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EVALUATION OF JOB-SHOPS WITH RANDOM MANUFACTURING TIMES : A PETRI NET APPROACH

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EVALUATION OF JOB-SHOPS WITH RANDOM MANUFACTURING TIMES : A PETRI NET APPROACH

EVALUATION D'ATELIERS AVEC TEMPS DE FABRICATION ALEATOIRES : UTILISATION DES RESEAUX DE PETRI

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ABSTRACT :

In this paper, we use Event Graphs, a particular type of Petri Nets, to evaluate job-shops when manufacturing times are random variables and the control is defined as the sequence of product types at the entrance of each machine. Under very general assumptions, we prove that the cycle time of the system converges in probability to a constant and asymptotically tends in distribution to a normal distribution whose standard deviation tends to zero. We also provide a lower bound of the main cycle time. We finally derive from the previous results a heuristic algorithm which leads to a near optimal solution of the problem (an optimal solution being the minimal work-in-process which allows the maximal productivity, i.e. the minimal cycle time, knowing the control).

Key words : Petri Nets, Event Graphs, Discrete Event Systems, System Evaluation, Job-Shop, Random Manufacturing Times

RESUME :

Dans ce papier, nous utilisons les graphes d'évènements pour évaluer un atelier lorsque les temps de fabrication sont aléatoires et que le contrôle du système se fait par le truchement de la séquence d'entrée des types de produits dans chaque machine. Sous des hypothèses très générales, nous montrons que le temps de cycle moyen du système converge en probabilité vers une constante et tend assymptotiquement vers une distribution normale dont l'écart-type tend vers zéro. Nous fournissons également une borne inférieure au temps de cycle moyen. Finalement, nous déduisons des résultats précédents une heuristique qui conduit à une solution proche de l'optimum, une solution optimale étant l'en-cours minimal qui autorise la productivité maximale (i.e. le temps de cycle maximal) connaissant le contrôle.

Mots clefs : Réseaux de Pétri, Graphe d'évènements, Systèmes à évènements discrets, Evaluation de Systèmes, Atelier, Temps de Production aléatoires

PAPIER RECUPERE ET RECYCLE

1. INTRODUCTION

Job-shop systems form a class of production systems in which each product has to visit a given sequence of machines, spending a given amount of time on each machine. The sequence of machines to visit may be different from one product to another. A sequence of machines along with the times spend by the related product on these machines is known as the manufacturing process of this product. Products having the same manufacturing process belong to the same product type.

In the problem considered in this paper, the mix of the product types to manufacture is known and constant. The control of the system is based on the sequence of product types entering each machine. These sequences are known as the input sequences. They have, of course, to be consistant with the required mix. In other words, if a given P_i-type product has to be produced n times more than a P_j-type product, then P_i has to appear n times more than P_j in each input sequence where both product types appear. Our aim is to maximize the productivity of the system when manufacturing times are random variables, the input sequences being known. This problem has been solved using Event Graphs when manufacturing times are constant ([5] and [6]). It has been proven that it is always possible to fully utilize the bottleneck machines providing that the initial state of the system (i.e. the initial location of the in-process products) satisfy some conditions. In terms of Event-Graph, the previous result means that the critical circuits are those corresponding to the command circuits of the bottleneck machines. A fast heuristic algorithm has been provided in [5] and [6] to compute the initial state of the system (i.e. the initial marking of the Event Graph) which verifies the previous conditions with a minimal work-in-process (i.e. with a minimal number of tokens in the Event Graph).

The fact that manufacturing times are random variables makes the previous approach invalid because the critical circuits change over the time, depending on the values taken by the random variables representing the manufacturing times. As a consequence, the approach proposed in this paper consists in studying the behavior of the average cycle time when the running period tends to be infinite. We prove that the average cycle time tends in probability to a constant and tends asymptotically in distribution to a normal distribution whose standard deviation tends to zero. This result vouches for the unicity of the mean value of the cycle time. Based on this result, we then propose a fast algorithm which leads to a near optimal solution to the problem, e.g. an initial location of work-in-process allowing the maximal productivity knowing the input sequences.

