Model following control for nonlinear systems

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Suivi de modèle en commande non linéaire

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Abstract

In this paper we deal with the problem of finding a dynamic state feedback controller under which the output of a given nonlinear plant follows that of a nonlinear reference model. A sufficient condition is given, as an application of the structure algorithm for characterizing the input-output structure properties of a nonlinear system. Our design procedure is illustrated by two examples.

Résumé

Ce rapport étudie le problème de la synthèse d'une commande d'état dynamique telle que le système non linéaire bouclé suive un modèle de référence donné. Une condition suffisante d'existence est donnée comme application de l'algorithme de structure et on illustre la méthode par deux exemples simples d'application.

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1 Introduction

A model following control system consists of two systems :
a physical system or plant and a reference model which implicitly defines the desired input-output behavior or the desired output of the compensated plant. The model following control problem, the so-called "asymptotic model matching problem", is that of finding a controller such that the output of the plant follows the output of the model. Shima and Isurugi [1] proposed design methods of the model following control for nonlinear systems with a full rank decoupling matrix.

On the other hand, for nonlinear systems, the model matching problem which is the problem of compensating the plant in order to match the model, has been studied by several authors in the last decade. Isidori [2] gave a solution to the problem of matching a linear model, which extends the result developped by Moore and Silverman [3]. Benedetto and Isidori [4] gave a general formulation and proposed a geometric condition for the existence of a solution. Benedetto [5] gave a solution based on the use of a zero-dynamics algorithm. He also pointed out that the compensated system is the asymptotical model matching control system under the assumption that the model is linear and asymptotically stable.

In the model matching control, considering the error between the output of the plant and that of the model, we have an error-dynamics such that all of their characteristic roots are zero. Such an error dynamics is not desirable since, in a practical control, there are errors in implementation of the controller and noise. Therefore, from an engineering view point it is necessary that the error dynamics is asymptotically stable.

For nonlinear systems decouplable by static state feedback, the model following controller can be easily constructed by a slight modification of a model matching algorithm, as reported in [1]. However, for a larger class of systems, where the decoupling matrix has not full rank and its relation of dependency is not constant, it is not straightforward to obtain the model following controller, though the model matching problem is solvable.

In this paper, we present a design method of the model following control system with a linear error dynamics based on a structure algorithm. Section 2 is devoted to a definition of model following control systems and a nonlinear structure algorithm. In section 3 we give a sufficient condition as an application of the structure algorithm. In section 4 the obtained design procedure is applied to two examples.

2 Preliminaries and problem statement

We consider a nonlinear plant described by differential equations of the form

$$\begin{aligned} \dot{x}_p &= f_p(x_p) + G_p(x_p)u_p & (2.1a)\\ y_p &= h_p(x_p) & (2.1b) \end{aligned}$$

with state $x_p = [x_p^1, \ldots, x_p^{n_p}]^T \in X_p \subset \mathbb{R}^{n_p}$, input $u_p = [u_p^1, \ldots, u_p^r]^T \in \mathbb{R}^r$ and output $y_p = [y_p^1, \cdots, y_p^m]^T \in \mathbb{R}^m$. f and the r columns $g_{p_1}, \ldots, g_{p_r}$ of the matrix G_p are real analytic vector fields on $\mathbb{R}^n$ and the m functions $h_p^1, \ldots, h_p^m$ of h_p are real analytic functions.

The given reference model is also described by differential equations of the form

$$\begin{aligned} \dot{x}_M &= f_M(x_M) + G_M(x_M)u_M & (2.2a)\\ y_M &= h_M(x_M) & (2.2b) \end{aligned}$$

with state $x_M \in X_M \subset \mathbb{R}^{n_M}$, input $u_M \in \mathbb{R}^r$ and output $y_M \in \mathbb{R}^m$, and real analytic f_M, G_M, h_M.

