission unit, called packet. We assume zero propagation delay. A node can transmit a packet at the outgoing link at the same time that it receives another packet in the incoming link. A node receiving a packet with destination another node in the ring, may retransmit the packet in the outgoing link *in the same slot*, i.e., the ring has *cut-through* capabilities.

In [5], [4], a policy is proposed for the operation of the ring. Each node is assigned a number called "quota" The policy operates in cycles. A node is allowed to transmit during a cycle as long as the number of transmitted packets does not exceed its assigned quota. An analysis of the throughput characteristics of this policy is provided in [6], where it is also shown that that if the end-to-end throughput requirements result in aggregate traffic load for each link of the network less than one, then the quotas can be selected to achieve these throughput requirements. In order to compute the appropriate quota values, however, it is necessary to know the end-to-end transmission rates a priori.

In this paper, we propose an adaptive version of the policy in [6]. The adaptive policy does not need to preselect the quota based on the end-to-end traffic requirements. During the operation of the system, it continuously readjusts the quotas based on the backlogs. In this manner it achieves any achievable traffic requirements. We denote such a policy as Π.

More precisely, the proposed policy operates in cycles and is based on the idea of allocating quotas to the nodes. At the beginning of a cycle it allocates "quota" to every user that is a function of the number of messages at the user buffer. During a cycle a node can transmit no more messages than the quota allocated to it. A cycle ends when *all* nodes transmit their quota number of packets as well as all the packets that they may have received through their incoming link and are destined to another node. The proposed policy requires a distributed mechanism by which every node realizes that all the other nodes completed their quota and thus a cycle ends. Such a mechanism is provided in [5].

Policy Π is shown to stabilize the network for any arrival rate vector in region R (section

2); this region contains all the arrival rate vectors which result in link utilization less than one. A similar study has been done in [2]. The stability of the ring was studied for stationary arrival processes and a policy was proposed that stabilized the ring under the same conditions for the arrival rates. The analysis that is done here is for a more restricted arrival process but the policy Π is considerably simpler than the one proposed in [2].

This paper is organized as follows. In the next section we present our main results and their consequences. In particular, we establish stability region for the adaptive policy, and show that it achieves the maximum possible node throughput. We delay all proofs until section 3. It contains a fairly detailed analysis of the policy from the stability view point. We use the Lyapunov test function method (cf. [7]) together with regenerative approach (cf. [1]) to establish our results.

2 The system model the policy and the main result

We consider a ring with spatial reuse, i.e., a ring in which multiple simultaneous transmissions are allowed as long as they take place over different links. Time is divided in slots and each slot is equal to the smallest transmission unit, called packet. We assume zero propagation delay. A node can transmit a packet at the outgoing link at the same time that it receives another packet in the incoming link. A node receiving a packet with destination another node in the ring, may retransmit the packet in the outgoing link *in the same slot*, i.e., the ring has *cut-through* capabilities.

Let M be the number of nodes and set $\mathcal{M} = \{1, \ldots, M\}$. The operations $i \oplus j$ and $i \ominus j$ denote respectively, addition and subtraction modulo M, with the convention that index 0 refers to node M. Furthermore, when i, j refer to node indices we denote $\sum_{k=i}^{j} x_k := x_i + x_{i\oplus 1} + \ldots + x_{j\ominus 1} + x_j$. We assume that the nodes are arranged on the ring according to their index so that the outgoing link to node i is the incoming link for node $i \oplus 1$. Node i may receive external traffic with destination any other node j in the system. Let $R_{ij}(t)$ be the number of packets that arrive at node i from the outside with destination node j. If $i = j$, then it is assumed that the packet has to cross all the nodes in the ring until it is received by the originating node, i.

Transmission policy Π

The proposed policy operates in cycles and is based on the idea of allocating quotas to the nodes, proposed in [5],[4]. Let τ_k be the beginning of the kth cycle and set $\tau_1 = 1$. At time τ_k each node allocates itself "quota" $\nu_i(k) = Q_i(\tau_k)$, where $Q_i(t)$ is the queue size of node i at time t. Node i can transmit up to $\nu_i(k)$ packets during cycle k according to *any fixed nonidling* policy, i.e., the only restriction that is imposed on the transmissions is that the node puts a packet in its outgoing link whenever either its queue is nonempty, or a message is received in the same slot in its incoming link with destination another node on the ring. Cycle k ends when *all* nodes transmit $\nu_i(k)$ packets as well as all the packets that they may have received through their incoming link and are destined to another node.

