Mean field annealing using compound Gauss-Markov random fields for edge detection and image restoration

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UNITÉ DE RECHERCHE
INRIA-SOPHIA ANTIPOLIS

Rapports de Recherche

N° 1295

Programme 6
Robotique, Image et Vision

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Octobre 1990

RR-1295

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RECUIT PAR CHAMPS MOYENS : UTILISATION DE CHAMPS DE MARKOV COMPOSES POUR LA DETECTION DE CONTOURS ET LA RESTAURATION D'IMAGE

MEAN FIELD ANNEALING USING COMPOUND GAUSS-MARKOV RANDOM FIELDS FOR EDGE DETECTION AND IMAGE RESTORATION

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Septembre 1990

abstract

In this report, we consider the problem of edge detection and image restoration with images corrupted by an additive Gaussian noise.

We propose a deterministic relaxation method based on Mean Field Annealing with a Compound Gauss-Markov Random (CGMRF) Field model. We present a set of iterative equations for the mean values of the intensity and both horizontal and vertical line-processes with or without taking into account some interaction between them. We show the relationship between this technique and two other methods : the one recently described by Geiger and Girosi and the one proposed by Simchony et al. . We emphasize on the need of an optimal step-descent method to get a robust algorithm.

Lastly, we present edge detection and image restoration results on a noisy aerial image (SNR = 5 dB) with line-process interaction and compare them with those obtained without such an interaction.

résumé

Dans ce rapport, nous considérons le probléme de la détection de contours et de la restauration d'image lorsque celle-ci est perturbée par un bruit additif Gaussien.

Nous proposons une méthode de relaxation déterministe basée sur le recuit par champs moyens et utilisant comme modéle un champ de Markov composé. Nous présentons un ensemble d'équations itératives pour obtenir les valeurs moyennes de l'intensité et des processus de ligne horizontaux et verticaux avec ou sans interaction entre eux. Nous montrons qu'il existe une relation entre cette technique et deux autres méthodes présentées respectivement par Geiger et Girosi et par Simchony et al.. Nous insistons sur la nécessité d'utiliser une méthode de descente avec un pas optimal afin d'obtenir un algorithme robuste.

Enfin, nous présentons les résultats obtenus pour la détection de contours et la restoration d'image sur une image aérienne bruitée (rapport S/B = 5 dB) avec une interaction entre les processus de ligne et nous les comparons avec ceux obtenus sans une telle interaction.

key words

Markov Random Fields,deterministic relaxation, edge detection, image restoration.

mots clefs

Champs de Markov, relaxation déterministe, détection de contours, restauration d'image.

Contents

1 INTRODUCTION

We consider herein the problem of edge detection and image restoration in an image corrupted by additive Gaussian noise. For many years, researchers have worked on this problem as edge detection and image restoration in noisy data are important preliminary steps for various high level vision processes.

Many solutions have been proposed. Among them, the method using line-processes, first introduced by Geman and Geman [8], seems the most promising. They explicitly take into account the cost of creating edges in the form of a line-process. The problem is then to minimize the energy function which is non-convex with respect to the image intensity and the line-processes. One way to obtain the minimum is to use the well-known stochastic technique called "simulated annealing" [8], [12], [17]. This method is optimal (you are able to get the global minimum asymptotically) but it is very time-consuming. Another way to deal with the problem is to search for sub-optimal deterministic algorithms. From a theoretical point of view, the main drawback is that these techniques are not guaranted to give the global minimum. But from a practical point of view, the shape of the energy function obtained for images does not have too many minima and the results are usually good enough even if not optimal. Deterministic algorithms yield solutions at a much lower computational cost.

An interesting deterministic technique is the Graduated Non-Convexity (GNC) algorithm introduced by Blake and Zisserman [3]. An extension of this technique using a Compound Gauss-Markov Random Field (CGMRF) [10] has been derived in [21], [22] for image restoration (the edges are given as a by-product).

Another class of methods coming from equilibrium statistical mechanics [18], [19] relies on the mean field approximation. The basic idea is to substitute the fields at the neighbor-sites by their respective statistical mean value. This approximation is valid only if the fluctuations around the mean are small. It has been used for both image processing and vision problems such as pattern recognition [4], surface reconstruction [6], [7], [16], [24] or stereovision [9], [23]. There is no proof of convergence for this technique but it reaches an equilibrium at a given temperature much faster than simulated annealing [2].

