Preliminary results on time-varying feedback stabilization of a nonholonomic car-like mobile robot

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RESULTATS PRELIMINAIRES SUR LA STABILISATION PAR RETOUR D'ETAT INSTASTIONNAIRE D'UN ROBOT MOBILE NON-HOLONOME DE TYPE VOITURE

PRELIMINARY RESULTS ON TIME-VARYING FEEDBACK STABILIZATION OF A NONHOLONOMIC CAR-LIKE MOBILE ROBOT

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Abstract

Although classical nonholonomic mobile robots are controllable in their configuration space, they usually fail to be stabilizable to a desired configuration by using smooth state feedback controls. To overcome this difficulty, stabilizing *discontinuous* feedbacks have then been proposed by some authors. However, there is still another interesting possibility consisting of *time-varying* smooth feedbacks, i.e. feedbacks which depend explicitly on the time index. This possibility, studied in [15] to stabilize a cart equipped with two independent motorized wheels on the same axis, is here applied to a front-wheel driven cart for which a set of globally stabilizing *time-varying* smooth feedback controls is derived.

Key words: mobile robots, nonholonomy, nonlinear systems, controllability, feedback stabilization.

Résumé

Bien que les robots mobiles soient commandables dans leur espace de configuration, ils ne peuvent généralement pas être stabilisés vers une configuration désirée en utilisant des commandes par retour d'état *continues*. Afin de résoudre le problème, certains auteurs ont proposé des commandes par retour d'état *discontinues*. Il existe cependant une autre possibilité intéressante consistant à utiliser des retours d'état continus *instationnaires*, c'est à dire dépendant explicitement de la variable temporelle. Cette possibilité, étudiée dans [15] dans le cas d'un chariot mobile équipé de deux roues motrices indépendantes montées sur le même axe, est ici appliquée au cas d'une plateforme de type voiture (avec roue motrice et directrice) pour laquelle un ensemble de retours d'état *instationnaires* globalement stabilisants est obtenu.

Mots clés: robots mobiles, non-holonomie, systèmes non-linéaires, contrôlabilité, commande par retour d'état.

Contents

1 Introduction

Feedback control of nonholonomic mobile robots has recently motivated an increasing number of studies and publications in the Robotics and Automatic Control communities [1]-[15]. While the topic may be seen as a logical extension of the much studied case of holonomic robot manipulators, it turns out that this extension is not as straightforward as it may first seem. In particular, although mobile robots subject to classical nonholonomic constraints are usually (when the number of actuators is equal to the number of degrees of freedom) completely controllable in their configuration space [1] [4] [16]-[21], they cannot be stabilized to a desired configuration by using smooth state feedback controls [1] [4] [13]. Both results are in fact rather direct consequences of nonholonomy. The case where stabilizing smooth state-feedbacks do not exist , is a well known problem in nonlinear control theory [22] [23], and it usually steers the conclusion that discontinuous feedback is the alternative. This idea has been exploited in [2], where a discontinuous feedback strategy is proposed to stabilize a knife edge moving in point contact on a plane surface. However, there is another alternative which consists of considering a possible dependency of the feedback control law on the exogeneous time index. This possibility, briefly studied in [15] to stabilize a cart equipped with two independent motorized wheels on the same axis, is here applied to a front-wheel driven cart and a set of globally stabilizing *non-stationary* smooth feedback controls is derived.

In this paper, the control inputs are assumed to be the two motorized wheel's angular velocities (advancement velocity and steering angle velocity). Extension to the more realistic case where the control variables are the torques applied to the motorized wheel, has been considered in [15] for the two-wheel driven cart.

In order to avoid any misenterpretation of the results proposed in this paper, it may be useful to forewarn the reader that we are here more concerned with conceptual new possibilities than with realistic modelling and implementation aspects. These will be the subject of forthcoming studies.

Although the control of nonholonomic WMR (Wheeled Mobile Robots) so as to track a desired path has been the subject of several research works ([3] [5] [6] [7] [8] [9] [12]...), it seems that the feedback stabilization problem treated here has not been solved before.

2 Problem statement

A view from above of the considered cart, a three-wheeled vehicle, is depicted in Fig.1.