Remarks:

1. The most important nonidling transmission policy for applications, is the policy where a node always gives nonpreemptive priority to the packets that arrive at the incoming link with destination another node. This way, only a single buffer capable of holding a maximum packet size message is needed to hold the traffic that arrives at the incoming link of a node. For details see [4].

2. The proposed policy requires a distributed mechanism by which every node realizes that all the other nodes completed their quota and thus a cycle ends. Such a mechanism is provided in [5].

Throughout this section we adopt the following assumption.

(A1) The vector process $\{\mathbf{R}(t)\}_{t=1}^{\infty}$, where $\mathbf{R}(t) = \{R_{ij}(t),\ i,j \in \mathcal{M}\}$, consists of i.i.d. vectors. We denote $R_{ij} := R_{ij}(1)$. Note that do not make any independence assumptions for the work arriving in various nodes at the same slot. To avoid technical difficulties we will also assume that $\Pr(R_{ij}(t) = 0,\ i,j \in \mathcal{M}) > 0$.

In section 4 we will see that the above assumption can be relaxed in certain ways without affecting significantly the validity of our results. In order to formulate our main results in a compact form, we need some additional notation. Let

$$\rho_{ij} := \lim_{n\to\infty} \frac{\sum_{t=1}^{n} R_{ij}(t)}{n} = ER_{ij}$$

be the traffic rate. We also define $\alpha_{im} := \sum_{m\oplus 1}^{i} \rho_{ij}$ and $r_m = \sum_{i\in\mathcal{M}} \alpha_{im}$. Note that α_{im} is the average number of packets per slot that are generated by node i and cross node m, and r_m is the average number of packets that cross node m during a slot. Finally, we set $r = \max\{r_m : \ m \in \mathcal{M}\}$.

It has been shown in [6] that for a fairly large class of policies for the ring, the throughput vector should lie within the following region

$$\mathcal{R} = \{\boldsymbol{\rho} : \max_{1\le m\le M} r_m < 1\} \tag{1}$$

In this work we show that as long as the arrival rates belong to R, policy Π stabilizes the network in a strong sense. Specifically we show that (i) the queue length $Q_i(t)$ possesses a limiting distribution; (ii) its lth moment $EQ_i^l(t)$ as $t \to \infty$ exists provided that $ER_{ij}^{l+1} < \infty$; (iii) the queue length $Q_i(t)$ as $t \to \infty$ has an exponential tail (i.e., large backlogs are very unlikely), provided that the same is true for R_{ij}. We summarize our main results in the following proposition. Its proof is presented in the next section.

Proposition 1 (i) *Under policy Π, the process of queue lengths $\{Q_i(t),\ i \in \mathcal{M}\}_{t=1}^{\infty}$ converges in distribution to a random finite vector $\{\tilde{Q}_i,\ i \in \mathcal{M}\}$ having a honest distribution, if and only if $r < 1$.*

(ii) *If and only if $ER_{ij}^{l+1} < \infty$ for some $l \ge 1$ and all $i, j \in \mathcal{M}$, then*

$$\lim_{t\to\infty} EQ_i^l(t) = E\tilde{Q}_i^l < \infty \tag{2}$$

(iii) *If for $\vartheta > 0$ the moment generating function of R_{ij} exists, that is, $E\exp(\vartheta R_{ij}) < \infty$ for all $i, j \in \mathcal{M}$, then there exists $\vartheta' > 0$ such that*

$$E\exp(\vartheta' \tilde{Q}_i) < \infty \tag{3}$$

for every $i \in \mathcal{M}$. ∎

As a direct consequence of Proposition 1(iii), we have the following corollary concerning the tail of the queue length.

Corollary 1 (i) *If* $ER_{ij}^{l+1} < \infty$ *for all* $i, j \in \mathcal{M}$, *then the tail of the queue length distribution decays polynomially fast, that is, for some constant* $C > 0$

$$\Pr\{\tilde{Q}_i > k\} \leq \frac{C}{k^l} . \tag{4}$$

(ii) *Under the hypothesis of Proposition 1(iii), the queue length* $\tilde{Q}_i$ *has an exponential tail, that is, there exists a constant* $C > 0$ *and* $\vartheta > 0$ *such that for all* $k \geq 0$

$$\Pr\{\tilde{Q}_i > k\} \leq Ce^{-\vartheta k} . \tag{5}$$

Proof. It suffices to apply Markov inequality. For example, for part (ii) we have $\Pr\{\tilde{Q}_i > k\} = \Pr\{e^{\vartheta \tilde{Q}_i} > e^{\vartheta k}\} \leq e^{-\vartheta k} E e^{\vartheta \tilde{Q}_i}$ provided (3) holds. ∎

3 Stability Analysis: Proofs

The proof of Proposition 1 is based on the analysis of the so called *node degree* at time t denoted as $N_i(t)$. It is defined as the total number of packets on the ring at time t that have to cross node i in order to reach their destination. Let $N(t) := \max\{N_i(t) : \; i \in \mathcal{M}\}$. It was shown in [6] that

$$T_k := \tau_{k+1} - \tau_k = N(\tau_k) \tag{6}$$

Now we establish an important asymptotic property of $N(\tau_2)$ that is used in the proof of Proposition 1.