Recently, we have proposed a method using mean field annealing for a CGMRF model with [26] or without [27] interaction between the line-processes. We show the relationship between this method and the one proposed by Geiger and Girosi in [6], [7] as well as the one described by Simchony et al. in [21].

2 MEAN FIELD ANNEALING USING A CGMRF MODEL

2.1 Compound GMRF model

The edges in gray level images play an important role in human interpretation. In order to preserve the sharpness of such edges, a Compound GMRF was initially introduced in [10]. The CGMRF model used in this paper has been suggested in [21]. Basically it is an extension of the weak membrane model [3]: the GMRF [11] breaks when edges occur which in turn creates homogeneous GMRF patches separated by the line processes.

We consider the following model for the image intensity corrupted by an additive Gaussian noise :

$$x = y + n \tag{1}$$

Using a first order neighborhood, a Compound GMRF model can be defined [21] which has the following conditional distribution [1] :

$$p(y(s)|y(s+\tau), y(s-\tau), l(s,\tau), l(s-\tau,\tau), \tau \in N^*) = \frac{\exp - E(y(s)|y(s+\tau), y(s-\tau), l(s,\tau), l(s-\tau,\tau), \tau \in N^*)}{Z} \tag{2}$$

where N^* is a set of shift vectors which corresponds to the neighborhood of the GMRF model. the line process is defined on the edges that connect the nodes (pixels) that are neighbors of a given pixel.

We suppose that the statistical properties of the original image y and the degradation are known. As the image has been degraded by an additive Gaussian noise, it is easy to construct the a posteriori density function $p(y|x)$ from the observation x using Bayes law:

$$p(y|x) = \frac{p(x|y)p(y)}{p(x)} \tag{3}$$

The MAP estimate is given by the maximization of $p(y|x)$ in order to find the most probable value of y. As the observation x is known, $p(x)$ can be considered as a constant.

Furthermore, $p(x|y)$ is known because $n(s)$ is a spatially independent Gaussian noise:

$$p(x|y) = \frac{1}{(2\pi\sigma^2)^{\frac{M^2}{2}}} \exp\{-\frac{1}{2\sigma^2} \sum_{s\in\Omega} (x(s) - y(s))^2\} \tag{4}$$

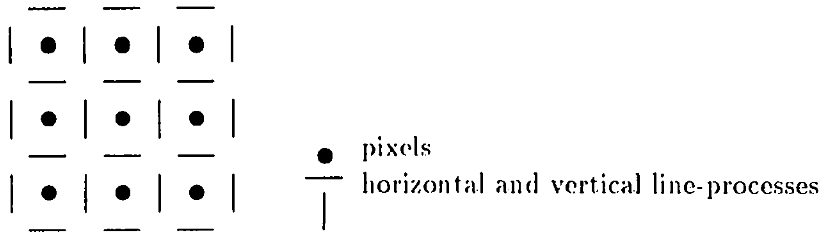

Figure 1: image lattice

where Ω is a finite lattice on which the image is defined :

$$\Omega = \{s = (i,j), 1 \leq i,j \geq M\} \tag{5}$$

As for the prior $p(y)$, it is in fact expressed in terms of a CGMRF that is $p(y,l)$:

$$p(y,l) = \exp\frac{-E(y,l)}{Z} \tag{6}$$

where $E(y,l)$ is expressed in terms of GMRF parameters and the line process (see [21] and [22] for more details).

Using horizontal and vertical line processes (cf Fig. 1) , the global energy for the Compound GMRF model disturbed by an additive noise is given by:

$$\begin{aligned} E &= \frac{1}{2\sigma^2}\sum_{i,j}\Big\{(y_{i,j} - x_{i,j})^2 + \lambda^2(1 - 2(\theta_x + \theta_y))y_{i,j}^2 \\ &\quad +\theta_x(\lambda^2(y_{i,j} - y_{i-1,j})^2(1 - l_{i,j}) + \alpha l_{i,j}) \\ &\quad +\theta_y(\lambda^2(y_{i,j} - y_{i,j+1})^2(1 - m_{i,j}) + \alpha m_{i,j})\Big\} \end{aligned} \tag{7}$$

with $(1 - 2(\theta_x + \theta_y)) > 0$ to ensure the positivity of the spectral density of the GMRF [21] and where σ^2 is the variance of the Gaussian noise, θ_x and θ_y are the GMRF parameters, l and m respectively stand for the horizontal and the vertical line processes, λ^2 corresponds to the regularization term which reflects the confidence we have in the data and α is the penalty to be paid to create an edge.

