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Sur l'amélioration des charactéristiques de stabilité du trafic asynchrone dans l'anneau à jeton FDDI

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Résumé

Le protocole "Fiber Distributed Data Interface" (FDDI) de l'anneau à jeton est capable de traiter deux types de trafic: synchrone et asynchrone. Le mécanisme de contrainte temporelle du FDDI garantit le délai du trafic synchrone: une durée cible de rotation du jeton (Target Token Rotation Time – TTRT) étant fixée, le protocole FDDI garantit que le temps de cycle du jeton est majoré par deux fois le TTRT. Dans cet article nous considérons la stabilité du trafic asynchrone dans le protocole FDDI. Nous obtenons des conditions nécessaires et suffisantes de stabilité qui indiquent comment le choix du TTRT influence la stabilité du système. Nous proposons une modification du protocole FDDI pour laquelle le temps de cycle du jeton est aussi majorée par deux fois le TTRT. Nous montrons que les conditions de stabilité du nouveau protocole pour le trafic asynchrone sont plus faibles (ce qui permet donc de transmettre plus de messages asynchrones). Ce nouveau protocole est aussi plus facile à implémenter.

Mots-clés: Anneau à jeton temporisé, FDDI, condition de stabilité.

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Abstract

The Fiber Distributed Data Interface (FDDI) token ring protocol supports two classes of traffic: synchronous and asynchronous. The time constraint mechanism of FDDI garantees the transmission delay of synchronous traffic: A Target Token Rotation Time (TTRT) being fixed, the FDDI protocol ensures that the token rotation time is always bounded above by twice TTRT. In this paper, we consider the stability issues of the asynchronous traffic in the FDDI protocol. We obtain sufficient and necessary stability conditions, which indicates how the choice of TTRT affects the stability of the system. We suggest an improvement of the FDDI protocol for which the token rotation time is also bounded by twice TTRT. We show that the stability conditions of the new protocol are weaker (and thus it enables to transmit more asynchronous packets). This protocol is also easier to implement.

Keywords: Timed token ring, FDDI, stability condition.

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1 Introduction

The Fiber Distributed Data Interface (FDDI) [2, 3] is designed for a 100 Mbit/s local area network (LAN) using fiber optics as the transmission medium. The media access control (MAC) protocol of FDDI is a timed token ring protocol which supports two classes of traffic: synchronous and asynchronous. The synchronous traffic is assigned guaranteed bandwidth so that real-time applications are allowed; whereas the asynchronous traffic is controlled by the station timers. Such a timed token scheme was first suggested by Grow [9]. Ulm [20] analyzed the influence of different parameters on the utilization of the ring. Johnson [10] and Sevcik and Johnson [16] proved cycle time properties in the FDDI token ring protocol.

Although the ability to deal with synchronous traffic is one of the particularities of the FDDI protocol, most of the current implementations of the protocol uses the asynchronous transmission mechanism alone. Performance analysis of asynchronous traffic in timed token rings were carried out for the computation (or approximation) of throughput (cf. Dykeman and Bux [7], Pang and Tobagi [14] and Tangemann and Sauer [18]) and waiting times (cf. Takagi [17], Nakamura et al. [13], Tangemann and Sauer [18] and Altman [4]).

In this paper, we investigate the stability conditions of the asynchronous traffic in both the original FDDI protocol and a modified version of the protocol. In FDDI, in order to bound the token rotation time, each station possesses a timer that determines whether a message may be transmitted at the actual visit of the token. In the original FDDI protocol there are constraints on the cycle time: a message is allowed to be transmitted at the current visit of the token if and only if the time elapsed from the last time when the token arrives at the station till the current time does not exceed a threshold, referred to as Target Token Rotation Time (TTRT). We suggest to use constraints on intervisit-time: a message is allowed to transmit at the current visit of the token if and only if the time elapsed from the last time when the token leaves the station till the current time does not exceed TTRT.

We first show that our new protocol satisfies the same bound on the cycle time as the standard FDDI, i.e. the token rotation time is bounded by twice TTRT. We then establish the stability conditions of both the original and the improved FDDI protocol.

Some previous work has been done on stability of token ring systems without cycle time constraints (see Zhdanov and Saksonov [21], Greorgiadis and Szpankowski [8] and Altman, Konstantopoulos and Liu [5]). In all these papers the state space was countable. The presence of time constraints forces us to use a non-countable state space. We use a generalization of

Foster's criterion to uncountable spaces, due to Tweedie [19], to obtain the stability conditions.

The paper is organized as follows. In the next section, we describe the FDDI protocol as well as the modification of the protocol. We also define notation used in the paper. In Section 3, we show that the modified FDDI protocol verifies the cycle time constraint, viz., the maximal cycle time is smaller or equal to twice the target token rotation time (TTRT). In Section 4, we provide stability conditions for both protocols.

2 Protocol Description

The FDDI MAC uses a timed token rotation protocol. Let there be K stations in a ring, numbered as $1, 2, \cdots, K$. At the ring initialization or reinitialization, the stations on the ring negotiate a value for the target token rotation time (TTRT) which is equal to the lowest bid by any of the stations. Let T^* be the value of TTRT. Each station is assigned a percentage of the TTRT for its synchronous traffic, say f_k for station k, where $1 \leq k \leq K$, $0 \leq f_k < 1$, $\sum_{k=1}^{K} f_k < 1$. Every time the token arrives at station k, the synchronous frames can be transmitted (exhaustively) for at most $f_k T^*$ units of time.

