An approximation to the optimum check-point interval using integer programming

M. Magazine

CENTRE DE ROCQUENCOURT

Institut National
de Recherche
en Informatique
et en Automatique

Domaine de Voluceau
Rocquencourt
B.P.105
78153 Le Chesnay Cedex
France
Tél.:954 90 20

Rapports de Recherche

N° 139

AN APPROXIMATION TO THE OPTIMUM CHECKPOINT INTERVAL USING INTEGER PROGRAMMING

Michael MAGAZINE

Juin 1982

AN APPROXIMATION TO THE OPTIMUM CHECKPOINT INTERVAL USING INTEGER PROGRAMMING

Michael MAGAZINE
Dept. of Management Sciences
University of Waterloo
Waterloo, Ont. N2L 3G1
CANADA

ABSTRACT

We find an approximation to the optimum checkpoint interval utilizing an integer programming model. This discrete model is deterministic over a finite horizon. We show that optimal intervals are essentially equally spaced and that the results are similar to previous efforts when the horizon goes to infinity.

RESUME

Nous déterminons une approximation fondée sur la programmation en nombres entiers pour le caldul de l'espacement optimal entre les points de reprise. Nous developpons un modèle discret et déterministe à horizon fini. Nous montrons que l'espacement optimal est essentiellement constant et nous retrouvons les résultats établis en horizon infini.

Key Words and Phrases : Database maintenance, checkpoint interval, rollback recovery and integer programming.

1. INTRODUCTION

The use of maintenance operations on systems with unreliable components is a common procedure for improving overall performance. In this paper, we use checkpoint operations to introduce redundancy of a data base system and improve performance. Our model, however, differs from previous efforts by treating the problem as a deterministic, discrete time finite horizon decision problem leading to a combinatorial optimization problem. The solution to the optimization problem is given along with the implications of the results in an infinite horizon and the comparison with previous results.

The rationale for using checkpoints to increase system availability or reduce response time has been given by several authors (e.g. [2], [3]). The decision problem is the tradeoff between the time to perform the checkpoint (copying of the entire data base onto secondary storage) and the time to perform rollback and recovery in the event of system failure. At the point of failure, the copy of the data base at the most recent checkpoint is reloaded into primary storage (rollback) and all transactions, which have been stored on the audit trail, are reprocessed in order (recovery). During these operations the system is unavailable.

Our objective is to find the optimal checkpoint intervals, i.e. to minimize total costs over a finite horizon. Typically, the surrogate for costs is system availability. The checkpoint interval which maximizes availability was found by Young [8] and Chandy [3] using a continuous time model over an infinite horizon and random system failures. In [5], Gelenbe generalizes this result to include queueing delays. Baccelli [1] showed how minimizing expected response time may lead to different interval and in [6] and [7] dynamic checkpointing was considered. That is, decisions were made based on the load to the system rather than the time since the most recent checkpoint.

Our combinatorial approach leads to results consistent with other authors and moreover, is able to show that the assumption of a stationary optimal interval is correct.

2. MODEL ASSUMPTIONS AND FORMULATION

The assumptions in our model are similar to those of Model A of Chandy [2].

Assume we are interested in minimizing "costs" over a finite horizon consisting of T periods. At the beginning of any period, when the system is available, we have the option of performing a checkpoint. We assume there is a fixed cost of this operation (again, this may refer to the expected time the system is unavailable given a checkpoint is performed, or possibly the increase in response time to this operation). That is, this cost is independent of the time or the number of errors since most recent checkpoint. The "cost of not performing a checkpoint" given the current state (number of periods from most recent checkpoint) will be assumed to be a linear function of the state. This may be interpreted as the expected unavailability time if a failure occurs in the current state.

Let A be the cost to perform a checkpoint and B + iC the cost of not doing the checkpoint when the observed state is i. Assume k - 1 checkpoints are performed dividing the original in $[0,T]$ into k intervals $[t_i, t_{i+1}]$ $i = 0,1,\ldots k-1$ where $t_o = 0$ and $t_k = T$. We wish to minimize the total cost, $F(k,t)$ where

$$(1)\ F(k,t) = (k-1)\,A + \sum_{i=i_o}^{i_o+t_1-1} (B+iC) + \sum_{j=2}^{k} \sum_{i=0}^{t_j-t_{j-1}-1} (B+iC)$$

where i_o is the state at time 0 (If $i_o = 0$ the last two terms simplify to $\sum_{j=1}^{k} \sum_{i=0}^{t_j-t_{j-1}-1} (B+iC)$). If A,B and C are units of unavailability then

$T-F(t,k)$ is the total available time which is to be maximized. We wish to find the value of k and the lengths of intervals $\{t_j-t_{j-1}\}$ which minimizes $F(t,k)$.