It is assumed that the cart moves on a horizontal ground. The model equations of the cart's motion are derived under the usual rolling-without-slippage assumption. They are thus simply obtained by expressing the fact that the point of each wheel in contact with the ground has zero velocity.

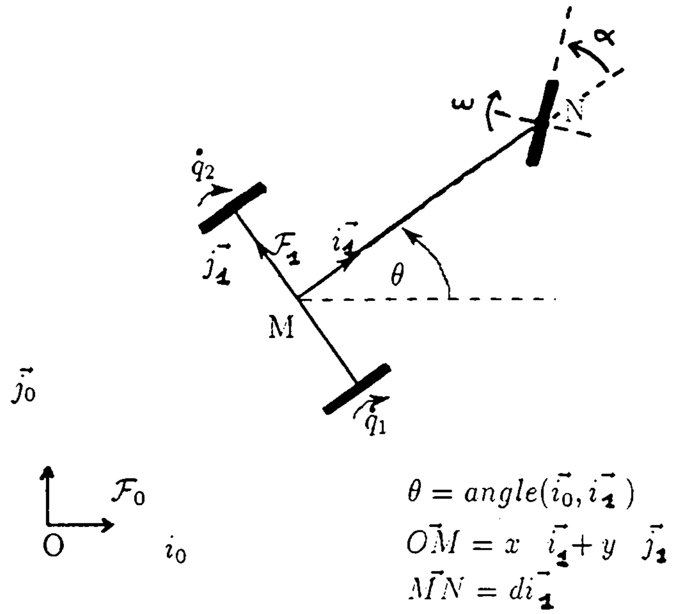

Fig.2 : Cart's parameters

The following notations are used:

- M: the cart's point located at mid-distance of the two rear wheels
- r: radius of the steering wheel
- d: distance between M and the steering wheel
- $\mathcal{F}_0 = (O; \vec{i}_0, \vec{j}_0)$: a fixed reference frame such that the point M belongs to the plane $(O; \vec{i}_0, \vec{j}_0)$
- $\mathcal{F}_1 = (M; \vec{i}_1, \vec{j}_1)$: a frame rigidly linked to the cart
- α: the steering wheel angle with respect to the cart's body
- ω: the motorized wheel's advancement angular velocity
- θ: the angle $(\vec{i}_0, \vec{i}_1)$ which characterizes the cart's orientation with respect to the fixed frame $\mathcal{F}_0$
- x, y: the coordinates of the vector $\vec{OM}$ in the basis of the frame $\mathcal{F}_1$
- v: the cart's advancement velocity ($\frac{d\vec{OM}}{dt} = v\vec{i}_1$)

Under the rolling-without-slippage assumption, the following equations hold (see [7] [13] [15] for example):

$$\begin{aligned} \dot{x} &= \dot{\theta} y + v \\ \dot{y} &= -\dot{\theta} x \end{aligned} \tag{1}$$

In the case where the rear wheels are motorized and the front wheel is a free rotating wheel, it is possible to take the rear wheels' angular velocities $\dot{q}_i$ $(i = 1, 2)$ as control variables. Moreover, since $(v, \dot{\theta})^T = D(\dot{q}_1, \dot{q}_2)^T$, where D is a known nonsingular matrix that depends on the wheels' radius and the distance between the wheels, it is equivalent to take v and $\dot{\theta}$ as control variables. This cart is *completely controllable* in the variables x, y and θ, as it may be shown either by expliciting simple open-loop control strategies [18] or by applying classical techniques of nonlinear control theory [13]. However there is no *stabilizing* smooth (or even continuous) feedback control, depending on these variables, able to make them converge to zero whatever the initials conditions. This result, given in [13], may be established by applying a theorem due to Brockett [22]. Nevertheless, stabilizing *non-stationary* smooth feedbacks exist and have been derived in [15]. By "non-stationary" it is meant that the feedback control law also depends on the exogeneous time index t.

A difference between this case and the car-like case where the front steering wheel is motorized is that v and $\dot{\theta}$ no longer correspond to physical control variables. Instead, ω and $\dot{\alpha}$ may be considered. Other control laws also have to been derived because the vectors $(v, \dot{\theta})^T$ and $(\omega, \dot{\alpha})^T$ are not isomorphic to each other. Indeed, it is immediate to verify that v and $\dot{\theta}$ are related to ω and α by the following equations:

$$\begin{aligned} v &= r\omega \;\cos\alpha \\ \dot{\theta} &= r\omega/d \;\sin\alpha \end{aligned} \tag{2}$$

We already notice that the angular velocity $\dot{\alpha}$ does not even appear in these equations.