The paper is organized as follows. In section 2, we set the problem and show how to model it using an Event Graph. The properties of the average cycle time when the running period tends to be infinite are proved in section 3. A heuristic algorithm which takes advantage of the previous properties is proposed in section 4. Finally, the previous approach is illustrated in section 5 by some numerical examples.

2. PROBLEM SETTING AND MODELLING

2.1. SETTING THE PROBLEM

A job-shop is composed of m machines $M_1, M_2, ..., M_m$. A product type P_i (i = 1, 2, ..., n) is defined by the sequence $M^i_{s_1}, M^i_{s_2}, ..., M^i_{s_i}$ of machines it has to visit, where $M^i_k \in \{M_1, M_2, ..., M_m\}$ for

$k \in \{s_1, s_2, ..., s_i\}$. The time spent by a part belonging to P_i on M_k^i is the value taken by a random variable X_k^i. All these random variables are independent of each other. A sequence of machines associated with a product type along with the related manufacturing times is called a manufacturing process. In the following, we assume that there is no transportation time. Taking into account the transportation times would not modify the following approach but only make the explanations much more complicated.

The production mix is given. It is reached by fixing an input sequence which is a sequence of part types indicating the types of the parts which have to be successively launched in the system. The ratios of the part types in the sequence correspond to the mix desired. We also fix a machine sequencing for each machine. It is a sequence of part types providing the types of parts which have to successively enter the machine. The part types in a machine sequencing are restricted to those which use the machine and the number of each part type in the machine sequencing has to fit with the production mix.

The problem we deal with consists in analyzing the dynamics of the job-shop and, in particular, in evaluating its productivity according to the work-in-process (WIP) inventories and their initial locations in the system. The goal of the study is to maximize the productivity of the system using a minimal total WIP inventory. In the case of an FMS (Flexible Manufacturing System) it means that we aim at maximizing the productivity with a minimal number of transportation devices (cost, pallets, ...), which are usually very expensive.

The following small example will be used all over the paper to illustrate our approach. We consider a job-shop composed of three machines M_1, M_2 and M_3. It can manufacture three types of products P_1, P_2 and P_3.

The following production mix is required :

25 % of products of type P_1

50 % of products of type P_2

25 % of products of type P_3

We choose as input sequence :

$S = (P_1, P_2, P_3, P_2)$

which contains 25 % of P_1-items, 50 % of P_2-items and 25 % of P_3-items.

The manufacturing processes are as follows :

$P_1 : M_1 (X_1^1), M_2 (X_2^1), M_3 (X_3^1)$

$P_2 : M_3 (X_1^2), M_1 (X_2^2)$

$P_3 : M_2 (X_1^3), M_3 (X_2^3)$

The random variables in brackets provide the times that the products is spending on the machines. We choose the following distribution densities :

$X_1^1 : f_{11}(x) = 0{,}5e^{-0{,}5x}$ for $x \in [0, +\infty)$

$X_2^1 : f_{12}(x) = 1/3e^{-1/3x}$ for $x \in [0, +\infty)$

$$X_3^1 : f_{13}(x) = \begin{cases} 1/2 \text{ if } x \in [1,3] \\ 0 \text{ otherwise} \end{cases}$$

$$X_1^2 : f_{21}(x) = \begin{cases} 1/4 \text{ if } x \in [1,5] \\ 0 \text{ otherwise} \end{cases}$$

$$X_2^2 : f_{22}(x) = 0{,}5e^{-0{,}5x} \text{ for } x \in [0, +\infty)$$

$$X_{21}^3 : f_{31}(x) = e^{-x} \text{ for } x \in [0, +\infty)$$

$$X_2^3 : f_{3,2}(x) = \begin{cases} 1/20 \text{ if } x \in [0,20] \\ 0 \text{ otherwise} \end{cases}$$

Furthermore, we choose the following machine sequencing:

$M_1 : S_1 = (P_1, P_2, P_2)$

$M_2 : S_2 = (P_1, P_3)$

$M_3 : S_3 = (P_1, P_2, P_2, P_3)$

Note that :

(i) S_1 does not contain P_3 because a P_3-type product does not use M_1. Similarly, S_2 does not contain P_2 because a P_2-type product does not use M_2. But M_3 being used by all the product types, S_3 contains P_1, P_2 and P_3.