Lemma 1 *If for some* $l \geq 1$ *we have* $ER_{ij}^l < \infty$ *for all* $i, j \in \mathcal{M}$, *then*

$$\lim_{n \to \infty} E\left(\left(\frac{N(\tau_2)}{T_1}\right)^l \middle| T_1 = n\right) = \max\left\{r_m^l : \; m \in \mathcal{M}\right\} .$$

Proof. According to the policy, the queue size at node i at time τ_2 consists of all the external packets that arrive in the interval $(1, \tau_2]$ to node i. From the definition of the degree of a node it follows that if $T_1 = n$,

$$N_m(\tau_2) = \sum_{t=1}^{n} \sum_{i \in \mathcal{M}} \sum_{j = m \oplus 1}^{i} R_{ij}(t). \tag{7}$$

Using the strong law of large numbers we conclude that

$$\lim_{n\to\infty} \frac{N_m(\tau_2)}{n} = \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} r_{ij} = \sum_{i\in\mathcal{M}} \alpha_{im}.$$

Now let $F(\cdot)$ be a nondecreasing continuous function. In view of the above, we have for almost all sample paths,

$$\begin{aligned}
\lim_{n\to\infty} F(N(\tau_2)/n) &= \lim_{n\to\infty} F\left(\max\{N_m(\tau_2)/n:\ m\in\mathcal{M}\}\right) \\
&= \lim_{n\to\infty} \max\{F(N_m(\tau_2)/n):\ m\in\mathcal{M}\} \\
&= \max\left\{\lim_{n\to\infty} F(N_m(\tau_2)/n):\ m\in\mathcal{M}\right\} \\
&= \max\{F(r_m):\ m\in\mathcal{M}\}. \qquad (8)
\end{aligned}$$

The lemma will follow from (8) with $F(x) = x^l$, $l \geq 1$, if we show that the sequence $\{(N(\tau_2)/n)^l\}$ is uniformly integrable (u.i.).

Using the *mean inequality*

$$\left(\frac{\sum_{i=1}^{k} |a_i|}{k}\right)^l \leq \frac{\sum_{i=1}^{k} |a_i|^l}{k}, \quad l \geq 1,$$

we have

$$\begin{aligned}
N_m^l(\tau_2) &= \left(\sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \left(\sum_{t=1}^{n} R_{ij}(t)\right)\right)^l \\
&\leq M^{2(l-1)} \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \left(\sum_{t=1}^{n} R_{ij}(t)\right)^l
\end{aligned}$$

Therefore,

$$\begin{aligned}
\left(\frac{N_m(\tau_2)}{n}\right)^l &\leq M^{2(l-1)} \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \left(\frac{\sum_{t=1}^{n} R_{ij}(t)}{n}\right)^l \\
&\leq M^{2(l-1)} \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \frac{\sum_{t=1}^{n} R_{ij}^l(t)}{n} \qquad (9)
\end{aligned}$$

Since by assumption $ER_{ij}^l(1) < \infty$ and the variables $\{R_{ij}(t)\}_{t=1}^{\infty}$ are i.i.d, it follows (see [3, exercise 4.2.7] that the sequence $\{(\sum_{t=1}^{n} R_{ij}^l(t))/n\}_{n=1}^{\infty}$ is uniformly integrable. Therefore, the sequence

$$\sum_{i \in \mathcal{M}} \sum_{j=m+1}^{i} \left(\sum_{t=1}^{n} \frac{R_{ij}(t)}{n} \right)^l$$

is uniformly integrable since it is the sum of uniformly integrable sequences (see [3, page 94]). From (9) it follows that the sequence $\{(N_m(\tau_2)/n)^l\}_{n=1}^{\infty}$ is uniformly integrable and since $N(\tau_2) \le \sum_{m \in \mathcal{M}} N_m(\tau_2)$, the same holds for the sequence $\{(N(\tau_2)/n\}_{n=1}^{\infty}$. Finally, from (8) and the uniform integrability of the sequence $\{(N(\tau_2)/n\}_{n=1}^{\infty}$ it follows that we can interchange limits and expectations, i.e,

$$\lim_{n \to \infty} E\left(\frac{N_m(\tau_2)}{n}\right)^l = \max\{r_m^l : \ m \in \mathcal{M}\}.$$

for all $l \ge 1$. ∎

A corresponding property established in Lemma 1 for the exponential tail is presented below. It is used to prove part (iii) of Proposition 1.

