On the representation of Petri nets as state difference equations

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ABSTRACT

Petri nets have been accepted as an excellent model for asynchronous concurrent information systems, both hardware and software. Several variants of these nets have been reported in the literature to suit specific requirements. In a recent paper MURATA [1] has successfully demonstrated the possibility of using the discrete state space approach to represent Petri nets. Murata's paper models Petri nets as a linear state difference equation and thus makes it possible to use several concepts of systems theory for the analysis of Petri nets . In this paper we examine critically Murata's approach and extend it to cover the "inibitor arc" concept .This paper also reviews a few relevant publications on Petri nets .

RESUME

Les réseaux de Petri ont été acceptés comme étant l'une des meilleures méthodes de modélisation des systèmes informationnels concurrents, tant matériel que logiciel. Des variantes des réseaux de Petri ont également été publiées pour pouvoir s'adapter aux cas particuliers. Dans une récente publications, MURATA [1] a démontré la possibilité de se servir de l'approche d'espace d'état discret pour représenter les réseaux de Petri. L'article de MURATA modélise les réseaux de Petri par des équations de différence afin de faciliter l'application des méthodes de la théorie de commande pour l'analyse des réseaux de Petri. Dans notre article nous examinons de près l'approche de MURATA et nous proposons une extension pour prendre en compte les "arcs inhibiteurs". Par ailleurs, cet article présente un survol des publications qui nous sont pertinentes.

1. WHAT ARE PETRI NETS ?

A Petri net is a network representation of concurrent information systems (both hardware and software) , and derives its name from that of its author : C.A. Petri [2] . Judging by the number of publications this method has provoked , Petri nets have proved to be an immensely important tool for specifying , modeling , analysing , testing and metricising information systems [3]. The surveys by PETERSON [4] and MOALLA et al [5] give an excellent background to the method of Petri nets . An overview of network theory as applicable in this context may be found in [6].

Amongst the major advantages of Petri nets we may cite :

Mathematical abstraction : A Petri net is an abstract and formal model .It thus frees the user from having to worry about details of the physical system and allows him the use of powerful and generalised analysis techniques .

Graphical Representation : Petri nets are almost invariably described using a graphical symbolism . This allows the user to " see " the evolution of a dynamic system. We are thus able to use the methods of applied graph theory (connectivity , reachability , equivalence etc.) for resolving some complex questions on the dynamic behaviour of systems [7]. It has also enabled the use of CAD methods for proving the correctness concurrent systems [8] [9].

2. INHIBITOR ARCS

Petri nets per se have certain limitations in terms of modeling power . This has provoked several extensions , most of which are summarised in [4] and [5] . Although it would be impossible to analyse them all ,we shall particularly focus on one of these extensions - the inhibitor arc - because of its practical value[10]. An "inhibitor arc" may be defined (in the sense of HACK) , as follows : An inhibitor arc is a special kind of arc . It has no size (i.e. It carries no tokens), and is only directed from a place to a transition . The transition is firable if and only if it was firable in the old sense and all inhibiting places (having inhibitor arcs directed at the transition) have zero tokens . Petri nets with inhibitor arcs are also called inhibitor nets .

The addition of inhibitor arcs significantly extends the modeling power of Petri nets and can resolve several decision problems . HACK [10] has established the equivalence between inhibitor nets and " priority nets " (transitions associated with priority). It is also known that inhibitor nets are at least as powerful as Turing machines , and can atmost be more convenient.

Our experience has shown that ,inhibitor arcs are of immense help in modeling synchronisation problems involving priority considerations , and also help to reduce drastically the graphism involved in depicting real life systems .

3. STATE REPRESENTATION OF PETRI NETS

Petri nets , we observe , have invariably been used as a tool for representing concurrent information systems in a graphical fashion .This is obvious from the profusion of graph theoretic publications which discuss Petri nets . The influence of graph theory is also evidenced by the nomenclature used (places, arcs etc.).Although this approach is justified to a large extent , we have observed that it does have a serious handicap : Petri net graphs , even for moderately sized systems, become highly involved and confusing . Besides being inconvenient to conceive , Petri net graphs have to be in any case transformed into some other formal description (incidence matrices , linked lists etc.), for computerised analysis purposes .It is in this context that MURATA's model marks a milestone in Petri net theory .