(ii) The P_2-type products being manufactured two times more than the P_1-type products on the same period, P_2 appears two times more than P_1 in S_1 and S_3. On the contrary, the P_1-type products being manufactured in the same proportions than the P_3-type products, P_1 appears as many times as P_3 in S_2 and S_3.

The last constraint on the job-shop is that any machine performs only one operation at time.

2.2. MODELLING THE JOB-SHOP

A timed Event Graph is used to model the job-shop. Remind that an Event Graph is a Petri Net whose transitions have only one input place and one output place. An Event Graph is said timed if some amount of time, representing the time needed to fire the transition, is assigned to each transition.

The modelling process is twofold.

a. Design of the manufacturing circuits

Each item of the input sequence is modelled using an elementary event graph circuit called manufacturing circuit (see fig. 1).

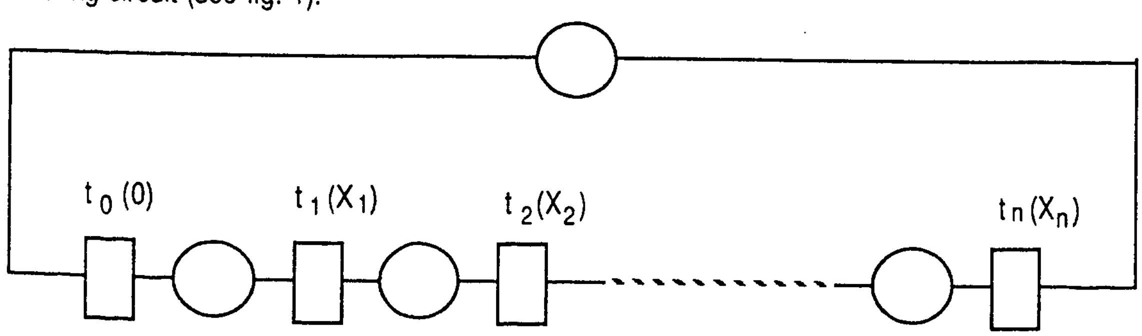

Fig. 1 : Modelling of one input sequence item

The first transition t_0 of this circuit is the input transition which represents the entrance of the job-shop. The time associated to this transition is 0. Each further transition represents an operation of the production process, and we associate to this transition the random variable providing the manufacturing time. For instance, figure 2 represents the manufacturing circuit related to the item P_1 of the example introduced in the previous section.

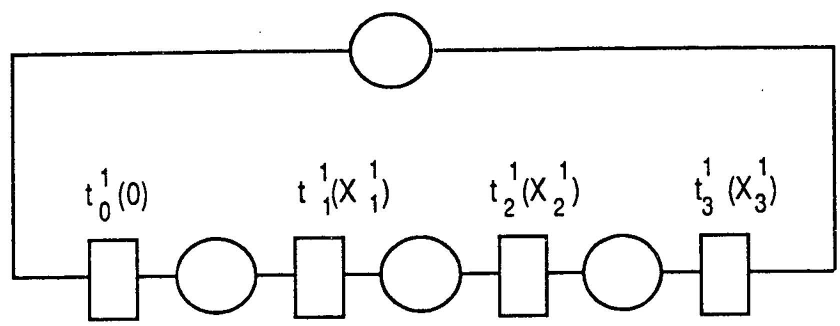

Fig. 2 : Manufacturing circuit related to P_1

Each token in a manufacturing circuit related to an item P represents one unit of P-type product. The number of tokens in such a circuit is unbounded. A token which leaves the last transition of a manufacturing process (transition t_n in figure 1, for instance) indicates the completion of one unit of P-type product. Such a token immediately recirculate in the circuit ; this can be interpreted as the reutilization of a transportation resource for a new product unit as soon as the product unit it carries is completed.

