

# Group technology in production management : the short horizon planning level

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### **GROUP TECHNOLOGY IN PRODUCTION MANAGEMENT: THE SHORT HORIZON PLANNING LEVEL**

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GROUP TECHNOLOGY IN PRODUCTION MANAGEMENT :

THE SHORT HORIZON PLANNING LEVEL

Software TEC - G1

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ABSTRACT

We consider a set of parts divided into subsets called part types, determined in such a way that the parts belonging to the same part type are manufactured using the same sequence of tasks (i.e. the same working process). We are looking for a partition of the set of part types in subsets called part families, and for a partition of the set of tasks in subsets called production subsystems defined as follows :

- 1) the number of part families and the number of production subsystems are equal
- 2) one (and only one) production subsystem corresponds to each part family
- 3) one (and only one) part family corresponds to each production subsystem
- 4) the previous partitions minimize the number of tasks performed in a production subsystem different from the one which corresponds to the part family containing the part involved.

We give a fast algorithm which leads to a good solution depending on the initial set of part families. We also propose an algorithm to find a "good" initial set of part families.

## RESUME

Nous considérons un ensemble de pièces divisé en sous-ensembles appelés types de pièces et déterminés de manière que les pièces qui appartiennent au même type de pièces soient fabriquées en subissant la même séquence de tâches (i.e. la même gamme de fabrication). Nous cherchons à partitionner l'ensemble des types de pièces en familles de pièces et l'ensemble des tâches en sous-systèmes de production tels que les deux partitions contiennent le même nombre d'éléments, qu'il existe une application biunivoque entre l'ensemble des familles de pièces et l'ensemble des sous-systèmes de production et que ces partitions minimisent le nombre de tâches accomplies dans un sous-système de production différent du sous-système correspondant à la famille de pièces de la pièce concernée. Nous donnons un algorithme rapide qui conduit à une bonne solution qui dépend de l'ensemble initial de familles de pièces. Nous proposons également un algorithme qui conduit à un "bon" ensemble initial de familles de pièces.

## I. INTRODUCTION

In this paper, we call working process a sequence of tasks to perform in order to manufacture a part. We say that two parts belong to the same part type if their working processes are the same. Note that we don't take the processing times into account.

We are looking for a partition of the set of part types in subsets called part families, and for a partition of the set of tasks in subsets called production subsystems defined as follows : 1) the number of part families and the number of production subsystems are equal 2) one (and only one) production subsystem corresponds to each part family 3) one (and only one) part family corresponds to each production subsystem 4) these partitions minimize the number of tasks performed in a production subsystem different from the one which corresponds to the part family containing the part involved.

The objective is then to obtain production subsystems which are as independant as possible at the point of view of the production management, according to the set of parts to produce : following the above definition, each production subsystem can be considered approximately as an independant production system dedicated to perform the parts belonging to the corresponding part family. This reduce the size of the scheduling problem. The case of parts which visit production subsystems different from the ones corresponding to their part families is taken into account at the second step of the management process.

This paper is divided into five paragraphs. The problem is set in paragraph 2. In the paragraph 3, we give an algorithm to obtain a good solution of the problem starting from an initial partition of the set of part types. In the paragraph 4, we show how to obtain a good initial partition of the set of part types. Finally, the paragraph 5 gives an example with 60 part types and 12 tasks.

## II. SETTING OF THE PROBLEM

We consider a set  $X$  of  $n$  part types and a set  $Y$  of  $m$  tasks.

We define the matrix :

$$A = [a_{i,j}], \quad i=1,2,\dots,n \text{ and } j=1,2,\dots,m$$

as follows :

$$a_{i,j} = \begin{cases} 1 & \text{if the working process of a part} \\ & \text{belonging to the part type } i \text{ con-} \\ & \text{tains the task } j. \\ 0 & \text{if not.} \end{cases} \quad (1)$$

Let  $p$  be the number of part families and of production subsystems to consider. This integer is chosen by the user.

If  $\mathcal{X} = \{X_1, \dots, X_p\}$  and  $\mathcal{Y} = \{Y_1, \dots, Y_p\}$  are respectively a partition of  $X$  and a partition of  $Y$ , we define the sets  $W_{r,s}$ ,  $r=1, \dots, p$  and  $s=1, \dots, p$  as follows :

$$W_{r,s} = \{(i,j) \in (1, \dots, n) \times (1, \dots, m) / i \in X_r \text{ and } j \in Y_s\} \quad (2)$$

For a given value of  $p$ ,  $\mathcal{X}^* = \{X_1^*, \dots, X_p^*\}$  and  $\mathcal{Y}^* = \{Y_1^*, \dots, Y_p^*\}$  are said to be the optimal solution of the problem if they verify :

$$\begin{aligned} & \sum_{r=1}^p \sum_{(i,j) \in W_{r,r}^*} u_i a_{i,j} + \sum_{r=1}^p \sum_{\substack{s=1 \\ s \neq r}}^p \sum_{(i,j) \in W_{r,s}^*} u_i (1 - a_{i,j}) = \\ & = \text{Max}_{W \in \mathcal{D}} \left[ \sum_{r=1}^p \sum_{(i,j) \in W_{r,r}} u_i a_{i,j} + \sum_{r=1}^p \sum_{\substack{s=1 \\ r \neq s}}^p \sum_{(i,j) \in W_{r,s}} u_i (1 - a_{i,j}) \right] \end{aligned} \quad (3)$$

where :

$u_i$  is the weight of the part type  $i$  (for instance, the proportion of parts of type  $i$  to produce during a given period)

$W_{r,s}^*$ ,  $r=1,2,\dots,p$  and  $s=1,2,\dots,p$  are built using (2), starting from  $\mathcal{X}^*$  and  $\mathcal{Y}^*$ .

$$\mathcal{W} = \{W_{r,s}\}_{r=1,\dots,p; s=1,\dots,p}$$

$\mathcal{D}$  is the set of admissible partitions  $\mathcal{W}$  (i.e. the partitions of  $(1,\dots,n) \times (1,\dots,m)$  which have  $p^2$  éléments).

Note that the maximal value of this criterion is  $m \sum_{i=1}^n u_i$ .