Lemma 2 *If $r < 1$ and $E\exp(\vartheta R_{ij}) < \infty$ for some $\vartheta > 0$ and all $i, j \in \mathcal{M}$, then there exists $\vartheta' > 0$ such that*

$$\lim_{n \to \infty} E\left(\left(\frac{\exp(\vartheta' N(\tau_2))}{\exp(\vartheta' T_1)}\right) | T_1 = n\right) = 0.$$

Proof. Observe first that if X_i, $i = 1, \ldots, K$ are random variables such that $E(\exp(\vartheta X_i)) < \infty$, $i = 1, \ldots, K$ for some ϑ, then

$$E\exp(\vartheta_1 \sum_{i=1}^{K} X_i) < \infty,$$

where $\vartheta_1 = \vartheta/K$. This follows by taking expectations in the following inequality that is a consequence of the convexity of the exponential function

$$\exp(\vartheta_1 \sum_{i=1}^{K} X_i) = \exp\left(\frac{\sum_{i=1}^{K} K\vartheta_1 X_i}{K}\right) \le \frac{1}{K}\sum_{i=1}^{K} \exp(\vartheta_1 K X_i) \tag{10}$$

Applying the previous observation to the random variables

$$\tilde{R}_m(t) := \sum_{i \in \mathcal{M}} \sum_{j=m \oplus 1}^{i} R_{ij}(t), \quad m \in \mathcal{M},$$

we see that there is a $\vartheta_2 > 0$ such that

$$E \exp(\vartheta_2 \tilde{R}_m(t)) < \infty, \quad m \in \mathcal{M}.$$

Consider now the function $\Phi_m(\vartheta) = E \exp(\vartheta \tilde{R}_m(t) - \vartheta)$, $0 \leq \vartheta \leq \vartheta_2$. ¿From the previous discussion we see that this function is well defined, continuous and differentiable in $[0, \vartheta_2]$.

Since $\Phi_m(0) = 1$, $\Phi'_m(0) = E\tilde{R}_m(t) - 1 = r_m - 1 \leq r - 1 < 0$, and $M < \infty$, it follows that there is a $\vartheta' > 0$ and a $\epsilon > 0$, such that for $m \in \mathcal{M}$, $\Phi_m(\vartheta') < 1 - \epsilon$, or equivalently,

$$\frac{E \exp(\vartheta' \tilde{R}_m(t))}{\exp \vartheta'} < (1 - \epsilon)$$

¿From (7) we see that

$$N_m(\tau_2) = \sum_{t=1}^{n} \tilde{R}_m(t)$$

and since the random variable $R_m(t)$, $t = 1, \ldots$ are i.i.d, we conclude that

$$\frac{E \exp(\vartheta' N_m(\tau_2))}{\exp(\vartheta' n)} = \left(\frac{E \exp(\vartheta' R_m(t))}{\exp(\vartheta')} \right)^n \leq (1 - \epsilon)^n. \tag{11}$$

Since

$$\begin{aligned} E \ \exp(\vartheta' N(\tau_2)) &= E \exp(\max\{\vartheta' N_m(\tau_2) : \ m \in \mathcal{M}\}) \\ &= E \max\{\exp(\theta' N_m(\tau_2)) : \ m \in \mathcal{M}\} \\ &\leq \sum_{m \in \mathcal{M}} E \exp(\vartheta' N_m(\tau_2)), \end{aligned}$$

taking into account (11) we conclude

$$0 \leq \lim_{n \to \infty} \frac{E \exp(\vartheta' N(\tau_2))}{\exp(\vartheta' n)} \leq M \lim_{n \to \infty} (1 - \epsilon)^n = 0$$

which completes the proof. ∎

¿From Lemmas 1 and 2 we easily conclude our next result.

