Hyperbolic systems in diagonal form with dissipation,in several spaces variables: existence and uniqueness results

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HYPERBOLIC SYSTEMS IN DIAGONAL FORM WITH DISSIPATION, IN SEVERAL SPACES VARIABLES : - EXISTENCE AND UNIQUENESS RESULTS

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Abstract

In this paper we consider a class of hyperbolic systems in diagonal form with dissipation. We study the Cauchy problem for which we obtain the existence and uniqueness results of generalized solution i.e. solution in $W^{1,\infty}$; assuming the dissipation term large enough.

Key words : Hyperbolic - Dissipation - Generalized solution;

Résumé

Dans ce papier on considère une classe de systèmes hyperboliques sous forme diagonale avec terme de dissipation. On établit pour le problème de Cauchy l'existence et l'unicité de solution généralisée, c'est-à-dire de solution dans $W^{1,\infty}$; ceci sous la condition que le terme de dissipation est assez grand.

Mots clefs : Hyperbolique - Dissipation - Solution généralisée.

HYPERBOLIC SYSTEMS IN DIAGONAL FORM WITH DISSIPATION, IN SEVERAL SPACES VARIABLES : EXISTENCE AND UNIQUENESS RESULTS

O. Bennouna

Introduction

It is well known that for the Cauchy problem of first order hyperbolic systems in diagonal form :

$$(*)\quad \left|\begin{array}{l} \dfrac{\partial u_k}{\partial t} + \sum\limits_{i=1}^{N} \lambda_k^i(t,x,u)\,\dfrac{\partial u_k}{\partial x_i} + c_k(t,x,u)u_k = 0 \quad \text{in } \Pi_T \\ u_k(0,x) = v_k(x) \quad \text{in } \mathbb{R}^N \;, \quad 1 \le k \le M \end{array}\right.$$

where $u = (u_1,\dots,u_M)$

$$\Pi_T =]0,T[\times \mathbb{R}^N ,$$

$v_k(x)$, $\lambda_k^i(t,x,v)$, $c_k(t,x,v)$ are given smooth data; a smooth solution exists only locally in time and singularities i.e. formation of shocks develop in a finite time.

In this note we prove that the addition of dissipation term permit us to obtain the existence and uniqueness of global generalized solution i.e. a Lipschitz continuous solution which satisfies (*) in the almost everywhere sense.

To obtain the necessary a priori estimates we use a successive approximation method.

For $N = 1$, similar results are obtained under other types of dissipation condition in [1], [2].

I. PRELIMINARY RESULTS

I.1. Notations, assumptions and definitions

We make the following assumptions about the data :

(1) The functions $\lambda_k^i(t,x,v)$, $c_k(t,x,v)$ are bounded and continuous and have bounded derivatives with respect to x and v.

(2) $v_k(x)$ is bounded continuous and has bounded derivatives.

(3) $c_k(t,x,v) \leq -\alpha < 0 \qquad \forall\ (t,x,v) \in \Pi_T \times \mathbb{R}^M$.

Let us give now the definition of a generalized solution of (*).

Définition 1.1 : u is a generalized solution of (*) if $u \in [W^{1,\infty}(\Pi_T)]^M$ and satisfies (*) in the almost everywhere sense.

Let $y^t_{x,k}(\tau)$ be the k-th characteristic curve passing through (t,x), which satisfies

(4) $$y^t_{x,k}(\tau) = x - \int_\tau^t \lambda_k(s, y^t_{x,k}(s), u(s, y^t_{x,k}(s)))ds$$

$$\forall\ \tau \quad , \quad 0 \leq \tau \leq t \leq T .$$

We shall prove some useful lemma concerning $y^t_{x,k}(\tau)$.