We also can set the problem as follows :

Starting from matrix  $A$ , build a matrix  $B$  by permutation of the rows and the columns of  $A$  in order to obtain  $p$  diagonal blocks  $E_1^*, \dots, E_p^*$  such that :

$$\sum_{\substack{(i,j) \in U \\ r=1}}^p u_i b_{i,j} + \sum_{\substack{(i,j) \notin U \\ r=1}}^p u_i (1-b_{i,j}) \quad (4)$$

where :

$B = (b_{i,j})$ ;  $i=1,2,\dots,n$  and  $j=1,2,\dots,m$ .

$u_i$  is the weight of the part type which corresponds to the row  $i$  of the matrix  $B$ .

### III. A GOOD SOLUTION STARTING FROM A PARTITION OF THE PART TYPES

#### III.1. A basic result

We first prove the following result.

#### THEOREM I

Let  $\mathcal{X}^k = \{x_1^k, x_2^k, \dots, x_p^k\}$  and  $\mathcal{Y}^k = \{y_1^k, y_2^k, \dots, y_p^k\}$  be a partition of  $X$  and  $Y$  respectively.



1. For  $j=1,2,\dots,m$ , we consider :

$$a_r^k(j) = \sum_{i \in X_r^k} u_i a_{i,j} + \sum_{i \notin X_r^k}^n u_i (1-a_{i,j}) \text{ for } r=1,\dots,p.$$

$r^k(j)$  is the integer which verifies : (5)

$$a_{r^k(j)}^k(j) = \text{Max}_{r=1,\dots,p} a_r^k(j)$$

We assign the task  $j$  to the production subsystem  $Y_{r^k(j)}^{k+1}$

Finally, we denote by  $\mathcal{Y}^{k+1}$  the partition :

$$\mathcal{Y}^{k+1} = \{Y_1^{k+1}, Y_2^{k+1}, \dots, Y_p^{k+1}\}$$

(some of the production subsystems  $Y_r^{k+1}$  may be empty).

2. For  $i=1,2,\dots,n$ , we consider :

$$b_r^k(i) = \sum_{j \in Y_r^{k+1}} a_{i,j} + \sum_{j \notin Y_r^{k+1}}^m (1-a_{i,j}) \text{ for } r=1,\dots,p.$$

$r^k(i)$  is the integer which verifies : (6)

$$b_{r^k(i)}^k(i) = \text{Max}_{r=1,\dots,p} b_r^k(i)$$

We assign the part type  $i$  to the part family  $X_{r^k(i)}^{k+1}$

Finally, we denote by  $\mathcal{X}^{k+1}$  the partition :

$$\mathcal{X}^{k+1} = \{X_1^{k+1}, X_2^{k+1}, \dots, X_p^{k+1}\}$$

Then, either the solution  $(\mathcal{X}^{k+1}, \mathcal{Y}^{k+1})$  is better than the solution  $(\mathcal{X}^k, \mathcal{Y}^k)$ , or the value of the criterion (see(3)) is the same for both solutions.

Proof :

1. Let us first consider the way to obtain  $\mathcal{Y}^{k+1}$  starting from  $\mathcal{X}^k$  (see (5)). We see that the partition  $\mathcal{X}^{k+1}$  is one of the partitions which lead to the greatest value of the criterion (see(3)) knowing  $\mathcal{X}^k$ . Then the value of the criterion for  $(\mathcal{Y}^{k+1}, \mathcal{X}^k)$  is greater than or equal to the value of the criterion for  $(\mathcal{Y}^k, \mathcal{X}^k)$ .

2. Let us now consider (6). We see that the partition  $\mathcal{X}^{k+1}$  is one of the partitions which lead to the greatest value of the criterion knowing  $\mathcal{Y}^{k+1}$ . Then the value of the criterion for  $(\mathcal{Y}^{k+1}, \mathcal{X}^{k+1})$  is greater than or equal to the value of the criterion for  $(\mathcal{Y}^{k+1}, \mathcal{X}^k)$ .

3. Finally, the value of the criterion for  $(\mathcal{Y}^{k+1}, \mathcal{X}^{k+1})$  is greater than or equal to the value of the criterion for  $(\mathcal{Y}^k, \mathcal{X}^k)$ .  $\square$

### III.2. The algorithm

We suppose that we know a partition of the part types (i.e. a set of part families) denoted by  $\mathcal{X}^0$ . We first compute  $\mathcal{Y}^0$  using (5) with  $k=0$ , then  $\mathcal{X}^1$  using (6) with  $k=0$ , then  $\mathcal{Y}^1$  using (5) with  $k=1$ , then  $\mathcal{X}^2$  using (6) with  $k=1$  and so on.

We stop the computation when two consecutive solutions lead to the same value for the criterion.

Remark :

This algorithm leads to a solution which depends on the initial set of part families.

For instance, consider the matrix A (fig.1).

|       |   |         |   |         |   |   |   |   |   |
|-------|---|---------|---|---------|---|---|---|---|---|
|       |   | $Y_1$   |   | $Y_2$   |   |   |   |   |   |
| $X_1$ | { | 1       | 1 | 1       | 1 | 0 | 0 | 0 | 0 |
|       |   | 1       | 1 | 1       | 1 | 1 | 0 | 0 | 0 |
|       |   | 1       | 1 | 1       | 1 | 0 | 0 | 0 | 0 |
|       |   | 1       | 0 | 1       | 0 | 0 | 0 | 1 | 1 |
|       |   | 1       | 1 | 0       | 0 | 0 | 1 | 1 | 0 |
| $X_2$ | { | 0       | 0 | 0       | 0 | 1 | 1 | 1 | 1 |
|       |   | 0       | 0 | 0       | 0 | 1 | 1 | 1 | 1 |
|       |   | 0       | 0 | 0       | 0 | 0 | 1 | 1 | 1 |
|       |   | 0       | 0 | 0       | 1 | 0 | 1 | 1 | 1 |
|       |   | $Y_1^*$ |   | $Y_2^*$ |   |   |   |   |   |

Fig.1

Let be :

$$\mathcal{Y} = \{Y_1, Y_2\}, \mathcal{X} = \{X_1, X_2\}$$

$$\mathcal{Y}^* = \{Y_1^*, Y_2^*\}, \mathcal{X}^* = \{X_1^*, X_2^*\}$$

The solution  $(\mathcal{X}, \mathcal{Y})$  is stable. It means that applying (5) with  $\mathcal{X}$  leads to  $\mathcal{Y}$  and applying (6) with  $\mathcal{Y}$  leads to  $\mathcal{X}$ . The value of the criterion is 57 for this solution. The criterion takes the value 60 for the solution  $(\mathcal{X}^*, \mathcal{Y}^*)$ : therefore  $(\mathcal{X}, \mathcal{Y})$  is not optimal.