Each station has a token rotation timer (TRT), which measures the difference of successive token arrival times. When the token arrives to a station, the contents of its TRT is compared to TTRT. If the token is late, i.e., $TRT > T^*$, then the TRT is reset and restarts counting, and only the synchronous traffic can be transmitted (for at most $f_k T^*$ units of time). Otherwise, the value of TRT is copied to the token holding timer (THT), which starts counting upward, and the TRT is reset. In such a case, the synchronous traffic can be transmitted for at most $f_k T^*$ units of time, whereas the asynchonous frames can be transmitted until the THT reaches the TTRT level. The transmission of the last frame can be completed before the token is passed to the next station (the so-called asynchronous overrun). Multiple levels (up to eight) of priority for asynchronous frames may be provided within a station by specifying additional (more restrictive) time thresholds for token rotation. The interested reader is referred to [2, 3, 15] for more detailed description.

In this paper, we propose a slight modification to the above control scheme. In the new scheme, the TRT of a station is reset only when the token leaves the station. When the token arrives to a station, the contents of its TRT is compared to TTRT. If the token is late, i.e., $TRT > T^*$, then (as in the previous one), only the synchronous traffic can be transmitted.

Otherwise, the synchronous traffic can be transmitted for at most $f_k T^*$ unit of time, whereas the asynchonous frames can be transmitted until the TRT reaches the TTRT level. The transmission of the last frame can be completed before the token is passed to the next station. Multiple levels of priority for asynchronous frames may still be provided within a station by specifying additional time thresholds for token rotation. Note that the modified scheme is simpler since it does not require the extra timer THT.

The difference between these two schemes is thus the constraint on the transmission of asynchronous traffic. In the first one, asynchronous traffic can be transmitted when the cycle time (CT) [1] is less than TTRT, whereas in the second one, asynchronous traffic can be transmitted when the inter-visit time (IT) [2] is less than TTRT. In the remainder of the paper, the original FDDI MAC scheme will be referred to as FDDI with respect to (w.r.t.) CT. and the new one as FDDI w.r.t. IT.

3 Cycle Time Constraint

It is shown in [10, 16] that in FDDI-CT, the maximum cycle time is bounded above by 2 times TTRT. We will show in this section that the same property holds for the modified FDDI protocol, viz. FDDI-IT.

Theorem 3.1 *Consider FDDI-IT. Unless initiation of ring recovery interrupts the cycle, the maximum cycle time is bounded above by $2T^*$.*

Proof: Let ψ_k, $1 \leq k \leq K$, be the medium propagation delay in between stations k and $(k \bmod K) + 1$. Let $\bar{\omega}_k$, $1 \leq k \leq K$, be the maximum delay due to the overheads at station k (which include for instance the token transmission time, the station latency, and the capture delay, etc. see [10, 15, 16]). Let $\bar{\sigma}_k$, $1 \leq k \leq K$, be the maximum synchronous transmission time at station k. Denote by $\psi = \sum_{k=1}^{K} \psi_k$, $\bar{\omega} = \sum_{k=1}^{K} \bar{\omega}_k$, $\bar{\sigma} = \sum_{k=1}^{K} \bar{\sigma}_k$.

Let F be the frame time, i.e., the time required to transmit a frame of maximum length. In order for a FDDI MAC protocol to operate properly, it is necessary [10] that

$$F + \psi + \bar{\omega} + \bar{\sigma} \leq T^*. \quad (1)$$

[1] The cycle time is the time elapsed between two successive arrivals of the token at a station.

[2] The inter-visit time is the time elapsed between a departure of the token from a station and the next arrival of the token at the same station.

Assume that the assertion of the theorem does not hold. Then, there are a station, say station 1, and a cycle time defined by the duration between two successive arrivals of the token at that station, such that the cycle time is strictly greater than $2T^*$. Let $\omega_k \geq 0$ (resp. $\sigma_k \geq 0$, $\alpha_k \geq 0$), $1 \leq k \leq K$, be the overhead delay (resp. synchronous transmission time, asynchronous transmission time) at station k during that token visit cycle. It then follows that

$$\sum_{k=1}^{K}(\psi_k + \omega_k + \sigma_k + \alpha_k) > 2T^*. \tag{2}$$

Let T^k, $1 \leq k \leq K$, be the value of TRT of station k when the token arrives at the station in that token visit cycle (T^1 is defined w.r.t. the first token arrival at station 1 in the cycle).