From 2, we also have:

$$\begin{aligned} \omega &= (v \;\cos\alpha \;+\; d\dot{\theta} \;\sin\alpha)/r \\ \tan\alpha &= d\dot{\theta}/v \end{aligned} \tag{3}$$

From these equations, it may be tempting to set ω and the steering angle α (rather than its derivative $\dot{\alpha}$) as the new control variables, and to compute them from values that are determined for v and $\dot{\theta}$. However, this solution is not very satisfactory for at least two reasons: i) the steering angle α is not determined when v and $\dot{\theta}$ are both equal to zero (yielding possible discontinuous variations of α when v and $\dot{\theta}$ pass thru zero simultaneously), and ii) due to dynamics effects, instantaneous monitoring of α (which may for example correspond to the position of the rotor of an electric motor) is not physically possible. Concerning this last point, the choice of $\dot{\alpha}$, as control variable, is already more satisfying since velocity regulation of an electric motor is common practice. As a matter of fact, it would be even better to consider the torque applied to the motor, i.e. the current going thru the motor in the case of a D.C. motor. But this would involve additional complexities which are out of the scope of this paper.

Let us then consider the following state vector:

$$X = \begin{bmatrix} x \\ y \\ \theta \\ \alpha \end{bmatrix} \tag{4}$$

and the control vector:

$$U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \omega \\ \dot{\alpha} \end{bmatrix} \tag{5}$$

From 1 and 2, we obtain the following nonlinear system's equation:

$$\dot{X} = F(X, U) \tag{6}$$

with:

$$F(X, U) = \begin{bmatrix} ru_1(x_2/d \ \sin x_4 + \cos x_4) \\ -(r/d)u_1 x_1 \ \sin x_4 \\ (r/d)u_1 \ \sin x_4 \\ u_2 \end{bmatrix} \tag{7}$$

It is simple to verify that the *controllability rank condition* [24] associated with this system is satisfied and thus that this system is completely controllable (meaning that for any set (X_i, X_f), there exists a control $U(t)$ that brings the system from X_i to X_f). Controllability in x, y and θ was also established in [18] [16] when considering α, instead of $\dot{\alpha}$, as a control variable. As a matter of fact, controllability in these variables is fairly intuitive since otherwise car drivers would experience serious difficulties.

On the other hand, since no vector $(0, \epsilon, 0, 0)^T$ ($\epsilon \neq 0$) can be reached by the function $F(X, U)$, we may also conclude from Brockett's theorem [22] that there is no stabilizing smooth feedback control law $U(X)$ able to make X converge to zero whatever the initial conditions. In fact, no such control is even able to stabilize the first three variables x, y and θ.

The aim of this paper is to show that stabilizing smooth feedbacks $U(X, t)$ can be derived.

3 A set of stabilizing smooth feedbacks

Let:

- $\mathcal{S}$ denote the set of matrix valued functions $f(.,t)$ defined on $R^k \times R^+$ ($k \in N$), of class C^∞, uniformly bounded with respect to the independent time variable t, and with successive partial derivatives also uniformly bounded with respect to t.

- $k(y, \theta, t)$ denote a scalar function in $\mathcal{S}$

A first result is stated in the following theorem:

Theorem 3.1
If the following control:

$$\begin{aligned} \omega &= -g_3[(1+\alpha h)(x+k) + (1/d)(g_2\theta - g_1 xy)\sin\alpha] - (1/r)\tfrac{\partial k}{\partial t} \\ \dot{\alpha} &= \tfrac{\partial k}{\partial t}[h(x+k) + (1/d)(g_2\theta - g_1 xy)\tfrac{\sin\alpha}{\alpha}] - g_4\alpha \end{aligned} \tag{8}$$

where g_i $(i = 1, 4)$ are strictly positive numbers and:

$$h(y, \theta, \alpha, t) = (1/d)\frac{\sin\alpha}{\alpha}(y - \frac{\partial k}{\partial y}x + \frac{\partial k}{\partial \theta}) + \frac{\cos\alpha - 1}{\alpha} \tag{9}$$

is applied to the car-like mobile robot, then:

- *α, $(x+k)$, $\dot{y}$, $\dot{\theta}$ and $\frac{\partial k}{\partial t}(g_2\theta + g_1 k y)$ asymptotically converge to zero, whatever the initial conditions*

- *$|x(t)|$, $|y(t)|$ and $|\theta(t)|$ are uniformly bounded with respect to the initial conditions*

Proof of Theorem 3.1:
Let X denote the following vector:

$$X = \begin{bmatrix} x+k \\ {g_1}^{1/2} y \\ {g_2}^{1/2}\theta \\ \alpha \end{bmatrix} \tag{10}$$

Deriving X with respect to time, and using 1 and 2, gives:

$$\dot{X} = A(X,t)X + B(X,t)V \tag{11}$$

with:

$$A(X,t) = \frac{\partial k}{\partial t}\begin{bmatrix} 0 & 0 & 0 & -h \\ 0 & 0 & 0 & ({g_1}^{1/2}/d)x\frac{\sin\alpha}{\alpha} \\ 0 & 0 & 0 & -({g_2}^{1/2}/d)\frac{\sin\alpha}{\alpha} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

$$B(X,t) = \begin{bmatrix} 1+h\alpha & 0 \\ -(g_1{}^{1/2}/d)x \ \sin\alpha & 0 \\ (g_2{}^{1/2}/d) \ \sin\alpha & 0 \\ 0 & 1 \end{bmatrix} \tag{13}$$

$$V = \begin{bmatrix} r & 0 \\ 0 & 1 \end{bmatrix} U + \begin{bmatrix} \frac{\partial k}{\partial t} \\ 0 \end{bmatrix} \tag{14}$$

Choosing the auxiliary control vector V as follows:

$$V = \frac{\partial k}{\partial t} \begin{bmatrix} 0 & 0 & 0 & 0 \\ h & -(g_1{}^{1/2}/d)x\frac{\sin\alpha}{\alpha} & (g_2{}^{1/2}/d)\frac{\sin\alpha}{\alpha} & 0 \end{bmatrix} X - (R(X,t)+\tilde{C}_1(X,t))B(X,t)^T X \tag{15}$$

with:

- $R(X,t)$: (2×2) positive symmetric definite (*p.s.d.*) matrix function in $\mathcal{S}$ (positivity meaning here that $Y^T R(X,t) Y > 0$ for all (X,t) and all vectors Y in R^2 with norm equal to one)
- $\tilde{C}_1(X,t)$: (2×2) skew-symmetric (*s.s.*) matrix function in $\mathcal{S}$ (meaning that $Y^T \tilde{C}_1(X,t) Y = 0$ for all vectors Y in R^2)

yields the closed-loop equation:

$$\dot{X} = -(Q(X,t) + \tilde{C}(X,t))X \tag{16}$$

with:

$$\begin{aligned} Q(X,t) &= B(X,t)R(X,t)B(X,t)^T \qquad : \textit{p.s. matrix function in } \mathcal{S} \\ \tilde{C}(X,t) &= A(X,t)^T - A(X,t) + B(X,t)\tilde{C}_1(X,t)B(X,t)^T \qquad : \textit{s.s. matrix function in } \mathcal{S} \end{aligned} \tag{17}$$

From 14 and 15, it is simple to verify that, when choosing:

$$\begin{aligned} \tilde{C}_1 &= 0 \\ R &= diag(rg_3, g_4) \end{aligned} \tag{18}$$

one obtains the control of Theorem 3.1. Slightly more general results could thus be derived by considering other choices for R and $\tilde{C}_1$.

The analysis of system 16 will now yield the announced results.

From the regularity of all functions involved in 16, we are already ensured of the local existence and uniqueness of the solutions starting at t=0.

Consider now the Lyapunov-like function $W(X) = 1/2X^T X$. From 16:

$$\dot{W}(X) = -X^T B(X,t) R B(X,t)^T X \quad (\leq 0) \tag{19}$$

Since W is positive and decreasing, it is bounded. From the definition of X, and using the fact that $k(y,\theta,t)$ belongs to $\mathcal{S}$, this in turn yields the uniform boundedness of x, y, θ and α with respect to the initial conditions. This also ensures the existence of the solutions over R^+ and the boundedness of V and U.

