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AN ANALYTICAL MODEL FOR THE HIGH SPEED PROTOCOL QPSX

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Abstract. *We give an analytical evaluation of the high speed protocol, QPSX, under a simple model. We suppose that the number of connected nodes is (virtually) infinite, providing a global load $\lambda < 1$; the distances between the nodes are large and randomly distributed. An analogy is developed with queueing systems with two files in tandem, which lead to analytical evaluation via closed formulas.*

UN MODELE ANALYTIQUE POUR LE PROTOCOLE A HAUT DEBIT QPSX

Résumé. *Nous donnons une évaluation analytique du protocole à haut débit, QPSX, sous une modèlisation simple. Nous supposons que le nombre de stations connectées est (virtuellement) infini, sous réserve d'une charge finie, $\lambda < 1$; les distances séparant les stations sont importantes et suivent une distribution aléatoire. On développe une analogie avec des systèmes avec deux files d'attente en tandem, qui se prêtent à une évaluation quantitative par des formules closes.*

PAPIER RECUPERE ET RECYCLE

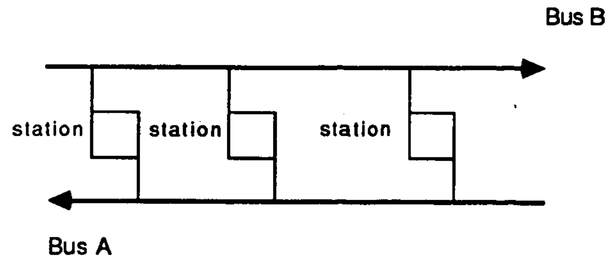

Figure 1: QPSX, two parallel busses in stream opposition.

1 The protocol QPSX

High speed communications call for a new generation of communication protocols. QPSX is based on two parallel busses. These busses are unidirectionnal (fiber optics) and set in opposition (see figure 1). Each node is connected on both busses and can read or write at each tap. Periodic signals are propagating from upstream to down stream on each busses, slicing the time in *slot*, all identical in length. Each slot is divided into two fields, the first one is the field of control bits and the second one is the data field. The packet sizes are supposed to fit the data fields of slots. The control field may be basically reduced to a single bit, namely the *request bit.*

The communication process relies on the two busses together; let us call them respectively, bus A and bus B. For convenience of exposition, let us suppose that packets are only transmitted on bus A. Each station, say station i, deals with a local integer counter, F_i. This counter evolves as following.

1 If the request bit of a slot on bus B is read equal to 1 by station i therefore 1 is added to F_i.

2 If $F_i > 0$ and station i lets pass on bus A a slot with its the data field empty, therefore 1 is substracted from F_i.

Note that we always have $F_i \geq 0$. It is not necessary that beginnings of slots on bus A coincide with beginnings of slots on bus B.

When station i wants to transmit a packet it initializes a temporary counter C_i with the current value of F_i and follows, in parallel, the two following additional rules.

3 Station i lets pass, on bus A, C_i slots with their data field read as empty, and then transmits its packet on the next slot coming on bus A with an empty data field.

4 Station i waits for the next available slot, on bus B, coming with a request bit equal to 0 (or empty), and sets it to 1.

Note that if the propagation delays between nodes are all negligible, these rules are equivalent to a distributed management of a $M/D/1$ queue. In our model we will suppose that the propagation delays between nodes are not negligible and follow some random distribution.

2 The model

We suppose that the traffic is Poisson on every station. Let us consider station i, let x such that λx is the global load of all the downstream stations (according to bus A) and let $\mathrm{d}x$ be such that $\lambda \mathrm{d}x$ is the specific load on station i. Therefore $(1 - x - \mathrm{d}x)\lambda$ is the global load of all the upstream stations (always according to bus A). In other words for station i, the rate of requests read on bus B is exactly λx, and the rate of data free slots read on bus A is exactly $1-(1-x-\mathrm{d}x)\lambda$.