It follows from relations (1) and (2) that for all $i = 1, \cdots, K$,

$$\begin{aligned}
& T^i + \omega_i + \sigma_i + \alpha_i \\
& \quad \geq \psi + \sum_{k=1}^{i}(\omega_k + \sigma_k + \alpha_k) \\
& \quad > 2T^* - \sum_{k=i+1}^{K}(\omega_k + \sigma_k + \alpha_k). \\
& \quad \geq 2T^* - \bar{\omega} - \bar{\sigma} + \bar{\omega}_i + \bar{\sigma}_i - \sum_{k=i+1}^{K}\alpha_k. \\
& \quad \geq T^* + F + \bar{\omega}_i + \bar{\sigma}_i - \sum_{k=i+1}^{K}\alpha_k.
\end{aligned}$$

Therefore,

$$T^i + \omega_i + \sigma_i + \alpha_i > T^* + F + \bar{\omega}_i + \bar{\sigma}_i - \sum_{k=i+1}^{K}\alpha_k. \tag{3}$$

In particular,

$$T^K + \omega_K + \sigma_K + \alpha_K > T^* + F + \bar{\omega}_K + \bar{\sigma}_K. \tag{4}$$

If $T^K \leq T^*$, then, by the definition of the protocol, $T^K + \alpha_K \leq T^* + F$, so we have necessarily,

$$T^K + \omega_K + \sigma_K + \alpha_K \leq T^* + F + \bar{\omega}_K + \bar{\sigma}_K.$$

which contradicts (4). Thus, we should have $T^K > T^*$, so that $\alpha_K = 0$. The same arguments can be used to prove by induction that for $i = K, K-1, \cdots, 1$, $T^i > T^*$, so that $\alpha_i = 0$.

As a consequence, (2) can be written as

$$\sum_{k=1}^{K} (\psi_k + \omega_k + \sigma_k) > 2T^*.$$

which contradicts (1).

Thus, relation (2) can not hold. ■

It is shown in [16] that in FDDI-CT, the average cycle time of arbitrarily n first cycles is bounded above by T^*, provided some restrictions on asynchronous overrun is added in the protocol. The similar arguments can be used to show that in FDDI-IT, the same property holds (under the same restriction on asynchronous overrun).

4 Stability Conditions

In this section, we consider the stability conditions of these token ring systems. More precisely, we provide conditions under which the vector of numbers of asynchronous messages in the stations converge in total variation.

4.1 Notation and Assumptions

Without loss of generality, we assume that the token is initially at station 1. Thus, the n-th ($n \geq 1$) station that the token visits is station $I(n) = (n-1) \bmod K + 1$, where $n \bmod K$ means the remainder of the division of n by K.

The time the token takes between the departure from the n-th station that the token visits and the arrival to the next station is a random variable, called the n-th walking time, and is denoted by D_n. For any given k, $1 \leq k \leq K$, the random variables D_{nK+k} are independent and identically distributed (i.i.d.), and their first moment, assumed finite, is denoted by d_k. Let $d = \sum_{i=1}^{K} d_i$. The walking times are used to model the overheads like the ring latency, station latency and token capture delay. They will also include the transmission times of synchronous traffic (see below). It is assumed (cf. Eq. (1) that for all $n \geq 1$, $\sum_{i=n}^{n+K-1} D_i \leq T^* - F$ *a.s.* (recall that F is the frame time).

Since we are only interested in the asynchronous traffic, we will assume that stations store and transmit asynchronous messages alone. The transmission delay due to synchronous traffic, if any, is considered to be included in the token walking times. Let T_n^k be the random variable that represents the value of TRT of station k when the token arrives at the n-th station that it visits, $1 \leq k \leq K$, $n \geq 1$. Let $\vec{T}_n = (T_n^1, \cdots, T_n^K)$.

The messages arrive to station k in accordance with a Poisson process of parameter λ_k. The transmission times of the messages at station k are i.i.d. with the same distribution as a random variable B_k. Denote by b_k the first moment of B_k, which is assumed to be finite. Let $\mathcal{A}(T)$ be the total workload (i.e., the sum of transmission times) of arrived messages at station k, $1 \leq k \leq K$, during a (possibly random) time interval of length T.

Let Q_n^k, $1 \leq k \leq K$, $n \geq 1$, be the random variable representing the number of waiting messages of station k when the token arrives at the n-th station that it visits. Let $\vec{Q}_n = (Q_n^1, ..., Q_n^K)$.

Let W_n^k, $1 \leq k \leq K$, $n \geq 1$, be the random variable representing the workload (i.e., the total transmission time of the Q_n^k waiting messages) of station k when the token arrives at the n-th station that it visits. Let $\vec{W}_n = (W_n^1, ..., W_n^K)$.

Let $\vec{S}_n = (\vec{Q}_n, \vec{W}_n, \vec{T}_n)$ be the state of the system when the token arrives at the n-th station that it visits, and let $\mathbf{S}$ denote the state space. It is easy to see that $\{\vec{S}_n\}$ is a (nonhomogeneous) Markov chain, and that for all $1 \leq k \leq K$, $\{\vec{S}_{nK+k}\}_n$ is a homogeneous Markov chain.

Let V_n be the time the token spends at the n-th visited station. Denote by $\tau(n)$ the time epoch when the token starts the n-th visit of a station.

Let $\rho_k = \lambda_k b_k$, $1 \leq k \leq K$, $\rho = \sum_{k=1}^{K} \rho_k$.

We define some notation on Markov chains. Let $\{X_n\}$ be a homogeneous Markov chain defined on a general state space $(\mathbf{X}, \mathcal{F})$, where the σ-field $\mathcal{F}$ of subsets of $\mathbf{X}$ is assumed to be countably generated. Denote by $P_{(x,\cdot)}^n$ the n-th step transition probabilities of $\{X_n\}$ with initial state x.