We first introduce an extended system associated with the plant (2.1a) and the model (2.2a)

$$\dot{x} = f(x) + G(x)\bar{u} \tag{2.3}$$

with state $x^T = [x_p^T x_M^T], \bar{u} = [u_p^T u_M^T]^T$, and

$$\begin{aligned} f(x) &= \begin{bmatrix} f_p(x_p) \\ f_M(x_M) \end{bmatrix}, G(x) = [G_1(x), G_2(x)],\\ G_1(x) &= \begin{bmatrix} G_p(x_p) \\ 0 \end{bmatrix},\\ G_2(x) &= \begin{bmatrix} 0 \\ G_M(x_M) \end{bmatrix}. \end{aligned}$$

The error between the output of the plant and that of the model is defined by

$$e = h(x) = h_M(x_M) - h_p(x_p) \tag{2.4}$$

where $e = [e^1, \ldots, e^m]^T$.

The compensator used to control the plant (2.1) is assumed to be a dynamic state feedback of the following form

$$u_p = \alpha(x) + \beta(x)u_M. \tag{2.5}$$

Then, we define model following systems as follows :

Definition 2.1 *If the compensated plant (2.3) and (2.5) with the output (2.4) satisfy the following properties, it is called a model following control system.*

P1 *The input u_M does not influence the error (2.4) of the system (2.3), (2.5).*

P2 *The error dynamics is globally asymptotically stable with the origin as a critical point.*

It is known [6], [7] that the input u_M does not influence the output e of the overall system (2.3) (2.5), if and only if the functions $\alpha(x)$ and $\beta(x)$ are such as to make the conditions

$$L_{(G_1\beta+G_2)}L^k_{(f+G_1\alpha)}h(x) = 0 \tag{2.6}$$

satisfied for all $k \geq 0$. Here, $L_{(f+G_1\alpha)}h(x)$ stands for the Lie derivative of h with respect to $(f + G_1\alpha)$ and $L^0_{(f+G_1\alpha)}h(x) = h(x)$, $L^k_{(f+G_1\alpha)}h = L_{(f+G_1\alpha)}(L^{k-1}_{(f+G_1\alpha)}h)$.

In general, differentiating the output with respect to time, we need relative orders. For the plant (2.1) the relative orders p_i of the output y_p^i are defined as follows :

$$p_i = \min\{j : L_{G_p}L^{j-1}_{f_p}h^i(x_p) \neq 0\}. \tag{2.7}$$

If there exist no such j ,then we define $p_i = \infty$. In this paper we assume $p_i < \infty$. Then, we can define the matrix $Q^0(x_p)$ consisting of non zero rows as follow :

$$Q^0(x_p) = \begin{bmatrix} L_{G_p}L^{p_1-1}_{f_p}h^1(x_p) \\ \cdots \\ L_{G_p}L^{p_m-1}_{f_p}h^m(x_p) \end{bmatrix}, \tag{2.8}$$

which is called the decoupling matrix of the system (2.1).

The following is the structure algorithm, as introduced in [8], for the extended system (2.3), (2.4).

Step 1.

Set $\hat{e}_0 = e$ and $\hat{a}_0(x) = h(x)$. Calculate

$$\dot{\hat{e}}_0 = \frac{\partial \hat{a}_0}{\partial x}(f(x) + G(x)\bar{u})$$

and write it as

$$\dot{\hat{e}}_0 = a_1(x) + b_1(x)\bar{u}.$$

Set $\rho_1 = \mathrm{rank} b_1(x)$. Let

$$P_1 = \begin{bmatrix} \tilde{P}_1 \\ \hat{P}_1 \end{bmatrix}$$

be a $m \times m$ permutation matrix, such that $\tilde{P}_1 b_1(x)$ has full rank ρ_1. Decompose $\dot{\hat{e}}_0$ as

$$\begin{aligned} \dot{\tilde{e}}_1 &= \tilde{a}_1(x) + \tilde{b}_1(x)\bar{u} \\ \dot{\hat{e}}_1 &= \hat{a}_1(x, \dot{\tilde{e}}_1), \end{aligned}$$

where $[\dot{\tilde{e}}_1^T \quad \dot{\hat{e}}_1^T]^T = P_1\dot{\hat{e}}_0$. Set $\tilde{A}_1(x) = \tilde{a}_1(x)$ and $\tilde{B}_1(x) = \tilde{b}_1(x)$.