I.2. Some lemma

We have :

<u>Lemma 1</u> : Assume that (1) holds and let $u \in [W^{1,\infty}(\Pi_T)]^M$.
Then

(5) $$||y^t_{x,k}(\tau)-y^t_{x+h,k}(\tau)|| \le ||h|| \exp[(t-\tau)\omega_{\lambda_k}(u)] \qquad \forall\ h \in \mathbb{R}^N$$

where

$$\omega_{\lambda_k}(u) = ||\frac{\partial\lambda_k}{\partial x}||_\infty + M\ ||\frac{\partial\lambda_k}{\partial u}||_\infty\ ||\frac{\partial u}{\partial x}||_\infty$$

$$||\frac{\partial\lambda_k}{\partial x}||_\infty = \max_{s\in[0,T]}\ \sup_{v\in\mathbb{R}^M}\ \sup_{x,x'\in\mathbb{R}^N} (|\lambda_k(s,x,v)-\lambda_k(s,x',v)|/||x-x'||)$$

$$||\frac{\partial\lambda_k}{\partial u}||_\infty = \max_{1\le\ell\le M}\ ||\frac{\partial\lambda_k}{\partial u_\ell}||_\infty$$

$$||\frac{\partial\lambda_k}{\partial u_\ell}|| = \max_{s\in[0,T]}\ \sup_{x\in\mathbb{R}^N}\ \sup_{v_\ell,v'_\ell\in\mathbb{R}} (|\lambda_k(s,x,u_1,\dots,v_\ell,u_{\ell+1},\dots)$$

$$- \lambda_k(s,x,u_1,\dots,v'_\ell,u_{\ell+1},\dots)|/|v_\ell-v'_\ell|)$$

$$||\frac{\partial u}{\partial x}||_\infty = \max_{1\le\ell\le M}\ ||\frac{\partial u_\ell}{\partial x}||_\infty\ .$$

<u>Proof</u> : We consider the function $||x||^2$ on the curve $y^t_{x,k}(\tau)-y^t_{x+h,k}(\tau)$, thus we get by integration by part that

(6) $$||y_x(\tau)-y_{x+h}(\tau)||^2 = 2\int_\tau^t (y_{x+h}(s)-y_x(s)).[\lambda_k(s,y_x(s),u(s,y_x(s)))$$

$$- \lambda_k(s,y_{x+h}(s),u(s,y_{x+h}(s))]ds + ||h||^2$$

$$= 2\int_\tau^t (y_{x+h}(s)-y_x(s)).[\lambda_k(s,y_x(s),u(s,y_x(s)))$$

$$- \lambda_k(s,y_{x+h}(s),u(s,y_x(s)))]ds$$

$$+ 2 \int_\tau^t (y_{x+h}(s)-y_x(s)).[\lambda_k(s,y_{x+h}(s),$$

$$u(s,y_x(s)))-\lambda_k(s,y_{x+h}(s),u(s,y_{x+h}(s)))]$$

$$ds + ||h||^2$$

$$= I + II + ||h||^2 .$$

We estimate separately I and II. We have

$$I \le 2 \; ||\frac{\partial\lambda_k}{\partial x}||_\infty \int_\tau^t ||y_{x,k}(s)-y_{x+h,k}(s)||^2 ds \tag{7}$$

$$II \le 2 \sum_\ell ||\frac{\partial\lambda_k}{\partial u_\ell}||_\infty \int_\tau^t ||y_{x,k}(s)-y_{x+h,k}(s)|| \tag{8}$$

$$|u_\ell(s,y_x(s))-u_\ell(s,y_{x+h}(s))|ds$$

$$\le 2M \; ||\frac{\partial\lambda_k}{\partial u}||_\infty \; ||\frac{\partial u}{\partial x}||_\infty \int_\tau^t ||y_{x,k}(s)-y_{x+h,k}(s)||^2 ds .$$

Combining (7) and (8) we get that

$$||y_x(\tau) - y_{x+h}(\tau)||^2 \le$$

$$||h||^2 \exp[2(t-\tau)(||\frac{\partial\lambda_k}{\partial x}||_\infty + M \; ||\frac{\partial\lambda_k}{\partial u}||_\infty \; ||\frac{\partial u}{\partial x}||_\infty)]$$

which completes the proof.