Let n_j be the state at the j^{th} checkpoint. Specifically,

$n_1 = i_0 + t_1$

$n_j = t_j - t_{j-1} \quad j = 2,\ldots,k - 1$

$n_k = T - t_{k-1}$

Our objective function (1) becomes

$$(k-1)A + \sum_{i=i_0}^{n_1-1} (B+iC) + \sum_{j=2}^{k} \sum_{i=0}^{n_j-1} (B+iC)$$

$$= (k-1)A + [(n_1-i_0) + \sum_{j=2}^{k} n_j]B$$

$$+\left[\frac{(n_1-1)n_1}{2} - \frac{(i_0-1)i_0}{2} + \sum_{j=2}^{k} \frac{(n_j-1)n_j}{2}\right] C$$

Noting that $\sum_{j=1}^{k} n_j = i_0 + T$, we get $(k-1)A + TB + \frac{C}{2}\left[\sum_{j=1}^{k} n_j^2 - T - i_0^2\right] =$

$$A k + \frac{C}{2} \sum_{j=1}^{k} n_j^2 + \left[TB - \frac{TC}{2} - \frac{i_0^2 C}{2-A} -A\right] \qquad (2)$$

This result in an integer programming problem of the form :

$$F = \min_{(n_1,n_2,\ldots,n_k\, k)} \left[\sum_{j=1}^{k} n_j^2 + \alpha k\right]$$

Subject to : $\sum_{j=1}^{k} n_j = \beta(=T + i_0)$ (3)

$n_j \geq o$ integer $j = 1,2,\ldots,k$ $\quad k \geq [0,T]$ integer

3. ANALYSIS

Rather than assuming that checkpoint intervals are equally spaced we will show that the discrete analog over a finite horizon of this intuitive result holds.

Proposition 1 : $n = (n_1, n_2, \ldots, n_k)$ are optimal checkpoint intervals for (3) if and only if

$$|n_j - n_i| \le 1 \quad \forall\, i,j \in \{1,2,\ldots,k\} \tag{4}$$

Proof : Consider a feasible solution $n^{(1)}$ such that $n^{(1)}_{max} - n^{(1)}_{min} > 1$ and another solution $n^{(2)}$, the same as $n^{(1)}$ excepting $n^{(2)}_{max} = n^{(1)}_{max} - 1$ and $n^{(2)}_{min} = n^{(1)}_{min} + 1$. The differences in their objective functions is :

$$F^{(2)} - F^{(1)} = n^{(2)2}_{max} + n^{(2)2}_{min} - \left(n^{(1)2}_{max} + n^{(1)2}_{min}\right)$$

$$= (n^{(1)}_{max} - 1)^2 + (n^{(1)}_{min} + 1)^2 - (n^{(1)2}_{max} + n^{(1)2}_{min})$$

$$= 2(n^{(1)}_{min} - n^{(1)}_{max} + 1) < 0$$

and $n^{(2)}$ is better than $n^{(1)}$. Hence, any solution not containing (4) can-be improved and solutions satisfying (4) cannot be improved. ∎

Hence, any feasible solution to (3) with (4) holding will yield optimal intervals.

Proposition 2 : Let $n(\beta,k) = k - (\beta - k\left\lfloor\frac{\beta}{k}\right\rfloor)$.

Then $n_1 = n_2 = \ldots = n_{n(\beta,k)} = \left\lfloor\frac{\beta}{k}\right\rfloor$

and $n_{n(\beta,k)+1} = \ldots = n_k = \left\lfloor \beta/k \right\rfloor + 1$

is an optimal solution to (3) for any given k.

