Marginal throughtput of a stack algorithm for CSMA/CD random length packet communication when the load is over the channel efficiency

Philippe Jacquet, Paul Muhlethaler

UNITÉ DE RECHERCHE
INRIA-ROCQUENCOURT

Institut National
de Recherche
en Informatique
et en Automatique

Domaine de Voluceau
Rocquencourt
B.P.105
78153 Le Chesnay Cedex
France
Tél.:(1) 39 63 55 11

Rapports de Recherche

N° 1275

Programme 3
Réseaux et Systèmes Répartis

MARGINAL THROUGHTPUT OF A STACK ALGORITHM FOR CSMA/CD RANDOM LENGTH PACKET COMMUNICATION WHEN THE LOAD IS OVER THE CHANNEL EFFICIENCY

Philippe JACQUET
Paul MUHLETHALER

Août 1990

MARGINAL THROUGHTPUT OF A STACK ALGORITHM FOR CSMA/CD RANDOM LENGTH PACKET COMMUNICATION WHEN THE LOAD IS OVER THE CHANNEL EFFICIENCY

Philippe JACQUET and Paul MÜHLETHALER
INRIA
Domaine de Voluceau
78153 Le Chesnay Cedex
France

Abstract—We give an exact evaluation of the marginal throughput of the free access stack collision resolution algorithm when the load is over the maximum channel efficiency, under the hypotheses of CSMA-CD local area network communication, where packets are of different length. This study shows that the free access tree algorithm still has a good behaviour for a load larger than the maximum throughput. Contrary to Aloha whose marginal throughput tends very quickly to zero, the marginal throughput of the free access stack collision resolution algorithm is slowly decreasing and all the more so as the packet duration is several times the duration of a blank.

DEBIT MARGINAL DU PROTOCOLE EN ARBRE POUR CSMA/CD AVEC DES TAILLES VARIABLES DE MESSAGE QUAND LA CHARGE EXCEDE LA CAPACITE MAXIMALE

Résumé–Nous donnons une évaluation exacte du débit d'un protocole en arbre à arrivée libre quand la charge est supérieure à la capacité maximale et sous l'hypothèse d'un réseau local CSMA-CD où les paquets sont de différente taille. Cette étude montre que le protocole en arbre à arrivée libre a encore un bon comportement pour une charge supérieure à la capacité maximale. Contrairement à Aloha dont le débit résiduel tend très rapidement vers zéro, le débit marginal du protocole en arbre décroit lentement et ceci est d'autant plus vrai que les paquets sont longs devant la taille des blancs.

I. Introduction

This paper analyses the performance of a protocol for managing the use of a *single-channel packet switching communication network* like the one used in the Ethernet [8], [10].

We consider the following model, commonly taken as the basis of mathematical studies of the multiple-access channel [3]. The time is slotted and stations can start transmitting only at the beginning of slots. A packet may have a length of several slots. Each transmission is within the reception range of every user. When more than one user transmit simultaneously, packets are said to *collide*, none is correctly transmitted and the colliding users abort their transmission at the end of the slot. For the next slots of a given packet, the other users are aware that a packet is in current transmission and wait for its final slot. Therefore, collision can occur only on the first slot of packets. These features are classical procedures in available standardized CSMA-CD ([10]).

Thus the status of a slot is simply a *blank* if no user transmit on it, a *success* if there is only one user transmitting and a *collision* when two or more users transmit. The collision resolution algorithm is clearly a major determinant of the behaviour of the communication process. In this paper we focus on an adaptation of Capetanakis-Tsybakov-Mikhailov protocol [5], [6], [7] with free access. This protocol enjoys nice properties such as simplicity and robustness, stability under a large population of users, and last but not least, according to our point of view, tractability to analysis.

In 1985, Fayolle, Flajolet, Hofri and Jacquet [1] published a complete analysis of this protocol with an exhaustive determination of the channel utilization and of packet delay moments. This analysis was done under the simple hypothesis that all packets be of the same length, namely one slot. In 1990 Jacquet and Merle [9] published an analysis of the free access tree algorithm with the more realistic hypothesis of variable length packets. The difficulty met in the analysis of the protocol with this hypothesis as showed by Tsybakov and Fedortsov in [4] is counterbalanced by a slight modification of the protocol.

In 1990, Jacquet and al [11] published an analysis of this protocol when the load is over the maximum throughput of the protocol (*i.e.* the maximum admissible arrival rate before the channel destabilizes) with the simple assumption that the duration of all the packets is one slot . Our aim in this article is to give the same analysis in the more realistic hypothesis of variable length packets. To have a tractable analysis, we will also use the modification indicated in [9]. In the following, we are giving a very short recall of this very protocol.

A. Specifications of the protocol

The protocol we consider is free access. Free access means that every station, with a new packet generated, immediately senses the channel for an eventual immediate transmission. Indeed, if no carrier is detected on the channel, the station transmits its packet (we include in the packet the standardized 96 bit silent overhead). Otherwise it defers its transmission to immediately after the end of the currently sensed transmission. Termed in the slotted model, packets generated during a blank or collision slot are immediately transmitted at the beginning of the next slot, packets generated during a successful packet transmission is immediately transmitted after the final slot of that packet. When transmitting its packet for the first time, say on slot number s, the station initializes a counter $C(s)$ at zero. This counter helps the station to schedule eventual retransmission, and depends, as a function of slots, s, on the succession of events detected on the channel. Below are the rules.

I1 If $C(s) = 0$, then station initiates transmission on slot s. If a collision occurs, station aborts transmission on the same slot and $C(s+1)$ assumes one of the values 0 and 1 with respective probabilities p and q (of course, $p+q=1$). Otherwise, station transmits the next slots of its packet and removes counter $C(s)$ (or becomes idle).

I2 When $C(s) > 0$, the station modifies the counter only when slot s is a collision, and therefore $C(s+1) = C(s) + 1$, or when slot s is a blank, and $C(s+1) = C(s) - 1$.