Since $\dot{W}$ is bounded and W decreases, $\dot{W}$ tends to zero. Thus $B(X,t)^T X$ tends to zero. Due to the structure of $B(X,t)$, it is fairly simple to verify that this yields the convergence of α and $(x+k)$ to zero.

Now, in view of 1 and 2, the convergence of α to zero and the boundedness of $U = (\omega, \dot{\alpha})^T$ yield the convergence of $\dot{\theta}$ and $\dot{y}$ to zero.

Since 16 may also be written: $\dot{X} = F(X,t)$, with $F(X,t)$ having the property of belonging to $\mathcal{S}$, the boundedness of $\dot{X}$ also implies the boundedness of the second derivative $\ddot{X}$. In particular $\ddot{\alpha}$ is bounded and, since α tends to zero, $\dot{\alpha}$ also tends to zero. From the expression of $\dot{\alpha}$, and the convergence of $(x+k)$ and α to zero, we finally obtain that $\frac{\partial k}{\partial t}(g_1 k y + g_2 \theta)$ converges to zero.

(end of proof).

Since the choice of the function $k(y,\theta,t)$ is not specified in Theorem 3.1, there remains to show that an adequate choice of this function yields the convergence of x, y and θ to zero. To this purpose, we will use the following technical lemma:

Lemma 3.1
Let:

- $a_i(t)$ $(i = 1,n)$ *denote* C^0 *periodic (with period* T*) bounded scalar functions such that, for all integers* l*:*

$$\begin{aligned} \int_{lT}^{(l+1)T} a_i(s) a_j(s) ds &= 0 \qquad \forall i,j \;;\; i \neq j \\ \int_{lT}^{(l+1)T} a_i(s)^2 ds &> c_i > 0 \qquad \forall i \end{aligned} \tag{20}$$

- $b_i(t)$ $(i = 1,n)$ *denote* C^1 *bounded scalar functions such that:*

$$\lim_{t\to\infty} \frac{d}{dt} b_i(t) = 0 \tag{21}$$

then:

$$\lim_{t\to\infty} \sum_{i=1}^{n} a_i(t) b_i(t) = 0 \quad \Rightarrow \quad \lim_{t\to\infty} b_i(t) = 0, \quad i = 1,n \tag{22}$$

Proof of Lemma 3.1:
Let:

$$e(l) = \int_{lT}^{(l+1)T} (\sum_{i=1}^{n} a_i(s)b_i(s))^2 ds \tag{23}$$

We have:

$$e(l) = \sum_i e_i(l) + \sum_i \sum_j e_{ij}(l) \quad ; i \neq j \tag{24}$$

with:

$$\begin{aligned} e_i(l) &= \textstyle\int_{lT}^{(l+1)T} a_i(s)^2 b_i(s)^2 ds \\ e_{ij}(l) &= \textstyle\int_{lT}^{(l+1)T} a_i(s)a_j(s)b_i(s)b_j(s)ds \end{aligned} \tag{25}$$

Integration by parts of $a_i a_j b_i b_j$ gives:

$$\begin{aligned} e_{ij}(l) = \ & (\textstyle\int_{lT}^{(l+1)T} a_i(s)a_j(s)ds) \ \ b_i((l+1)T)b_j((l+1)T) \\ & - \textstyle\int_{lT}^{(l+1)T} (\int_{lT}^{s} a_i(u)a_j(u)du) \ \ (b_i(s)\dot{b}_j(s) + \dot{b}_i(s)b_j(s)) \ ds \end{aligned} \tag{26}$$

and, by using the properties of the functions $a_i(t)$ and $b_i(t)$:

$$\lim_{l \to \infty} e_{ij}(l) = 0 \tag{27}$$

Integration by parts of $a_i{}^2 b_i{}^2$ gives:

$$\begin{aligned} e_i(l) &= f_i(l) + g_i(l) \\ f_i(l) &= (\textstyle\int_{lT}^{(l+1)T} a_i(s)^2 ds) b_i((l+1)T)^2 \\ g_i(l) &= -2 \textstyle\int_{lT}^{(l+1)T} (\int_{lT}^{s} a_i(u)^2 du) b_i(s)\dot{b}_i(s)ds \end{aligned} \tag{28}$$