Proof : Clearly, $\forall\, i,j \in \{1,2,\ldots,k\}$, $|n_i - n_j| \le 1$. It is only necessary to show that

$$\sum_{j=1}^{k} n_j = \beta$$

$$\sum_{j=1}^{k} n_j = n(\beta,k) \left\lfloor \frac{\beta}{k} \right\rfloor + (k - n(\beta,k)) \left(\lfloor \beta/k \rfloor + 1 \right)$$

$$= k \left\lfloor \frac{\beta}{k} \right\rfloor + k - n(\beta,k)$$

$$= k \left\lfloor \frac{\beta}{k} \right\rfloor + k - (k - \beta + k \left\lfloor \frac{\beta}{k} \right\rfloor) = \beta \qquad \blacksquare$$

Clearly, any permutation of the indices is also an optimal solution.

Problem (3) is now reduced to :

$$F = \min_{k \in \{1,\ldots,T\}} F(k) = \min \left[\alpha k + \sum_{j=1}^{n(\beta,k)} \left\lfloor \frac{\beta}{k} \right\rfloor^2 + \sum_{j=n(\beta,k)+1}^{k} \left(\left\lfloor \frac{\beta}{k} \right\rfloor + 1 \right)^2 \right]$$

which on simplification becomes

$$F(k) \doteq \alpha k - \frac{(k-n(\beta,k))^2}{k} + \frac{\beta^2}{k} + (k - n(\beta,k)) \tag{5}$$

In order to perform this minimization we would like to eliminate the greatest integer function ($\lfloor\ \rfloor$) from (5).

Realizing that for a given value of β this function takes on the same value for several values of k we can do this. For instance, if $\beta = 100$, then for $k = 51,\ldots,100, \left\lfloor \frac{\beta}{k} \right\rfloor = 1$. Consider the intervals of k where $\left\lfloor \frac{\beta}{k} \right\rfloor$ take the same value. That is,

$$\left\lfloor \frac{\beta}{k} \right\rfloor = 1 \qquad \frac{\beta}{2} < k \le \beta, \text{ integer}$$

$$\left\lfloor \frac{\beta}{k} \right\rfloor = 2 \quad \frac{\beta}{3} < k \leq \frac{\beta}{2} \text{ , integer}$$

or in general $\left\lfloor \frac{\beta}{k} \right\rfloor = l$, $\frac{\beta}{l+1} < k \leq \frac{\beta}{l}$, integer.

(Note, that some intervals, $\left(\left[\frac{\beta}{l+1}, \frac{\beta}{l}\right]\right.$, may contain no integer points).

On each of these intervals

$$n(\beta,k) = k -(\beta - kl) = (l + 1)\, k - \beta$$

and (5) becomes :

$$F(k) = \alpha k - \frac{(\beta - lk)^2}{k} + \frac{\beta^2}{k} + (\beta - lk)$$

$$= \alpha k - \frac{\beta^2+2\beta\, lk-l^2k^2}{k} + \frac{\beta^2}{k} + \beta - lk$$

$$= k(\alpha - l^2 - l) + (\beta + 2\beta l) \tag{6}$$

Since (6) is linear, the minimum for each interval is obtained at one of the endpoints depending on the sign of $\alpha - l^2 - l$. Let

$$l^* = - \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\alpha}$$

be the positive root of $l^2 + l - \alpha = 0$. For each $l \leq l^*$, $\alpha - l^2 - l \geq 0$ and the minimum is at the lowest endpoint ; conversely, for $l \geq l^*$, $\alpha - l^2 - l \leq 0$ and the minimum is at the highest endpoint. That is, for

$$\begin{array}{ll} l \leq l^* & k = \left\lfloor \frac{\beta}{l+1} \right\rfloor + 1 \\ l \geq l^* & k = \left\lfloor \frac{\beta}{l} \right\rfloor \end{array} \tag{7}$$

The problem then reduces to finding which of these endpoints give a global minimum. Let F_l be the value of F_k when (7) is used. For $l \leq l^*$,

$$F_l - F_{l-1} = (\alpha - l^2 - l)\left(\left\lfloor\frac{\beta}{l+1}\right\rfloor + 1\right) + \beta + 2\beta l$$

$$- (\alpha - (l-1)^2 - (l-1))\left(\left\lfloor\frac{\beta}{l}\right\rfloor + 1\right) + \beta + 2\beta(l-1)$$

$$= (\alpha - l^2 - l)\left(\left\lfloor\frac{\beta}{l+1}\right\rfloor - \left\lfloor\frac{\beta}{l}\right\rfloor\right) - 2l\left(\left\lfloor\frac{\beta}{l}\right\rfloor + 1\right) + 2\beta$$

$$\leq 0$$

Similarly, it can be shown that $F_l - F_{l+1}$ for $l \geq l^*$ is ≤ 0. Hence, we have shown :

<u>Proposition 3</u> : $F_{l^*} = \min F_l$ if l^* is an integer or at either the left endpoint of the interval defined by $\lfloor l^* \rfloor$ or the right endpoint of the interval defined by $\lceil l^* \rceil$.