It is easy to show [21] that $\lambda^2 = \sigma^2/\nu$ where ν is the variance of the GMRF model. Therefore when the data are noisy, λ^2 is set high to reflect the low confidence in the data and to effect a great deal of smoothing.

2.2 Mean field approximation

The energy of the model given in (7) is non-convex. Therefore it could exhibit local minima. The problem of minimization of (7) can be solved with two kinds of methods. The classical stochastic relaxation technique which asymptotically gives the global minimum but requires a large convergence time. To avoid that computational burden, other algorithms - namely the deterministic relaxation techniques - have been developed [3, 5, 9, 10, 13, 14, 16, 20, 21]. Among them, the mean field annealing from statistical mechanics [18], [19] has proven to give good results much faster than simulated annealing [2]. The technique of mean field approximation is derived below to get an iterative algorithm which gives the restored image as well as the edge map.

Using (7), the partition function [18], [19] is given by :

$$Z = \sum_{y,l,m} \exp\{-\beta^* E(y,l,m|x)\} \tag{8}$$

The idea is to define an effective potential E^{β}_{eff} so that the summation in (8) is only dependent on y :

$$Z = \sum_{y} \exp\{-\beta(D(y,x) + E^{\beta}_{eff}(y))\} \tag{9}$$

where

$$D(y,x) = \sum_{i,j} (y_{i,j} - x_{i,j})^2 + \lambda^2(1 - 2(\theta_x + \theta_y))y^2_{i,j}$$

and

$$\beta = \frac{\beta^*}{2\sigma^2} \propto \frac{1}{T}$$

Therefore, we get a deterministic equation for y as follows :

$$\frac{\partial(D + E^{\beta}_{eff})}{\partial y_{i,j}} = 0 \qquad \forall i,j \tag{10}$$

If β tends to infinity, we obtain y which minimizes E. Otherwise, we have a mean field approximation $\bar{y}$ of y which is sufficient because the fluctuations of the field are small.

In a similar way, the mean field value of the line processes l and m can be derived after some algebraic manipulations by noting that the line process term in the partition function (8) can be viewed as the partition function of 2 spin systems - l and m - in an external field [6], respectively $G^l_{i,j}$ and $G^m_{i,j}$, without interaction between neighboring sites :

$$\bar{l}_{i,j} = 1 - T\frac{\partial \ln Z}{\partial G^l_{i,j}} \tag{11}$$

with

$$G^l_{i,j} = \alpha - \lambda^2(u_{i,j} - u_{i-1,j})^2 \tag{12}$$

A similar equation can be derived for $\bar{m}_{i,j}$

After some calculus, we get the following set of deterministic equations :

$$\bar{l}_{i,j} = \sigma_\beta\{\theta_x(\lambda^2(\bar{y}_{i,j} - \bar{y}_{i-1,j})^2 - \alpha)\} \tag{13}$$

and

$$\bar{m}_{i,j} = \sigma_\beta\{\theta_y(\lambda^2(\bar{y}_{i,j} - \bar{y}_{i,j+1})^2 - \alpha)\} \tag{14}$$

and

$$\begin{aligned}\bar{y}_{i,j} \quad = \quad & \frac{1}{1 + \lambda^2(1 - 2(\theta_x + \theta_y))}\Big\{x_{i,j} - \lambda^2\theta_y(\bar{y}_{i,j} - \bar{y}_{i,j+1})(1 - \bar{m}_{i,j}) \\ & +\lambda^2\theta_y(\bar{y}_{i,j-1} - \bar{y}_{i,j})(1 - \bar{m}_{i,j-1}) \\ & -\lambda^2\theta_x(\bar{y}_{i,j} - \bar{y}_{i-1,j})(1 - \bar{l}_{i,j}) \\ & +\lambda^2\theta_x(\bar{y}_{i+1,j} - \bar{y}_{i,j})(1 - \bar{l}_{i+1,j})\Big\}\end{aligned} \tag{15}$$

where σ_β is the sigmoid function. Therefore, the line-processes take continuous values in $[0, 1]$. It is only when β tends to infinity that the line-processes become boolean values.

Considering these equations, a threshold h for creating discontinuities can be exhibited. This threshold defines the resolution of the system.

$$h = \sqrt{\frac{\alpha}{\lambda^2}} \tag{16}$$

As λ^2 is fixed in the CGMRF case, the only free parameter available to choose the threshold is α.