Step $k+1$

Suppose that in Step 1 through $k, \dot{\tilde{e}}_1, \ldots, \tilde{e}_k^{(k)}, \hat{e}_k^{(k)}$ have been defined so that

$$\begin{array}{rcl} \dot{\tilde{e}}_1 & = & \tilde{a}_1(x) + \tilde{b}_1(x)\bar{u} \\ \ddot{\tilde{e}}_2 & = & \tilde{a}_2(x, \dot{\tilde{e}}_1, \ddot{\tilde{e}}_1) + \tilde{b}_2(x, \dot{\tilde{e}}_1)\bar{u} \\ & \cdots & \\ \tilde{e}_k^{(k)} & = & \tilde{a}_k(x, \{\tilde{e}_i^{(j)} : 1 \le i \le k-1, i \le j \le k\}) \\ & & + \tilde{b}_k(x, \{\tilde{e}_i^{(j)} : 1 \le i \le k-1, i \le j \le k-1\})\bar{u} \\ & & \\ \hat{e}_k^{(k)} & = & \hat{a}_k(x, \{\tilde{e}_i^{(j)} : 1 \le i \le k, i \le j \le k\}) \end{array} \tag{2.9}$$

Suppose also that the matrix $\tilde{B}_k = [\tilde{b}_1^T, \ldots, \tilde{b}_k^T]^T$ has full rank equal to ρ_k. Then calculate

$$\hat{e}_k^{(k+1)} = \frac{\partial \hat{a}_k}{\partial x}(f(x) + G(x)\bar{u}) + \sum_{i=1}^{k} \sum_{j=i}^{k} \frac{\partial \hat{a}_k}{\partial \tilde{e}_i^{(j)}} \tilde{e}_i^{(j+1)}$$

and write it as

$$\begin{array}{rcl} \hat{e}_k^{(k+1)} & = & a_{k+1}(x, \{\tilde{e}_i^{(j)} : 1 \le i \le k, i \le j \le k+1\}) \\ & & + b_{k+1}(x, \{\tilde{e}_i^{(j)} : 1 \le i \le k, i \le j \le k\})\bar{u}. \end{array}$$

Set $\rho_{k+1} = \text{rank} \begin{bmatrix} \tilde{B}_k \\ b_{k+1} \end{bmatrix}$, where the rank is evaluated on the set of functions $(x, \tilde{e}_i^{(j)} : 1 \le i \le k, i \le j \le k)$ which satisfy (2.9). Let

$$P_{k+1} = \begin{bmatrix} \tilde{P}_{k+1} \\ \hat{P}_{k+1} \end{bmatrix}$$

be a $(m - \rho_k) \times (m - \rho_k)$ permutation matrix, such that $\tilde{P}_{k+1} b_{k+1}$ has full rank ρ_{k+1}. Decompose $\hat{e}_k^{(k+1)}$ as

$$\begin{array}{rcl} \tilde{e}_{k+1}^{(k+1)} & = & \tilde{a}_{k+1}(x, \{\tilde{e}_i^{(j)} : 1 \le i \le k, i \le j \le k+1\}) \\ & & + \tilde{b}_{k+1}(x, \{\tilde{e}_i^{(j)} : 1 \le i \le k, i \le j \le k\})\bar{u} \\ & & \\ \hat{e}_{k+1}^{(k+1)} & = & \hat{a}_{k+1}(x, \{\tilde{e}_i^{(j)} : 1 \le j \le k+1, i \le j \le k+1\}), \end{array}$$

where $[\tilde{e}_{k+1}^{(k+1)T}, \hat{e}_{k+1}^{(k+1)T}]^T = P_{k+1} \hat{e}_k^{(k+1)}$. Set $\tilde{A}_{k+1} = \begin{bmatrix} \tilde{A}_k \\ \tilde{a}_{k+1} \end{bmatrix}$, and $\tilde{B}_{k+1} = \begin{bmatrix} \tilde{B}_k \\ \tilde{b}_{k+1} \end{bmatrix}$.