The counter F_i is nothing else than the height of a single FIFO local queueing. The inputs of the file are the requests read by station i on bus B, and as output process we may consider a cyclic server which visits the file each time a data free slot is detected on bus A. Therefore each time that station i has a packet to transmit the application of rule **3** is equivalent to virtually disposing the packet in the file (at height F_i) and waiting for its service. Therefore, computing the delay of a packet is equivalent to computing its delay in such a local queue. See figure 2 for an illustration of the local queue.

The stream of requests detected on bus B does not describe a Poisson input process, since two consecutive requests are necessarily separated by a slot duration. In fact this process is precisely equivalent to the output process of $M/D/1$ queue with Poisson input rate, $x\lambda$, and deterministic service time, 1 slot. The proof of this assertion is easy. The propagating slots on bus B lead to a virtual synchronization of nodes (We consider that every slot on bus B is sensed at the same "corrected" time by all nodes). Therefore, requests issued from downstream stations and which are waiting for transmission (applying rule 4) at a same given "corrected" time are served one by one, according to their rank on bus B, at every slot. The fact that the process which generates requests to be transmitted matches the process of Poisson generation of packets in stations, ends the proof of this assertion.

Thus we have our local model of two queues in *tandem*, and it remains to describe the service of the lower one. Unfortunately the stream of data free slots detected on bus A does not describe a tractable process. At this point we must refine our model. Our model relies on the two following points.

(i) We suppose that the propagation delay between nodes are random and large according to the renewing periods in the local queueings.

(ii) We suppose that the population of nodes is infinite and the traffic is uniform in the sense that $\mathrm{d}x \to 0$, for all x, with the global load, λ.

Note that point (i) implies that the cycle of the server at local queue is independent of the input process, which leads to more conventional queueing analysis. But this does not give a precise description of the stochastic behaviour of the cycles, in fact it is not readily that the cycles are at least independent from each other. Therefore we have to introduce a third point as the following approximation.

(iii) We suppose that the delays between consecutive data free slots detected by station i are i.i.d. and correspond to Bernoulli trials of rate $1-(1-x)\lambda$.

It is clear that (i) does not imply (iii) and the latter is certainly false in general. In fact (iii) becomes *asymptotically* true when $\lambda \to 1.$, with (i) and (ii). We defer the proof of this assertion in appendix.

3 The results

By *access* delay we mean the delay between the generation of the packet and its successful transmission on the bus by its station. For simplicity we suppose that packets are generated just at the beginning of slots on bus A.

Theorem 1

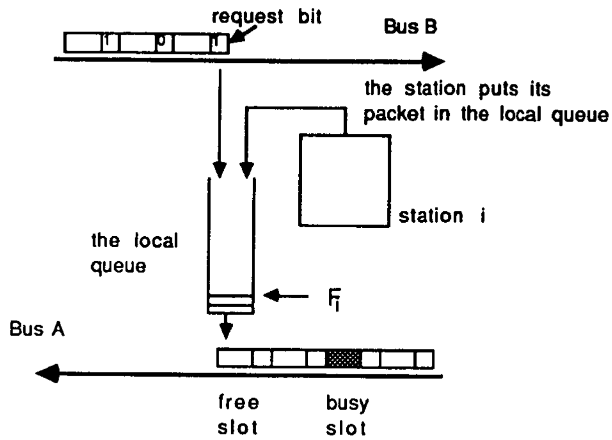

Figure 2: The local queue in station number i is filled with the requests read on bus B and served by the free slots on bus A.

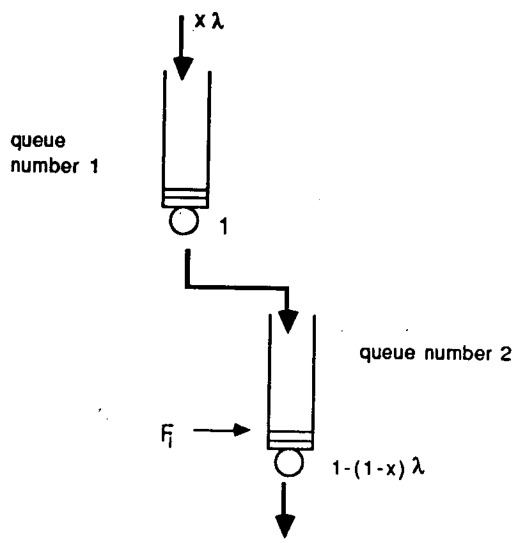

Figure 3: The model for station x, two queues in tandem.