2.3 Line-process interaction

A smoothness constraint on the discontinuity field (l or m) can be introduced by substracting an ϵ term to the energy function so that the price to be paid to create a discontinuity will be decreased when a discontinuity at a neighboring site is present :

$$\begin{aligned} E &= \frac{1}{2\sigma^2}\sum_{i,j}\{(y_{i,j}-x_{i,j})^2+\lambda^2(1-2(\theta_x+\theta_y))y_{i,j}^2 \\ &+\theta_x(\lambda^2(y_{i,j}-y_{i-1,j})^2(1-l_{i,j})+\alpha l_{i,j}-\epsilon\alpha\frac{l_{i,j-1}+l_{i,j+1}}{2}) \\ &+\theta_y(\lambda^2(y_{i,j}-y_{i,j+1})^2(1-m_{i,j})+\alpha m_{i,j}-\epsilon\alpha\frac{m_{i-1,j}+m_{i+1,j}}{2})\} \end{aligned} \tag{17}$$

where ϵ allows to control the amount of propagation of the line due to the interaction between the line-processes ($\epsilon \in [0,1]$).

Using mean field approximation as described in Section 2.2, we get :

$$\bar{l}_{i,j} = \sigma_\beta\{\theta_x(\lambda^2(\bar{y}_{i,j}-\bar{y}_{i-1,j})^2-\alpha+\epsilon\alpha\frac{\bar{l}_{i,j-1}+\bar{l}_{i,j+1}}{2})\} \tag{18}$$

and

$$\bar{m}_{i,j} = \sigma_\beta\{\theta_y(\lambda^2(\bar{y}_{i,j}-\bar{y}_{i,j+1})^2-\alpha+\epsilon\alpha\frac{\bar{m}_{i-1,j}+\bar{m}_{i+1,j}}{2})\} \tag{19}$$

The results presented above are derived after a few approximations. An exact calculus could be done but would not be of any use from a practical point of view(cf [6], [7] for details about the Transfer Matrix method and the need to obtain a local solution). In order to reduce the computational complexity of this algorithm, we have chosen to get $\bar{y}$ without taking into account the term due to the line-process interaction (i.e. to use equation (15)). This choice has been done after comparing the simulation results obtained with and without this approximation.

The threshold and suprathreshold for creating a contour are $h_0 = \sqrt{\frac{\alpha}{\lambda^2}(1-\epsilon)}$ and $h_1 = \sqrt{\frac{\alpha}{\lambda^2}}$. It means that, at zero temperature, if the intensity gradient is greater than h_1 a contour will be detected, if it is lower than h_0 a smoothing will be done and in-between the creation of an edge will depend on the presence of an edge at the neighbor-sites.

2.4 Relation with some other methods

The first algorithm considered herein is the one proposed by Geiger and Girosi in [6] and [7]. Basically, their model applied to edge detection and image restoration is a special case of the above model. When the CGMRF model is isotropic (with $\theta_x = \theta_y = \frac{1}{4}$), it is easy to get the same equations as in [6] (taking $\sigma^2 = \frac{1}{2}$ and the corresponding values for ν, α and λ^2). From a theoretical point of view, the model used by Geiger and Girosi is simpler. It does not require the estimation of the CGMRF parameters but needs the estimation of λ^2 (called α in [6] and [7]). The parameter α (called γ in [6] and [7]) has to be fixed by trial and error in both cases. We have implemented both algorithms the results of which are commented in Section 3.

The second technique is the GNC introduced by Blake and Zisserman [3]. They have formulated the problem of detecting discontinuities in terms of minimization of an energy (*cost function*) under *weak continuity constraints*. The extension of the GNC to the case of a CGMRF model has been described in [21]. As the cost function obtained using this model is non-convex, the following deterministic algorithm has been proposed : the minimization starts with a convex approximation of the cost function (global minimum). Then, a sequence of cost functions depending on a parameter p varying from 1 to 0 is constructed (the convex case is obtained at p=1). The GNC optimizes the whole sequence of cost functions, one after the other, using the optimal solution of the previous optimization step as a starting point for the next one while decreasing the value of p. This method is related to mean field annealing in the sense that the decrease on p plays the same role as the annealing of the temperature T and that the construction of the different cost functions is the parallel of the evolution of the effective potential with T (see also [5], [6] and [24]).