In this paper, the FDDI system will be said stable if for all $1 \leq k \leq K$, the Markov chain $\{\vec{S}_{nK+k}\}_n$ converges in total variation, i.e., there exists some probability measure π on $\mathcal{F}$, such that for every initial state $x \in \mathbf{X}$,

$$||P^n_{(x,\cdot)} - \pi|| \to 0, \quad n \to \infty,$$

where $||\cdot||$ denotes total variation of signed measures on $\mathcal{F}$. Note that if the FDDI system is stable, then, both $\{Q_{nK+k}\}_n$ and $\{W_{nK+k}\}_n$ converge in total variation.

4.2 Sufficient Conditions

In order to establish the sufficient conditions for the stability of the system, we need the following lemma which is proved in the appendix.

Lemma 4.1 *Let $M \geq 1$ be an arbitrary integer. If $\sum_{k=1}^{K} W_n^k > 2MT^*$, then under the FDDI-IT mechanism,*

$$\sum_{i=0}^{MK-1} E[V_{n+i}|\vec{S}_n] \geq (M-K)(T^* - d),$$

and under the FDDI-CT mechanism,

$$\sum_{i=0}^{MK-1} E[V_{n+i}|\vec{S}_n] \geq \lfloor \frac{M}{2} \rfloor (T^* - 2d)$$

where $\lfloor x \rfloor$ denotes the largest integer smaller or equal to x.

Theorem 4.1 *Assume that $\rho < 1$.*

(i) Under the FDDI-IT mechanism, if $T^* > d/(1-\rho)$, then the system is stable.

(ii) Under the FDDI-CT mechanism, if $T^* > 2d/(1-\rho)$, then the system is stable.

In order to prove Theorem 4.1 we need the following lemma due to Tweedie [19].

Lemma 4.2 *Consider a Markov chain X_n on a state space $\mathbf{X}$. Assume that there exists a set $\mathcal{H} \in \mathbf{X}$ and a function $g : \mathbf{X} \to \mathbb{R}$ such that*

(i) there exists some $\epsilon > 0$ such that $E[g(X_{n+1}) - g(X_n)|X_n] \leq -\epsilon$ for $X_n \in \mathcal{H}^c$:
(ii) $E[g(X_{n+1})|X_n] \leq \infty$ for $X_n \in \mathcal{H}$:
(iii) $\mathcal{H}$ is a small set.

Then X_n is stable.

In the above lemma, a set $\mathcal{H}$ is said to be small if there exists some positive measure ϕ on $\mathbf{X}$, such that for any $A \subseteq \mathbf{X}$ with $\phi(A) > 0$ there exists j such that

$$\inf_{x\in\mathcal{H}} \sum_{n=1}^{j} P^n_{(x,B)} > 0.$$

where $P_{(x,B)} \equiv P^1_{(x,B)}$ is the transition probabilities of the Markov chain.

Proof of Theorem 4.1: We first note that the following equality in law ($=_d$) holds:

$$W^k_{n+1} =_d W^k_n - V_n \mathbf{1}_{\{k=I(n)\}} + \mathcal{A}_k(D_n + V_n). \qquad 1 \leq k \leq K.$$

It then follows from the stochastic intensity integration formula [6, § III.5] that

$$E[W^k_{n+1}|\vec{S}_n] = W^k_n - E[V_n \mathbf{1}_{\{k=I(n)\}}|\vec{S}_n] + \lambda_k b_k(d_{I(n)} + E[V_n|\vec{S}_n]). \qquad 1 \leq k \leq K. \tag{5}$$

Summing over all k on both sides of (5), we obtain

$$\sum_{k=1}^{K} E[W^k_{n+1}|\vec{S}_n] = \sum_{k=1}^{K} W^k_n + \rho d_n + (\rho - 1)E[V_n|\vec{S}_n].$$

Thus,

$$\begin{aligned}
&\sum_{k=1}^{K} E\left[W^k_{n+2}|\vec{S}_n\right] \\
&= \sum_{k=1}^{K} E\left\{E\left[W^k_{n+2}|\vec{S}_{n+1}\right]|\vec{S}_n\right\} \\
&= \sum_{k=1}^{K} W^k_n + \rho d_n + \rho d_{n+1} + (\rho - 1)E\left[V_n + V_{n+1}|\vec{S}_n\right]
\end{aligned}$$

Iterating this procedure, we get for any integer M,

$$\sum_{k=1}^{K} E\left[W^k_{n+MK}|\vec{S}_n\right] - \sum_{k=1}^{K} W^k_n = M\rho d + (\rho - 1) \sum_{l=0}^{MK-1} E\left[V_{n+l}|\vec{S}_n\right] \tag{6}$$

Let $\epsilon > 0$ be an arbitrarily fixed constant. Let M^* be some integer that satisfies

$$M^* \geq \begin{cases} \dfrac{\epsilon + (1-\rho)KT^*}{(1-\rho)T^* - d} & \text{FDDI-IT case} \\ \\ \dfrac{2\epsilon}{(1-\rho)T^* - 2d} & \text{FDDI-CT case} \end{cases}$$

For FDDI-CT we shall assume for simplicity that M^* is even. It follows from the assumptions of the theorem on T^* that the denominator in the definition of M^* is strictly positive.