Let us denote

$$X^t_{x,k}(\tau) = \exp \int_\tau^t c_k(s,y^t_{x,k}(s),u(s,y^t_{x,k}(s))ds \tag{9}$$

which satisfies the integral equation

$$X^t_{x,k}(\tau) = 1 + \int_\tau^t X^t_{x,k}(s)c_k(s,y^t_{x,k}(s),u(s,y^t_{x,k}(s)))ds \tag{10}$$

$$\forall \tau , \quad 0 \le \tau \le t \le T .$$

We have :

<u>Lemma 2</u> : Assume that (1), (3), (5) hold. Then we have

(11)
$$|X^t_{x,k}(\tau)-X^t_{x+h,k}(\tau)| \le$$
$$\exp[-(t-\tau)\alpha \int_\tau^t |c_k(s,y_x(s),u(s,y_x(s)))$$
$$- c_k(s,y_{x+h}(s),u(s,y_{x+h}(s)))|ds]$$
$$\forall\ h \in \mathbb{R}^N .$$

The proof of this result is easy using the fact that

$$|\exp a - \exp b| \le \max(\exp a,\exp b)|a-b|$$
$$\forall\ a,b \in \mathbb{R} .$$

II. <u>EXISTENCE RESULT</u>

To prove the existence of a generalized solution of (*) we shall use the successive approximations method.

For $w \in [W^{1,\infty}(\Pi_T)]^M$ let us denote

$$(Tw)_k(t,x) = X^t_{x,k}(0)v(y^t_{x,k}(0)) \quad , \quad 1 \le k \le M ,$$

where $y^t_{x,k}(\tau)$ is the solution of the integral equation

$$y^t_{x,k}(\tau) = x - \int_\tau^t \lambda_k(s,y^t_{x,k}(s),w(s,y^t_{x,k}(s)))ds$$
$$\forall\ \tau \quad , \quad 0 \le \tau \le t \le T .$$

We define the successive approximations as follows :

$$(12)\qquad \left| \begin{array}{l} (S^0(t)v)_k = v_k(x) \\ \\ (S^{n+1}(t)v)_k(x) = (TS^n(.)v)_k(t,x) \quad , \quad n \geq 0 \, . \end{array} \right.$$

We shall establish some a priori estimates, more precisely we have :

<u>Lemma 3</u> : Under assumptions (1), (2), (3) and

$$(13)\qquad \alpha \geq \gamma_1 + \gamma_2 \, M||v||_{W^{1,\infty}}$$

we have

$$(14)\qquad \frac{1}{||h||} \; |S^{n+1}(t)v(x) - S^{n+1}(t)v(x+h)| \leq ||v||_{W^{1,\infty}}$$

$$\forall \, n \geq 0 \quad , \quad \forall \, t \in [0,T] \quad , \quad \forall \, h,x \in \mathbb{R}^N \, ,$$

where

$$\gamma_1 = \max(||\frac{\partial\lambda}{\partial x}||_\infty \, , \, ||\frac{\partial c}{\partial x}||_\infty)$$

$$\gamma_2 = \max(||\frac{\partial\lambda}{\partial u}||_\infty \, , \, ||\frac{\partial c}{\partial u}||_\infty)$$

$$|x| = \max_{1 \leq i \leq N} |x_i| \qquad \forall \, x \in \mathbb{R}^N \, .$$

<u>Proof</u> : Indeed we have

$$(15)\qquad |(S(t)^{n+1}v)_k(x) - (S(t)^{n+1}v)_k(x+h)| =$$

$$|X^t_{x,k,n}(0)v_k(y^t_{x,k,n}(0)) - X^t_{x+h,k,n}(0)v_k(y^t_{x+h,k,n}(0))|$$

$$\leq |(X^t_{x,k,n}(0) - X^t_{x+h,k,n}(0))v_k(y^t_{x,k,n}(0))| +$$

$$|(v_k(y^t_{x,k,n}(0)) - v_k(y^t_{x+h,k,n}(0)))X^t_{x+h,k,n}(0)|$$

$$= \mathrm{I} + \mathrm{II} \, .$$

Then, using lemma 1, 2, we get that

$$I \le ||v||_\infty \exp(-\alpha t) . \int_0^t |c_k(s,y_x(s),v(s,y_x(s))$$

$$- c_k(s,y_{x+h}(s),v(s,y_{x+h}(s)))|ds$$

$$\le ||v||_\infty \exp(-\alpha t) \ . \ ||\frac{\partial c}{\partial x}||_\infty \int_0^t ||y_{x,k}(s)-y_{x+h,k}(s)||ds$$

$$+ M||v||_\infty \exp(-\alpha t) \ . \ ||\frac{\partial c}{\partial u}||_\infty \ ||\frac{\partial S(.)^n v}{\partial x}||_\infty \ .$$