Corollary 2 (i) *If $r < 1$ and $ER_{ij}^l < \infty$ for all $i, j \in \mathcal{M}$ and some $l \geq 1$, then there exist $\delta > 0$ and $B > 0$ such that*

$$E(N^l(\tau_2)|N(\tau_1) = n) \leq (1-\delta)n^l \qquad n \geq B \ .$$

(ii) *If $r < 1$ and $E\exp(\vartheta R_{ij}) < \infty$ for some $\vartheta > 0$ and all $i, j \in \mathcal{M}$, then there exists $\vartheta' > 0$ such that*

$$E\left(\exp(\vartheta' N(\tau_2))|N(\tau_1) = n\right) \leq (1-\delta)\exp(\vartheta' n) \qquad n \geq B \ .$$

Proof. Since $r_m \leq r < 1$, we have that $\max\{r_m^l : m \in \mathcal{M}\} \leq r < 1-\delta$, $\delta > 0$. Using this observation, part (i) of the Corollary 2 follows directly from from (6) and Lemma 1. Part (ii) follows directly from (6) and Lemma 2. ∎

To proceed, we need the following theorem, due to Tweedie, [7, Theorem 3] , which we present in a form appropriate for the problem under consideration.

Theorem 1 (Tweedie.) *Suppose that $\{X_n\}_{n=1}^{\infty}$ is an aperiodic and irreducible Markov chain with countable state space S. Let $f(x)$ be a nonnegative real function on the state space. If A is a finite set such that $f(x) \geq \epsilon > 0$, $x \in A^c$,*

$$E(f(X_2)|X_1 = x) < \infty, \quad x \in A$$

and for some $\delta > 0$,

$$E(f(X_2)|X_1 = x) < (1-\delta)f(x), \quad x \in A^c,$$

then the Markov chain is ergodic and

$$Ef(\hat{X}) < \infty,$$

where $\hat{X}$ has the steady state distribution of the Markov chain $\{X_n\}_{n=1}^{\infty}$. ∎

Let $Q_{ij}(t)$ be the number of packets at node i with destination node j, including both external packets and packets received from the incoming link to node i. Clearly, $Q_i(t) = \sum_{j\in\mathcal{M}} Q_{ij}(t)$. Let also $\mathbf{Q}(t) := \{Q_{ij}(t) : \ i, j \in \mathcal{M}\}$. From the operation of the policy and assumption (A1) we conclude that the process $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$ is an imbedded Markov chain. Using Theorem 1 we shall prove that this imbedded Markov chain is ergodic if $r < 1$.

Moreover, using a regenerative structure of the queueing process, we extend this assessment to all t, hence proving our main result Proposition 1.

Let us first consider the imbedded Markov chain $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$. We prove that under the condition of Proposition 1 this process converges weakly to a honest random vector $\hat{\mathbf{Q}}$. Since by assumption $\Pr(R_{ij}(t) = 0,\ i,j \in \mathcal{M}) > 0$, it can be seen that the imbedded Markov chain has only one irreducibility set and if restricted to this set, the chain is aperiodic. Let now $\mathbf{Q}(\tau_1) = \mathbf{0}$, and we define two stopping times, namely: θ_k and $\mathcal{T}_k$. For the former we set $\theta_1 = 1$, and then

$$\theta_{k+1} := \inf\{n > \theta_k : \mathbf{Q}(\tau_n) = 0\} \ . \tag{12}$$

For the latter we set $\mathcal{T}_0 = 1$ and

$$\mathcal{T}_{k+1} = \min\{\tau_l :\ \tau_l > \mathcal{T}_k \ \text{ such that } \ \mathbf{Q}(\tau_l) = 0\} \ . \tag{13}$$

Note that $\mathcal{T}_k = \tau_{\theta_k}$. We also observe that by Theorem 2 proved below, the times $\mathcal{T}_j$ are well defined for all k since the system will empty infinitely often almost surely. Let also $d_k = \theta_{k+1} - \theta_k$, and $D_k = \mathcal{T}_{k+1} - \mathcal{T}_k$. Clearly, d_k and $\mathcal{T}_k$ for $k = 1, \ldots,$ are i.i.d.

The next result establish stability property for the imbedded Markov chain.

Theorem 2 (i) *If $r < 1$ then the Markov chain $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$ is ergodic and*

$$E \sum_{n=1}^{\theta_2 - 1} N(\tau_n) < \infty \ . \tag{14}$$

(ii) *If, in addition, for $l \geq 2$ we have $ER_{ij}^l < \infty$ for all $i, j, \in \mathcal{M}$, then*

$$E \sum_{n=1}^{\theta_2 - 1} N^l(\tau_n) < \infty \ , \tag{15}$$

(iii) *If for some $\vartheta > 0$ we have $E\exp(\vartheta R_{ij}) < \infty$ for all $i, j \in \mathcal{M}$, then there exists $\vartheta' > 0$ such that*