According to our model, the mean access delay, $W(x,\lambda)$, of a packet generated on station x is

$$W(x,\lambda) = 1/2\frac{\lambda(\lambda^2x^3 - \lambda^2x^2 + 2\lambda^2x - 2\lambda x^2 - 2\lambda + 2)}{(1-(1-x)\lambda)(1-\lambda)(1-x\lambda)} .$$

The unconditional mean access delay is therefore

$$W(\lambda) = \int_0^1 W(x,\lambda)\mathrm{d}x ,$$

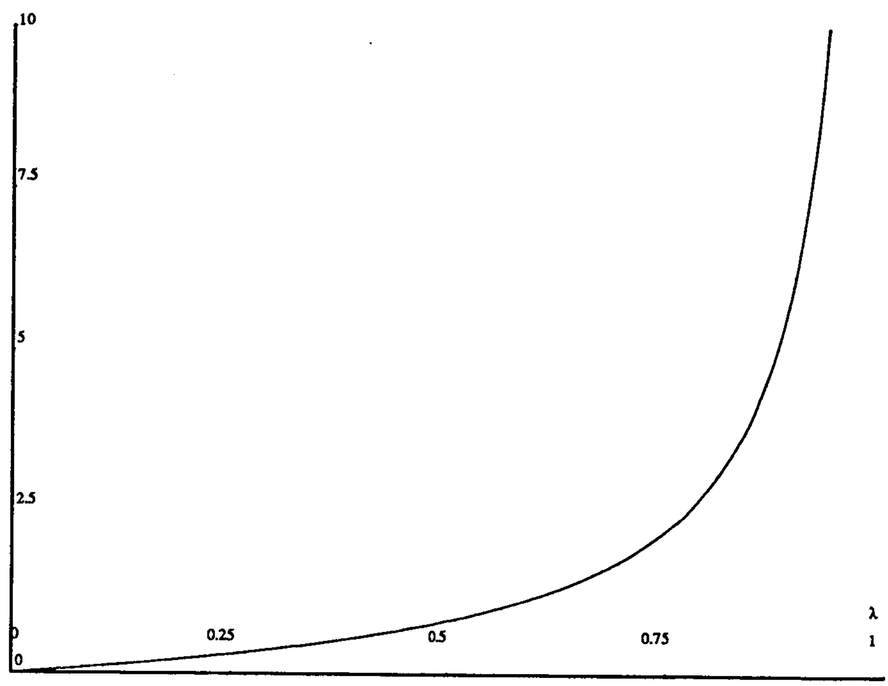

Figure 4: The mean access delay $W(\lambda)$ as a function of λ.

and

$$W(\lambda) = 1/4\frac{4-\lambda}{1-\lambda} + 1/2\frac{(4-5\lambda+2\lambda^2)\log(1-\lambda)}{\lambda(2-\lambda)} \ .$$

The variance and the other moments are obtained via similar close formulas.

These results lead to several data about the behaviour of the protocol. Figure 4 gives $W(\lambda)$ as a function of λ. Figure 5 gives the ratio $W(\lambda)$ on $\lambda/2(1-\lambda)$ (the last expression is the mean delay of a $M/D/1$ queue: *the perfect scheduler*) as a function of λ. Figure 6 gives the functions $W(x,\lambda)/W(\lambda)$ for some values of x (illustration of the "skewing"). Note that $W(0,\lambda) = \lambda/(1-\lambda)$ and $W(1,\lambda) = \lambda$.

4 The analysis

Our purpose is to prove theorem 1. First we characterize the input process of the requests in our local queueing. We use the following lemma.