Let us assume an " ordinary , self loop free " Petri net with n places and t transitions (see page 35 of Ref. 7 for

The above approach of MURATA is interesting in several ways. It provides a very compact representation (as a matrix equation) of a Petri net . Thus storage and analysis using a digital computer is made possible . The methods of matrix algebra can be used to derive properties of a given Petri net . MURATA's equation is in the form of a state difference equation , very well known to control theorists. Several concepts of control theory can be transposed to analyse Petri nets , as has been done in [1] for instance . However for any practical implementation of this technique , we would need a few additional checks (not foreseen by MURATA) .

MURATA assumes that the elementary firing or control vector U contains exactly one non zero entry , a " 1 " . This has been introduced obviously to avoid conflict . This, in our opinion, is not a restriction , since the case of firings of multiple (non conflicting) transitions can be broken down to a sequence of single transition firings.The case of conflicting transitions can be easily predicted by a simple examination of the matrix A : No column of A should contain more than one element of value -1 . By examining the rows of A one can deduce if a given transition is a source transition (creates one or more tokens without losing any) , or a sink transition (loses one or more tokens without creating any) . This test is very useful to detect possible runaway conditions or deadlock conditions .

MURATA's modeling assumes implicitely that only firable transitions will be fired by the control vector U_K , i.e. U_K will contain a " 1 " only for that transition whose upstream places have at least one token each . Computer implementation of this model can be facilitated if a test can be made to avoid illegal transition firings. The test would consist of verifying for each firing instant :

$$\left\{ m_{iK-1} = 0 \Rightarrow \sum_{j=1}^{t} a_{ij}\, u_{jk} \neq -1 \right\} \qquad \forall i = 1,2,\ldots p \qquad (2)$$

Attempts have been made by some authors to use MURATA's approach for modeling mathematically basic logical operations : AND, OR , NAND and NOT [11] .The approach of KHAN et al has however some weaknesses in the sense that they use non standard primitives and an adhoc symbolism. The basic Petri net model employed has very limited modeling power. We propose a very simple extension to MURATA's state equation, so that it can handle inhibitor arcs also and thus render modeling of logical operations a trivial affair . The advantages gained are enormous since we will be raising the power of these state equations to that of a Turing machine . We will also be able to model several practical applications , involving priorities for instance , by using appropriate transformations as explained in paragraph 2 .

4. STATE MODEL FOR INHIBITOR NETS (AND PRIORITY NETS)

Let us consider , as in paragraph 3 above , an ordinary, self loop free Petri net with the difference that the arcs connecting places and transitions may be the normal arcs or inhibitor arcs . All the earlier assumptions are still held valid. The inhibitor net will be said to contain a self loop even for cases where a place and a transition are connected by a pair of opposing arcs (normal arc and inhibitor arc) . In such cases the model will lead to ambiguities and will be mathematically incorrect . The concept of conflict will also remain the same i.e. If a place contains more than one outgoing arc (normal or inhibitor) the resultant situation may be a conflict unless the downstream transitions to which these arcs lead , are mutually exclusive . The matrix A (equation 1) will remain the same both for normal and for inhibitor arcs.

We define a matrix B (of order t x p) such that :

b_{ij} = 1 if place j is connected to transition i by an inhibitor arc .

= 0 otherwise .