3 SIMULATION

3.1 Implementation

Without entering into the details of the algorithm implementation, one should notice that the minimization of the energy for a given temperature has to be done with an optimal step descent method for $\bar{y}$. In their papers [6], [7] Geiger and Girosi have proposed to use the steepest descent method with a fixed step chosen by trial and error. We have implemented their method and we have observed that if the image presents low contrasts in grey level intensity, it is impossible to get a correct convergence of the energy . Of course, we have the same problem with our method. Therefore, we have tried both the steepest descent and the conjugate gradient methods using an optimal step. The first method is obviously faster but the second one is more robust to noise. We have used an extended version of the original conjugate gradient method (see [15], [22] for more details on the Polak and Ribiere extension).

3.2 Parameter estimation

For the estimation of the GMRF parameters, we have taken the results of the method developed by Simchony et al. [22]. The probabilistic distribution of the noise is assumed to be known and the original image is assumed to be available. Then, the problem is to estimate the GMRF parameters only in the homogeneous patches that do not include the edges. The solution proposed by Simchony et al. [22] is based on an Expectation Maximization (EM) algorithm. This technique ignores the intensity data in the strips centered at the edges and replaces these data by the conditional mean of the GMRF model given the model parameters and the neighboring pixel intensity data. Then, an estimation step based on a least square technique is performed on the smoothed image. Lastly, the estimated parameters are used to compute a new conditional mean for the pixels in the edge neighborhood and a new parameter estimation step is performed. The algorithm stops when the change in the parameter value is below a threshold.

As λ^2 is defined by σ^2 and ν, the only free parameter is α. In fact, we work on the gradient threshold h given in (16). The choice is done by trial and error according to the type of images we have to deal with (see hereafter).

3.3 Simulation results

Some general remarks about the simulations are given :

-**Temperature schedule** : we have tried different schedules proposed in the literature for annealing. We have found that the quality of the restored image as well as of the edge map was not good eventhough the energy seemed to have converged correctly. Now we use an ad-hoc temperature schedule which is slower and gives better results (start with $\beta = 0.0002$, then do $\beta = \beta * 4$ until $\beta > 1$).

-**Initial conditions for T fixed** : the first time (when $\beta = 0.0002$), we initialize $\bar{y}$ as the noisy data and the line processes to 0.5. Then, we take as initial conditions the values of $\bar{y}$, $\bar{l}$ and $\bar{m}$ obtained after convergence at the previous temperature.

-**Test of convergence** : the convergence of the algorithm for each temperature value is tested on ΔE, but we also imposed a maximum number of iterations per step.

-Choice of the boundaries : we work with free boundaries.

We present the results obtained on an aerial image (128,128) - see Fig. 2 - with a signal-to-noise ratio equal to 5 dB (see Fig. 3). The parameters of the CGMRF model are those obtained in [22]. We have chosen $\epsilon = 0.3$ (giving a threshold $h_0 = 4.70$ and a suprathreshold $h_1 = 5.65$) and have got the restored image (see Fig. 4) and the edge map (see Fig. 5) after 193 iterations. These results can be compared with those obtained without any line-process interaction (see Fig. 6 and 7) after 182 iterations ($h = 5.50$):

- The quality of the restored images is nearly the same (compare Fig. 6 with Fig. 4).

- The edge map is better with line-process interaction (compare Fig. 7 with Fig. 5).

It is worth pointing out that the introduction of a simple form of interaction between the line-processes gives an algorithm with a reduced sensibility to the choice of the free parameter α.

Comparing to the results obtained without a Compound GMRF model, we can say that :

- if the goal is only edge detection, then the algorithm described in [6] and [7] is sufficient (cf [25]) provided an optimal step descent method is used.

- if both the restored image and the edges are needed, it is better to use the proposed algorithm eventhough it involves more computation to estimate the CGMRF parameters. Especially, it should be possible (but not done here) to estimate the CGMRF parameters for each patch in order to have a more acute anisotropic model for restoration.

4 CONCLUSION

In this report, we have studied algorithms based on mean field approximation using a Compound GMRF model with and without line-process interaction for image restoration and edge detection. We have also shown how the algorithms proposed by Geiger and Girosi are a special case of these techniques. From an implementation point of view, we have emphasized the need , for all the methods, of an optimal step for the descent. Finally, it would be interesting to look at other kinds of line-process interactions in order to impose more constraints.

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ISSN 0249-6399