LEMMA 2.
Let us consider a $M/D/1$ queue with input load ν and service time 1. Let $A(z)$ be the probability generating function of the busy period. We have the equation $A(z) = \exp(\nu(z\,A(z)-1))$.
Proof. Let $B_n(z)$ be the p.g.f. of the busy period starting with n customers in the queue. We know that $B_n(z) = B^n(z)$, with $B(z) = B_1(z)$. Since

$$A(z) = \sum_{n=0}^{\infty} B_n(z)\frac{\nu^n}{n!}e^{-\nu} \ ,$$

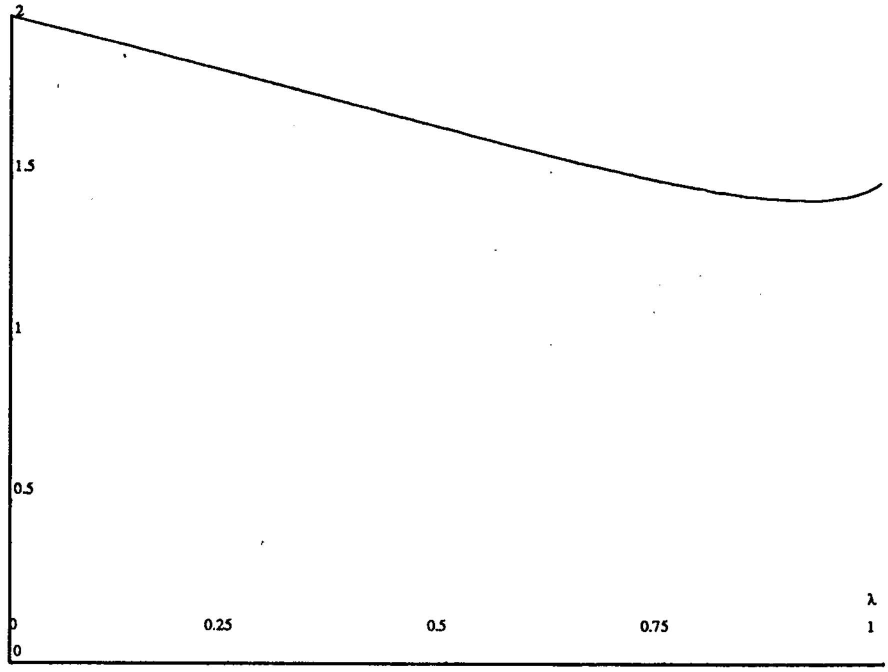

Figure 5: The ratio of the mean access delay $W(\lambda)$ with QPSX with the mean access delay, $\lambda/(2(1-\lambda))$, with the perfect scheduler, as a function of λ.

and

$$B_{n+1}(z) = z \sum_{m=0}^{\infty} B_{n+m}(z) \frac{\nu^m}{m!} e^{-\nu} ,$$

the proof is terminated. ∎

As obvious corollary, the mean busy period is $\nu/(1-\nu)$ and the variance $\nu/(1-\nu)^3$.

We know that our local queueing is equivalent to two queues in tandem (see figure 3). The upper queue (queue number 1) is an $M/D/1$ queue with input load $\nu = x\lambda$ and periodic service time 1 slot, the server of the lower queue (queue number 2) is cyclic with a geometric period of rate $\beta = 1-(1-x)\lambda$.

Let us consider the size of the queue number 2 at the beginning of each busy period (starting with an idle slot) of queue number 1. Since the busy periods are i.i.d., this queue length describes an embedded Markov process.

LEMMA 3.
Let $q(z)$ be the conditional p.g.f. of the size of queue number 2 at the beginning of each busy period on queue number 1, we have the expression:

$$q(z) = \left(\frac{\beta-\nu}{1-\nu}\right) \frac{(z-1)}{\frac{z}{A(\beta+(1-\beta)\,z)} - \beta - (1-\beta)\,z} .$$