The set of manufacturing processes corresponding to the input sequence items is insufficient to properly model the job-shop for two mean reasons :

(i) the output frequencies may not fit with the production mix

(ii) several transitions representing operations performed on the same machine may fire (i.e. may contain a token) at the same instant, which would mean that the same machine is used to manufacture simultaneously several units of products.

These remarks lead to the second step of the modelling process.

b. Design of the command circuits

Command circuits are designed in order to synchronize the behavior of the different manufacturing circuits. A command circuit is an elementary circuit which contains either the transitions related to one of the machines or the input transitions of the manufacturing circuits. A command circuit only contains one token ; as a consequence, the transitions related to the same machine fire successively and the flow in the different manufacturing circuits is controlled in order to meet the production mix.

The model of the example introduced in the previous section is partially given in figure 3. For clearity, only the command circuits related to the input transitions and to machine M_1 are represented. The locations of the tokens correspond to the input sequence and the M_1-machine sequencing of the example. The token in the manufacturing circuits are not represented.

The model obtained is a strongly connected random event graph.

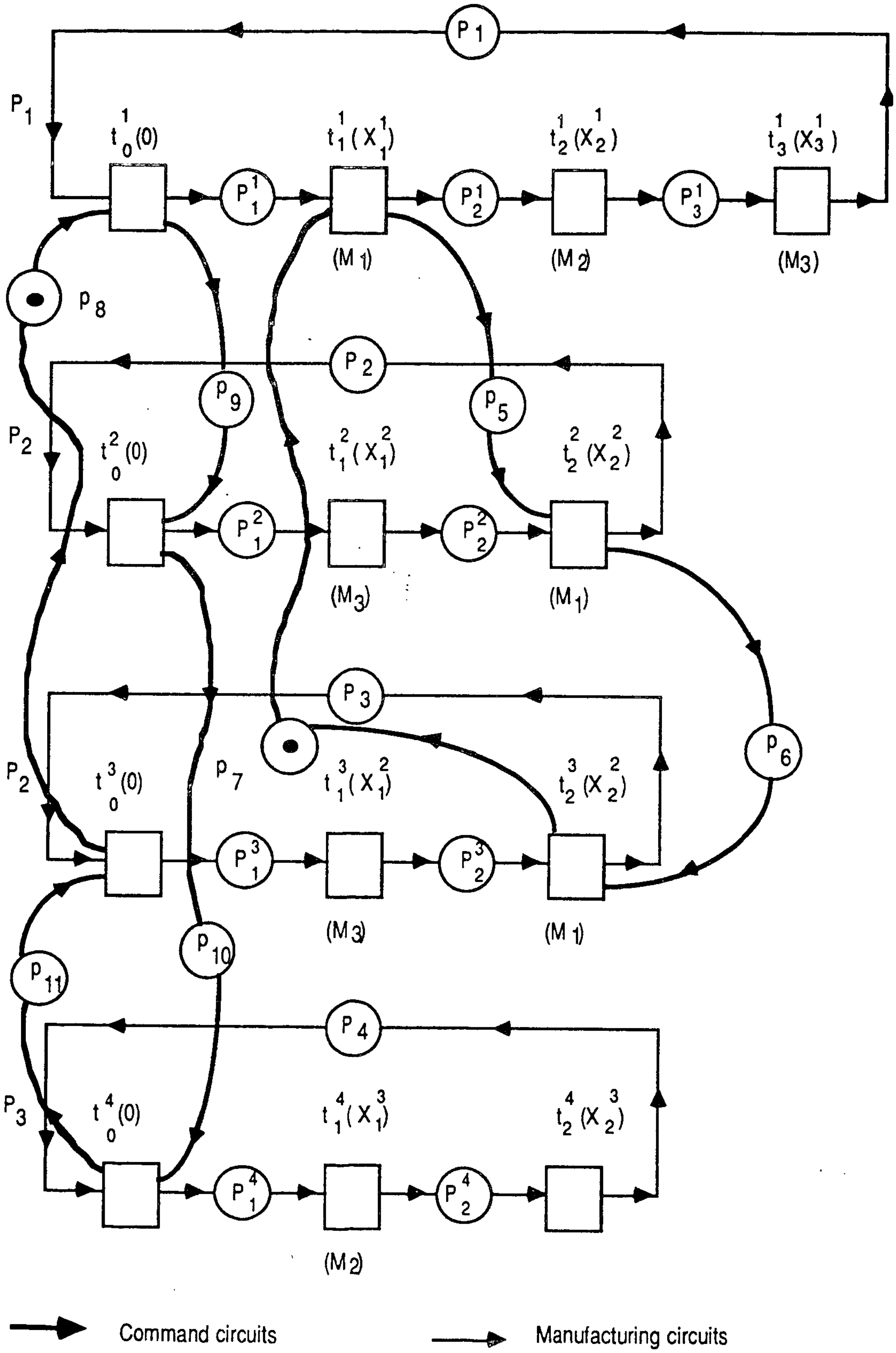

Fig. 3 : Example modelling

3. PERFORMANCE EVALUATION

3.1. STOCHASTIC CASE VERSUS DETERMINISTIC CASE

As we outlined in section 2, an elementary circuit of the event graph is a circuit in which each summit appears at most once. We have already identified two types of elementary circuits : the manufacturing circuits and the command circuits. The mixed circuits are the third and last type of elementary circuits. They are made of summits belonging to both manufacturing circuits and command circuits. For instance, in figure 3 :

$$(t_0^1, p_1^1, t_1^1, p_5, t_2^2, p_6, t_2^3, p_3, t_0^3, p_8)$$