End of Step $k+1$.

The last step of the algorithm, denoted by k^*, is the least integer k such that rank $\tilde{B}_k = m$. If there exists no such k then we define $k^* = \infty$.

Remark 2.1 *The matrices $P_1, P_2, \ldots$ of the structure algorithm are not unique in general. However, it is known [9] that the integers $\rho_1, \rho_2, \ldots$ do not depend on the choice of the matrices $P_1, P_2, \ldots$.*

3 Main Result

We assume, in the above procedure, that k^* is finite. Then we can obtain the following theorem.

Theorem 3.1 *If the following conditions are satisfied at each step k, there exists $\alpha(x)$ and $\beta(x)$ with which the system (2.3)-(2.5) is a model following control system.*

(i)

$$rank\ \tilde{B}_{k1} = rank\ \tilde{B}_k \tag{3.1}$$

where $\tilde{B}_k = [\tilde{B}_{k1}, \tilde{B}_{k2}]$ such that $\tilde{B}_k\bar{u} = \tilde{B}_{k1}u_p + \tilde{B}_{k2}u_M$.

(ii)

$$rank\ \tilde{B}_{k1}(x, \{\tilde{e}_i^{(j)} : 1 \le i \le k-1, i \le j \le k-1\}) = rank\ \tilde{B}_{k1}(x, \{0, \dots, 0\})\ |_{x\in M_k} \tag{3.2}$$

where $M_k = \{x|\hat{a}_i(x, \{0, \dots, 0\}) = 0 : 0 \le i \le k-1\}$.

Proof At the step k^* such as $\rho_{k^*} = m$, we have obtained

$$\begin{aligned}
\begin{bmatrix} \tilde{e}_1^{(1)} \\ \vdots \\ \tilde{e}_{k^*}^{(k^*)} \end{bmatrix} &= \tilde{A}_{k^*} + \tilde{B}_{k^*1}u_p + \tilde{B}_{k^*2}u_M \\
\begin{bmatrix} \hat{e}_1^{(1)} \\ \vdots \\ \hat{e}_{k^*-1}^{(k^*-1)} \end{bmatrix} &= \begin{bmatrix} \hat{a}_1(x, \tilde{e}_1^{(1)}) \\ \vdots \\ \hat{a}_{k^*-1}(x, \{\tilde{e}_i^{(j)} : 1 \le i \le k^*-1, i \le j \le k^*-1\}) \end{bmatrix}.
\end{aligned} \tag{3.3}$$

If the condition (3.1) is satisfied, then the matrix $\tilde{B}_{k^*1}$ has full rank. Moreover, from condition (3.2) the rank of the matrices $\tilde{B}_{k1}$ is invariant under the following relations :

$$\begin{aligned}
\tilde{e}_1^{(1)} &= Z^1(\hat{e}_0) \\
\tilde{e}_2^{(2)} &= Z^2(\hat{e}_0, \hat{e}_1^{(1)}) \\
&\cdots \\
\tilde{e}_k^{(k)} &= Z^k(\hat{e}_0, \hat{e}_1^{(1)}, \dots, \hat{e}_{k-1}^{(k-1)})
\end{aligned} \tag{3.4}$$

From the permutation matrices P_k in the algorithm, we get

$$\begin{bmatrix} \tilde{e}_{k+1} \\ \tilde{e}_{k+2} \\ \vdots \\ \tilde{e}_{k^*-1} \\ \tilde{e}_{k^*} \end{bmatrix} = \begin{bmatrix} \tilde{P}_{k+1} \\ \tilde{P}_{k+2}\hat{P}_{k+1} \\ \vdots \\ \tilde{P}_{k^*-1}\hat{P}_{k^*-2}\dots\hat{P}_{k+1} \\ \hat{P}_{k^*-1}\hat{P}_{k^*-2}\dots\hat{P}_{k+1} \end{bmatrix} \hat{e}_k, \quad \text{for } 0 \le k \le k^*-1,$$

which yield

$$\tilde{e}_k^{(k)} = Z^k(\{\hat{e}_j^{(i-1)} : 1 \le i \le k, i \le j \le k^*\}). \tag{3.5}$$