A consequence of the previous remark is that we have to execute a number of trials to be sure to obtain a solution close to the optimal one.

We now give an algorithm which leads to an initial set of part families which seems to be a good initial set.

#### IV. THE INITIAL SET OF PART FAMILIES

Each part family is an element of  $\mathbb{R}^m$ . A distance  $d$  is given on  $\mathbb{R}^m$ . We denote by  $x_i, i=1,2,\dots,n$ , the part families, and  $u_i$  the weight of  $x_i$ .

The goal consists in partitioning  $E = \{x_1, x_2, \dots, x_n\}$  in  $p$  subsets. Two elements of the same subset are close in the sense of the distance  $d$ .

We try to minimize the sum of the inertia of the subsets, i.e. :

$$\sum_{k=1}^p \sum_{i/x_i \in e^k} u_i d^2(x_i, y^k) \quad (7)$$

where :

$y^k$  is the inertia center of  $e^k$ ,  $k=1,2,\dots,p$

and :  $\{e^1, e^2, \dots, e^p\}$  is the partition.

THEOREM II

Let  $x_1^0, x_2^0, \dots, x_p^0$  ( $p \leq n$ ) be some points of  $\mathbb{R}^m$ , belonging to E or not.

We consider the following algorithm, called A1 :

1. Set  $k=0$ .
2. Compute a partition  $e_1^k, e_2^k, \dots, e_p^k$  of E as follows :
  - for  $i=1,2,\dots,n$  :
    - a. Find the set  $J_i \subset \{1,2,\dots,p\}$  defined as follows :
 
$$J_i = \{j \in \{1,2,\dots,p\} / d(x_i, x_j^k) = \text{Min}_{s=1,\dots,p} d(x_i, x_s^k)\}$$
    - b. If  $\text{card}(J_i)=1$ ,  $x_i$  is assigned to  $e_{j_1}^k$ , if not  $x_i$  is assigned to  $e_{j_1}^k$  where  $j_1 = \text{Min}_{j \in J_i} j$ .
3. For  $i=1,2,\dots,p$ , compute  $x_j^{k+1}$ , inertia center of  $e_j^k$  (see(7)).
4. Test.
  - 4.1. If  $x_j^{k+1} = x_j^k$  for  $j=1,2,\dots,p$ , end of the process.
  - 4.2. Else, set  $k=k+1$  and go to 2.

The algorithm A1 converges.

Proof :

We define :

$$I(y, E) = \sum_{x \in E} u_x d^2(y, x), \text{ where } u_x \text{ is the weight of } x \text{ and } E \text{ a}$$

finite set.

If  $x_j^{k+1} \neq x_j^k$  for at least one  $j \in \{1, 2, \dots, p\}$ , then :

$$\sum_{j=1}^p I(x_j^{k+1}, e_j^k) < \sum_{j=1}^p I(x_j^k, e_j^k) \quad (8)$$

(see step 3 of the algorithm)

Furthermore :

$$\sum_{j=1}^p I(x_j^k, e_j^k) \leq \sum_{j=1}^p I(x_j^k, e_j^{k-1}) \quad (9)$$

(see step 2 of the algorithm and the relation (7))

Finally, considering the relations (8) and (9) :

$$\sum_{j=1}^p I(x_j^{k+1}, e_j^k) < \sum_{j=1}^p I(x_j^k, e_j^{k-1}) \quad (10)$$

This complete the proof. □

Using theorem II, we can easily obtain an algorithm which leads to a "good" partition of  $E$  starting from a set  $x_1^0, x_2^0, \dots, x_p^0$  of initial points.

We summarize the algorithm which leads to this set of points.

It consists in finding  $p$  points which are :

1. located in high density zones of  $\mathbb{R}^m$  (i.e. in zones where are located a great number of points).
2. far one from the others.

V. AN EXAMPLE

The matrix to transform is given in figure 2. It concerns 60 part types and 12 tasks. The first column represents the part type numbers, and the last one contains the weights.