Let $\mathcal{H}$ be the subset of the state space defined by

$$\mathcal{H} = \{\vec{S} = (\vec{W}, \vec{Q}, \vec{T}) \mid \sum_{k=1}^{K} W^k \leq 2M^*T^*\}$$

We shall now use Lemma 4.2 to establish the stability of the Markov Chain $\vec{S}_{k+mM^*K}$, $m = 1, 2, \ldots$ for any $k = 1, \ldots, K$. Combining Lemma 4.1 with (6) we get that for all $\vec{S}_n \notin \mathcal{H}$.

$$\begin{aligned}
&\sum_{k=1}^{K} E\left[W^k_{n+M^*K} | \vec{S}_n\right] - \sum_{k=1}^{K} W^k_n \\
&= \; M^*\rho d + (\rho - 1) \sum_{i=0}^{M^*K-1} E\left[V_{n+i} | \vec{S}_n\right] \\
&\leq \; \begin{cases} M^*\rho d + (\rho-1)[(M^* - K)(T^* - d) - Kd] & \text{for FDDI-IT} \\ M^*\rho d + (\rho - 1)M^*(T^* - 2d)/2 & \text{for FDDI-CT} \end{cases} \\
&\leq \; -\epsilon
\end{aligned} \qquad (7)$$

where the last inequality follows from the definition of M^*. This establishes condition (i) of Lemma 4.2.

On the other hand, we have from (6) that for all $\vec{S}_n \in \mathcal{H}$.

$$\sum_{k=1}^{K} E\left[W^k_{n+M^*K} | \vec{S}_n\right] - \sum_{k=1}^{K} W^k_n \leq M^*\rho d \qquad (8)$$

This establishes condition (ii) of Lemma 4.2.

It remains to prove that the set $\mathcal{H}$ is small. Let P be the transition probability of the Markov Chain $\vec{S}_m$, $m = 0, 1, 2, \ldots$.

Let i be an integer such that

$$\inf_{\vec{S}_n \in \mathcal{H}} P(\vec{Q}_{n+(i+3)M^*K-2} = 0 \mid \vec{S}_n, \text{ no arrivals after } \tau(n)\ ) = 1.$$

which can also be written as

$$\inf_{y \in \mathcal{H}} \hat{P}^{(i+3)M^*K-2}_{y,(0,0,[0,2T^*])} = 1 \tag{9}$$

where $\hat{P}_{yB}$ is the probability to move from state $y \in \mathbf{S}$ to the set $B \subseteq \mathbf{S}$ given that no arrivals occurred during the transition. The existence of such a finite i is established in Lemma A.3 in the Appendix. Define

$$\phi(B) \stackrel{\text{def}}{=} \int P_{\{(0,0,t),B\}} G(dt). \qquad B \in \mathbf{S}.$$

where $G(\bullet)$ is the distribution of the total walking time in a cycle. Choose any $y \in \mathcal{H}$. Then for all B such that $\phi(B) > 0$,

$$\begin{aligned}
P^{(i+3)M^*K}_{yB} &\geq \int_{t,s} P^{(i+3)M^*K-2}_{\{y,(0,0,ds)\}} P_{\{(0,0,s),(0,0,dt)\}} P_{\{(0,0,t),B\}} \\
&\geq \int_s P^{(i+3)M^*K-2}_{\{y,(0,0,ds)\}} \int_t [\epsilon^{-2\lambda T^*} \hat{P}_{(0,0,s),(0,0,dt)} P_{\{(0,0,t),B\}}] \\
&= \epsilon^{-2\lambda T^*} \phi(B) \int_s P^{(i+3)M^*K-2}_{\{y,(0,0,ds)\}} \\
&\geq \epsilon^{-2\lambda T^*} \phi(B) \int_s \hat{P}^{(i+3)M^*K-2}_{\{y,(0,0,ds)\}} \epsilon^{-2\lambda T^*((i+3)M^*K-2)} \\
&= \epsilon^{-2\lambda T^*((i+3)M^*K-1)} \phi(B) \int_s \hat{P}^{(i+3)M^*K-2}_{\{y,(0,0,ds)\}} \\
&= \epsilon^{-2\lambda T^*((i+3)M^*K-1)} \phi(B) > 0
\end{aligned}$$

uniformly in y. The second and the last inequality follow since the cycles time are bounded by $2T^*$. This establishes the smallness of $\mathcal{H}$ and hence stability of the Markov Chain $\vec{S}_{k+mM^*K}$, $m = 1, 2, \ldots$ for any $k = 1, \ldots, K$. Since $\vec{S}_{k+nK}$ is itself a communicating Markov chain, it follows that it is stable as well. ■

It can be shown that for FDDI-CT, when walking times are deterministic then the sufficient condition for stability can be improved. While Theorem 4.1 provides sufficient conditions for stability, we present some necessary stability conditions below.