Therefore, we can determine $\alpha(x)$ and $\beta(x)$ such that the equations

$$\begin{array}{rcl} \tilde{B}_{k^\bullet 1}\beta(x) + \tilde{B}_{k^\bullet 2} & = & 0 \\ \tilde{A}_{k^\bullet} + \tilde{B}_{k^\bullet 1}\alpha(x) & = & [Z^1, \dots, Z^{k^\bullet}]^T \end{array} \tag{3.6}$$

are satisfied.

Since the functions Z^k are arbitrary functions of $\{\tilde{e}_j^{(j-1)} : 1 \le i \le k, i \le j \le k^*\}$, it is always possible to choose them as linear functions :

$$Z^k = \sum_{i=1}^{k}\sum_{j=i}^{k^*} \alpha_{kj}^{i-1}\tilde{e}_j^{(i-1)}$$

where the coefficients α_{kj}^{i-1} are $(\rho_k - \rho_{k-1}) \times (\rho_j - \rho_{j-1})$ matrices. Then we have a linear error dynamics as follows :

$$\begin{bmatrix} \dot{\tilde{e}}_1 \\ \dot{\tilde{e}}_2 \\ \ddot{\tilde{e}}_2 \\ \vdots \\ \dot{\tilde{e}}_{k^\bullet} \\ \vdots \\ \tilde{e}_{k^\bullet}^{(k^\bullet)} \end{bmatrix} = \begin{bmatrix} * & * & 0 & \dots & * & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & I & & 0 & \dots & \dots & \dots & \dots & 0 \\ * & * & * & & * & * & 0 & \dots & \dots & 0 \\ & & & \dots & & & & & & \\ 0 & 0 & 0 & & 0 & I & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & & & \ddots & & & \\ 0 & 0 & 0 & & 0 & \dots & \dots & \dots & 0 & I \\ * & * & * & & * & * & \dots & \dots & * & * \end{bmatrix} \begin{bmatrix} \tilde{e}_1 \\ \tilde{e}_2 \\ \dot{\tilde{e}}_2 \\ \vdots \\ \tilde{e}_{k^\bullet} \\ \vdots \\ \tilde{e}_{k^\bullet}^{(k^\bullet-1)} \end{bmatrix}$$

As a result, the choice of α_{kj}^{i-1} such that the error dynamics is asymptotically stable is possible.

Remarks

1) If condition (3.1) of the theorem is not satisfied, then the solution $\alpha(x)$ and $\beta(x)$ satisfying the equations (3.3) for all $u_M(t)$ do not exist, since $\tilde{B}_{k^\bullet 1}$ has not full rank. Therefore, the condition (3.1) is a necessary condition.

2) M_k are submanifolds in the Zero Dynamics Algorithm. As a result, it is obvious that, if the conditions (3.1) and (3.2) are satisfied, then a model matching problem is solvable, as reported in [5]. However, since in general,

$$\rho_k \ge \text{ rank } \tilde{B}_{k_1}(x\{0\dots 0\}) \mid_{x \in M_k} \triangleq \rho'_k,$$

the model following problem is not always solvable, even if the model matching problem is solvable.