From a global point of view this protocol consists in monitoring a virtual *stack* containing the stations which are waiting for (re)transmission. The stack is a sequence of cells, numbered from 0 to ∞ and during the slot s, the elements of the cell number i ($i \geq 0$) are the stations such that $C(s) = i$. According to the statements of the protocol, the stack levels are incremented when detecting a collision and decremented when detecting a blank slot.

B. The probabilistic Model

First of all, it is assumed that the number N of stations is large, so that the assumption $N = \infty$ is valid. Moreover, the number of newly created active users in each slot is supposed to be independent of the state of the stack and its history, and to be a Poisson arrival process with a fixed rate λ. Thus, if this number is denoted by X,

$$\Pr(X = n) = \frac{\lambda^n}{n!} e^{-\lambda}.$$

At last, we assume that the length T of packets is independent of X, of the stack and its history. $\Pr(T = n)$ will be denoted by T_n, and the mean of T, by M. Thus,

$$M = \sum_{n=0}^{\infty} n T_n.$$

The following is divided into three parts. In the second one, we introduce the functional equations which are arising in our problem. The third part gives the techniques to solve them. We finally derive the marginal output stream we are looking for in the last part.

II The functional equations

A. Equations about CRI length

When $\lambda > \lambda_{\max}$, a session has a non zero probability to be infinite, but also a non zero probability to be finite. Let $\mathbf{L}_n$ be the CRI length of a session starting with a collision of multiplicity n; obviously $\mathbf{L}_0 = 1$. Let $P_n(u)$ be the characteristic function of the (finite) CRI lengths, namely $P_n(u) = \sum_{k=0}^{\infty} \Pr\{\mathbf{L}_n = k\} u^k$. We have $P_0(u) = u$ and

$$P_1(u) = \sum_{n=0}^{\infty} T_n u^n P(n\lambda).$$

Of course $P_n(1) = 1 - \Pr\{\mathbf{L}_n = \infty\}$. Let $P_n = P_n(1)$ be the probability for $\mathbf{L}_n$ be finite. When $\lambda < \lambda_{\max}$ we have $P_n(1) = 1$, otherwise, when $\lambda \geq \lambda_{\max}$, we generally have $P_n < 1$ (except $P_0 = 1$).

The classic recursion formula for $n \geq 2$, $\mathbf{L}_n = 1 + \mathbf{L}_{n_1 + A_1} + \mathbf{L}_{n_2 + A_2}$, (with A_i of Poisson λ) holds, even with infinite value. Introducing $P(z, u) = \sum_{n=0}^{\infty} P_n(u) z^n e^{-z}/n!$, we get the functional equation:

$$\frac{P(z,u)}{u} = (1 + zP_1(u))e^{-z} + P(pz+\lambda, u)P(qz+\lambda, u) - P(\lambda, u)(P(\lambda, u)(1+z) + P_z(\lambda, u)z)e^{-z}, \quad (1)$$

with P_z expressing the first derivative of $P(z, u)$ with respect to variable z and

$$P_1(u) = \sum_{n=0}^{\infty} T_n u^n P(n\lambda).$$

Expressing $P(z) = P(z,1)$ we get

$$P(z) = P(pz+\lambda)P(qz+\lambda) + (1+zP_1(1))e^{-z} - P(\lambda)(P(\lambda)(1+z) + P'(\lambda)z)e^{-z} , \qquad (2)$$

($P'(z)$ is the first derivative of $P(z)$). Note that $P(z) = 1$ (the P_ns are all equal to 1) is a trivial solution to (2); in fact this is the *real* solution when $\lambda < \lambda_{\max}$. The problem is to find the other solution which becomes the real one when $\lambda \geq \lambda_{\max}$. This will be treated in section 2.

Let us introduce $X_n = \mathrm{E}[\mathbf{L}_n$ when finite$]$, or in other words $X_n = P'_n(1)$. Using the generating function $X(z) = \sum_{n=0}^{\infty} X_n z^n e^{-z}/n! = P_u(z,1)$, we get the functional equation

$$\begin{aligned} X(z) =& P(z) + P(qz+\lambda)X(pz+\lambda) + P(pz+\lambda)X(qz+\lambda) - \\ & - X(\lambda)(2P(\lambda)(1+z) + P'(\lambda)z)e^{-z} - X'(\lambda)P(\lambda)ze^{-z} + ze^{-z}(P_{1,u}(1) - P_1(1)) . \end{aligned} \qquad (3)$$

The knowledge of $P(z,u)$ or, in particular of P_n and X_n, allows us to determine the temporary behaviour of the protocol before meeting an infinite CRI. But, of course some works remain to determine the behaviour of the algorithm during an infinite CRI, namely its marginal output. This is the object of the following section.

B. Equations about the marginal output stream

Let us introduce $S_n(u)$ as the mean *output polynomial* of a session starting with a collision of multiplicity n, quantity u being a complex number of modulus strictly less than 1 (to ensure $S_n(u)$ to be finite). To be precise, $S_n(u) = \sum_{k=1}^{\infty} S_n^k u^k$, with $S_n^k = \Pr\{\mathbf{L}_n \geq k$ and there is a success at slot $k\}$, we call a success only the slot at the beginning of a packet. Of course $S_0(u) = 0$ and $S_1(u) = u + \sum_{k=1}^{\infty} u^k T_k S(k\lambda, u)$. Quantity $S_n(u)$ is build in order to capture the behaviour of the protocol when $\mathbf{L}_n = \infty$. For example, by Cesaro, $\lim_{u\to 1}(1-u)S_n(u) = M\lambda_o(1-P_n)$ where λ_o is the marginal output stream allowed by the protocol when $\lambda \geq \lambda_{\max}$. Thus determining $S_n(u)$ determines λ_o. When $n \geq 2$, an easy recursion leads to

$$S_n(u) = u \sum_{\substack{n_1+n_2=n \\ A_1,A_2}} S_{n_1+A_1}(u) + P_{n_1+A_1}(u)S_{n_2+A_2}(u) . \qquad (4)$$