is an elementary mixed circuit. The computation of the elementary circuits of a strongly connected event graph can be performed using an algorithm proposed by H. HILLION [(1)].

The cycle time of an elementary circuit γ is defined as :

$c(\gamma) = t(\gamma) / n(\gamma)$

where $t(\gamma)$ is the sum of the times assigned to the transitions of γ and $n(\gamma)$ the number of tokens in γ. An important property of the event graphs is that $n(\gamma)$ is constant whatever the state of the system (i.e. whatever the sequence of transitions which have been previously fixed) (see [7]). But $t(\gamma)$ is constant only in the deterministic case (i.e. the case when the manufacturing times are deterministic); otherwise, $t(\gamma)$ depends on the realisations of the random variables associated to the transitions of the elementary circuit γ. As a consequence, in non deterministic cases (see [6]), the critical circuits (i.e. the circuits having the greatest cycle time) are no more the key points of the behavior of the system and another approach is required to evaluate the productivity of the system.

Let $T = \{t_1, t_2, ..., t_n\}$ be the set of transitions belonging to the event graph under study. We also denote X_i the random variables associated to t_i, $i = 1, 2, ..., n$.

Let M° be the initial marking of the event graph, i.e. the location of the tokens in the event graph at time zero :

$M° : \mathcal{P} \rightarrow \mathcal{N}$

where $\mathcal{P}$ si the finite set of places and $\mathcal{N}$ the infinite set of non negative integers.

$\overset{o}{M}_{t_i, t_j}$ is the number of tokens in the unique place located between t_i and t_j (if any). We also denote $f_t(k)$ the instant at which the k-th firing of transition t occurs.

---

(1) This algorithm has been developed in the Laboratory for Information and Decision Systems at MIT. It has been supported by the US office of Naval Research under grands N 00014-85-K-0782 and N 00014-84-K-0519.

The following relation holds :

$$f_{t_j}(k) = \mathcal{M}ax_{t \in E_j} \{f_t(k - \overset{\circ}{M}_{t_i, t_j}\} + X_j \quad (1)$$

where E_j is the set of transitions preceding t_j (i.e. the set of transitions such that there exists a path connecting any element of E_j and t_j which contains only one place).