|    |                            |    |                            |
|----|----------------------------|----|----------------------------|
| 1  | 0.1.0.1.0.0.0.0.0.0.1.3.   | 35 | 0.0.1.0.1.0.0.0.0.0.0.1.   |
| 2  | 1.0.0.0.1.0.0.0.0.1.1.0.3. | 36 | 1.0.1.0.0.0.1.0.0.1.0.1.2. |
| 3  | 0.0.0.1.0.1.0.0.1.0.0.0.1. | 37 | 0.0.0.0.0.0.0.1.0.1.0.0.4. |
| 4  | 1.0.0.0.0.1.0.0.1.1.0.0.2. | 38 | 0.0.0.0.1.0.0.0.0.0.0.5.   |
| 5  | 1.1.0.1.0.0.0.1.0.0.0.1.2. | 39 | 0.0.0.0.0.1.0.0.0.0.0.1.   |
| 6  | 0.0.0.0.0.0.0.1.0.0.0.0.2. | 40 | 1.0.1.0.1.0.0.0.0.0.0.3.   |
| 7  | 0.1.0.0.0.0.1.0.1.1.0.1.2. | 41 | 1.0.0.0.0.1.0.0.0.0.1.0.3. |
| 8  | 0.0.0.0.0.0.0.0.0.1.0.1.4. | 42 | 0.1.0.1.0.0.0.1.0.0.0.0.1. |
| 9  | 1.0.0.0.0.0.0.0.0.0.1.1.1. | 43 | 0.0.1.0.1.0.0.0.0.1.0.0.2. |
| 10 | 0.0.0.0.0.0.0.0.0.1.0.0.1. | 44 | 0.1.0.0.0.0.0.1.0.1.0.0.1. |
| 11 | 0.0.0.0.0.0.0.0.1.0.0.0.3. | 45 | 0.0.0.1.0.1.0.1.0.0.0.0.1. |
| 12 | 0.0.0.0.0.1.0.0.1.1.0.0.1. | 46 | 0.0.1.0.1.0.0.0.0.1.0.0.3. |
| 13 | 0.1.0.1.0.0.0.1.0.0.0.0.1. | 47 | 0.1.0.0.0.0.0.1.0.0.0.0.1. |
| 14 | 0.0.0.0.0.1.1.0.1.0.1.0.2. | 48 | 0.1.0.1.0.0.0.1.1.0.0.0.4. |
| 15 | 0.1.0.0.0.0.0.1.0.0.0.0.1. | 49 | 0.0.0.0.0.0.1.0.0.1.0.0.1. |
| 16 | 0.0.0.1.0.0.0.1.0.0.0.1.2. | 50 | 0.0.0.0.0.1.0.0.1.0.0.0.1. |
| 17 | 0.1.0.0.0.0.1.0.0.1.0.1.1. | 51 | 0.1.0.1.0.0.0.0.0.0.0.1.3. |
| 18 | 0.1.0.0.1.1.0.0.1.0.0.0.1. | 52 | 1.0.0.0.1.0.0.0.0.1.1.0.3. |
| 19 | 0.1.0.0.0.1.0.1.0.0.0.0.5. | 53 | 0.0.0.1.0.1.0.0.1.0.0.0.1. |
| 20 | 0.0.0.0.0.1.1.1.1.0.0.0.2. | 54 | 1.0.0.0.0.1.0.0.1.1.0.0.2. |
| 21 | 0.0.0.0.1.0.1.1.0.1.0.1.4. | 55 | 1.1.0.1.0.0.0.1.0.0.0.1.2. |
| 22 | 0.0.0.0.0.1.0.0.1.0.1.0.3. | 56 | 0.0.0.0.0.0.0.1.0.0.0.0.2. |
| 23 | 1.0.1.0.0.0.1.0.1.0.0.1.3. | 57 | 0.1.0.0.0.0.1.0.1.1.0.1.2. |
| 24 | 0.1.0.0.0.0.0.0.1.0.0.0.3. | 58 | 0.0.0.0.0.0.0.0.0.1.0.1.4. |
| 25 | 0.0.1.0.1.0.1.0.0.0.0.0.2. | 59 | 1.0.0.0.0.0.0.0.0.0.1.1.1. |
| 26 | 1.0.0.0.0.0.0.0.0.0.1.0.4. | 60 | 0.0.0.0.0.0.0.0.0.1.0.0.1. |
| 27 | 0.1.1.0.1.0.1.0.0.0.0.0.2. |    |                            |
| 28 | 0.0.0.0.0.0.0.0.0.1.0.1.3. |    |                            |
| 29 | 0.0.1.0.0.0.1.0.1.1.1.0.3. |    |                            |
| 30 | 0.1.0.1.0.0.0.1.0.0.0.1.2. |    |                            |
| 31 | 0.0.0.1.0.0.0.1.0.0.0.0.1. |    |                            |
| 32 | 0.0.0.0.0.1.0.0.1.0.0.1.3. |    |                            |
| 33 | 0.1.0.1.0.0.0.1.0.0.0.0.1. |    |                            |
| 34 | 0.0.0.0.0.0.1.0.0.1.1.1.2. |    |                            |

Figure 2

The algorithm leads to the matrix of figure 3 if 5 part families are requested. The first column and the first row contain the class numbers. The second column contains the part type numbers.

The second row contains the column numbers of the initial matrix.

Value of the criterion : 1369

|   |    | 1  | 1  | 1  | 2  | 2  | 2  | 3  | 3  | 4  | 4  | 5  | 5  |    |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|   |    | 8  | 4  | 2  | 12 | 10 | 7  | 9  | 6  | 5  | 3  | 11 | 1  |    |
| 1 | 1  | 0. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3. |
| 1 | 5  | 1. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 2. |
| 1 | 6  | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 2. |
| 1 | 13 | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 1 | 15 | 1. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 1 | 16 | 1. | 1. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 2. |
| 1 | 19 | 1. | 0. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 5. |
| 1 | 30 | 1. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 2. |
| 1 | 31 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 1 | 33 | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 1 | 42 | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 1 | 44 | 1. | 0. | 1. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 1 | 45 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. |
| 1 | 47 | 1. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 1 | 48 | 1. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 4. |
| 1 | 51 | 0. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3. |
| 1 | 55 | 1. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 2. |
| 1 | 56 | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 2. |
| 2 | 34 | 0. | 0. | 0. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 2. |
| 2 | 36 | 0. | 0. | 0. | 1. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 1. | 2. |
| 2 | 37 | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4. |
| 2 | 7  | 0. | 0. | 1. | 1. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 2. |
| 2 | 21 | 1. | 0. | 0. | 1. | 1. | 1. | 0. | 0. | 1. | 0. | 0. | 0. | 4. |
| 2 | 23 | 0. | 0. | 0. | 1. | 0. | 1. | 1. | 0. | 0. | 1. | 0. | 1. | 3. |
| 2 | 28 | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3. |
| 2 | 29 | 0. | 0. | 0. | 0. | 1. | 1. | 1. | 0. | 0. | 1. | 1. | 0. | 3. |
| 2 | 49 | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 2 | 8  | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4. |
| 2 | 10 | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 2 | 17 | 0. | 0. | 1. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 2 | 57 | 0. | 0. | 1. | 1. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 2. |

|   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 2 | 58 | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4. |
| 2 | 60 | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 3 | 11 | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 3. |
| 3 | 12 | 0. | 0. | 0. | 0. | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 1. |
| 3 | 18 | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 1. | 1. | 0. | 0. | 0. | 1. |
| 3 | 4  | 0. | 0. | 0. | 0. | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 2. |
| 3 | 32 | 0. | 0. | 0. | 1. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 3. |
| 3 | 50 | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 1. |
| 3 | 20 | 1. | 0. | 0. | 0. | 0. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 2. |
| 3 | 53 | 0. | 1. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 1. |
| 3 | 54 | 0. | 0. | 0. | 0. | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 2. |
| 3 | 14 | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. | 0. | 0. | 1. | 0. | 2. |
| 3 | 22 | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 1. | 0. | 3. |
| 3 | 3  | 0. | 1. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 1. |
| 3 | 39 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. |
| 3 | 24 | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 3. |
| 4 | 38 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 5. |
| 4 | 46 | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 3. |
| 4 | 35 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 1. |
| 4 | 40 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 1. | 3. |
| 4 | 27 | 0. | 0. | 1. | 0. | 0. | 1. | 0. | 0. | 1. | 1. | 0. | 0. | 2. |
| 4 | 43 | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 2. |
| 4 | 25 | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 1. | 1. | 0. | 0. | 2. |
| 5 | 26 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 4. |
| 5 | 2  | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 1. | 1. | 3. |
| 5 | 52 | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 1. | 1. | 3. |
| 5 | 9  | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. |
| 5 | 59 | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. |
| 5 | 41 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 1. | 1. | 3. |

Figure 3



The figure 4 gives the result obtained in the case of 10 part families are requested. Note that the algorithm leads to only 7 part families and production subsystems.