3) In the case of $\rho_k > \rho'_k$, we have a relation

$$\phi_k : \{\phi_k(\{\tilde{e}_i^{(j)} : 1 \le i \le k-1, i \le j \le k-1\}, \{\hat{e}_i^{(i)} : 0 \le i \le k-1\}) = 0\},$$

which reduces the rank. Then, we may proceed further with the procedure by adding the following operations : Set $\rho_k = \rho'_k$; find ϕ_k, and test the condition

(3.1), i.e., ρ_k = rank $\tilde{B}_k|\phi_k$. From the next step, the rank is always evaluated under the relation ϕ_k. When the algorithm terminates at step k^*, we have a constraint condition ϕ_k for the construction of the error dynamics. Consequently, if there exist Z_k such that the error dynamics is asymptotically stable under the restriction $\bar{q}_k$, then we can construct the model following controller from (3.6).

4) It is quite easy to see that, if the given plant (2.1) is such that the decoupling matrix (2.8) has full rank, then the condition (3.2) is always satisfied since the elements of $\tilde{B}_{k^*1}$ are functions of only x_p, i.e., $\tilde{B}_{k^*1} = Q^0(x_p)$. In this case the condition (3.1) is equivalent to

$$p_i \leq \pi_i \quad \text{for } 1 \leq i \leq m,$$

where π_i are the relative orders of the model.

4 Examples

Two examples are given below to illustrate the algorithm.

Example 1

First we consider the example proposed in [4] and [5], where the given nonlinear plant is described by

$$\begin{aligned}\dot{x}_p &= \begin{pmatrix} 0 \\ x_p^4 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} x_p^3 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} u_p \\ y_p &= \begin{pmatrix} x_p^2 - x_p^3 \\ x_p^1 \end{pmatrix}.\end{aligned}$$

The reference model is a linear system :

$$\begin{aligned}\dot{x}_M &= \begin{pmatrix} x_M^2 \\ 0 \\ x_M^4 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} u_M \\ y_M &= \begin{pmatrix} x_M^1 \\ x_M^3 \end{pmatrix}\end{aligned}$$

Step 1 : We have

$$\hat{e}_0 = e = \begin{pmatrix} x_M^1 - x_p^2 + x_p^3 \\ x_M^3 - x_p^1 \end{pmatrix}.$$

Differentiating $\hat{e}_0$ with respect to time, we obtain

$$\dot{\hat{e}}_0 = \begin{pmatrix} x_M^2 - x_p^4 \\ x_M^4 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ -x_p^3 & 0 \end{pmatrix} u_p$$

and $\rho_1 = 1$. Then,

$$\begin{aligned}\dot{\hat{e}}_1 &= x_M^2 - x_p^4 + [1\ 0]u_p \\ \dot{\tilde{e}}_1 &= x_M^2 x_p^3 - x_p^3 x_p^4 + x_M^4 - x_p^3 \dot{\hat{e}}_1\end{aligned}$$

Step 2 : We differentiate $\dot{\tilde{e}}_1$ further to obtain

$$\ddot{\tilde{e}}_1 = -x_p^3 \ddot{\hat{e}}_1 + [x_p^3 \quad 1]u_M + [x_M^2 - x_p^4 - \dot{\hat{e}}_1 \quad x_M^2 - x_p^3 - x_p^4 - \dot{\hat{e}}_1]u_p$$

and $\rho_2 = 2$. The matrix

$$\tilde{B}_{21}(x, 0) = \begin{pmatrix} 1 & 0 \\ x_M^2 - x_p^4 & x_M^2 - x_p^3 - x_p^4 \end{pmatrix}$$

has full rank for all $x \in M_2$, where $M_2 = \{x | x_M^1 - x_p^2 + x_p^3 = 0, x_M^3 - x_p^1 = 0, x_M^2 x_p^3 - x_p^3 x_p^4 + x_M^4 = 0\}$. Hence the algorithm terminates at $k^* = 2$. We have

$$\tilde{A}_2 = \begin{pmatrix} x_M^2 - x_p^4 \\ -x_p^3 \ddot{\tilde{e}}_1 \end{pmatrix},$$

$$\tilde{B}_2 = \left(\begin{array}{cc:cc} 1 & 0 & 0 & 0 \\ x_M^2 - x_p^4 - \dot{\tilde{e}}_1 & x_M^2 - x_p^3 - x_p^4 - \dot{\tilde{e}}_1 & x_p^3 & 1 \end{array} \right).$$