and, by using the properties of the functions $a_i(t)$ and $b_i(t)$:

$$\lim_{l \to \infty} g_i(l) = 0 \tag{29}$$

Thus:

$$\begin{aligned} e(l) &= \textstyle\sum_i f_i(l) + \eta(l) \\ \eta(l) &= \textstyle\sum_i g_i(l) + \sum_i \sum_j e_{ij}(l) \quad (i \neq j) \\ \lim_{l \to \infty} \eta(l) &= 0 \end{aligned} \tag{30}$$

Now, if $\sum a_i(t)b_i(t)$ tends to zero, then $e(l)$ also tends to zero when l tends to infinity. Thus, from what precedes, $\sum f_i(l)$ tends to zero, and, since $f_i(l)$ is positive:

$$\lim_{l \to \infty} f_i(l) = 0 \qquad i = 1, n \tag{31}$$

Assume now that $b_i(t)$ does not tend to zero, then, for some positive number ϵ_i:

$$\forall t_0,\ \exists t > t_0 \ :\ |b_i(t)| > 2\epsilon_i \tag{32}$$

and, since $\dot{b}_i(t)$ tends to zero:

$$\forall l_0,\ \exists l > l_0 \ :\ |b_i((l+1)T)| > \epsilon_i \tag{33}$$

Therefore, by using the assumption made on $a_i(t)$:

$$\forall l_0,\ \exists l > l_0 \ :\ f_i(l) > c_i{\epsilon_i}^2 \tag{34}$$

Since this contradicts the convergence of $f_i(l)$ to zero, $b_i(t)$ must tend to zero. **(end of proof).**

In this lemma, the conditions put on the functions $a_i(t)$ are clearly sufficient, but not necessary to the convergence of the functions $b_i(t)$ to zero, and weaker conditions may certainly be derived. However, this lemma is all that is needed, with Theorem 3.1, to establish the following result:

Lemma 3.2
An example of function $k(y, \theta, t)$ that can be used in the control 8 to ensure convergence of x, y and θ to zero is:

$$k(y,\theta,t) = g_5 y \sin(\beta t) + g_6 \theta \cos(\beta t) \qquad ;\ g_5 > 0,\ g_6 > 0,\ \beta \neq 0 \tag{35}$$

Proof of Lemma 3.2:
From 35, we have:

$$\frac{\partial k}{\partial t}(g_2\theta + g_1 k y) = \sum_{i=1,4} a_i b_i \tag{36}$$

with:

$$\begin{aligned}
a_1 &= \cos(\beta t) & b_1 &= \beta g_2 g_5 y \theta \\
a_2 &= \sin(\beta t) & b_2 &= -\beta g_2 g_6 \theta^2 \\
a_3 &= \sin(2\beta t) & b_3 &= \tfrac{1}{2}\beta g_1 y({g_5}^2 y^2 - {g_6}^2\theta^2) \\
a_4 &= \cos(2\beta t) & b_4 &= \beta g_1 g_5 g_6 y^2 \theta
\end{aligned} \tag{37}$$

and we know, from Theorem 3.1, that $\sum a_i(t)b_i(t)$ tends to zero, and that the above functions $b_i(t)$ satisfy the conditions imposed in Lemma 3.1.

It is also immediate to verify that the functions $a_i(t)$ satisfy the conditions of Lemma 3.1 with $T = 2\pi/\beta$.

We may thus deduce from this lemma that $b_i(t)$ $(i = 1,4)$ asymptotically tends to zero. Now, the convergence of $b_2(t)$ to zero implies the convergence of $\theta(t)$ to zero, and the convergence of $b_3(t)$ to zero in turn implies the convergence of $y(t)$ to zero. Thus $k(t)$ also tends to zero, and, since (from Theorem 3.1) $(x(t) + k(t))$ tends to zero, $x(t)$ converges to zero.

(end of proof).

We will leave to the interested reader the task of finding other adequate functions $k(y, \theta, t)$ and stabilizing non-stationary smooth feedbacks. Simulation of the proposed controls, study of the convergence rate, robustness to modelling errors and generalisation of the approach will be the subject of future research.

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