Value of the criterion : 1378

|   |    | 2  | 2  | 3  | 3  | 4  | 4  | 5  | 6  | 9  | 9  | 10 | 10 |    |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|   |    | 12 | 10 | 4  | 2  | 5  | 3  | 7  | 8  | 11 | 1  | 9  | 6  |    |
| 2 | 7  | 1. | 1. | 0. | 1. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 2. |
| 2 | 8  | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4. |
| 2 | 10 | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 2 | 17 | 1. | 1. | 0. | 1. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 1. |
| 2 | 21 | 1. | 1. | 0. | 0. | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 4. |
| 2 | 28 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3. |
| 2 | 34 | 1. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 1. | 0. | 0. | 0. | 2. |
| 2 | 36 | 1. | 1. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 1. | 0. | 0. | 2. |
| 2 | 57 | 1. | 1. | 0. | 1. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 2. |
| 2 | 58 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4. |
| 2 | 60 | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 3 | 5  | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 1. | 0. | 0. | 2. |
| 3 | 1  | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3. |
| 3 | 42 | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. |
| 3 | 48 | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 1. | 0. | 4. |
| 3 | 51 | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3. |
| 3 | 55 | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 1. | 0. | 0. | 2. |
| 3 | 13 | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. |
| 3 | 30 | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 2. |
| 3 | 33 | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. |
| 4 | 46 | 0. | 1. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 3. |
| 4 | 35 | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 4 | 25 | 0. | 0. | 0. | 0. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 2. |
| 4 | 38 | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 5. |
| 4 | 40 | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 3. |
| 4 | 27 | 0. | 0. | 0. | 1. | 1. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 2. |
| 4 | 43 | 0. | 1. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 2. |
| 5 | 49 | 0. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 1. |
| 5 | 29 | 0. | 1. | 0. | 0. | 0. | 1. | 1. | 0. | 1. | 0. | 1. | 0. | 3. |
| 5 | 23 | 1. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 1. | 1. | 0. | 3. |
| 6 | 6  | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 2. |

|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 6  | 47 | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. |
| 6  | 37 | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 4. |
| 6  | 31 | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. |
| 6  | 19 | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 5. |
| 6  | 15 | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. |
| 6  | 56 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 2. |
| 6  | 16 | 1. | 0. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 2. |
| 6  | 44 | 0. | 1. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. |
| 6  | 45 | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 1. |
| 9  | 2  | 0. | 1. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 3. |
| 9  | 26 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 4. |
| 9  | 41 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 1. | 3. |
| 9  | 9  | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 1. |
| 9  | 59 | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 1. |
| 9  | 52 | 0. | 1. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 3. |
| 10 | 12 | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. |
| 10 | 18 | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. |
| 10 | 3  | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. |
| 10 | 50 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. |
| 10 | 39 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. |
| 10 | 20 | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 1. | 1. | 2. |
| 10 | 53 | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. |
| 10 | 54 | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. | 2. |
| 10 | 14 | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 1. | 0. | 1. | 1. | 2. |
| 10 | 22 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 1. | 1. | 3. |
| 10 | 32 | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 3. |
| 10 | 4  | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. | 2. |
| 10 | 24 | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 3. |
| 10 | 11 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 3. |

Figure 4

The last trial has been made with 15 requested part families.  
 The figure 5 gives the results : finally, only 7 part families and  
 production subsystems have been obtained.

Value of the criterion : 1371.

|   |    | 1  | 1  | 2  | 2  | 4  | 4  | 5  | 8  | 8  | 11 | 15 | 15 |    |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|   |    | 12 | 10 | 8  | 4  | 11 | 1  | 7  | 9  | 6  | 2  | 5  | 3  |    |
| 1 | 7  | 1. | 1. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 1. | 0. | 0. | 2. |
| 1 | 8  | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4. |
| 1 | 10 | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 1 | 17 | 1. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 1. | 0. | 0. | 1. |
| 1 | 21 | 1. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 4. |
| 1 | 28 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3. |
| 1 | 34 | 1. | 1. | 0. | 0. | 1. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 2. |
| 1 | 36 | 1. | 1. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 1. | 2. |
| 1 | 57 | 1. | 1. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 1. | 0. | 0. | 2. |
| 1 | 58 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4. |
| 1 | 60 | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 2 | 31 | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 2 | 33 | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 1. |
| 2 | 16 | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 2. |
| 2 | 6  | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 2. |
| 2 | 37 | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4. |
| 2 | 42 | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 1. |
| 2 | 45 | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. |
| 2 | 48 | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 0. | 1. | 0. | 0. | 4. |
| 2 | 55 | 1. | 0. | 1. | 1. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 2. |
| 2 | 56 | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 2. |
| 2 | 5  | 1. | 0. | 1. | 1. | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 0. | 2. |
| 2 | 13 | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 1. |
| 2 | 30 | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 2. |
| 4 | 41 | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 1. | 0. | 0. | 0. | 3. |
| 4 | 2  | 0. | 1. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 3. |
| 4 | 26 | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 4. |
| 4 | 9  | 1. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |
| 4 | 59 | 1. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. |