Introducing $S(z,u) = \sum_{n=0}^{\infty} S_n(u)z^n e^{-z}/n!$ we get $\lim_{u\to 1}(1-u)S(z,u) = \lambda_o(1-P(z))$ and the following functionnal equation

$$\begin{aligned} \frac{S(z,u)}{u} - ze^{-z}S_1(u) = & S(pz+\lambda,u) + P(pz+\lambda,u)S(qz+\lambda,u) - \\ & - S(\lambda,u)((1+P(\lambda,u))(1+z) + P_z(\lambda,u)pz)e^{-z} - \\ & - S_z(\lambda,u)z(p+qP(\lambda,u))e^{-z} . \end{aligned} \qquad (5)$$

with :

$$S_1(u) = u + \sum_{k=1}^{\infty} u^k T_k S(k\lambda, u).$$

III RESOLUTION OF THE FUNCTIONAL EQUATIONS

A. Resolution for $P(z)$

Let us call T the non-linear operator defined on analytical functions $f(z)$ by $Tf(z) = f(pz+\lambda)f(qz+\lambda) - f(\lambda)(f(\lambda)(1+zP_1(1)) + f'(\lambda)z)e^{-z} + (1+z)e^{-z}$. According to (2), the function $P(z)$

is a fixed point of the operator T. The function 1 is also a fixed point of T. We use *brute force* to obtain the result: we start with $(1+z)e^{-z}$, which has all its coefficients less or equal to those of $P(z)$ (the formal identity $P(z) = e^{-z}$ should lead to all P_n's equal to 0 when $n \geq 2$), and then to iterate T. We get

$$\lim_{n\to\infty} T^n(1+z)e^{-z} = P(z) . \tag{6}$$

As a numerical illustration, it is interesting to notice that when $\lambda < \lambda_{\max}$ we get $\lim_{n\to\infty} T^n(1+z)e^{-z} = 1$ as expected. When $\lambda > \lambda_{\max}$ the iterations converge to something else which is suspected to be $P(z)$. Note that the iterations converge with difficulty when λ is close to $\lambda_{\max}$, since it is easy to check that the gradient of T at $f = 1$ when $\lambda = \lambda_{\max}$ is exactly the identity: $T(1+h) = 1 + h + O(h^2)$ when function $h \to 0$.

B. Resolution for $X(z)$

Now we have function $P(z)$. Equation (3) is simpler to deal with since it is linear with a more familiar form. Let us note σ_1 and σ_2 the linear operators defined by $\sigma_1 f(z) = f(pz+\lambda)$ and $\sigma_2 f(z) = f(qz+\lambda)$. Let us introduce the linear operator Π_2 defined on analytical functions $f(z)$ by $\Pi_2 f(z) = f(z) - f(0) - zf'(0)$. We define the operator R by $Rf(z) = \sigma_2 P(z)\Pi_2\sigma_1 f(z) + \sigma_1 P(z)\Pi_2\sigma_2 f(z)$ [2]. For instance $\Pi_2\sigma_1 f(z) = f(\sigma_1(z)) - f(\lambda) - f'(\lambda)pz$. The interesting fact about the operator R is the fact that $\sum_{n=0}^{\infty} R^n$ converges like a geometric sequence of rate $p^2+q^2 < 1$ (basically the square comes from the application of operator Π_2) whatever be λ. Therefore equation $f - Rf = g$ of unknown f and parameter g has the unique solution $f = \sum_{n=0}^{\infty} R^n g = (1-R)^{-1}g$.

The equation (3) can be tuned to the form

$$X(z) - RX(z) = g_0(z) + X(\lambda)g_1(z) + X'(\lambda)g_2(z) + Y(\lambda)g_3(z) .$$

For instance

$$\begin{cases} g_0(z) = P(z) + \displaystyle\sum_{n=2}^{\infty} T_n(n-1)P(\lambda n)ze^{-z} \\ g_1(z) = P(pz+\lambda) + P(qz+\lambda) - (2P(\lambda)(1+z) + P'(\lambda)z)e^{-z} \\ g_2(z) = z(qP(pz+\lambda) + pP(qz+\lambda) - P(\lambda)e^{-z}) \\ g_3(z) = ze^{-z} . \end{cases}$$

In this perspective we can use the operator $(1-R)^{-1}$, and obtain $X(z) = (1-R)^{-1}g_0(z) + X(\lambda)(1-R)^{-1}g_1(z) + X'(\lambda)(1-R)^{-1}g_2(z) + Y(\lambda)(1-R)^{-1}g_3(z)$.

By using elementary identification at $z = \lambda$ we get the following linear system in $X(\lambda)$ and $X'(\lambda)$:

$$\begin{cases} a_{11}X(\lambda) + a_{12}X'(\lambda) + a_{13}Y(\lambda) = x_1 \\ a_{21}X(\lambda) + a_{22}X'(\lambda) + a_{23}Y(\lambda) = x_2 \\ a_{31}X(\lambda) + a_{32}X'(\lambda) + a_{33}Y(\lambda) = x_3 \end{cases}$$

with

$$\begin{array}{lll} a_{11} = 1 - (1-R)^{-1}g_1(\lambda) & a_{12} = -(1-R)^{-1}g_2(\lambda) & a_{13} = -(1-R)^{-1}g_3 \\ a_{21} = -((1-R)^{-1}g_1)'(\lambda) & a_{22} = 1 - ((1-R)^{-1}g_2)'(\lambda) & a_{23} = -((1-R)^{-1}g_3)'(\lambda) , \end{array}$$

and

$$a_{31} = -\sum_{n=0}^{\infty} T_n(1-R)^{-1} g_1(\lambda n)$$

$$a_{32} = -\sum_{n=0}^{\infty} T_n(1-R)^{-1} g_2(\lambda n)$$

$$a_{33} = 1 - \sum_{n=0}^{\infty} T_n(1-R)^{-1} g_3(\lambda n) ,$$

and $x_1 = (1-R)^{-1} g_0(\lambda)$, $x_2 = ((1-R)^{-1} g_0)'(\lambda)$ and $x_3 = \sum_{n=0}^{\infty} T_n((1-R)^{-1} g_0)'(\lambda n)$.