If in (3.5), we set

$$Z = \begin{bmatrix} -\alpha_0 \tilde{e}_1 \\ -\beta_0 \tilde{e}_2 - \beta_1 \dot{\tilde{e}}_2 \end{bmatrix},$$

from (3.6), we can determine $\alpha(x)$ and $\beta(x)$ as follows :

$$\alpha(x) = \begin{pmatrix} -x_M^2 + x_p^4 - \alpha_0(x_M^1 - x_p^2 + x_p^3) \\ \frac{1}{x_M^2 - x_p^4 - x_p^3 - \alpha_0(x_M^1 - x_p^2 + x_p^3)} \{(-x_M^2 + x_p^4 - \alpha_0(x_M^1 - x_p^2 + x_p^3))^2 \\ +(\alpha_1)^2(x_M^1 - x_p^2 + x_p^3)x_p^3 - \beta_0(x_M^3 - x_p^1) \\ -\beta_1(x_M^2 x_p^3 - x_p^3 x_p^4 + x_M^4 + \alpha_0 x_p^3(x_M^1 - x_p^2 + x_p^3))\} \end{pmatrix}$$

$$\beta(x) = \frac{1}{x_M^2 - x_p^4 - x_p^3 - \alpha_0(x_M^1 - x_p^2 + x_p^3)} \begin{pmatrix} 0 & 0 \\ -x_p^3 & -1 \end{pmatrix}$$

Thus, the dynamic state feedback control law yields the error dynamics

$$\begin{pmatrix} \dot{e}^1 \\ \dot{e}^2 \\ \ddot{e}^2 \end{pmatrix} = \begin{pmatrix} -\alpha_0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -\beta_0 & -\beta_1 \end{pmatrix} \begin{pmatrix} e^1 \\ e^2 \\ \dot{e}^2 \end{pmatrix},$$

where $e = [e^1 \; e^2]^T$. Since the condition of theorem is a structural condition, in this example, the controller is not defined whenever $x_M^2 - x_p^4 - x_p^3 - \alpha_0(x_M^1 - x_p^2 + x_p^3) = 0$. If we choose $\alpha_0 = \beta_0 = \beta_1 = 0$, then we have the model matching controller, which coincides with the one derived in [5] using the zero dynamics algorithm.

Example 2

The following example shows how the error dynamics can be constructed by means of the modified algorithm, i.e. remarks (3), when the condition (ii) of theorem is not satisfied.

The nonlinear plant is described by

$$\dot{x}_p = \begin{pmatrix} x_p^1 \\ x_p^2 x_p^4 - x_p^1 x_p^3 \\ 0 \\ x_p^4 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ x_p^3 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} u_p, \quad y_p = \begin{pmatrix} x_p^1 \\ x_p^2 \end{pmatrix},$$

and the model is a linear system

$$\dot{x}_M = \begin{pmatrix} x_M^2 \\ x_M^3 \\ -x_M^3 + x_M^4 + x_M^5 \\ -x_M^4 \\ x_M^4 - x_M^5 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} u_M, \quad y_M = \begin{pmatrix} x_M^1 \\ x_M^2 \end{pmatrix}.$$

Step 1 : We have $\rho_1 = 1$ and

$$\begin{aligned} \dot{\tilde{e}}_1 &= x_M^2 - x_p^1 + [1 \quad 0]u_p \\ \dot{\hat{e}}_1 &= x_M^2 x_p^3 + x_M^3 - x_p^2 x_p^4 - x_p^3 \dot{\tilde{e}}_1 \end{aligned}$$