|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 4  | 52 | 0. | 1. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 3. |
| 5  | 29 | 0. | 1. | 0. | 0. | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 1. | 3. |
| 5  | 49 | 0. | 1. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 1. |
| 5  | 23 | 1. | 0. | 0. | 0. | 0. | 1. | 1. | 1. | 0. | 0. | 0. | 1. | 3. |
| 8  | 18 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. | 1. | 0. | 1. |
| 8  | 20 | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 1. | 1. | 0. | 0. | 0. | 2. |
| 8  | 32 | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 3. |
| 8  | 50 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 1. |
| 8  | 12 | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 1. |
| 8  | 53 | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 1. |
| 8  | 54 | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 2. |
| 8  | 22 | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 3. |
| 8  | 3  | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 0. | 1. |
| 8  | 14 | 0. | 0. | 0. | 0. | 1. | 0. | 1. | 1. | 1. | 0. | 0. | 0. | 2. |
| 8  | 39 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. |
| 8  | 4  | 0. | 1. | 0. | 0. | 0. | 1. | 0. | 1. | 1. | 0. | 0. | 0. | 2. |
| 8  | 11 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 3. |
| 11 | 47 | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 1. |
| 11 | 15 | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 1. |
| 11 | 24 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 1. | 0. | 0. | 3. |
| 11 | 1  | 1. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 3. |
| 11 | 51 | 1. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 3. |
| 11 | 44 | 0. | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 1. |
| 11 | 19 | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 0. | 0. | 5. |
| 15 | 40 | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 0. | 1. | 1. | 3. |
| 15 | 27 | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 1. | 1. | 1. | 2. |
| 15 | 25 | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 0. | 0. | 1. | 1. | 2. |
| 15 | 43 | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 2. |
| 15 | 38 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 0. | 5. |
| 15 | 35 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 1. |
| 15 | 46 | 0. | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 1. | 1. | 3. |

## Main Program

```

1 c
2 c     programme mumu -> remplissage manuel du fichier 70
3 c     programme alea -> remplissage aleatoire du fichier 70
4 c     programme umum -> verification du contenu du fichier 70
5 c
6 c     le fichier 70 est deja rempli pour l'exemple demande (15,55)
7 c     le fichier 50 -> resultats
8 c
9 c     CONTRAINTES ACTUELLES DU PROGRAMME
10 c----Nombre maxi. d'observations : 1999
11 c----Nombre maxi. de classes      : 50
12 c----Nombre maxi. de parametres   : 19
13     dimension a(50,20),aini(50,20),iz(2000),ds(50),it(500),b1(20),b2(20)
14     dimension kz(20),jz(20),bj(50),kr(2000),kw(20)
15     open(70,access="direct",form="formatted",recl=2000)
16     open(63,access="direct",form="formatted",recl=2000)
17     open(50,form="formatted")
18     read(70,1,rec=1,err=27)n,m
19 c
20     eps=.001
21     write(0,70)n,m
22     70 format("_Nombre d'observations",i4," _Nombre de parametres",i4)
23 c
24     1 format(2i4)
25     21 write(0,5)
26     5 format(2x,"_Nombre de classes ? ")
27     read(0,3)ip
28     3 format(v)
29     write(0,200)
30     200 format(12x,"<< CHOIX DES DISTANCES POUR LA CONSTITUTION DES CLASSES
31     &>>",//)
32     write(0,201)
33     201 format(2x,"_Distance euclidienne ;.....tapez 1 ",//)
34     read(0,3)idis
35     write(0,400)
36     400 format(12x,"<< CHOIX DES REPRESENTANTS >>")
37     write(0,401)
38     401 format(2x,"centre d'inertie; tapez 1")
39     read(0,3)irep
40 c
41 c
42 c     *****
43 c     *           PHASE D'INITIALISATION           *
44 c     *****
45 c
46 c     Recherche du premier point.
47 c
48     write(0,2)
49     2 format(2x,"_Choix du rayon pour le calcul des densites. ")
50     read(0,3)r
51     8 write(0,4)
52     4 format(2x,"_Choix de la distance minimale entre points initiaux. ")

```

```

53      read(0,3)d1
54      if(d1.lt.r) goto 8
55      call random_$uniform(x)
56      if(x.gt.(1.-1.e-4))x=1.-1.e-4
57      k=x*n+1
58      k1=(k-.5)/50+2
59      k2=k-(k1-2)*50
60      k3=n-(k1-2)*50
61      if(k3.gt.50) k3=50
62      read(70,6,rec=k1,err=13)((a(k,j),j=1,m),k=1,k3)
63      6 format(1000f2.0)
64      jj=1
65      do 7 j=1,m
66      b1(j)=a(k2,j)
67      7 aini(1,j)=a(k2,j)
68 c
69 c
70 c
71 c
72      do 12 j=1,50
73      12 ds(j)=0
74      ii=0
75      do 9 k=1,n
76      k1=(k-.5)/50+2
77      k2=k-(k1-2)*50
78      if(k2.gt.1)go to 10
79      k3=n-(k1-2)*50
80      if(k3.gt.50) k3=50
81      read(70,6,rec=k1,err=26)((a(i,j),j=1,m),i=1,k3)
82      10 do 11 j=1,m
83      b2(j)=a(k2,j)
84      11 continue
85      call dist(b1,b2,m,idis,y)
86      if(y.le.r) ds(jj)=ds(jj)+1
87      if(y.lt.d1) goto 9
88      ii=ii+1
89      it(ii)=k
90      9 continue
91 c
92 c
93 c
94 c
95 c
96 c
97 c
98 c
99      18 call rando._$uniform(x)
100     if(x.gt.(1.-1.e-4))x=1.-1.e-4
101     i11=x*ii+1
102     k=it(i11)
103     k1=(k-.5)/50+2
104     k2=k-(k1-2)*50
105     k3=n-(k1-2)*50
106     if(k3.gt.50) k3=50
107     read(70,6,rec=k1,err=28)((a(k,j),j=1,m),k=1,k3)
108     jj=jj+1
109     do 14 j=1,m
110     b1(j)=a(k2,j)
111     aini(jj,j)=a(k2,j)
112     14 continue

```

Recherche de la densite du premier  
point et des points suivants.

```

*****
*                ITERATIONS                *
*****