$$E \sum_{n=1}^{\theta_2 - 1} \exp(\vartheta' N(\tau_n)) < \infty \ . \tag{16}$$

Proof. Define first $f_l(\mathbf{Q}(\tau_n)) = N^l(\tau_n),\ \ l \geq 1$. Clearly, the set $A := \{\mathbf{Q} : f_l(\mathbf{Q}) < B\}$ is finite. Also, if $\mathbf{Q}(\tau_1) \in A$, then since $T_1 = N(\tau_1) < B^{(1/l)}$, using arguments similar to those used in the proof of Lemma 1 it can be easily seen that

$$E\left(N^l(\tau_2)|\mathbf{Q}(\tau_1) = \mathbf{Q} \in A\right) < \infty,$$

provided that $ER^l_{ij} < \infty$. From the above discussion, Corollary 2 and Theorem 1 we conclude that $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$ is ergodic, and provided that $ER^l_{ij} < \infty,\ i,j \in \mathcal{M}$ for some $l \geq 1$,

$$E\hat{N}^l < \infty,$$

where $\hat{N}^l = f_l(\hat{\mathbf{Q}})$, and $\hat{\mathbf{Q}}$ has the steady state distribution of $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$. Now observe that the sequence $\{N(\tau_n)\}_{n=1}^{\infty}$ is regenerative with respect to the renewal sequence $\{\theta_n\}_{n=1}^{\infty}$. Since the ergodicity of $\{\mathbf{Q}(\tau_n)\}$ implies $Ed_1 < \infty$, from the regenerative theorem, [1, Corollary 1.4] and the fact that $N(t)$ is nonnegative, we have that

$$\frac{E\sum_{n=1}^{\theta_2-1} N^l(\tau_n)}{Ed_1} = E\hat{N}^l < \infty.$$

for every $l \geq 1$.

For part (iii), the proof is along the same lines with $f(\mathbf{Q}(\tau_n)) = \exp(\vartheta N(\tau_n))$. ■

We now turn our attention to the process $\{\mathbf{Q}(t)\}_{t=1}^{\infty}$ for all $t = 0, 1, \ldots,$. We establish stability of this process proving our main result in Proposition 1. Assume that $\mathbf{Q}(1) = \mathbf{0}$. Consider the times $\mathcal{T}_k,\ k = 0, 1, ..$ defined in (13). The process $\{\mathbf{Q}(t)\}_{t=1}^{\infty}$ is regenerative with respect to the renewal process $\{\mathcal{T}_n\}_{n=1}^{\infty}$. From (6) and Theorem 2 we have

$$ED_1 = E\sum_{n=1}^{\theta_2-1} N(\tau_n) < \infty.$$

Since the assumption $\Pr(R_{ij}(t) = 0,\ i,j \in \mathcal{M}) > 0$ implies that D_k is aperiodic, applying the regenerative theorem we conclude that $\{\mathbf{Q}(t)\}_{t=1}^{\infty}$ converges in distribution to a honest random variable $\tilde{\mathbf{Q}}$. Let now $F(\cdot)$ be a nonnegative nondecreasing function (i.e., in our case either $F(x) = x^l$ or $F(x) = \exp(\vartheta x)$). Using now the nonnegativity of $Q_i(t),\ N(t)$, and

$$\lim_{t\to\infty} EF(Q_i(t)) = EF(\tilde{Q}_i) \leq EF(\tilde{N}) = \frac{E\sum_{t=1}^{\mathcal{T}_1-1} F(N(t))}{ED_1} \tag{17}$$

(see also problem 1.4, chapter 5 in [1]), Proposition 1 will follow from (17) if we show that

$$E \sum_{t=1}^{\mathcal{T}_1 - 1} F(N(t)) < \infty,$$

under conditions of Proposition 1. This is shown in the following lemma.

Lemma 3 (i) *If* $ER_{ij}^{l+1} < \infty, \quad i, j \in \mathcal{M}, \quad l \geq 1$, *then,*

$$E \sum_{t=1}^{\mathcal{T}_1 - 1} N^l(t) < \infty \ . \tag{18}$$

(ii) *If* $E \exp(\vartheta N(t)) < \infty$ *for some* $\vartheta > 0$, *then there exists* $\vartheta' > 0$ *such that*

$$E \sum_{t=1}^{\mathcal{T}_1 - 1} \exp(\vartheta' N(t)) < \infty \ . \tag{19}$$

Proof. Observe that we can write

$$\begin{aligned} \sum_{t=1}^{\mathcal{T}_1 - 1} N^l(t) &\leq \sum_{k=1}^{\theta_2 - 1} T_k \left[N(\tau_k) + A(\tau_{k+1}) - A(\tau_k)\right]^l , \\ &\leq 2^{l-1} \sum_{k=1}^{\theta_2 - 1} T_k \left(N^l(\tau_k) + (A(\tau_{k+1}) - A(\tau_k))^l\right), \end{aligned}$$

and, in view of (10),

$$\sum_{t=1}^{\mathcal{T}_1 - 1} \exp(\vartheta N(t)) \leq \sum_{k=1}^{\theta_2 - 1} T_k \left[\exp(2\vartheta N(t)) + \exp(2\vartheta (A(\tau_{k+1} - A(\tau_k)))\right] \ . \tag{20}$$