We are interested in computing the cycle time of the system in steady state. Thus, we have to compute :

$$\pi_{t_j} = \lim_{k \to +\infty} (f_{t_j}(k) / k)$$

According to the fact that the event graph is strongly connected and contains a finite number of tokens, π_{t_j} does not depend on t_j and :

$$\pi = \lim_{k \to +\infty} (f_t(k) / k) \quad (2)$$

whatever $t \in T$.

Finally, if the limit (2) exists, π is the cycle time of the event graph in steady state.

3.2. CONVERGENCE IN PROBABILITY OF THE CYCLE TIME

The following proposition holds :

PROPOSITION 1 :

If second moment of X_i is finite whatever $i \in \{1, 2, ..., n\}$ (i.e. $E[X_i^2] < +\infty$) and if the event graph is strongly connected and bounded, then $f_t(k) / k$ tends in probability to a constant π.

Proof :

$$f_t(k) = f_t(k - 1) + U_t(k) \quad (3)$$

where $U_t(k)$ is a random variable :

(i) independent of $f_t(k - 1)$

(ii) such that :

$$0 \leq U_t(k) \leq \sum_{i=1}^{n} X_i \quad (4)$$

From (3) we derive :

$$f_t(k) = \sum_{r=1}^{k} U_t(r) \quad (5)$$

and :

$$\sigma^2 (f_t(k)) = \sum_{r=1}^{k} \sigma^2 (U_t(r)) \quad (6)$$

because random variables $U_t(r)$ are independent each other.

We set :

$$E[(\sum_{i=1}^{n} X_i)^2] = M$$

Thus, from (4) :

$$\sigma^2(U_t(r)) \leq M$$

and from (6) :

$$\sigma^2(f_t(k)) \leq k\,M$$

which leads to :

$$\sigma^2(f_t(k)/k) \leq M/k$$

Thus, $\sigma^2(f_t(k)/k)$ tends to 0 as k tends to because infinite.

Q.E.D.

The following proposition also holds :

PROPOSITION 2 :

Under the same assumptions as for proposition 1, the random variables $f_t(k)/k$ asymptotically tends in distribution to a normal distribution $\mathcal{N}(\pi, \sigma_k^2)$, where σ_k tends to 0 as k tends to be infinite.

Proof :

We set :

$$V_t(r) = U_t(r) - E[U_t(r)],\ \forall\, t \in T,\ r \in \{1, 2, 3, ...\} \qquad (7)$$

and :

$$Z_t(k) = \sum_{r=1}^{k} V_t(r) \qquad (8)$$

According to the central-limit theorem due to Lindeberg (see [1]) :

$Z_t(k) \,/\, [\sum_{r=1}^{k} \sigma^2(V_t(r))]^{1/2}$ tends in distribution to $\mathcal{N}(0,1)$, normal distribution with a 0-mean value and a 1-standard-deviation.

But according to (5), (7) and (8) :

$$Z_t(k) = f_t(k) - \sum_{r=1}^{k} E[U_t(r)]$$

Thus :

$Z_t(k)/k = f_t(k)/k - E[\sum_{r=1}^{k} U_t(r)/k]$ tends in distribution to :

$$\mathcal{N}(0, [\sum_{r=1}^{k} \sigma^2(V_t(r))]/k^2) \qquad (9)$$

And considering (5) :

$E[\sum_{r=1}^{k} U_t(r)/k] = E[f_t(k)/k]$ tends in probability to π (10)

(see proposition 1)

Thus, from properties (9) and (10), and the following lemma due to Bickel and Doksum (see [2]) :

if Y_n tends in distribution to Y as n tends to be infinite
if A_n and B_n tends in probability respectively to A and B as n tends to be infinite
then :
$A_n + B_n\ Y_n$ tends in distribution to $A + BY$ as n tends to be infinite

we derive :

$f_t(k) / k$ tends to $\mathcal{N}(\pi, \sigma_k^2)$

where $\sigma_k^2 = \sum_{r=1}^{k} \sigma^2 (V_t(r)) / k^2$ which tends to 0 as k tends to be infinite.