Theorem 4.2 *For both FDDI-CT and FDDI-IT, if the system is stable, then $\rho < 1$ and*

$$T^* \geq \max\left(d, \frac{d}{2(1-\rho)}\right).$$

Proof: It follows from [16] and Theorem 3.1 that the token cycle time is bounded by $2T^*$ so that the expected cycle time is finite. According to [5], this expected cycle time is equal to $d/(1-\rho)$. Thus, $\rho < 1$ and $2T^* \geq d/(1-\rho)$. The relation $T^* \geq d$ follows from (1). ■

Theorem 4.3 *Assume that the walking times $D_i = d_i$ are deterministic. If the system is stable, then*

(i) $T^* + e \geq d/(1-\rho)$ for FDDI-IT:

(ii) $T^* + e \geq d(1 + \max_{1 \leq i \leq K} \rho_i)/(1-\rho)$ for FDDI-CT.

where e is the maximal asynchronous overrun.

Proof: We note that in steady state, the expected cycle duration $E[C]$ satisfies ([5])

$$E[C] = \sum_{i=1}^{K} E[V_{n-i}] + d = \frac{d}{1-\rho} \tag{10}$$

Consider FDDI-IT. We have

$$V_n \leq \max\left\{0, T^* + e - d - \sum_{i=1}^{K-1} V_{n-i}\right\} \tag{11}$$

(we shall understand $V_n = 0$ for $n < 0$). Denote $A_n := T^* + e - d - \sum_{i=1}^{K-1} V_{n-i}$. It follows that $A_0 \geq 0$, and thus

$$V_n \leq A_n \tag{12}$$

for $n = 0$. We show by induction that $A_n \geq 0$ for all n and hence (12) holds for all n. Suppose $A_{n-1} \geq 0$, for some n. Then

$$V_{n-1} \leq A_{n-1} \leq T^* + e - d - \sum_{i=2}^{K} V_{n-i}$$

and hence $A_n \geq 0$. By taking expectation of (12) we get in steady state

$$E[V_n] \leq T^* + e - d - \sum_{i=1}^{K-1} E[V_{n-i}].$$

Combining this with (10) we get (i).

Consider FDDI-TC. By arguments similar to the FDDI-IT above, we get in steady state

$$E[V_n] \leq T^* + e - E[C]$$

Since $E[V_n] = \rho_{I(n)} E[C]$ (see ([5]) we get (ii), using again (10). ∎

Remark: In general, the maximal asynchronous overrun e equals the frame time F. Trivially, $e = 0$ if asynchronous overrun never occurs (in which case, the message transmissions may be either preempted when TRT reaches TTRT, or not allowed if this transmission will not finish before TRT reaches TTRT.

5 Concluding Remarks

We established in this paper necessary and sufficient conditions for stability for both the original and the modified FDDI protocols (Theorems 4.1, 4.2 and 4.3).

Comparing these conditions we may conclude the following: For the case that the asynchronous overrun e is negligible and D_n are deterministic, the sufficient condition for FDDI-IT is also necessary. This condition has a simple interpretation: the expected cycle duration should be smaller than T^*, and $\rho < 1$. If the asynchronous overrun is not negligible there is a gap between the necessary and the sufficient conditions that we obtained for T^*. The exact necessary and sufficient condition would be somewhere in that gap and would be more complicated. The intuitive sufficient condition that expected cycle duration should be smaller than T^* may be quite far from the necessary condition, and in fact the system may be stable with expected cycle duration considerably larger than T^*. This is illustrated by the trivial case of a single queue. In that case the sufficient condition for stability under the FDDI-IT are simply $\rho < 1$ and $D + F \leq T^*$ w.p.1. Thus the system can be stable even if $d(1-\rho)^{-1} > T^*$.

Consider the case that D is deterministic and the asynchronous overrun is negligible. Since the sufficient condition for stability of the new protocol is weaker than the necessary stability condition of the original protocol, it follows that the new FDDI protocol (FDDI-IT) enables to transmit more asynchronous messages. This improvement does not affect the strict bound on the token rotation time, which is kept smaller than twice TTRT, as we showed in Theorem 3.1.

Another advantage of the FDDI-IT that we propose over the existing FDDI-CT, is that FDDI-IT does not require to have the THT timer in each station.

Similar stability analysis can be carried out for other timed token ring protocols such as the IEEE 802.4 [1], or the protocol analysed by Sevcik and Johnson [16] which includes also a latency counter.

Appendix: Lemmas for the proof of Theorem 4.1

In this appendix, we will first prove the Lemmas A.1 and A.2 below, so that Lemma 4.1 holds. We then prove Lemma A.3 to show the existence of a finite i fulfilling equation (9).

Lemma A.1 *Consider FDDI-IT, and let $M \geq 1$ be an arbitrary integer. If $\sum_{k=1}^{K} W_n^k > 2MT^*$, then*

$$\sum_{i=0}^{MK-1} E[V_{n+i}|\vec{S}_n] \geq (M-K)(T^*-d) - Kd.$$

Proof: Without loss of generality assume that $I(n) = 1$, so that the token starts at station 1. Let $m_k \geq 0$ denote the number of cycles the token takes after $\tau(n)$ in order to transmit the Q_n^k messages present at time $\tau(n)$ at station k, $1 \leq k \leq K$.