<u>Corollary</u> : The value of k which minimizes F(k) is either $\left\lfloor\frac{\beta}{\lceil l^* \rceil}\right\rfloor$ or $\left\lceil\frac{\beta}{\lceil l^* \rceil}\right\rceil$.

<u>Proof</u> : The right endpoint of the interval defined by $\lceil l^* \rceil$ is $\left\lfloor\frac{\beta}{\lceil l^* \rceil}\right\rfloor$. The left endpoint of the interval defined by $\lfloor l^* \rfloor$ is :

$$\left\lfloor\frac{\beta}{\lfloor l^* \rfloor + 1}\right\rfloor + 1 = \left\lceil\frac{\beta}{\lceil l^* \rceil}\right\rceil .$$

4. COMPARISON WITH PREVIOUS RESULTS

The optimal checkpoint interval in [8] was found assuming that over an infinite horizon we would like to minimize a long run cost per period. Using our notation, this yields,

$$\min_T \; F(T) = \min_T \left[A + \sum_{i=0}^{T-1} (B + Ci) \right]$$

$$= \min_T \left[\frac{A}{T} + \frac{C}{2} \; (T-1) \right]$$

Since F(T) is convex, treating T as a continuons variable yields

$$T^* = \left(\frac{2A}{C}\right)^{1/2} = \frac{1}{2}\left(\frac{8A}{C}\right)^{1/2}$$

When T is not an integer, the solution to the discrete problem is found by choosing the best of $\lfloor T^* \rfloor$ or $\lceil T^* \rceil$. Alternatively we can invoke difference arguments on F(T). Noting that second differences are positive we want an adjacent integer to the value of T where

$$F(T) - F(T-1) = 0$$

This yields,

$$T_D^* = \frac{1}{2} + \frac{1}{2}\left(1 + \frac{8A}{C}\right)^{1/2}$$

The third approach is to extend the previous result from the finite analysis, i.e. treating $\lfloor 1^* \rfloor$ or $\lceil 1^* \rceil$ as the determining factor we get,

$$1^* = -\frac{1}{2} + \frac{1}{2}\left(1 + \frac{8A}{C}\right)^{1/2}$$

These three values will yield intervals that are close in size : $T_D^* - 1^* = 1$

$T_D^* - T^* \leq 1$ as is $T^* - 1^*$. In addition, T^* and either 1^* or T_D^* will lead to the same integer number of periods. Intuitively, T_D^* may be most appealing as this represents the value of T where the cost per period of performing a checkpoint balances the cost of rollback and recovery due to failures.

REFERENCES

1. BACCELLI F.
Analysis of a Service Facility with Periodic Checkpointing,
Acta Informatica 15, pp 67-81, 1981.

2. CHANDY K.M
A survey of Analytic Models of Roll back Recovery Strategies,
Computer 8, pp 40-47, 1975.

3. CHANDY K.M., BROWNE J.C., DISSLY C.W and Uhrig W.R
Analytical Models for Roll back Recovery Strategies in Data Base Systems,
IEEE Trans. Software Eng, 1, pp 100-110, 1975.

4. GELENBE E.
On the Optimum Checkpoint Interval,
JACM, 26, pp 259-270, 1979.

5. GELENBE E. and DE ROCHETTE D.
Performance of Roll back Recovery Systems Under Intermittent Failures,
Comm. of the ACM, 21-6, pp 493-499, 1978.

6. Lohman G.M and MUCKSTADT J.A.
Optimal Policy for Batch Operations : Backup , Checkpointing, Reorganization and Updating,
ACM Trans. on Database Systems, 2-3, pp 209-222, 1977.

7. MIKOU N. and TUCCI S.
Analyse et Optimisation d'une Procédure de Reprise dans un Système de Gestion de Données Centralisées,
Acta Informatica, 12, pp 321-338, 1979.

8. YOUNG J.W.
A First Order Approximation to the Optimum Checkpoint Interval,
Comm. ACM , 17-9, pp 530-531, 1974.

Imprimé en France

par

l'Institut National de Recherche en Informatique et en Automatique