Q.E.D.

3.3. LOWER BOUND OF THE CYCLE TIME

The following proposition provides a lower bound of the cycle time.

PROPOSITION 3 :

The cycle time π verifies

$$\pi \geq \mathcal{M}ax_{t \in T}\ E\,\beta_t$$

where :

$$\beta_t = \mathcal{M}ax_{\gamma \in \Gamma_t} \frac{l(\gamma)}{n(\gamma)}$$

$l(\gamma)$ is the sum of the random times associated to the transitions of γ

$n(\gamma)$ is the total number of tokens in γ

Γ_t is the set of elementary circuits which contains the transition t

Proof :

Obvious if we observe that the cycle time of a strongly connected event subgraph composed of elementary circuits belonging to Γ_t is smaller than or equal to the cycle time of this event subgraph plunged in a strongly connected event graph and that, in turn, the cycle time of the event subgraph is smaller than or equal to the cycle time of the event graph.

Q.E.D.

4. A NEAR-OPTIMAL SOLUTION TO THE PROBLEM

Propositions 1 and 2 allow to approximate the cycle time with a precision ε, ε being given by the user. We use this approximation in the following algorithm.

ALGORITHM

We denote Γ the set of elementary cycle times, excluding the command circuits.

First phase :

1. Compute the optimal solution to the problem when the manufacturing times are equal to the average values of the related distribution. The computation is made as showed in [6] and the cycle time is π^* which minimizes the cycle time of the system.

2. Simulate the system in order to obtain the cycle time Q_0 corresponding to this solution.

Second phase :

3. Let γ_0 the elementary circuit of Γ having the greastest cycle time, γ_1 the elementary circuit having the greatest cycle time among the circuits belonging to Γ-{γ_0} which have at least one place in common with γ_0, γ_2 the elementary circuit having the greatest cycle time among the circuits belonging to Γ-{$\gamma_0 \cup \gamma_1$} which have at least one place in common with $\gamma_0 \cap \gamma_1$, and so on until we reach γ_q such that $\gamma_1 \cap \gamma_2 \cap ... \cap \gamma_q = \varnothing$
4. Add one token in a place P belonging to $\gamma_1 \cap \gamma_2 \cap ... \cap \gamma_{q-1}$
5. Simulate the system in order to obtain the cycle time Q_1 related to this solution
6. Test :
    6.1. If $Q_0 - \varepsilon \leq Q_1 \leq Q_0 + \varepsilon$:
        6.1.1. Remove the last token introduced in the system
        6.1.2. Go to 7
    6.2. If $Q_1 < Q_0 - \varepsilon$:
        6.2.1. Set $Q_0 = Q_1$
        6.2.2. Go to 3

Third phase :

7. Let γ_0 be the elementary circuit having the greatest cycle time among the circuits belonging to Γ, γ_1 the elementary circuit having the greatest cycle time among the circuits belonging to Γ-{γ_0} the elementary circuit having the greatest cycle time among the circuits belonging to Γ-{$\gamma_0 \cup \gamma_1$} and so on until we reach γ_q^c such that $\gamma_1^c \cap \gamma_2^c \cap \gamma_q^c = \varnothing$
8. Remove one token from one of the places belonging to $\gamma_0^c \cap \gamma_1^c \cap \gamma_{q-1}^c$
9. Simulate the system in order to obtain the cycle time Q_1 corresponding to this solution
10. Test :
    10.1. If $Q_0 - \varepsilon \leq Q_1 \leq Q_0 + \varepsilon$, go to 7
    10.2. If $Q_1 > Q_0 + \varepsilon$:
        10.2.1. Replace the last token removed
        10.2.2. End of the process

The following proposition holds :

PROPOSITION 4 :

The algorithm converges.