Since in each cycle, the total transmission time is bounded above by $2T^*$, we have that

$$\max_{1\leq k\leq K} m_k \geq M. \tag{13}$$

Let γ be a permutation on $\{1, \cdots, K\}$ such that

$$m_{\gamma(1)} \leq m_{\gamma(2)} \leq \cdots \leq m_{\gamma(K)}.$$

It can be checked by the definition of FDDI-IT, that for all $l \geq 1$,

$$\begin{aligned} T_{l+1}^{I(l)} &= D_l, \\ T_{l+1}^{I(l+k)} &= T_l^{I(l+k)} + V_l + D_l, \qquad 1 \leq k \leq K-1. \end{aligned}$$

It then follows that for all $i \geq 1$,

$$T^k_{n+iK+k-1} = \sum_{j=2}^{K} V_{n+iK+k-j} + \sum_{j=2}^{K+1} D_{n+iK+k-j}, \quad 1 \leq k \leq K.$$

The above equality implies that if at the $n + iK + k - 1$-st visit of the token, station $n + k$ has enough messages to transmit, then $V_{n+iK+k-1} \geq \max(0, T^* - \sum_{j=2}^{K} V_{n+iK+k-j} - \sum_{j=2}^{K+1} D_{n+iK+k-j})$.

Therefore,

$$\begin{array}{rcll}
\sum_{j=1}^{K} V_{n+iK+\gamma(1)-j} & \geq & T^* - \sum_{j=2}^{K+1} D_{n+iK+\gamma(1)-j}, & 1 \leq i \leq m_{\gamma(1)} - 1; \\
\sum_{j=1}^{K} V_{n+iK+\gamma(2)-j} & \geq & T^* - \sum_{j=2}^{K+1} D_{n+iK+\gamma(2)-j}, & m_{\gamma(1)} + 1 \leq i \leq m_{\gamma(2)} - 1; \\
 & \vdots & & \\
\sum_{j=1}^{K} V_{n+iK+\gamma(K)-j} & \geq & T^* - \sum_{j=2}^{K+1} D_{n+iK+\gamma(K)-j}, & m_{\gamma(K-1)} + 1 \leq i \leq m_{\gamma(K)} - 1.
\end{array} \tag{14}$$

Since $m_{\gamma(K)} \geq M$, we obtain from relations (14) that in the MK visits of the token after time $\tau(n)$, we can find at least $M - 2K - 1$ cycles (which do not necessarily start from the same station) such that the total transmission time of each of these cycles is greater or equal to T^* minus the total walking time in that cycle. More precisely, let h be the index such that $m_{\gamma(h-1)} < M \leq m_{\gamma(h)}$, where by convention, $\gamma(0) \equiv 0$ and $m_0 \equiv 0$. Then,

$$\begin{aligned}
&\sum_{i=0}^{MK-1} V_{n+i} \\
&\geq \sum_{k=1}^{h-1} \sum_{i=m_{\gamma(k-1)}+1}^{m_{\gamma(k)}-1} \sum_{j=1}^{K} V_{n+iK+\gamma(k)-j} + \sum_{i=m_{\gamma(h-1)}+1}^{M-1} \sum_{j=1}^{K} V_{n+iK+\gamma(h)-j} \\
&\geq \sum_{k=1}^{h-1} \sum_{i=m_{\gamma(k-1)}+1}^{m_{\gamma(k)}-1} \left(T^* - \sum_{j=2}^{K+1} D_{n+iK+\gamma(k)-j}\right) + \sum_{i=m_{\gamma(h-1)}+1}^{M-1} \left(T^* - \sum_{j=2}^{K+1} D_{n+iK+\gamma(h)-j}\right) \\
&= (M-h)T^* - \sum_{i=0}^{MK-1} D_{n+i} + \sum_{k=0}^{h-1} \sum_{j=2}^{K+1} D_{n+m_{\gamma(k)}K+\gamma(k)-j} \\
&\geq (M-K)T^* - \sum_{i=0}^{MK-1} D_{n+i}.
\end{aligned}$$

By taking the expectation in the above inequality, we obtain

$$\sum_{i=0}^{MK-1} E[V_{n+i}|\vec{S}_n] \geq (M-K)(T^* - d) - Kd.$$

The proof is thus completed. ■

Lemma A.2 *Consider FDDI-CT and let $M \geq 1$ be an arbitrary integer. If $\sum_{k=1}^{K} W_n^k > 2MT^*$, then*

$$\sum_{i=0}^{MK-1} E[V_{n+i}|\vec{S}_n] \geq \lfloor \frac{M}{2} \rfloor (T^* - 2d).$$

Proof: The proof is similar to the previous one. Assume $I(n) = 1$. Let m_k and γ be defined as in the previous case. We have as before that $m_{\gamma(K)} \geq M$.