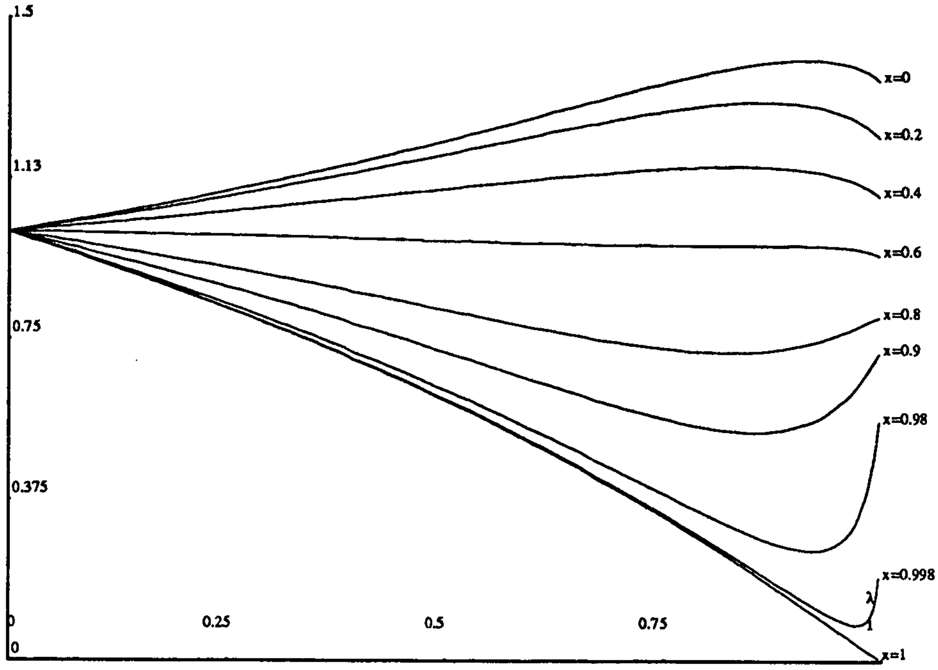

Figure 6: Illustration of the skewing, perturbation of the mean access delay depending on the location of the station on the bus. Here the ratio $W(x,\lambda)/W(\lambda)$ for some value of x, as a function of λ.

Proof. We have the identity

$$q(z) = ((1-\beta)\,q(z) + \beta\,(\frac{q(z)-q(0)}{z} + q(0))) \times A(\beta + (1-\beta)\,z)\ .$$

The factor $((1-\beta)\,q(z) + \beta\,(\frac{q(z)-q(0)}{z} + q(0)))$ in the right hand side means that the first slot of the busy period is idle (no arrival in queue 2) and the probability that a service occurs at this very slot is β (and nothing happens with probability $1-\beta$). The last factor $A(\beta+(1-\beta)\,z)$ in the right hand side means that at each of the remaining slots of the busy period, the probability that the queue size increases is $1-\beta$ (one arrival but no service) and the probability that the queue size remains the same is β (one arrival and one service). Therefore

$$q(z) = \beta q(0)\,\frac{z-1}{\dfrac{z}{A(\beta+(1-\beta)\,z)} - \beta - (1-\beta)\,z}\ .$$

We remove the indetermination on $q(0)$ by identifying $z = 1$ and $q(1) = 1$ and using Liouville's theorem:

$$1 = \frac{\beta q(0)}{1-(1-\beta)(A'(1)+1)}\ ,$$

where $A'(1)$ is the first moment of the busy period (not including the idle first slot). Estimates issued from lemma 2 end the proof. ∎

As immediate corollary the mean queue length, $q'(1)$, at embedded points, is therefore:

$$q'(1) = \frac{1}{2}\frac{(1-\beta)\,\nu\,(2-\beta\nu+2\nu^2-3\nu)}{(1-\nu)^2(\beta-\nu)} .$$

LEMMA 4.
The unconditional mean size of queue number 2 at the beginning of each slot, Eq, has the following expression:

$$\begin{aligned} Eq =& 1/2\,\frac{(1-\beta)\,\nu\,(2-\beta\nu+2\nu^2-3\nu)}{(1-\nu)^2(\beta-\nu)}+ \\ &+\nu^2(1-\beta)(-\frac{1}{(1-\nu)}+1/2\frac{(3-2\nu)}{(1-\nu)^2})+\nu . \end{aligned}$$