```

Choix du point suivant

Recherche de la densite du point  
et des points a retenir.

```
113 c
114 c
115 c
116 c
117 c
118     i11=0
119     do 15 k5=1,ii
120     k=it(k5)
121     k1=(k-.5)/50+2
122     k2=k-(k1-2)*50
123     k3=n-(k1-2)*50
124     if(k3.gt.50) k3=50
125     read(70,6,rec=k1,err=29)((a(i,j),j=1,m),i=1,k3)
126     do 17 j=1,m
127 17  b2(j)=a(k2,j)
128     call dist(b1,b2,m,idis,y)
129     if(y.le.r) ds(jj)=ds(jj)+1
130     if(y.lt.dl) goto 15
131     i11=i11+1
132     it(i11)=k
133 15  continue
134     ii=i11
135     if(ii.gt.0) goto 18
136     if(jj.ge.ip) goto 22
137     write(0,19)
138 19  format(2x,"_Nombre de points initiaux insuffisant .")
139     write(0,20)
140 20  format(2x,"_Abandon = 0",/,2x,"_Reprise = 1",/)
141     read(0,3)is
142     if(is.eq.0) goto 1664
143     goto 21
144 c
145 c
146 c
147 c
148 c
149 c
150 c
151 c
152 22  do 23 i=1,jj-1
153     do 23 j=i+1,jj
154     if(ds(i).gt.ds(j))goto 23
155     do 24 k=1,m
156     x=aini(i,k)
157     aini(i,k)=aini(j,k)
158 24  aini(j,k)=x
159     x=ds(i)
160     ds(i)=ds(j)
161     ds(j)=x
162 23  continue
163 c
164 c
165 c
166 c
167 c
168 c
169     index=0
170 c
171 c
172 c
```

```
*****
*   CLASSEMENT DES POINTS INITIAUX   *
*                   POSSIBLES         *
*****
```

```
*****
*   FIN DE L'INITIALISATION           *
*****
```

```
*****
```

```
173 c          *      CONSTITUTION DES CLASSES      *
174 c          *****
175 c
176 c
177 333 do 40 i=1,n
178 40 iz(i)=0
179 do 41 k=1,n
180 k1=(k-.5)/50+2
181 k2=k-(k1-2)*50
182 if(k2.gt.1) goto 42
183 k3=n-(k1-2)*50
184 if(k3.gt.50) k3=50
185 read(70,6,rec=k1,err=39)((a(i5,j),j=1,m),i5=1,k3)
186 42 do 43 j=1,m
187 43 b1(j)=a(k2,j)
188 do 44 i=1,ip
189 do 45 j=1,m
190 45 b2(j)=aini(i,j)
191 call dist(b1,b2,m,idis,y)
192 if(i.gt.1) goto 46
193 z=y
194 iz(k)=i
195 goto 44
196 46 if(y.gt.z) goto 44
197 z=y
198 iz(k)=i
199 44 continue
200 41 continue
201 c
202 c
203 c          *****
204 c          *      RECHERCHE DES REPRESENTANTS      *
205 c          *****
206 c
207 c          Recherche des representants suivants.
208 c
209 call rep(ip,iz,aini,irep,n,m,idis,itel,eps)
210 index=index+1
211 if(index.lt.30) goto 600
212 write(0,31)
213 31 format(2x,"_Nombre d'iterations superieur a 30 .")
214 goto 700
215 600 if(itel.eq.1) goto 333
216 c
217 c          *****
218 c          *      MAXIMISATION DU POIDS DU TABLEAU*
219 c          *      *****
220 c
221 c          *****
222 c          *      REGROUPEMENT DES OPERATIONS      *
223 c          *****
224 c
225 700 pod=0.
226 do 900 i=1,m-1
227 do 900 j=1,ip
228 900 aini(i,j)=0.
229 do 901 k=1,n
230 k1=(k-0.5)/50+2
231 k2=k-(k1-2)*50
232 if(k2.gt.1)go to 904
```



```

233      k3=n-(k1-2)*50
234      if(k3.gt.50)k3=50
235      read(70,6,rec=k1,err=30)((a(i5,j),j=1,m),i5=1,k3)
236  904      do 905 j=1,ip
237          if(iz(k).eq.j)go to 906
238          do 907 i=1,m-1
239  907      aini(i,j)=aini(i,j)+(1-a(k2,i))*a(k2,m)
240          go to 905
241  906      do 909 i=1,m-1
242  909      aini(i,j)=aini(i,j)+a(k2,i)*a(k2,m)
243  905      continue
244  901      continue
245          do 910 i=1,m-1
246          zz=0.
247          nn=1
248          do 911 j=1,ip
249          if(aini(i,j).lt.zz)go to 911
250          zz=aini(i,j)
251          nn=j
252  911      continue
253          pod=pod+zz
254  910      jz(i)=nn
255          write(0,1115)pod
256  1115   format(5x,"Poids de la matrice :",f5.0)
257  c
258  c          *****
259  c          *   REGROUPEMENT DES PRODUITS   *
260  c          *****
261  c
262          poc=0.
263          do 921 k=1,n
264          k1=(k-0.5)/50+2
265          k2=k-(k1-2)*50
266          if(k2.gt.1)go to 924
267          k3=n-(k1-2)*50
268          if(k3.gt.50)k3=50
269          read(70,6,rec=k1,err=30)((a(i5,j),j=1,m),i5=1,k3)
270  924      do 928 j=1,ip
271          bj(j)=0.
272          do 75 ii=1,m-1
273          if(jz(ii).eq.j) go to 76
274  75      continue
275          go to 928
276  76      do 929 i=1,m-1
277          if(jz(i).eq.j)go to 927
278          bj(j)=bj(j)+(1-a(k2,i))*a(k2,m)
279          go to 929
280  927      bj(j)=bj(j)+a(k2,i)*a(k2,m)
281  929      continue
282  928      continue
283          zz=0.
284          nn=1
285          do 930 j=1,ip
286          if(bj(j).lt.zz)go to 930
287          zz=bj(j)
288          nn=j
289  930      continue
290          poc=poc+zz
291  921      iz(k)=nn
292          write(0,1115)poc