So, it suffices to show that

$$E \sum_{k=1}^{\theta_2 - 1} T_k F(N(\tau_k)) \ < \ \infty \tag{21}$$

and

$$E \sum_{k=1}^{\theta_2 - 1} T_k F(A(\tau_{k+1}) - A(\tau_k)) \ < \ \infty. \tag{22}$$

where either $F(x) = x^l$ or $F(x) = \exp(2\vartheta x)$.

¿From (6) we have

$$E\left(\sum_{k=1}^{\theta_2-1} T_k N^l(\tau_k)\right) = E\sum_{k=1}^{\theta_2-1}(N(\tau_k))^{l+1}\ , \tag{23}$$

and

$$E\left(\sum_{k=1}^{\theta_2-1} T_k \exp(2\vartheta N(\tau_k))\right) \le C(\vartheta) E\sum_{k=1}^{\theta_2-1}\exp(3\vartheta N(\tau_k))\ , \tag{24}$$

where in the latter inequality we use the fact that $T_k \le C(\vartheta)\exp(\vartheta T_k)$, for some constant $C(\vartheta)$. Based on (23) and (24) it is easy to see from Theorem 2 that (21) holds for both choices of the function $F(x)$ and appropriate choice of ϑ'.

We now concentrate on proving (22) for $F(x) = x^l$. Let $\mathcal{G}_k$ denote the sigma-field generated by $\mathbf{Q}(\tau_k)$, $k = 1, 2, \cdots$ and observe that θ_2 is a $\mathcal{G}_k$-stopping time. Using successively the facts that $\{\theta \ge k+1\} \in \mathcal{G}_k$, the process $\{\mathbf{Q}(\tau_k)\}_{k=1}^{\infty}$ is Markov and $T_k = N(\tau_k)$ is $\mathcal{G}_k$-measurable, we get

$$\begin{aligned} E\sum_{k=1}^{\theta_2-1} T_k(A(\tau_{k+1})-A(\tau_k))^l &= \sum_{k=1}^{\infty} E\left[T_k(A(\tau_{k+1})-A(\tau_k))^l 1_{\{\theta_2-1\ge k\}}\right] \\ &= \sum_{k=1}^{\infty} E\left[E\left[T_k(A(\tau_{k+1})-A(\tau_k))^l \mid \mathcal{G}_k\right] 1_{\{\theta_2\ge k+1\}}\right] \\ &= \sum_{k=1}^{\infty} E\left[E\left[T_k(A(\tau_{k+1})-A(\tau_k))^l \mid \mathbf{Q}(\tau_k)\right] 1_{\{\theta_2\ge k+1\}}\right] \\ &= \sum_{k=1}^{\infty} E\left[E\left[(A(\tau_{k+1})-A(\tau_k))^l \mid \mathbf{Q}(\tau_k)\right] T_k 1_{\{\theta_2\ge k+1\}}\right] \end{aligned} \tag{25}$$

Arguments similar to those used in the proof of Lemma 1, show that

$$(A(\tau_{k+1})-A(\tau_k))^l \le C_1 \sum_{i,j\in\mathcal{M}}\left(\sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t)\right)^l ,$$

where C_1 depends only on M and l. Therefore,

$$E\left[(A(\tau_{k+1})-A(\tau_k))^l \mid \mathbf{Q}(\tau_k)\right] \le C_1 \sum_{i,j\in\mathcal{M}} E\left[\left(\sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t)\right)^l \mid \mathbf{Q}(\tau_k)\right].$$

Since by assumption (A.1) $R_{ij}(t),\ t = 1,2,\cdots$ are i.i.d and $ER_{ij}^l < \infty$, using Corollary 10.3.2 in [3], we conclude that for $l \geq 2$,

$$E\left[\left(\sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t)\right)^l \mid \mathbf{Q}(\tau_k)\right] \leq C_2 T_k^l,$$

where C_2 depends only on M, l and ER_{ij}^l. Clearly, the same inequality is true for $l = 1$. Using these estimates in (25), we finally have,

$$\begin{aligned}
E\sum_{k=1}^{\theta_2-1} T_k[A(\tau_{k+1}) - A(\tau_k)]^l &\leq CE\left(\sum_{k=1}^{\infty} T_k^{l+1} 1_{\{\theta_2 \geq k+1\}}\right) \\
&= CE\left(\sum_{k=1}^{\theta_2-1} T_k^{l+1}\right) \\
&= CE\left(\sum_{k=1}^{\theta_2-1} (N(\tau_k))^{l+1}\right) \\
&< \infty,
\end{aligned} \tag{26}$$

where the last iequality follows from Theorem 2.