Step 2 : We obtain $\rho_2 = 2$ and

$$\begin{aligned} \tilde{e}_2^{(2)} &= x_M^3(x_p^3 - 1) + x_M^4 + x_M^5 - x_p^4(x_p^2 x_p^4 - x_p^1 x_p^3 + x_p^2) \\ &- x_p^3 \ddot{\tilde{e}}_1 + [-x_p^3 x_p^4 - x_p^2 \quad x_M^2 - x_p^2 - \dot{\tilde{e}}_1]u_p \end{aligned}$$

and

$$\tilde{B}_{21} = \begin{pmatrix} 1 & 0 \\ -x_p^3 x_p^4 - x_p^2 & x_M^2 - x_p^2 - \dot{\tilde{e}}_1 \end{pmatrix}$$

The condition (3.2) is not met since $\rho_2' = 1$. Then, we set $\rho_2 = \rho_2'$ again and we get $\phi_2 = \{\dot{\tilde{e}}_1 = [0, 1]\hat{e}_0\}$ according to remarks (3).

$$\begin{aligned} \hat{e}_2^{(2)} &= x_M^2(x_p^2 + x_p^3 x_p^4) + x_M^3(x_p^3 - 1) + x_M^4 + x_M^5 \\ &- x_p^2(x_p^1 + (x_p^4)^2 + x_p^4) - x_p^3 \ddot{\tilde{e}}_1 - (x_p^3 x_p^4 + x_p^2)\dot{\tilde{e}}_1 \end{aligned}$$

Step 3 : An easy computation shows that $\rho_3 = 2$ and the condition is satisfied under the restriction ϕ_2. Since $\rho_3 = 2$, the algorithm terminates at $k^* = 3$.

We have the error dynamics

$$\begin{aligned} \dot{\tilde{e}}_1 &= Z_1(e) \\ \tilde{e}_3^{(3)} &= Z_2(e, \dot{\hat{e}}_1, \ddot{\hat{e}}_2) \\ \phi_2 : \dot{\tilde{e}}_1 &= [0 \quad 1]\hat{e}_0. \end{aligned}$$

From $\tilde{e}_1 = [1 \quad 0]\hat{e}_0$ and $\tilde{e}_3 = [0 \quad 1]\hat{e}_0 = \hat{e}_1 = \hat{e}_2$, the error dynamics can be written in the form

$$\begin{aligned} \dot{\tilde{e}}_1 &= Z_1(\tilde{e}_3) = \tilde{e}_3 \\ \tilde{e}_3^{(3)} &= Z_2(\tilde{e}_1, \tilde{e}_3, \dot{\tilde{e}}_3, \ddot{\tilde{e}}_3). \end{aligned}$$

If we choose

$$Z_2 = -\beta_0 \tilde{e}_1 - \beta_1 \tilde{e}_3 - \beta_2 \dot{\tilde{e}}_3 - \beta_3 \ddot{\tilde{e}}_3$$

then the error dynamics becomes

$$\begin{pmatrix} \dot{e}^1 \\ \dot{e}^2 \\ \ddot{e}^2 \\ \dddot{e}^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\beta_0 & -\beta_1 & -\beta_2 & -\beta_3 \end{pmatrix} \begin{pmatrix} e^1 \\ e^2 \\ \dot{e}^2 \\ \ddot{e}^2 \end{pmatrix}$$

where $e = [e^1 \; e^2]^T$. Since the rank of $\tilde{B}_{k_1}$ is invariant with respect to the error dynamics, we can design the model following controller by solving (3.6).

$$u_p = \tilde{B}_{31}^{-1}\{\tilde{B}_{32}u_M - \begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_3 \end{pmatrix} - \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}\}.$$

In this example, the solution is defined for $x_p^2 \neq 0$.

5 Conclusions

In this paper, we have proposed a condition for the existence of a dynamic state feedback under which the output of a given nonlinear plant follows that of a reference modal.

The given condition is a structural condition corresponding to the fact the model following control problem is solvable almost everywhere ; at every point where $\tilde{B}_{k^*1}$ is full rank.

We have restricted our attention to the error dynamics based on an input-output approach. Hence, the proposed solution is such as to make the system maximally unobservable. Since some of the internal dynamics, which is refered to as the zero dynamics, is automatically assigned, the given plant is necessarily minimum phase nonlinear systems.

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