$$\int_0^t ||y_{x,k}(s)-y_{x+h,k}(s)||ds \ .$$

Thus,

(16)
$$I \le ||v||_\infty \exp(-\alpha t).(||\frac{\partial c}{\partial x}||_\infty + M \ ||\frac{\partial c}{\partial u}||_\infty \ ||\frac{\partial S(.)^n v}{\partial x}||_\infty) \ .$$

$$\int_0^t ||y_{x,k}(s)-y_{x+h,k}(s)||ds \ .$$

On the other hand, we have

(17)
$$\int_0^t ||y_{x,k}(s)-y_{x+h,k}(s)||ds \le$$

$$||h|| \int_0^t \exp[(t-s)(||\frac{\partial \lambda_k}{\partial x}||_\infty + M||\frac{\partial \lambda_k}{\partial u}||_\infty \ ||\frac{\partial S(.)^n v}{\partial x}||_\infty \ ]ds \le$$

$$||h|| \int_0^t \exp[s(\gamma_1 + M\gamma_2 \ ||\frac{\partial S(.)^n v}{\partial x}||_\infty)]ds \ .$$

Hence,

(18)
$$\int_0^t ||y_{x,k}(s)-y_{x+h,k}(s)||ds \le ||h||(\gamma_1 + M \ \gamma_2 \ ||\frac{\partial S(.)^n v}{\partial x}||_\infty)^{-1} \ .$$

$$\exp[t(\gamma_1+M\gamma_2 \ ||\frac{\partial S(.)^n v}{\partial x}||_\infty)]$$

and then, we get

$$I \le ||h||\ ||v||_\infty \exp[(-\alpha+\gamma_1+M\gamma_2\ ||\frac{\partial S(.)^n v}{\partial x}||_\infty)t] \quad (19)$$

We also have, in a similar way

$$II \le ||\frac{\partial v}{\partial x}||_\infty\ ||h||\ \exp[(-\alpha+\gamma_1+M\gamma_2\ ||\frac{\partial S(.)^n v}{\partial x}||_\infty)t] \quad (20)$$

Combining (19) and (20) we obtain finally

$$|S^{n+1}(t)v(x)-S^{n+1}(t)v(x+h)| \le$$

$$||h||\ ||v||_{W^{1,\infty}} \exp[(-\alpha+\gamma_1+M\gamma_2\ ||\frac{\partial S^n(.)v}{\partial x}||_\infty)t] .$$

Let us denote

$$\text{(21)} \quad \text{and} \quad \left| \begin{array}{l} L_n(t) = ||\frac{\partial S^n(t)v}{\partial x}||_\infty \\ \\ \hat{L}_n = \sup\limits_{0\le t\le T} L_n(t) \end{array} \right.$$

thus we have proved that

$$L_{n+1}(t) \le ||v||_{W^{1,\infty}} \exp[(-\alpha+\gamma_1+M\gamma_2\hat{L}_n)t] .$$

On the other hand, we have

$$L_0(t) = ||\frac{\partial v}{\partial x}||_\infty \le ||v||_{W^{1,\infty}}$$

$$\hat{L}_0 \le ||v||_{W^{1,\infty}}$$

since (13) holds, we have

$$-\alpha + \gamma_1 + M\gamma_2\hat{L}_0 \le 0$$

and then

$$\hat{L}_1 \le ||v||_{W^{1,\infty}} .$$

By induction we see that

$$L_n(t) \le ||v||_{W^{1,\infty}}$$

which is exactely (14).

We have :

<u>Lemma 4</u> : Under conditions (1), (2), (3) and (13), we have the estimate

$$\frac{1}{|\theta|} |S^{n+1}(t+\theta)v(x)-S^{n+1}(t)v(x)| \le \tag{22}$$

$$||v||_{W^{1,\infty}} \max(||\lambda||_\infty,||c||_\infty)\exp(-\alpha t)$$

$$\forall\ t,\theta \in [0,T] \quad , \quad n \ge 0 \ , \quad \forall\ x \in \mathbb{R}^N .$$