Now we focus on proving (22) for $F(x) = \exp(\vartheta x)$. We can use the same arguments as before together with (10) to obtain

$$\begin{aligned}
E\sum_{k=1}^{\theta_2-1} T_k \exp(\vartheta(A(\tau_{k+1}) - A(\tau_k))) &\leq C_1 E \sum_{k=1}^{\theta_2-1} \exp\left(\vartheta M \sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t)\right) \\
&\leq C_2 \sum_{k=1}^{\theta_2-1} E\exp(\vartheta' T_k) = C_2 \sum_{k=1}^{\theta_2-1} E\exp(\vartheta' N(\tau_k)) < \infty .
\end{aligned}$$

This completes the proof of the lemma, and also our main result Proposition 1. ■

4 Correlated arrival models

In the previous sections we assumed that packet arrivals are independent from slot to slot. In this section we show that the stability properties of the adaptive policy Π are maintained for other arrival models as well. Specifically we consider arrivals with bounded burstiness and Markov modulated arrivals.

In the arrival model with bounded burstiness we assume that for each arrival stream $\{R_{ij}(t)\}_{t=1}^{\infty}$ there are numbers ρ_{ij}, b_{ij} such that

$$\sum_{t=t_1}^{t_2} R_{ij}(t) \leq \rho_{ij}(t_2 - t_1) + b_{ij} \tag{27}$$

If the vector $\{\rho_{ij}\}$ lie in region $\mathcal{R}$ then the system is stable under Π in the sense that the backlogs are uniformly bounded over time. To see this notice that by the definition of $N_m(t)$, relation (6) and inequality (27), we have that

$$\begin{aligned} N_m(\tau_{k+1}) &\leq r_m T_k + \sum_{i,j} b_{ij} \\ &= r_m N(\tau_k) + \sum_{i,j} b_{ij}, \end{aligned}$$

where the r_m's are defined in terms of the ρ_{ij}'s in the same manner as in the definition of $\mathcal{R}$ and T_k, τ_k are the same as in (6). Therefore,

$$N(\tau_{k+1}) \leq rN(\tau_k) + B,$$

where $B = \sum_{i,j} b_{ij}$ and $r = \max\{r_m : m \in \mathcal{M}\} < 1$. We conclude that if the vector of ρ_{ij}'s lies in the region $\mathcal{R}$ then,

$$N(\tau_k) \leq \frac{B}{1-r} + r^k N(\tau_1),$$

Since $N_m(t) \leq N_m(\tau_k) + N_m(\tau_{k+1})$ whenever $\tau_k < t < \tau_{k+1}$, we can easily extend the previous bound for an arbitrary time t.

The proof of stability that we gave when the arrivals are i.i.d. goes through in the more general case where the arrivals are Markov modulated. Consider the following Markov modulated arrival model. There is a finite irreducible Markov chain $\{u(t)\}_{t=1}^{\infty}$ with state space $\mathcal{U}$ and a family of distributions $\{F_u : u \in \mathcal{U}\}$ such that the conditional distribution of $R_{ij}(t)$ given $u(t)$ is $F_{u(t)}$ Furthermore $R_{ij}(t)$ is independent of $\{R_{ij}(\tau) : \tau < t\}$ given $u(t)$. Assume finaly that $\{u(t)\}_{t=1}^{\infty}$ is stationary therefore $\{R_{ij}(t)\}_{t=1}^{\infty}$ is stationary as well. With the above assumptions parts (i) and (ii) of Proposition 1 holds with minor modifications in the proofs. The only difference is that the queue length process at the beginnings of a cycle is not a Markov chain any more. However, the combination $(Q(\tau_n), u(\tau_n))$ of the queue length vector with the modulating chain constitutes a Markov chain and the proofs can be carried through based on this chain.

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Unité de Recherche INRIA Rocquencourt
Domaine de Voluceau - Rocquencourt - B.P. 105 - 78153 LE CHESNAY Cedex (France)

Unité de Recherche INRIA Lorraine Technopôle de Nancy-Brabois - Campus Scientifique
615, rue du Jardin Botanique - B.P. 101 - 54602 VILLERS LES NANCY Cedex (France)
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