```

```
293      if(abs(poc-pod).gt.1)go to 700
294 c
295 c          *****
296 c          *      EDITION      *
297 c          *****
298 c
299 c          *****
300 c          *  CREATION DU FICHER FINAL  *
301 c          *****
302 c
303      write(50,1115)poc
304      nn=(n-0.5)/50+2
305      write(63,1,rec=1,err=27)n,m
306      do 1012 k=1,n
307 1012  kr(k)=k
308      do 1013 i=1,n-1
309      do 1013 j=i+1,n
310      if((iz(j)-iz(i)).ge.0)go to 1013
311      kk=iz(i)
312      iz(i)=iz(j)
313      iz(j)=kk
314      kk=kr(i)
315      kr(i)=kr(j)
316      kr(j)=kk
317 1013  continue
318      do 1301 j=1,m
319 1301  kw(j)=j
320      do 1300 i=1,m-2
321      do 1300 j=i+1,m-1
322      if(jz(j).gt.jz(i))go to 1300
323      kk=jz(i)
324      jz(i)=jz(j)
325      jz(j)=kk
326      kk=kw(i)
327      kw(i)=kw(j)
328      kw(j)=kk
329 1300  continue
330      kk=0
331      do 1020 ig=1,n
332      k=kr(ig)
333      k1=(k-0.5)/50+2
334      k2=k-(k1-2)*50
335      k3=n-(k1-2)*50
336      if(k3.gt.50)k3=50
337      read(70,6,rec=k1,err=30)((a(i5,j),j=1,m),i5=1,k3)
338      kk=kk+1
339      kk1=(kk-0.5)/50+2
340      kk2=kk-(kk1-2)*50
341      kk3=n-(kk1-2)*50
342      if(kk3.gt.50)kk3=50
343      do 1025 j=1,m
344 1025  aini(kk2,j)=a(k2,j)
345      if(kk2.ne.kk3)go to 1020
346      write(63,6,rec=kk1,err=9999)((aini(i5,kw(j)),j=1,m),i5=1,k3)
347 1020  continue
348 c
349 c
350 c          *****
351 c          *  IMPRESSION DES RESULTATS  *
352 c          *****
```

```
353      write(50,801)(jz(k),k=1,m-1)
354      write(50,801)(kw(k),k=1,m-1)
355 801  format(12x,20i3)
356 c
357      do 702 k=1,n
358      k1=(k-.5)/50+2
359      k2=k-(k1-2)*50
360      if(k2.gt.1) goto 704
361      k3=n-(k1-2)*50
362      if(k3.gt.50) k3=50
363      read(63,6,rec=k1,err=8888)((a(i5,j),j=1,m),i5=1,k3)
364 704  write(50,802)iz(k),kr(k),(a(k2,j),j=1,m)
365 802  format(2(2x,i4),2x,20(f2.0,lx))
366 702  continue
367 701  continue
368      go to 1664
369 30  write(0,32)
370 32  format("erreur label 30")
371      goto 1664
372 29  write(0,33)
373 33  format("erreur label 29")
374      goto 1664
375 28  write(0,34)
376 34  format("erreur label 28")
377      goto 1664
378 26  write(0,35)
379 35  format("erreur label 26")
380      goto 1664
381 27  write(0,36)
382 36  format("erreur label 27")
383      goto 1664
384 13  write(0,37)
385 37  format("erreur label 13")
386      go to 1664
387 39  write(0,38)
388 38  format("erreur label 39")
389      goto 1664
390 9999 write(0,3001)
391 3001 format(2x,"Erreur label 9999")
392      goto 1664
393 8888 write(0,3002)
394 3002 format(2x,"Erreur label 8888")
395 1664 close(70)
396      close(50)
397      close(63)
398      stop
399      end
```

## Subroutine rep

```

418 c
419 c
420 c
421   subroutine rep(ip,iz,aini,irep,n,m,idis,itel,eps)
422   dimension a(50,20),aini(50,20),iz(2000),b1(20),b2(20),co(50,20,3)
423   goto (1,100),irep
424   1 do 2 i=1,50
425     do 2,j=1,m-1
426       do 2 k=1,3
427         2 co(i,j,k)=0
428           do 12 i=1,50
429             do 12 j=1,20
430 12   aini(i,j)=0.
431           do 3 i=1,n
432             k1=(i-.5)/50+2
433             k2=i-(k1-2)*50
434             if(k2.gt.1) goto 4
435             k3=n-(k1-2)*50
436             if(k3.gt.50) k3=50
437 c
438   read(70,5,rec=k1,err=9)((a(k,j),j=1,m),k=1,k3)
439   5 format(1000f2.0)
440   4 i5=iz(i)
441     do 6 j=1,m-1
442       co(i5,j,1)=co(i5,j,1)+a(k2,m)
443       co(i5,j,2)=co(i5,j,2)-2*a(k2,m)*a(k2,j)
444   6 co(i5,j,3)=co(i5,j,3)+a(k2,m)*a(k2,j)**2
445   3 continue
446   itel=0
447   do 7 i=1,ip
448     if(co(i,1,1).gt.eps)go to 17
449     write(0,18)i
450 18   format(2x,"Representant num.",i4,"non utilise")
451     go to 7
452 17   do 7 j=1,m-1
453     a1=co(i,j,1)
454     a2=co(i,j,2)
455     a3=co(i,j,3)
456     pp=-a2/2./a1
457     uu=abs(pp-aini(i,j))
458     if(uu.gt,eps) itel=1
459     aini(i,j)=pp
460   7 continue
461     goto 1000
462 100 write(0,15)
463 15 format(2x,"_Coefficient de dissimilarite non prevu !")
464     go to 1000
465   9 write(*,8)
466   8 format(2x,"_Erreur lecture fichier70 dans sous-programme rep.")
467 1000 return
468   end

```

Subroutine dist

```
400 c
401 c
402 c
403     subroutine dist(b1,b2,m,idis,y)
404     dimension b1(20),b2(20)
405     y=0
406     goto (1,2),idis
407     1 do 10 j=1,m-1
408     10 y=y+(b1(j)-b2(j))**2
409     y=y**.5
410     goto 100
411     2 do 20j=1,m-1
412     i5=1
413     i5=b1(j)+b2(j)
414     if(i5.eq.2) i5=0
415     20 y=y+i5
416     100 return
417     end
```

## CONCLUSION

Number of trials have been made with the algorithm presented in this paper. It seems to be effective and well adapted to the problems of great dimension.

The problem of the choice of the distance to use for finding the initial part families remains open. We used the euclidian distance for solving the above example. A systematic study of the available distances is expected.

A final remark : the approach proposed in this paper not only leads to the part families, but also gives the subsystems which are responsible of this classification. Using the Data Analysis language, this algorithm gives then the classes and, for each of them, the parameters which explain these classes.

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