Therefore :

$$X(\lambda) = \frac{\det \begin{vmatrix} x_1 & a_{12} & a_{13} \\ x_2 & a_{22} & a_{23} \\ x_3 & a_{32} & a_{33} \end{vmatrix}}{\det \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} .$$

$$X'(\lambda) = \frac{\det \begin{vmatrix} a_{11} & x_1 & a_{13} \\ a_{21} & x_2 & a_{23} \\ a_{31} & x_3 & a_{33} \end{vmatrix}}{\det \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} .$$

$$Y(\lambda) = \frac{\det \begin{vmatrix} a_{11} & a_{12} & x_1 \\ a_{21} & a_{22} & x_2 \\ a_{31} & a_{32} & x_3 \end{vmatrix}}{\det \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} .$$

C. Resolution for $S(z,u)$

Let us call $H(u)$ the linear operator defined by $H(u)f(z) = u(\Pi_2\sigma_1 f(z) + P(pz+\lambda,u)\Pi_2\sigma_2 f(z))$. This operator has two interesting properties. First, the serie $\sum_{n=0}^{\infty} H^n(u)$ converges like a geometric serie of rate $u(p^2+q^2)$ thus is absolutely convergent when $|u| < (p^2+q^2)^{-1}$. Second, equation (5) translates into $S(z,u) - H(u)S(z,u) = h_0(z,u)S_1(\lambda,u) + S(\lambda,u)h_1(z,u) + S_z(\lambda,u)h_2(z,u)$ with

$$\begin{cases} h_0(z,u) = uze^{-z} \\ h_1(z,u) = u(1 + P(pz+\lambda,u) - ((1+P(\lambda,u))(1+z) + P_z(\lambda,u)pz)e^{-z}) \\ h_2(z,u) = uz(p + qP(pz+\lambda,u) - (p+qP(\lambda,u))e^{-z}) . \end{cases}$$

We use the operator $(1-H(u))^{-1} = \sum_{n=0}^{\infty} H^n(u)$, and obtain $S(z,u) = S_1(\lambda,u)(1-H(u))^{-1}h_0(z,u) + S(\lambda,u)(1-H(u))^{-1}h_1(z,u) + S_z(\lambda,u)(1-H(u))^{-1}h_2(z,u)$.

By using elementary identification at $z=\lambda$ and the definition of $S_1(\lambda,u)$ we get the following linear system in $S_1(\lambda,u)$, $S(\lambda,u)$ and $S_z(\lambda,u)$:

$$\begin{cases} b_{11}(u)S_1(\lambda,u) + b_{12}(u)S(\lambda,u) + b_{13}(u)S_z(\lambda,u) = 0 \\ b_{21}(u)S_1(\lambda,u) + b_{22}(u)S(\lambda,u) + b_{23}(u)S_z(\lambda,u) = 0 \\ b_{31}(u)S_1(\lambda,u) + b_{32}(u)S(\lambda,u) + b_{33}(u)S_z(\lambda,u) = u \end{cases}$$

with

$$\begin{array}{ll} b_{11}(u) = 1-(1-H(u))^{-1}h_0(\lambda,u) & b_{21}(u) = -((1-H(u))^{-1}h_0)_z(\lambda,u) \\ b_{12}(u) = -(1-H(u))^{-1}h_1(\lambda,u) & b_{22}(u) = 1-((1-H(u))^{-1}h_1)_z(\lambda,u) \\ b_{12}(u) = -(1-H(u))^{-1}h_3(\lambda,u) & b_{23}(u) = -((1-H(u))^{-1}h_1)_z(\lambda,u)\ , \end{array}$$

and

$$b_{31}(u) = 1-\sum_{n=0}^{\infty} T_n u^n (1-H(u))^{-1}h_0(n\lambda,u)$$

$$b_{32}(u) = -\sum_{n=0}^{\infty} T_n u^n ((1-H(u))^{-1}h_1)_z(n\lambda,u)$$

$$b_{33}(u) = -\sum_{n=0}^{\infty} T_n u^n ((1-H(u))^{-1}h_2)_z(n\lambda,u)\ ,$$

where $(f)_z(z,u) = f_z(z,u)$, the derivative with respect to variable z. Therefore $S_1(\lambda,u)$, $S(\lambda,u)$ and $S_z(\lambda,u)$ can be found as the result of a linear system of equations.

IV Analysis of the marginal output stream

We have $\lambda_o(1-P(z)) = \lim_{u\to1}(1-u)S(z,u)$, therefore, by identification $z=\lambda$ and using the results of the last section we obtain:

$$\lambda_o = \frac{1}{1-P(\lambda)} \lim_{u\to1} \frac{u(b_{11}(u)b_{13}(u) - b_{21}(u)b_{13}(u))}{\det\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}(u)},$$

where $\det(u)$ is the determinant of the linear system in $S_1(\lambda,u)$,$S(\lambda,u)$ and $S_z(\lambda,u)$. Since the functions, $b_{11}(u)$, $b_{12}(u)$, *etc*, are continuous for $|u| < (p^2+q^2)^{-1}$, and $\det(1) = 0$, we get, by Liouville,

$$\lambda_o = -\frac{(b_{11}(1)b_{13}(1) - b_{21}(1)b_{13}(1))}{(1-P(\lambda))\det'\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}(1)},$$

where $\det'(u)$ is the first derivative of function $\det(u)$ with respect to the variable u. Now we have an exact expression for λ_o, it remains to compute the result.