MURATA's state equation (Eqn. 1) will have to be modified as follows :

$$M_K = M_{k-1} + A^T U_K + B^T U_K \tag{3}$$

$$= M_{K-1} + (A + B)^T U_K \tag{4}$$

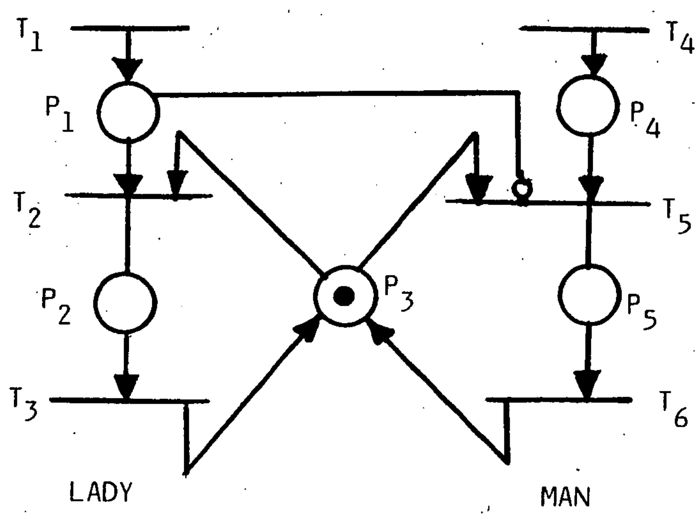
For any U_K, the product $B^T U_K$ will indicate if any transition containing an incident inhibitor arc is being fired . In such cases the element corresponding to the place from which the inhibitor arc originated will contain a " 1 " .

It will also be observed from Eqn. 3 that the test originally proposed for conflict situation (see paragraph 3) is still valid . The test for legality of a transition firing (see Eqn. 2) will however have to look for the absence of a token . We shall leave this as an exercise to the reader .

5. AN EXAMPLE

Let us imagine a narrow stairway leading out of a building . Whenever a person wants to go out , he (or she) looks if the stairway is vacant and promptly walks out. Let us assume that all men are chivalrous and they let a lady take the stairway first. Let us also assume that the stairway is always used in only one direction and that there can be atmost only one person using it at a time .

we have tried to depict this situation using a Petri net . The inhibitor arc ensures that a lady gets the right of way before a man .



we could have eliminated the inhibitor arc by assigning priorities to transitions and by giving transition T2 a higher priority than transition T5 .

we can transform this net into a state representation (Eqn. 4) with the following values for matrices A and B :

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix} \qquad (5)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (6)$$

By a simple observation of the rows and columns of A , we conclude the following :

- Place 1 and place 3 could lead to conflicts .
- Transition 1 and transition 4 could lead to runaway conditions if they fire too often .

Let us observe the evolution of the Petri net for the following scenario :

- Stairway free .
- A lady approaches the stairway .
- A gentleman approaches the stairway .
- Another lady approaches the stairway while the first lady is still on her way .

If we define Nk as the transpose of M for all k = 1,2,..... (that makes typing easier !). We can generate the following marking vectors by using equation 4 :

Firing sequence	Marking vector	Remarks
Initial Marking	N0 = [0 0 1 0 0]	Vacant stairway
T1 Fires (u1=1)	N1 = [1 0 1 0 0]	A lady enters
T2 Fires (u2=1)	N2 = [0 1 0 0 0]	Lady walks out
T4 Fires (u4=1)	N3 = [0 1 0 1 0]	Man enters,waits
T1 Fires (u1=1)	N4 = [1 1 0 1 0]	Another lady
T3 Fires (u3=1)	N5 = [1 0 1 1 0]	Vacant stairway
T2 Fires (u2=1)		
T5 is inhibited	N6 = [0 1 0 1 0]	Chivalry !
T3 Fires (u3=1)	N7 = [0 0 1 1 0]	Vacant stairway
T5 Fires (u5=1)	N8 = [0 0 0 0 1]	Man walks out

That brings the story to a happy ending !

6. CONCLUSIONS

The practical importance of MURATA's contribution to Petri nets has been detailed . MURATA's model has been extended to cover the concept of inhibitor arcs (and hence its dual - priority nets) , both of which have very important practical applications . It has been shown that the extension proposed conserves all the properties of the original model . A reference is also made to the mathematical modeling of Boolean functions AND , OR , NOT and NAND. We have also derived several checks for detecting a priori certain pathological conditions . These tests could be very important for any practical implementation of Petri nets as state difference equations .

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