<u>Proof</u> :

$$(S^{n+1}(t+\theta)v_k(x)-(S^{n+1}(t)v)_k(x) \tag{23}$$

$$= X^{t+\theta}_{x,k,n}(0)v_k(y^{t+\theta}_{x,k,n}(0)) - X^t_{x,k,n}(0)v_k(y^t_{x,k,n}(0))$$

$$= \int_t^{t+\theta} X^s_{x,k,n}(0) \left[\frac{\partial v_k}{\partial x}(y^s_{x,k,n}(0)).\lambda_k(s,y^s_{x,k,n}(0),(S^n(s)v)(y^s_{x,k,n}(0))\right)$$

$$+ c_k(s,y^s_{x,k,n}(0),(S^n(s)v)(y^s_{x,k,n}(0)))v_k(y^s_{x,k,n}(0))]ds .$$

Thus,

$$|(S^{n+1}(t+\theta)v)_k(x)-(S^{n+1}(t)v)_k(x)| \le$$

$$|\theta|(||\frac{\partial v_k}{\partial x}||_\infty \ ||\lambda_k||_\infty + ||v_k||_\infty \ ||c_k||_\infty) \exp(-\alpha t)$$

which implies (22).

Now, from the above a priori estimates we obtain the following existence result.

<u>Theorem 1</u> : Assume that (1), (2), (3) and (13) hold, then there exists a generalized solution to (*) with the initial condition $v(x)$ given by

$$u(t,x) = \lim_{n\to\infty} S^n(t)v(x) \tag{24}$$

uniformly on any compact subset of $\mathbb{R}^N$ and uniformly with respect to t.

<u>Proof</u> : Indeed, from the estimates (14), (22) and the uniform boundedness of the sequence $S^n(t)v(x)$, we get that, for some subsequence, there exists

$$u(t,x) \in C^0(\Pi_T)$$

such that (24) holds.

After some calculation, we have

$$\begin{aligned} &||y^t_{x,k,n}(\tau)-y^t_{x,k,m}(\tau)|| \le \\ &C_1T(\exp(C_2T))\ ||S^n(.)v-S^m(.)v||_\infty \\ &\forall\tau\ ,\ 0 \le \tau \le t \le T \end{aligned} \tag{25}$$

hence,

$$\lim_{m,n\to\infty} ||y^t_{x,k,m}(\tau)-y^t_{x,k,n}(\tau)|| = 0 \text{ uniformly in } (\tau,x)\ . \tag{26}$$

Thus, there exists $y^t_{x,k}(\tau)$ such that

(27) $$y^t_{x,k}(\tau) = x - \int_\tau^t \lambda_k(s,y^t_{x,k}(s),u(s,y^t_{x,k}(s)))ds$$

and we obtain that

(28) $$u_k(t,x) = X^t_{x,k}(0)v_k(y^t_{x,k}(0)) \quad , \quad 1 \le k \le M .$$

Finally from (28) and the fact that $u(t,x)$ is Lipschitz continuous we see that $u(t,x)$ is a generalized solution of (*).

III. UNIQUENESS RESULTS

We shall prove now that $u(t,x)$ is the unique generalized solution of (*).

Indeed, let $v(t,x)$ be another generalized solution of (*) and let

(29) $$y^t_{x,k,v}(\tau) = x - \int_\tau^t \lambda_k(s,y^t_{x,k,v}(s),v(s,y^t_{x,k,v}(s)))ds$$

then

(30) $$\begin{aligned} v_k(t,x) = {} & X^t_{x,k,v}(0)\, v_k(y^t_{x,k,v}(0)) + \\ & \int_0^t X^s_{x,k,v}(0)[\frac{\partial v_k}{\partial t}(s,y^s_{x,k,v}(0)) + \lambda_k(s,y^s_{x,k,v}(0),v(s,y^s_{x,k,v}(0))) \\ & \frac{\partial v_k}{\partial x}(s,y^s_{x,k,v}(0)) + c_k(s,y^s_{x,k,v}(0),v(s,y^s_{x,k,v}(0))) \\ & \qquad v_k(s,y^s_{x,k,v}(0))]\, ds \\ = {} & X^t_{x,k,v}(0) \end{aligned}$$

On the other hand, we have

(31) $$\begin{aligned} \|u(t)-v(t)\|_\infty \le {} & c_1 \int_0^t \|y^t_{x,k,u}(s)-y^t_{x,k,v}(s)\|ds + \\ & c_2 \int_0^t \|u(s)-v(s)\|_\infty\, ds \end{aligned}$$

and

(32) $$||y^t_{x,k,u}(\tau)-y^t_{x,k,v}(\tau)|| \leq$$

$$c_3 \int_\tau^t ||u(s)-v(s)||_\infty + c_4 \int_\tau^t ||y^t_{x,k,u}(s)-y^t_{x,k,v}(s)||ds \ .$$