The expressions of $b_{11}(1)$, $b_{11}(1)$, *etc*, include application of operator $H(1)$, which is well defined with function $P(z)$, iterated on functions $h_0(z,1)$, $h_1(z,1)$ and $h_2(z,1)$:

$$\begin{cases} h_0(z,1) = ze^{-z} \\ h_1(z,1) = 1+P(pz+\lambda) - ((1+P(\lambda))(1+z) + P'(\lambda)pz)e^{-z} \\ h_2(z,1) = z(p+qP(pz+\lambda) - (p+qP(\lambda))e^{-z})\ . \end{cases}$$

The derivative of $\det(u)$ includes the first derivatives of functions $a(u)$, $b(u)$, *etc.* The derivative of $(1-H(u))^{-1}f(z,u)$ with respect to u is exactly $(1-H(u))^{-1}H'(u)(1-H(u))^{-1}f(z,u) + (1-H(u))^{-1}f_u(z,u)$, where $H'(u)$ is the derivative of $H(u)$ with respect to u. We have $H'(1)f(z) = H(1)f(z) + X(pz+\lambda)\Pi_2\sigma_2 f(z)$ and

$$\begin{cases} \dfrac{\partial}{\partial u}h_0(z,1) = h_0(z,1) \\ \dfrac{\partial}{\partial u}h_1(z,1) = h_1(z,1) + X(pz+\lambda) - (X(\lambda)(1+z) + X'(\lambda)pz)e^{-z} \\ \dfrac{\partial}{\partial u}h_2(z,1) = h_2(z,1) + qz(X(pz+\lambda) - X(\lambda)e^{-z})\ . \end{cases}$$

This allows us to compute the numerical results which are illustrated by the following figures (1 to 9). We have tried different values for the length of the packets. The first computations have been done with packets twice as long as the duration of a blank. Then we have taken packets five times longer than a blank. The last results concern the case where the packets are ten times longer than a blank. As expected the computations show that the marginal throughput of the protocol is decreasing more slowly when the mean packet length is large.

V Conclusion

We have studied the marginal output stream of a free access algorithm. This paper shows that with packets whose length are several times larger than the duration of a slot the marginal output stream is slowly decreasing when the input is over the maximum channel capacity.

References

[1] G. Fayolle, P. Flajolet, M. Hofri, P. Jacquet [1985]. "Analysis of a Stack Algorithm for Random Multiple Access Communication", in Special Issue on Random-Access Communication, *IEEE Transactions on Information Theory* **IT-31**, 1985, 244-254.

[2] G. Fayolle, P. Flajolet, M. Hofri [1986]. "On a Functional Equation Arising in the Analysis of a Protocol for a Multi-Access Broadcast Channel", *Advances in Applied Probability* **18**, 1986, 441-472.

[3] J. L. Massey [1981]. "Collision Resolution Algorithms and Random-Access Communications", in *Multi-User Communication Systems*, G. Longo Editor, *CISM Courses and Lectures no.* **255**, Springer Verlag, Wien-New York, 1981, 73-137.

[4] B. S. Tsybakov, S. P. Fedortsov [1986]. "Local-area network with random-multiple-access stack algorithm", *Problems of Information Transmission* Translated from *Problemy Peredachi Informatsii* **22**, 1986, 49-58.

[5] J. I. Capetanakis [1979]. "Tree Algorithms for Packet Broadcast Channels", *IEEE Transactions on Information Theory* **IT-25**, 1979, 505-515.

[6] B. S. Tsybakov, V. A. Mikhailov [1979]. "Free Synchronous Packet Access in a Broadcast Channel with Feedback", *Probl. Inform. Transmission* **14**, 1979, 259-280.

[7] B. S Tsybakov, N. Vvedenskaya [1980] "Random multiple access stack algorithms", *Probl. Inform. Transmission* **16**, 1980, 230-243.

[8] R. M. Metcalfe, D. Boggs [1976]. "Ethernet: Distributed Packet Switching for Local Computer Networks", *Comm. ACM* **19**, 1976, 395-404.

[9] P. Jacquet, E. Merle [1990]. "Analysis of a stack algorithm for for CSMA/CD random length packet communication," *IEEE Transactions on Information Theory* , March 1990.

[10] "csma/cd Access method and physical layer specifications," *IEEE 802.3 or ISO 8802/3*, October 1984.

[11] P. Jacquet and al [1990]. "The Free Access Tree Algorithm Beyond Infinity " INRIA research report to be published.

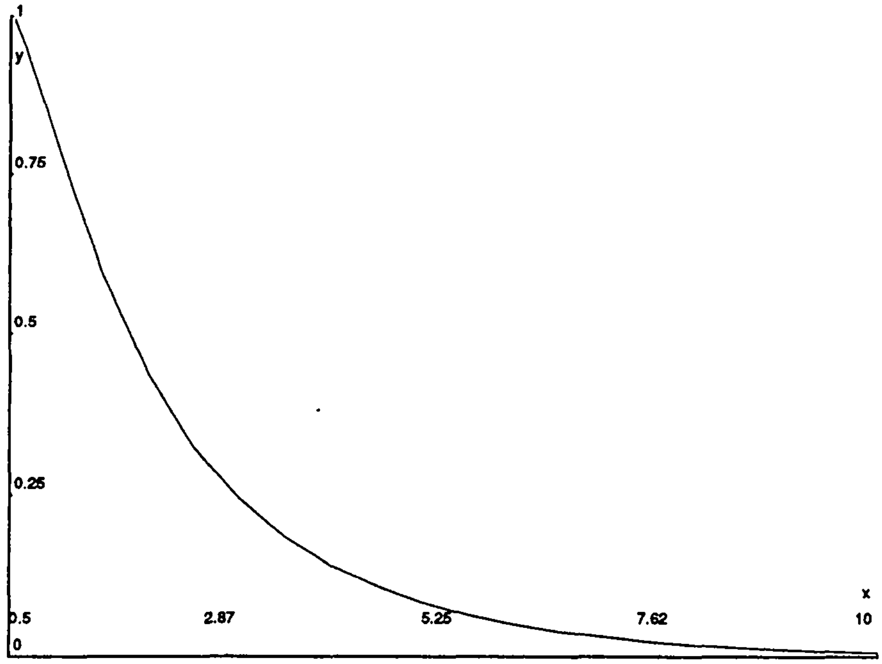

Figure 1: unconditional probability for a CRI to be finite ($P(\lambda)$) versus $M\lambda$, between 0.55 and 10, $M = 2$.