Proof. Looking inside a busy period of queue number 1, and taking care of side effect, leads to the identity

$$Eq = q'(1) + \frac{A'(1)\beta(q(0)-1)+(1-\beta)\frac{A''(1)}{2}+A'(1)}{1+A'(1)} ,$$

where $A'(1)$ and $A''(1)$ are respectively the first and second derivative of $A(z)$ at $z=1$. Quantity $1+A'(1)$ is simply the mean number of slots (including the first idle slot) of a busy period. $A'(1)$ is the number of *internal* slots of such a busy period. Let us suppose that the number of internal nodes of the busy period is k and the number of customers already in queue at the beginning of the period is $n>0$. Since after the first idle slot there a service occurs with probability β and the first arrival from queue 1 happens, the queue size becomes in average $n-\beta+1$. On each slot number $i \le k$ there is a new customer in the queue with probability 1 and a service with probability β, therefore the new customer stays in the queue during the $k-i$ remaining slots with probability $1-\beta$. Thus the average cumulation of the queue sizes during this k slots is $k(n-\beta+1)+(1-\beta)k(k-1)/2$, or, when $n=0$, $k+(1-\beta)k(k-1)/2$. Averaging on k and n leads to the result. ∎

Proof of theorem 1. We have $W(x,\lambda)=(Eq+1)\beta^{-1}-1$, the "$-1$" comes from side effect in the counting of the ultimate waiting slot. Identifying ν and β, leads to the expanded expression in the theorem. The unconditional delay, $W(\lambda)$, is deduced from the identity $W(\lambda)=\int_0^1 W(x,\lambda)\mathrm{d}x$, which leads to the close expression of the theorem by elementary integration (not for the author which made intensive use of the symbolic manipulations of Maple©[2]). ∎

References

[1] Z. BUDRIKIS *et al.*, "QPSX : a queued packet and synchronous circuit exchange," in *8th ICCC*, Munich, pp 288-293, 1986.

[2] B. W. CHAR *et al.*, *MAPLE, Reference Manual*, Ed. Warcom, Canada, 1988.

APPENDIX

Proof of the asymptotic accuracy of the model.
Our purpose is to prove that when $\lambda \rightarrow 1$ the stream of busy slots seen by station x on bus A tends to follow a Bernoulli distribution of parameter $(1-x)\lambda$. This will be more hand-waving than rigourous approach.

When $\lambda \rightarrow 1$ the utilizations $\rho(x) = x\lambda/(1-(1-x)\lambda$ of all the local queueings uniformly tend to 1. Therefore it is expected that almost all the F_i's tend to ∞ with a large dispersion as in classical queueings.

Let us consider stations between y and $y - \Delta y$ with $\Delta y \ll 1$. Their contribution to the global load is $\Delta y \lambda$. Our purpose is to evaluate the (small) perturbation they introduce between the two streams of busy slots respectively seen by station y and by station $y - \Delta y$ on bus A.

The probability that one of the stations between y and $y - \Delta y$ generates a new packet for transmission at a given slot is $1 - \exp(-\Delta y \lambda) \approx \Delta y \lambda$. Since the F_i of the station is expected to be large, the station will have to let pass a lot of empty slot on bus A before trying to insert its packet in the stream. In other words the choice of the empty slot on which the station will insert its packet will depend of events far in the past. Asymptotically all empty slots seen by station y are equiprobable to receive the packet generated by stations in the range $[y - \Delta y, y[$. Since the rate of this empty slots is $1-(1-y)\lambda$, the probability than one these empty slots receive such packet is $\Delta y \lambda/(1-(1-y)\lambda)$. Therefore the stream seen by station $y - \Delta y$ is simply the stream seen by station y where each empty slot has probability $\Delta y \lambda/(1-(1-y)\lambda)$ to become busy when detected by station $y - \Delta y$. Making y gliding between 1 (where the stream of busy slot is null) to x by small jumps of Δy each, we obviously build a Bernoulli stream. ∎

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