Proof :

Obvious if we consider that :

(i) the second phase of the algorithm is necessarily finite: the number w of times the process pass though instruction 6.1.2. is upper bounded by $\sum_{\gamma \in \Gamma} nn(\gamma)$ where nn (γ) is the smallest integer such that t (γ)/nn (γ) $\leq \theta^*$, θ^* being the maximal cycle time among those belonging to the set of command circuits.

(ii) the third phase is necessarily finite: the number of times the process pass though instruction 10.1. is upper bounded by $\sum_{\gamma \in \Gamma} nn(\gamma)$ - card (Γ) where :

a. nn (γ) is the number of tokens in γ at end of the second phase. This number is finite because it is upper bounded by nn° (γ) + w where :

a_1. nn° (γ) is the number of tokens in γ at the end of the first phase. We know that this number is finite.

a_2. w has been defined in (i).

b. card (Γ) is the number of elementary circuits of Γ.

Q.E.D.

4. EXAMPLE

We consider the example presented in section 2 and whose model is given in figure 3.

The first phase of the algorithm leads to $\pi^* = 18$ and to put one token in p_1, p_2, p_3 and p_2^4 (see figure 3 for the relations). The cycle time of that first configuration is $Q_0 = 18.3307$

The second phase of the operation provides successively the following results : ($\varepsilon = 0.005$).

Steps	Put one more token in	Cycle time
1	p_2^1	18.1853
2	p_2^2	18.1792
3	p_4	18.1194
4	p_3^1	18.1097
5	p_2^3	18.1039
6	p_1	18.004

The last phase does not improve the cycle time.

5. CONCLUSION

In this paper, we have provided some new results which permit to define how simulation converges to the real value of the cycle time, and so to control the precision of the simulation. At the same time, we have provided a lower bound of the cycle time. We have finally provided a step-by-step algorithm which converges to a near-optimal solution.

The next step of the research should be to find a way to avoid the simulation steps.

Appendix

Central-Limit Theorem (Lindeberg) . Let X_1, X_2... be mutually independent one-dimensional random variables with distributions F_1, F_2..., such that

$$E(X_k) = 0 \quad Var(X_k) = \sigma_k^2$$

and put $s_n^2 = \sigma_1^2 + ... + \sigma_n^2$

Assume that for each $t > 0$

$$s_n^2 \sum_{k=1}^{n} \int_{|y| \geq ts_n} y^2 F_k(dy) \rightarrow 0 \text{ when } n \rightarrow \infty \qquad (11)$$

then the distribution of the normalized sum

$$s_n^* = (X_1 + ... + X_n) / s_n$$

tends to the normal distribution $\mathcal{N}(0,1)$ with zero expectation and unit variance.

Note that condition (11) holds because :

$$0 < \underline{\sigma}^2 \leq \sigma^2(v_t(r)) \leq \overline{\sigma}^2$$

which leads to $k\underline{\sigma}^2 < s_k^2 \leq k\,\overline{\sigma}^2$

Furthermore $V_t(r) \leq \sum_{i=1}^{n} X_i$ which distribution is F

As a consequence :

$$\int_{y > ts_k} y^2 F_k (dy) \leq \int_{y > ts_k} y^2 F (dy) < +\infty$$

Thus :

$$s_k^{-2} \sum_{l=1}^{k} \int_{y > ts_k} y^2 F_k (dy) \leq s_k^{-2} \sum_{l=1}^{k} \int_{y > ts_k} y^2 F (dy)$$

$$= s_k^{-2} k \int_{y > ts_k} y^2 F (dy) \;)$$

But $\int_{y > ts_k} y^2 F (dy) \to 0$ as k tends to becom infinite because $\int y^2 F (dy) < +\infty$, and s_k tends to infinite

and since $ks_k^{-2} \leq A = \frac{1}{\sigma^2}$

We derive from the previous results :

$s_k^{-2} k \int_{y > ts_k} y^2 F (dy) \to 0$ as k tends to be infinite

Q.E.D.

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