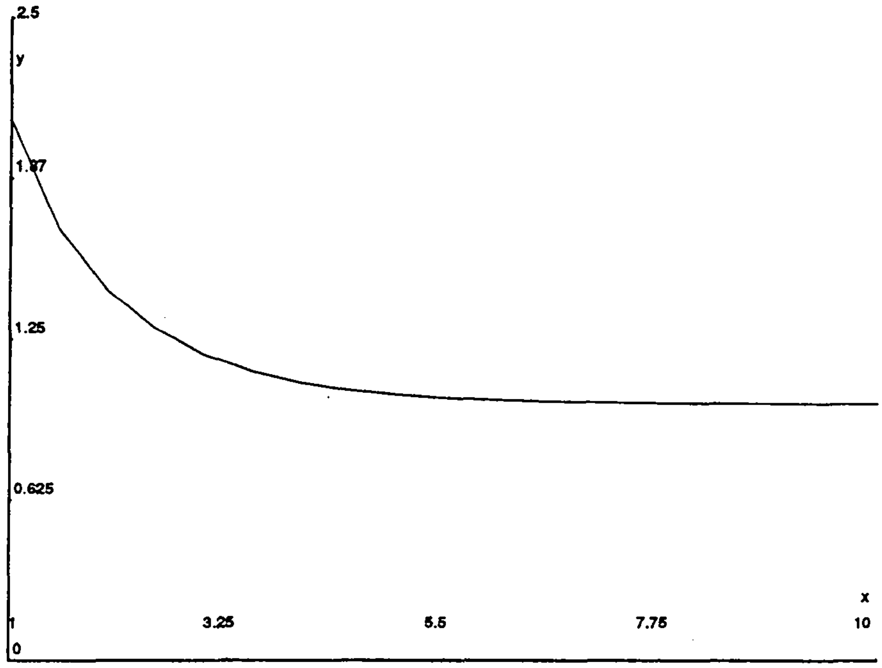

Figure 2: mean CRI length conditioned by the fact it is finite ($X(\lambda)/P(\lambda)$), versus $M\lambda$, between 1 and 10, $M = 2$.

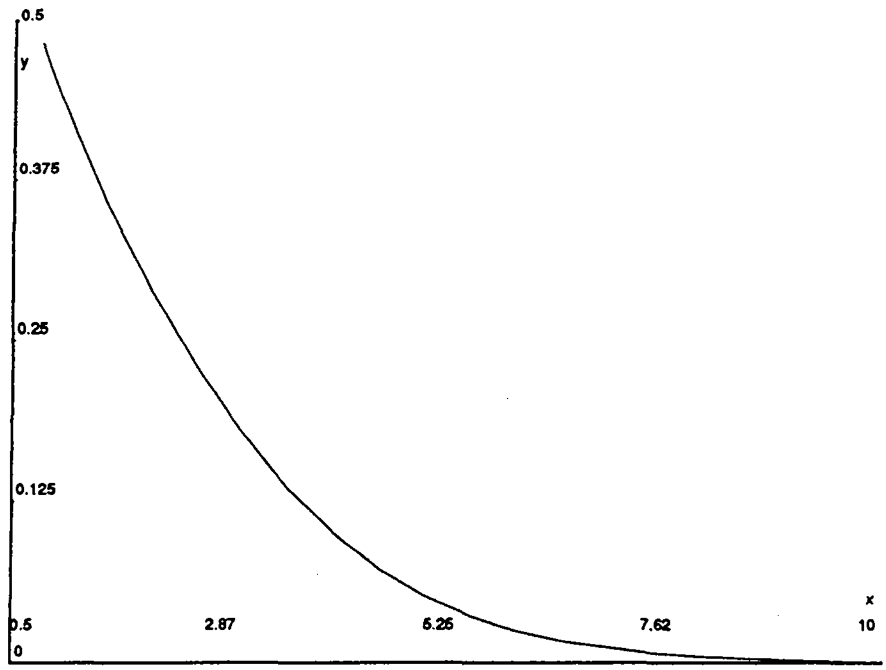

Figure 3: marginal output stream ($M\lambda_o$), versus $M\lambda$, between 0.55 and 10 , $M = 2$.