Hence

(33) $$||u(t)-v(t)||_\infty \leq c_1 \ e^{c_2 t} \int_0^t ||y^t_{x,k,u}(s)-y^t_{x,k,v}(s)||ds$$

and

(34) $$||y^t_{x,k,u}(\tau)-y^t_{x,k,v}(\tau)|| \leq c_3 \ e^{c_2 t} \int_\tau^t ||u(s)-v(s)||_\infty \ ds \ .$$

Combining (33) and (34) yields

$$||u(t)-v(t)||_\infty \leq c_1 c_3 e^{(c_2+c_4)t} \int_0^t \int_s^t ||u(\bar{s})-v(\bar{s})||_\infty \ d\bar{s} \ ds$$

and then,

$$||u(t)-v(t)||_\infty \leq c_5 t \ e^{c_6 t} \int_0^t ||u(s)-v(s)||_\infty \ ds$$

this yields

(35) $$||u(t)-v(t)||_\infty = 0 \quad \forall \ t \in [0,T]$$

which completes the proof.

Remark 1 : We can prove similar results for the problems :

(36) $$\left| \begin{array}{l} \dfrac{\partial u_k}{\partial t} + \sum\limits_{i=1}^{N} \lambda^i_k(t,x,u)\dfrac{\partial u_k}{\partial x_i} + \sum\limits_{\ell=1}^{M} c_{k\ell}u_\ell = 0 \\ \\ u_k(0,x) = v_k(x) \end{array} \right.$$

under the supplementary condition :

(37) the matrix $(c_{k\ell})$ is commuting with the diagonal matrices (λ_k^i), $1 \le i \le N$.

Then using the same technic as above yields the result.

Remark 2 : Let us consider the quasilinear hyperbolic system

(38)
$$\left|\begin{aligned} &\frac{\partial u_1}{\partial t} - \frac{\partial u_2}{\partial x} = 0 \\ &\frac{\partial u_2}{\partial t} - \frac{\partial \sigma}{\partial x}(u_1) = 0 \end{aligned}\right.$$

which arises in fluid mechanics, nonlinear elasticity, and other fields of continuum mechanics. With the change of variables

(39)
$$u_2 = \frac{\partial w}{\partial t} \quad , \quad u_1 = \frac{\partial w}{\partial x}$$

(38) is equivalent to the second-order nonlinear wave equation

$$\frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial x}\,\sigma\left(\frac{\partial w}{\partial x}\right) = 0 .$$

For (38) it is known that there exist functions r and s called Riemann invariants (see [3], [4]) (*)

(40)
$$\left|\begin{aligned} &r = u_2 + \int_0^{u_1} \sigma'(v)^{1/2}\,dv \\ &s = u_2 - \int_0^{u_1} \sigma'(v)^{1/2}\,dv . \end{aligned}\right.$$

As we can see, r and s satisfy

(*) we assume $\sigma' > 0$.

(41)
$$\left| \begin{array}{l} \frac{\partial r}{\partial t} - \sigma'(u_1)^{1/2} \frac{\partial r}{\partial x} = 0 \\ \\ \frac{\partial s}{\partial t} + \sigma'(u_1)^{1/2} \frac{\partial s}{\partial x} = 0 \ . \end{array} \right.$$

Hence if we consider the perturbed problem

(42)
$$\left| \begin{array}{l} \frac{\partial r_\varepsilon}{\partial t} - \sigma'(u_{1\varepsilon})^{1/2} \frac{\partial r_\varepsilon}{\partial x} - \varepsilon\, r_\varepsilon = 0 \\ \\ \frac{\partial s_\varepsilon}{\partial t} + \sigma'(u_{1\varepsilon})^{1/2} \frac{\partial s_\varepsilon}{\partial x} - \varepsilon\, s_\varepsilon = 0 \end{array} \right.$$

then theorem 1 implies the existence and uniqueness of a generalized solution of (42) with smooth Cauchy data for ε large enough.

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