It can be checked by the definition of FDDI-CT, that for all $l \geq 1$,

$$\begin{aligned} T_{l+1}^{I(l)} &= V_l + D_l, \\ T_{l+1}^{I(l+k)} &= T_l^{I(l+k)} + V_l + D_l, \qquad 1 \leq k \leq K-1. \end{aligned}$$

It then follows that for all $i \geq 1$,

$$T_{n+iK+k-1}^{k} = \sum_{j=2}^{K+1} V_{n+iK+k-j} + \sum_{j=2}^{K+1} D_{n+iK+k-j}, \quad 1 \leq k \leq K.$$

The above equality implies that if at the $n+iK+k-1$-st visit of the token, station k has enough messages to transmit, then $V_{n+iK+k-1} \geq \max(0, T^* - \sum_{j=2}^{K+1} V_{n+iK+k-j} - \sum_{j=2}^{K+1} D_{n+iK+k-j})$.

Therefore,

$$\begin{aligned} \sum_{j=1}^{K+1} V_{n+iK+\gamma(1)-j} &\geq T^* - \sum_{j=2}^{K+1} D_{n+iK+\gamma(1)-j}, \quad 1 \leq i \leq m_{\gamma(1)} - 1; \\ \sum_{j=1}^{K+1} V_{n+iK+\gamma(2)-j} &\geq T^* - \sum_{j=2}^{K+1} D_{n+iK+\gamma(2)-j}, \quad m_{\gamma(1)} + 1 \leq i \leq m_{\gamma(2)} - 1; \\ &\;\;\vdots \\ \sum_{j=1}^{K+1} V_{n+iK+\gamma(K)-j} &\geq T^* - \sum_{j=2}^{K+1} D_{n+iK+\gamma(K)-j}, \quad m_{\gamma(K-1)} + 1 \leq i \leq m_{\gamma(K)} - 1. \end{aligned} \tag{15}$$

Let h be the index such that $m_{\gamma(h-1)} < M \leq m_{\gamma(h)}$. (Note that (13) holds for FDDI-CT as well). Then,

$$
\begin{aligned}
&\sum_{i=0}^{MK-1} V_{n+i} \\
\geq\ & \sum_{k=1}^{h-1} \sum_{i=\lfloor m_{\gamma(k-1)}/2\rfloor+1}^{\lfloor m_{\gamma(k)}/2\rfloor} \sum_{j=1}^{K+1} V_{n+(2i-1)K+\gamma(k)-j} + \sum_{i=\lfloor m_{\gamma(h-1)}/2\rfloor+1}^{\lfloor M/2\rfloor} \sum_{j=1}^{K+1} V_{n+(2i-1)K+\gamma(h)-j} \\
\geq\ & \sum_{k=1}^{h-1} \sum_{i=\lfloor m_{\gamma(k-1)}/2\rfloor+1}^{\lfloor m_{\gamma(k)}/2\rfloor} \left(T^* - \sum_{j=2}^{K+1} D_{n+(2i-1)K+\gamma(k)-j}\right) \\
&+ \sum_{i=\lfloor m_{\gamma(h-1)}/2\rfloor+1}^{\lfloor M/2\rfloor} \left(T^* - \sum_{j=2}^{K+1} D_{n+(2i-1)K+\gamma(h)-j}\right) \\
\geq\ & \lfloor M/2\rfloor T^* - \sum_{i=0}^{MK-2} D_{n+i}
\end{aligned}
$$

Thus, by taking the expectation in the above inequality, we obtain

$$\sum_{i=0}^{MK-1} E[V_{n+i}|\vec{S}_n] \geq \lfloor \frac{M}{2} \rfloor (T^* - 2d).$$

The proof is thus completed. ■

Lemma A.3 *Under the conditions of Theorem 4.1, there exists some finite i such that*

$$\inf_{y\in\mathcal{H}} \hat{P}^{(i+3)M^*K-2}_{y,(0,0,[0,2T^*])} = 1.$$

Proof: We assume that from $\tau(n)$ on, there are no more arrivals. Let $\bar{d} := \inf\{x : P(D > x) = 0\}$. Define $\Delta = T^* - \bar{d}$. Choose some $k \in 1, \ldots, K$, and assume that $\vec{S}_n \in \mathcal{H}$. Let $\zeta(m)$ be the number of times that $T^k_{n+jK} > T^* - \Delta/2$, $j = 1, 2, \ldots, m$. If $T^k_{n+jK} > T^* - \Delta/2$ for some j, then the amount of service time spent in the other queues in the last cycle is at least $\Delta/2$. Thus the total time spent in serving in the different queues during $[\tau(n), \tau(n + mK)]$ is at least $\zeta(m)\Delta/2$. For any m

$$2M^*T \geq \sum_{k=1}^{K} W^k_n \geq \zeta(m)\frac{\Delta}{2}$$

(where the first inequality follows from the definition of H). Hence for any m,

$$\zeta(m) \le \frac{4M^*T}{\Delta}$$

We get

$$\begin{aligned} W^k_{n+(i+1)K} &\le \max\left\{0, W^k_n - [i - \zeta(i)]\frac{\Delta}{2}\right\} \\ &\le \max\left\{0, 2M^*T - \left[i - \frac{4M^*T}{\Delta}\right]\frac{\Delta}{2}\right\} \\ &= \max\left\{0, 4M^*T - \frac{i\Delta}{2}\right\} \end{aligned}$$

Thus for all i such that $i > 8M^*T/\Delta$, $W^k_{n+(i+1)K} = 0$. Since this argument holds for all k, this establishes the proof. ∎

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