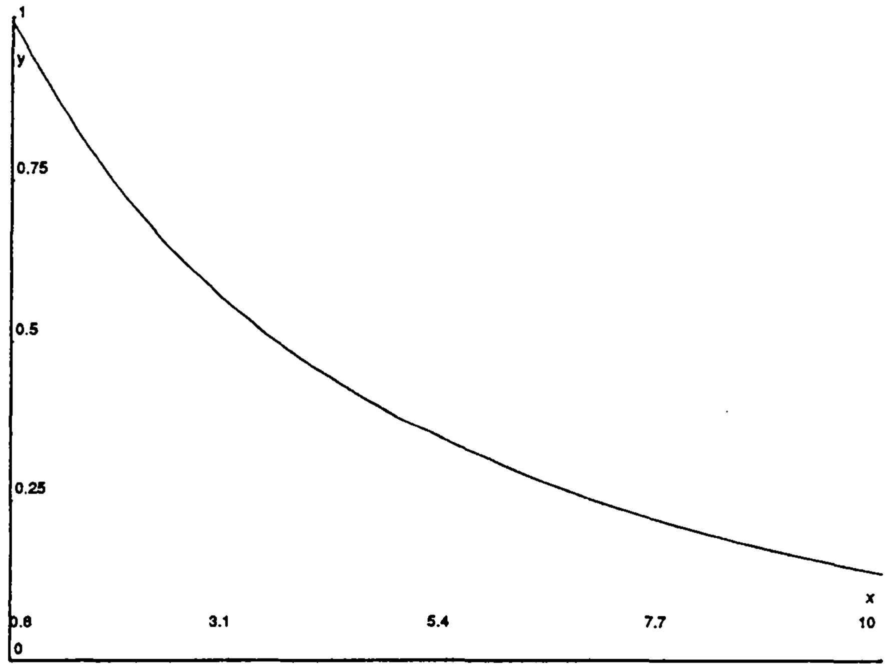

Figure 4: unconditional probability for a CRI to be finite ($P(\lambda)$) versus $M\lambda$, between 0.8 and 10, $M = 5$.

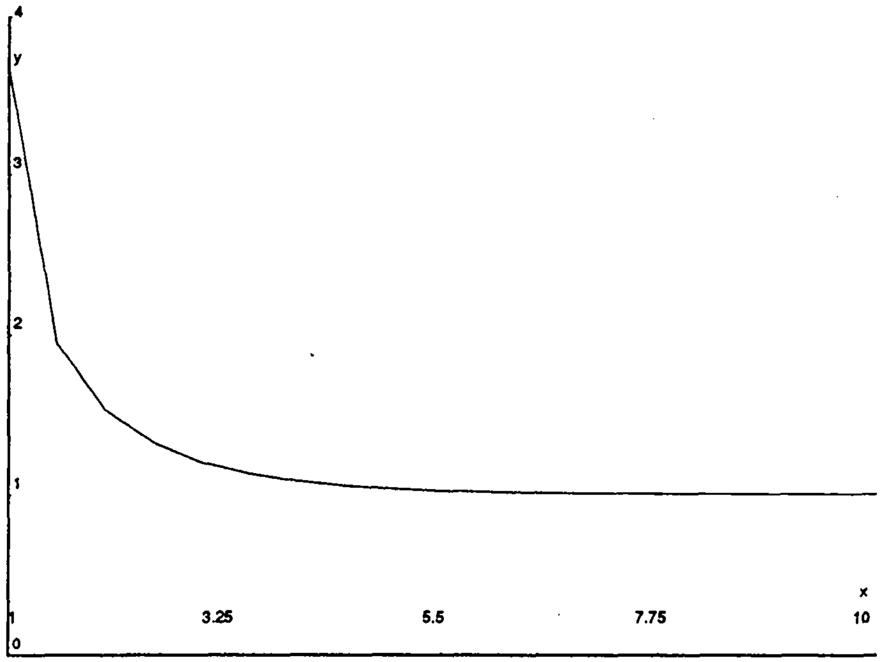

Figure 5: mean CRI length conditioned by the fact it is finite $(X(\lambda)/P(\lambda))$, versus $M\lambda$, between 1 and 10, $M = 5$.

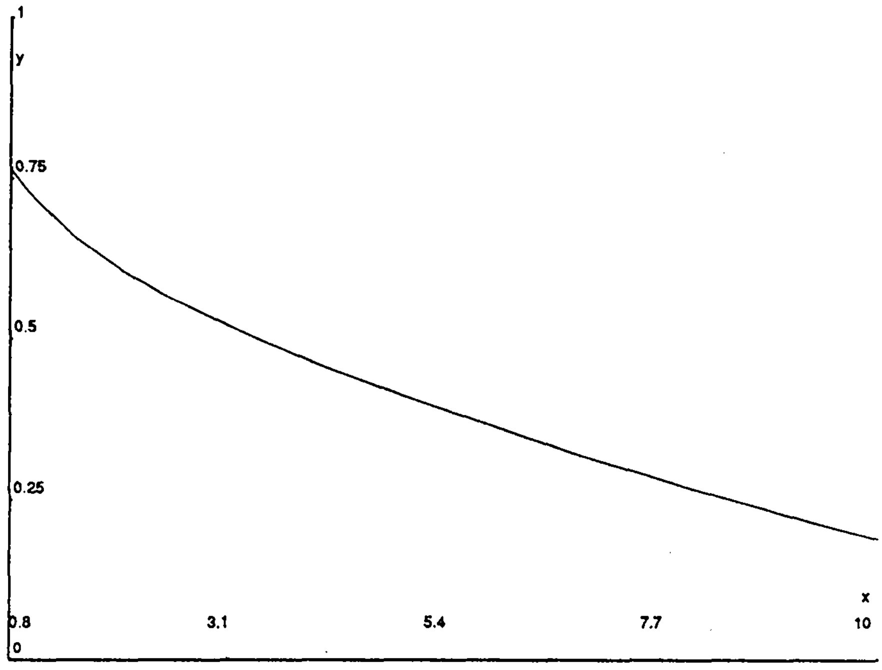

Figure 6: marginal output stream $(M\lambda_o)$, versus $M\lambda$, between 0.8 and 10 , $M = 5$.

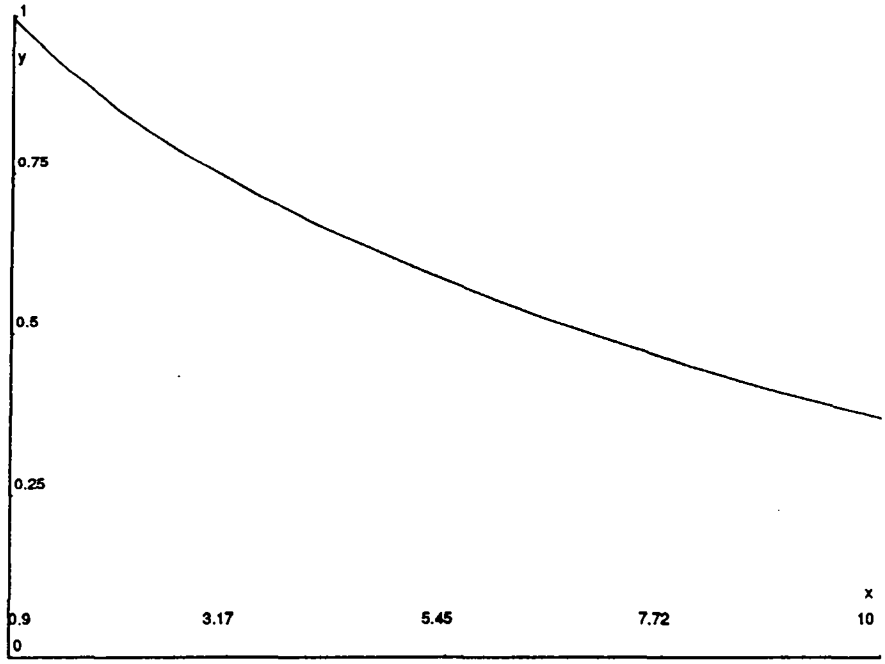

Figure 7: unconditional probability for a CRI to be finite ($P(\lambda)$) versus $M\lambda$, between 0.9 and 10, $M = 10$.

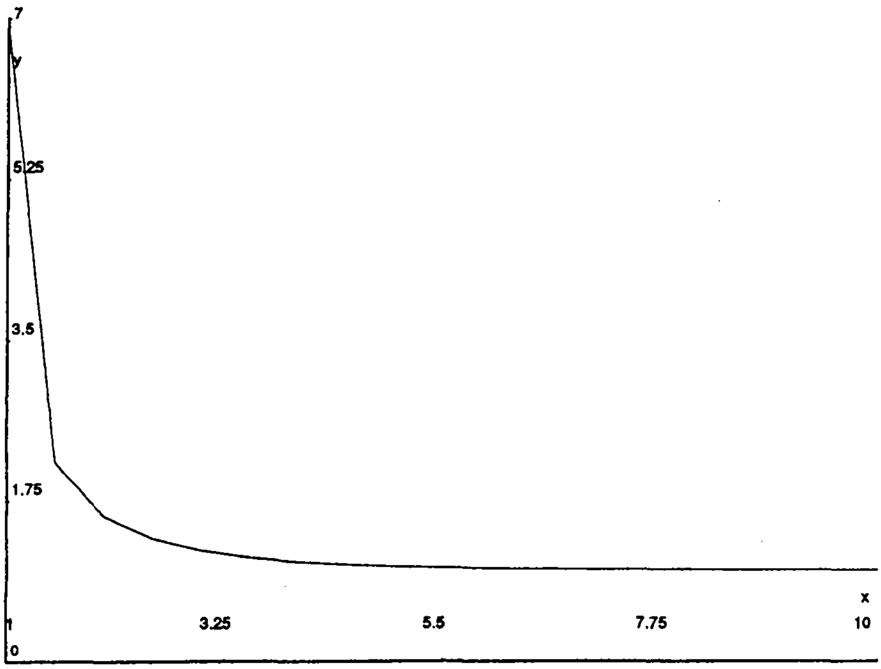

Figure 8: mean CRI length conditioned by the fact it is finite ($X(\lambda)/P(\lambda)$), versus $M\lambda$, between 1 and 10, $M = 10$.

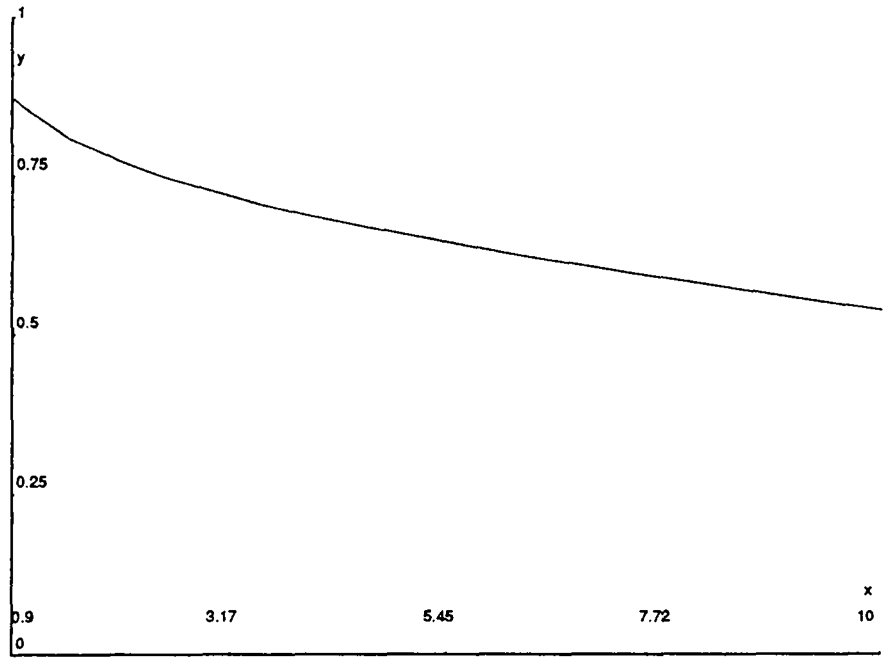

Figure 9: marginal output stream $(M\lambda_o)$, versus $M\lambda$, between 0.9 and 10 , $M = 10$.

Imprimé en France
par
l'Institut National de Recherche en Informatique et en Automatique

ISSN 0249-6399